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Variation of the magnetic moment of a charged particle during its nonadiabatic motion in a dipole field

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The law of variation of the magnetic moment of an electron located in a dipole trap and undergoing multiple reflection is investigated. It is shown that for certain initial conditions the measured particle lifetimes agree with the theoretical expression obtained for the change, $\Delta\mu$, occurring in the magnetic moment μ in one reflection under the assumption that $\Delta\mu$ is randomly accumulated. An expression is obtained for the coefficient of diffusion due to the nonadiabaticity of the motion.

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INTRODUCTION

The experimental investigation of the nonadiabatic mechanism underlying the losses in magnetic traps is of interest first and foremost from the purely physical point of view, since it is connected with the problem of the appearance of statistical laws in a dynamical system.[1] The statistical description arises in the case when the nonlinear oscillations are unstable (i.e., on account of the instability of the dynamical solutions). Analytic semiqualitative estimates for the stochastic instability of the nonlinear electron oscillations in a magnetic trap are given in Chirikov's paper.[2] From the standpoint of stochastic theory, the nonadiabatic motion can be presented as statistically irreversible, unstable motion in magnetic-moment space that leads to the diffusion of the particles along the magnetic field. The criteria for such a motion in magnetic fields of different configurations were found earlier,[3,4] but the nature of the variation of the invariant (the magnetic moment μ) has itself been little studied. Therefore, the investigation of the law of variation of μ during the nonadiabatic motion is important. Furthermore, the nonadiabatic effects are also of great importance from the standpoint of applications to the problem of the dynamics of charged particles in traps of cosmic dimensions, e.g., in the geomagnetic and Jupiter's magnetic traps, whose fields are nearly dipole fields.

The direct determination of the nature of the variation of the adiabatic invariant in multiple reflections through the measurement of the particle lifetimes, $\bar{\tau}$, due to the nonadiabaticity does not in itself give any information about the law of variation of μ. It is still necessary to prescribe the law of accumulation of $\Delta\mu$ (the stochastic law, for example), compute the change in μ per reflection, and compare the measured mean value of $\Delta\mu$, found through $\bar{\tau}$, with the theoretical value. The agreement of the theoretical value of $\Delta\mu$ with the value found with the aid of $\bar{\tau}$ will in this case be the criterion for the correctness of the chosen law of variation of μ.

The present paper is devoted to the investigation of the nature of the variation of the magnetic moment of the electron during the non-adiabatic motion of the electron in a dipole magnetic field.

MEASUREMENT PROCEDURE AND RESULTS

The experiments were carried out in a dipole trap formed by the field of a uniformly magnetized sphere of diameter 16 cm and magnetic moment $M = 2.6\times10^5$ G cm^2 in a ~2×10^{-10}-Torr ultimate vacuum. The energy of the injected electrons varied from ~20 to ~200 eV. The injection and registration of the electrons were accomplished in much the same way as were done in[3]. The quantity under investigation was the lifetime of the particles in the trap as a function of their energy W, the angle, α, between the velocity vector and the line of force, and the distance, R_e, to the drift shell. The quantity R_e lies in the median plane $\theta=\pi/2$ in the spherical coordinate system (R, θ, φ) with origin on the dipole and axis parallel to the magnetic moment **M**. As the magnitude of the angle α, we took its value in the median plane $(\alpha=\alpha(\theta=\pi/2))$. The coordinates of the location of the entrance window of the detector in the trap were varied in the range: $\theta\approx 80-50°$; $R_e = 18-22$ cm.

Since the nonadiabaticity parameter for given R_e and W varies along a line of force, the electron motion in the trap will be determined by the maximum value of the transverse or longitudinal component of this parameter. Let us write the expression for the adiabaticity parameter in the form

$$\chi=\frac{T}{2\pi H}\mathbf{v}\,\mathrm{grad}\,H,$$

where T is the period of the Larmor revolution. Then the transverse component will have the usual form:

$$\chi_\perp=\frac{\rho|(\nabla H)_\perp|}{H}=\frac{\rho}{R_c}=\frac{3pcR_e^2}{eH_0R_M^3}\frac{\sin\alpha\sin^2\theta(1+\cos^2\theta)}{(1+3\cos^2\theta)^{7/4}},\qquad(1)$$

where R_M is the radius of the magnet, ρ is the Larmor radius, R_c is the radius of curvature of the line of force, and H_0 is the magnetic field at the distance $R_e = R_M$. For the longitudinal component we obtain

$$\chi_\parallel=\frac{v_\parallel}{v_\perp}\frac{\rho(\nabla H)_\parallel}{H}=\frac{3pcR_e^2}{eH_0R_M^3}\frac{(1-\eta\sin^2\alpha)^{1/2}\sin^4\theta\cos\theta(3+5\cos^2\theta)}{(1+3\cos^2\theta)^2},\qquad(2)$$
$$\eta=(1+3\cos^2\theta)\sin^{-6}\theta.$$

It follows from (1) and (2) that $\chi_{\perp\max}=\chi_\perp(\theta=\pi/2)$ and that the nature of the motion for $\alpha\gtrsim 25°$ will be determined by the quantity $\chi_\perp$.

Under the conditions of the experiment, the nonadiabaticity parameter had the value

$$\chi_\perp=\left(\frac{\rho}{R_c}\right)_{\theta=\pi/2}\leqslant 2.5\chi_{\perp c},$$

where $\chi_{\perp c}$ is the critical value of the adiabaticity parameter. [3] The quantity $\bar\tau$ was determined from the expression

$$\bar\tau=\frac{1}{n(0)}\int_0^{t_1}n(t)\,dt,\qquad(3)$$

where $n(t)$ is the number of captured electrons at the moment of time t and $t_1\sim 10$ sec is the time required for the complete disappearance of signals from the counting system. Typical $\bar\tau$ curves are shown in Fig. 1, from which it can be seen that at large values of the parameter $\chi_\perp$ ($\chi_\perp\gtrsim 2\chi_{\perp c}$) the lifetime depends weakly on the nonadiabaticity. As the pressure increases, the boundary between the regions of strong and weak dependences of $\bar\tau$ on $\chi_\perp$ becomes smooth. The curve $\bar\tau(\chi_\perp)$ becomes smoother, with the point of inflection shifting toward the region of higher values of $\chi_\perp$. As the angle between the vectors **v** and **H** was decreased, the appearance of the nonadiabatic effects began at lower values of the parameters $\chi_\perp$ and the lifetime decreased accordingly. The minimum value of the initial angle α in the median plane was $\alpha_0\sim 30°$.

It should be noted that the quantity $\bar\tau$ defined by (3) is determined not only by the nonadiabaticity of the motion, but also by scattering by the residual gas. This is true especially for the region near the point of inflection. Consequently, we should have separated from $\bar\tau$ the time, τ_μ, of electron retention due to the nonadiabaticity, and subsequently use only it. This, however, is not possible, since the particle losses as a result of the nonadiabaticity and scattering by the gas are dependent. [4] In this case the inverse times are not additive quantities, and the well-known relation $\tau^{-1}=\tau_s^{-1}+\tau_\mu^{-1}$ (τ_s is the time of scattering by the gas) is not valid. Therefore, under the expression (3), it is natural to understand the particle lifetime due to magnetic scattering in the medium. The extent to which the quantity $\Delta\mu$, due to "pure" nonadiabaticity, differs from its value under real conditions will be seen below.

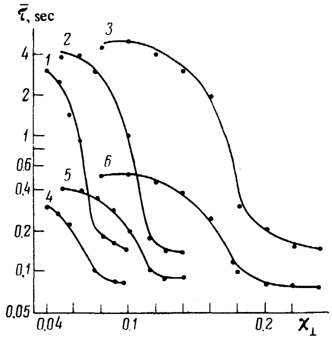

FIG. 1. The dependence $\bar\tau(\chi_\perp)$ for different pressures of the residual gases and for different values of the angle α. The curves 1–3 and 4–6 were measured at pressures of $\sim 2\times10^{-10}$ and $\sim 2\times10^{-9}$ Torr respectively. The curves 1, 4 are for $\alpha_0=31°$, $\alpha_c=25°$; 2, 5) for $\alpha_0=45°$, $\alpha_c=38°$; 3, 6) for $\alpha_0=90°$, $\alpha_c=50°$; the distance to the drift shell was $R_e=21.5$ cm.

VARIATION OF THE MAGNETIC MOMENT OF A PARTICLE

Let us estimate the change in the quantity μ in the region of sharp decrease of $\bar\tau$. Let us assume that stochastic magnetic scattering obtains in the region of nonadiabaticity-parameter values $\chi_{\perp c}\lesssim\chi_\perp\lesssim 2\chi_{\perp c}$. Then the total change in the magnetic moment after N reflections from the magnetic mirror will be $N^{1/2}\Delta\mu$. The number of reflections a particle undergoes before reaching the loss cone will be determined by the expression

$$N=\left(\frac{\sin^2\alpha_0-\sin^2\alpha_c}{\sin^2\alpha_0}\right)^2\left(\frac{\Delta\mu}{\mu_0}\right)^{-2},\quad \mu_0=\frac{W}{H_e}\sin^2\alpha_0,\qquad(4)$$

where H_e is the magnetic field in the median plane and $\alpha_c=\alpha_c(\theta=\pi/2)$ is the critical value, determined by the coordinates of the detector, of the quantity α. Hence we can express the lifetime in terms of the relative change in the magnetic moment per reflection if we take into account the fact that the period of the oscillatory motion between the points of reflection is equal to [5]

$$\tau_2\approx 7R_eW^{-1/2}(1.3-0.56\sin\alpha_0)\cdot10^{-8}\ [\mathrm{sec}],$$

where W is measured in electron volts. Thus, it follows from (4) that

$$\tau_\mu\approx\frac{3.5\cdot10^{-8}R_e(\sin^2\alpha_0-\sin^2\alpha_c)^2(1.3-0.56\sin\bar\alpha_0)}{W^{1/2}(\Delta\mu/\mu)^2\sin^4\alpha_0},\qquad(5)$$

where $\bar\alpha_0$ is the mean angle in the interval $[\alpha_0,\ \alpha_c]$. According to Dykhne and Chaplik, [6] the change in the adiabatic invariant of a charged particle in a nonuniform, axially symmetric magnetic field has the form

$$\frac{\Delta\mu}{\mu}\sim\exp\left(i\int\frac{\omega\,ds}{v_\parallel}\right), \tag{6}$$

where ω is the Larmor frequency in the complex plane, s is the length of the line of force, and s_0 is the integrand's discontinuity that gives the smallest value of the imaginary part of the integral.

The exponential function in Eq. (6) is of universal character, i.e., it will have a similar form for a dipole field. In a spherical coordinate system we have for the dipole field the expression

$$\frac{\omega\,ds}{v_\parallel}=\frac{eH}{mc}\frac{(R^2+R'^2)^{1/2}\,d\theta}{v[1-(H/H_e)\sin^2\alpha_0]},\qquad s_0=\frac{i}{\sqrt{3}}.$$

Calculation of the preexponential factor gives a value $\sim 0.5(\rho/R_e)^{1/4}$. With allowance for these quantities Eq. (6) can be reduced to the form

$$\frac{\Delta\mu}{\mu}\approx 0.5\left(\frac{\rho}{R_e}\right)^{1/4}\exp\left[-\psi(\alpha)\frac{R_e}{\rho}\right]\cos\varphi_0, \tag{7}$$

$$\psi(\alpha)=\int_0^{3^{-1/2}}\frac{1-3x^2}{(1+x^2)^3}\left[1-\sin^2\alpha\frac{(1-3x^2)^{1/2}}{(1+x^2)^3}\right]^{-1/2}dx,\qquad x=\cos\theta, \tag{7a}$$

where φ_0 is the phase of the particle. Under the conditions of the experiment the phase of the injection (i.e., the phase of the particle) corresponded to $\varphi_0=0$. Substituting (7) into (5), we obtain

$$\tau_\mu\sim\exp[2\psi(\alpha)R_e/\rho],$$

which qualitatively agrees with the dependence $\bar{\tau}(\chi_\perp)$ shown in Fig. 1. For a quantitative comparison, let us replace in (5) the value of τ_μ by the experimental $\bar{\tau}$ for $P\sim 2\times10^{-10}$ Torr and take (7) into account; we then obtain

$$\psi(\alpha)=-\frac{\rho}{R_e}\ln\left\{3.74\cdot10^{-4}\left(\frac{R_e}{\rho}\right)^{1/4}\frac{(\sin^2\alpha_0-\sin^2\alpha_c)}{\sin^2\alpha_0}\times\left[\frac{R_e(1.3-0.56\sin\bar{\alpha}_0)}{W^{1/2}\bar{\tau}}\right]^{1/2}\right\}. \tag{8}$$

The expressions (7) and (8) are the basic expressions for the verification of the stochastic hypothesis. In Fig. 2 we show the theoretical dependence $\psi(\alpha)$ (the curve 1) computed from the formula (7a) and the dependence found through the measurement of the lifetime $\bar{\tau}$ with the aid of (8) for different values of the nonadiabaticity parameter (the curves 2–5). It can be seen from Fig. 2 that for values of $\chi_\perp(\alpha)\lesssim 1.5\,\chi_{\perp c}(\alpha)$ the experimental quantity $\bar{\psi}$ is a function of tow variables, i.e., $\bar{\psi}=\bar{\psi}(\alpha,\chi_\perp)$. The difference between the measured, $\bar{\psi}$, and theoretical values of ψ is the greater, the closer $\chi_\perp$ is to $\chi_{\perp c}$. For values of $\chi_\perp\gtrsim 1.5\chi_{\perp c}$, the quantity $\bar{\psi}$ does not depend on $\chi_\perp$, and virtually coincides with the theoretical function $\psi(\alpha)$ for $\alpha\lesssim 50°$. The discrepancy between the functions $\psi(\alpha)$ and $\bar{\psi}(\alpha, \chi_\perp\gtrsim 1.5\chi_{\perp c})$ at large α is connected with the exactness of the asymptotic expansions with the aid of which the change in the adiabatic invariant was computed.[6]

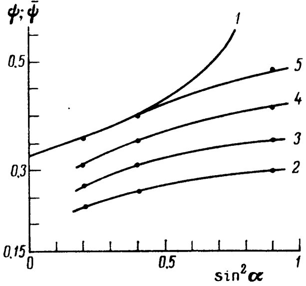

FIG. 2. Comparison of the empirical dependence of $\bar{\psi}$ on α and $\chi_\perp$ with the theoretical function $\psi(\alpha)$: 1) the theoretical curve, computed from the formula 7*a*; for $P\sim 2\times10^{-10}$ Torr, the curve 2 is the experimental dependence $\bar{\psi}(\alpha,\chi_\perp)$ for $\chi_\perp=\chi_{\perp c}$; 3) for $\chi_\perp=1.2\chi_{\perp c}$; 4) for $\chi_\perp=1.4\chi_{\perp c}$; 5) for $\chi_\perp=(1.6-2.0)\chi_{\perp c}$.

The improvement, observable in Fig. 2, in the agreement between the theory and experiment as $\chi_\perp$ increases in the interval $\chi_{\perp c}\lesssim\chi\lesssim 2\chi_{\perp c}$ is explained by the influence of two opposed effects. On the one hand the increase of $\chi_\perp$ should lead to a decrease in the accuracy of perturbation theory, with which, without allowance for the medium, Eq. (8) was derived. On the other hand, the growth of $\chi_\perp$ leads to the exponential enhancement of the role of the nonadiabaticity in comparison with the influence of the residual gas. The second effect is apparently stronger than the first, and, therefore, as $\chi_\perp$ is increased to values $\sim 2\chi_{\perp c}$, $\bar{\psi}$ is observed to tend to ψ. At higher $\chi_\perp$ ($\chi_\perp>2\chi_{\perp c}$) we again observe a growing discrepancy between $\bar{\psi}$ and ψ (see (8) and Fig. 1), which, possibly, is connected with an increase in the role of the subsequent terms in the expansion of the equation of motion in the parameter ρ/R_c.

Thus, the experimentally found change in the magnetic moment of the electron per reflection from the magnetic mirror under the condition $P\sim 2\times10^{-10}$ Torr and $1.5\chi_{\perp c}\lesssim\chi_\perp\lesssim 2\chi_{\perp c}$ agrees with the theoretical result for total vacuum ($\psi(\alpha,\ \chi_\perp\to 1.5\chi_{\perp c})\to\psi(\alpha)$). This also indicates that in the indicated region of $\chi_\perp$ values the changes in μ have a stochastic character, and are described by the Fokker–Planck equation

$$\frac{\partial n}{\partial t}=\frac{1}{2}\frac{\partial}{\partial\mu}\mu^2D(\mu)\frac{\partial n}{\partial\mu}, \tag{9}$$

in which D, the coefficient of diffusion in μ space, has the form

$$\frac{1}{2}D\approx\frac{0.5}{\tau_2}\left(\frac{\rho}{R_e}\right)^{1/2}\exp\left(-2\psi\frac{R_e}{\rho}\right),\qquad 2\psi(\alpha)\approx 0.65+0.35\sin^2\alpha. \tag{10}$$

The smallest value $\chi_\perp'$ at which the condition $\bar{\psi}(\alpha,\chi_\perp')\approx\psi(\alpha)$ is fulfilled depends on the pressure of the residual gas. The quantity $\chi_\perp'$ decreases with decreasing P, and, apparently, in the limit $\chi_\perp'(P\to 0)\to\chi_{\perp c}$.

As to the region $1.5\chi_{\perp c}>\chi_\perp\to\chi_{\perp c}$, for it the expression (10) with the corresponding functions $\bar{\psi}(\alpha,\chi_\perp)$ may be regarded not merely as a formal way of describing the particle lifetime. Indeed, since the stochasticity of the variation of the magnetic moment is realized at large $\chi_\perp$, it is natural to assume that it is also maintained at smaller $\chi_\perp$, when the influence of scattering by the residual gas becomes important. Then the empirical function $D[\psi(\alpha, \chi_\perp)]$ is a generalization of the coefficient of diffusion to the case of small $\chi_\perp$.

If in (9) we average μ^2 and take into account the fact that $2\psi R_e/\rho = a_0 + a_1 y$, $y = \sin^2\alpha$, then the solution to Eq. (9) with the obvious boundary condition $n(\mu_c, t) = 0$ will be

$$n(y,t) = \sum_{m=0}^{\infty} c_m \exp\left(-k_m^2 t + \frac{a_1 y}{2}\right) J_1\left\{\lambda_m \exp\left[\frac{a_1(y-y_c)}{2}\right]\right\},$$

$$k_m^2 = {}^1/_4 a_1^2 \lambda_m^2 \overline{y}^2 D(y_c),$$

where J_1 is the Bessel function of the first order. The escape rate (i.e., the counting rate) is determined by the function $(\partial n/\partial y)_{y=y_c}$, and its decreasing part is approximately described by the equation

$$j(y_c, t) \approx A \exp(-k_1^2 t), \quad k_1^2 \approx 3.68 a_1^2 \overline{y}^2 D(y_c).$$

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The change in the anisotropy of the Fermi surface in the p-type semiconducting alloy $Bi_{0.9}Sb_{0.1}$ upon going over into the gapless state under the action of pressure

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The oscillatory and galvanomagnetic effects arising in the p-type semiconducting alloy $Bi_{0.9}Sb_{0.1}$ on going over into the gapless state (GS) under pressure at liquid-helium temperatures have been investigated. As the GS was approached in samples of the alloy with "light"-hole concentrations $\sim 10^{15}$ cm^{-3}, Shubnikov oscillations were observed in the entire angle range as the magnetic field was rotated in the binary-bisector and bisector-trigonal planes, and this allowed the complete reestablishment of the shape of the hole Fermi surface. It is shown that, in the first approximation, the hole Fermi surfaces at the L point in the investigated alloy are highly anisotropic ellipsoids, the anisotropy of the ellipsoids increasing appreciably in the transition into the GS. The data obtained are discussed on the basis of the Abrikosov theory of the band spectrum of materials of the Bi type.

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1. INTRODUCTION

A characteristic property of the energy spectrum of Bi and the alloys $Bi_{1-x}Sb_x$ $(x < 0.2)$ is the smallness of the direct gap ε_{gL} at the L point of the reduced Brillouin zone. The band structures of Bi and Sb at the L point are mutually inverted[1–5]; in the $Bi_{1-x}Sb_x$ alloys the inversion of the terms at the L point is removed as x is increased, as a result of which a gapless state (GS: $\varepsilon_{gL} \approx 0$) is realized at some $x = x_0$. The most probable value of x_0 is roughly 0.02.[6]

The smallness of the gap ε_{gL} leads to highly nonparabolic electron and hole spectra at the L points in Bi and the alloys $Bi_{1-x}Sb_x$. Several models for the energy spectrum of the carriers at L have been coexisting right up to the present time.[7–9]

In the coordinate system fixed to the ellipsoid at L the Lax dispersion law has the form[7]

$$\sum_i p_i^2/2m_i = \varepsilon(1+\varepsilon/\varepsilon_{gL}), \tag{1}$$

where the m_i are the masses at the bottom of the band; these masses satisfy the relation

$$m_i/m_0 = (1+2|M_i|^2/\varepsilon_{gL})^{-1}, \tag{2}$$

where the M_i are the Kane matrix elements.

It is known from experiment that the electron masses in the directions of the short semiaxes (x, y) of the ellipsoids are much smaller than the free-electron mass,[10–12] from which it follows that $m_{x,y}/m_0 \approx \varepsilon_{gL}/2|M_{x,y}|^2$. Lax assumed that in the direction, z, of elongation of the electron ellipsoids the mass m_z at the bottom of the band depends similarly on the gap ε_{gL}.

According to the Lax model (see (1) and[7]), the electron and hole constant-energy surfaces at L are strictly