Spin waves in a randomly inhomogeneous anisotropic medium

V. A. Ignatchenko and R. S. Iskhakov

L. V. Kirenskiĭ Physics Institute, USSR Academy of Sciences (Siberian Branch)
(Submitted May 25, 1976)
Zh. Eksp. Teor. Fiz. **72**, 1005–1017 (March 1977)

The modification of the dispersion relation and the damping of the spin waves in a medium in which the magnetic-anisotropy parameters (the magnitude of the anisotropy and the orientation of its axis) are randomly fluctuating in space are calculated by the methods of the correlation theory of random functions. The spin waves are treated with neglect of the dipole-dipole interaction. The nonuniformity of the parameters of the medium is described by a random function with an arbitrary correlation length $1/k_0$ determined by the size of the inhomogeneities. The presence of anisotropic inhomogeneities leads to the appearance of a static stochastic magnetic structure with a correlation length $1/k_a$ determined by the exchange interaction and the magnetic field. Both of the characteristic wave numbers k_0 and k_a appear in the modified dispersion law. A shift of the frequency of the uniform oscillation ($k = 0$) and a decrease of the damping for $k \gg k_0$ are also characteristic for anisotropic inhomogeneities. The limiting case $k \ll k_0$ describes the modification of the dispersion relation and the damping in amorphous ferromagnets, in which $k_0 \sim \pi/a$ (a is the lattice constant).

PACS numbers: 75.30.Ds, 75.30.Gw

INTRODUCTION

The quasi-particle spectrum in an imperfect crystal has been investigated in a large number of papers, the results of which have been reflected in the monographs of Maradudin, Montroll and Weiss[1] and Maradudin[2] (for phonons) and Izyumov and Medvedev[3] (for magnons). The inhomogeneities considered in these papers were impurity atoms differing in their interaction constant, mass or spin magnitude from the atoms of the perfect crystal lattice; in papers by Kaneyoshi[4] and Edwards and Jones[5,6] another type of inhomogeneity was investigated—disordered alloys of magnetic with nonmagnetic materials. Now yet another type of inhomogeneity in magnetically ordered materials is being intensively investigated—amorphous magnets. In view of the great mathematical difficulties, two simplified models are considered. The first is the so-called stochastic lattice model, in which the lattice is ideal but the exchange-interaction constant fluctuates[7]; the second is a model with topological disorder, in which the angles between the bonds (the Bravais angles) fluctuate and all the interactions are assumed to be constant.[8]

It is clear that in the long-wavelength approximation ($ka \ll 1$) all microscopic inhomogeneities (whose size is of the order of the lattice constant a) will lead to the same modification of the dispersion relation. Therefore, besides the microscopic approach typical of[1–8], for long waves it is also correct to use the phenomenological approach in which the inhomogeneities are modeled by stationary random functions of the coordinates, reflecting the spatial fluctuations of the density of the material, the interaction "constant," etc.

The propagation of waves in media with parameters varying randomly in space is the subject of a vast literature (cf., e.g., the review[9]), the problematics of which, it is true, differs substantially from that of[1–8]. In the former case the phenomenological method is used to study light scattering in the atmosphere, diffuse reflection of radio waves from the ionosphere, etc. However, in the papers of Howe[10] and Henderson and de Graaf[11] such a method has already also been used to obtain the dispersion relation.

In the present paper the phenomenological approach, based on the correlation theory of random functions, is applied to calculate the dispersion relation and damping of the spin waves in a medium with spatially fluctuating magnetic anisotropy. So far as we know, this question has not been investigated before, either in a microscopic model or phenomenologically. We can mention only the following papers. For a discrete model of an amorphous magnet, Harris, Plischke and Zuckermann[12] have calculated the effect of inhomogeneities in the anisotropy on the Curie temperature and magnetization; the phenomenological approach has been widely used to calculate the effect of anisotropic inhomogeneities on the uniform ferromagnetic resonance (FMR) (for crystallites with no mutual exchange interaction, cf., e.g.,[13]; for strongly interacting crystallites, cf.[14,15]). However, the modification of the dispersion relation and the spin-wave damping were not considered in these papers.

At the same time, anisotropic inhomogeneities are characteristic for magnetically ordered materials. Thus, in a polycrystalline material the orientation of the anisotropy axis can be regarded as a random function of the space coordinate $\mathbf{r}$. True, if the crystallites are sufficiently large and are, therefore, relatively independent, then, besides the method of random functions, simpler methods can be used, e.g., averaging with the appropriate distribution function for physical characteristics calculated for one crystallite. However, for finely dispersed systems, in which the crystallites can no longer be treated independently since the range r_α of the exchange correlations becomes greater than the radius r_0 of a crystallite, only by representing the

 0038-5646/77/4503-0526$02.40

anisotropy in the form of a random function is it possible to formulate the problem correctly. The relationship $r_\alpha \gg r_0$ should also be well fulfilled for all amorphous magnets in which crystalline order is preserved only in the first coordination spheres. Nonuniform internal elastic stresses and certain defects can also be described in many cases by an effective anisotropy in which both the magnitude of the anisotropy and the orientation of its axis are random functions of the coordinates.

In the first section we discuss the approximations used in the paper, justify the choice of the form of the correlation function of the inhomogeneities, and give the dispersion relations for a medium with isotropic inhomogeneities (the results of the subsequent sections of the paper will be compared with these). In the second section the static stochastic magnetic structure that arises when the magnetic-anisotropy axis fluctuates is considered. In the third section the dispersion relations for spin waves propagating in a medium with anisotropic inhomogeneities are derived.

1. DISPERSION RELATIONS AND DAMPING OF WAVES IN A MEDIUM WITH ISOTROPIC INHOMOGENEITIES

We shall give an account of the approximations employed in the paper, using the simple example of a one-dimensional wave equation of the type of the elasticity equation for a medium with nonuniform density:

$$[G+\Delta G\rho(x)]\frac{\partial^2 u}{\partial t^2}-A\frac{\partial^2 u}{\partial x^2}=0, \tag{1.1}$$

where A is the interaction constant, G is the mean density, ΔG is the mean-square fluctuation of the density, and $\rho(x)$ is a dimensionless stationary random function with mathematical expectation equal to zero and dispersion $D=1$.

Expanding (1.1) in plane waves $\sim \exp[i(kx-\omega t)]$, we obtain

$$L(k)u(k)+\gamma\int u(k')\rho(k-k')dk'=0, \tag{1.2}$$

where

$$L(k)=1-Ak^2/G\omega^2,\ \gamma=\Delta G/G.$$

The dispersion relation corresponding to uniform density of the substance ($\gamma=0$) is $L(k)=0$, i.e.,

$$\omega=sk,\ s=(A/G)^{1/2}. \tag{1.3}$$

Averaging (1.2) over random realizations and decoupling the correlator $\langle u(k')\rho(k-k')\rangle$ that is formed, in lowest order in the parameter γ we obtain

$$L(k)\langle u(k)\rangle-\gamma^2\iint\frac{dk'\,dk''}{L(k')}\langle u(k'')\rangle\langle\rho(k-k')\rho^*(k''-k')\rangle=0. \tag{1.4}$$

Since $\rho(x)$ is a stationary random function, we have

$$\langle\rho(k)\rho^*(k')\rangle=S(k)\delta(k-k'), \tag{1.5}$$

where $S(k)$ is the spectral density of the correlation function. Substituting (1.5) into (1.4) and performing one integration, we obtain the dispersion relation in the form

$$L(k)-\gamma^2\int\frac{dk'\,S(k-k')}{L(k')}=0. \tag{1.6}$$

Thus, all that we need to know about the structure of the inhomogeneity in our approximation is $S(k)$. For each concrete model $S(k)$ can be calculated exactly if the autocorrelation function $K(x-x')$ of the inhomogeneities is known. In most cases, however, a knowledge of just two parameters of the autocorrelation function—the relative dispersion γ^2 of the fluctuating parameter, and the correlation length $1/k_0$—is sufficient; the details of the law describing the cutoff of the correlation do not play an important role, since the law appears inside the integral in the general dispersion relation (1.6).

For the estimates we choose the correlation function, and the spectral density related to its Fourier transform, in the form

$$K(x-x')=De^{-k_0|x-x'|},\quad S(k)=\frac{D}{\pi}\frac{k_0}{k_0^2+k^2}, \tag{1.7}$$

where k_0 is the characteristic wave number ($b=2/k_0$ is the characteristic size of an inhomogeneity and $r_0=1/k_0$ is the correlation length of the random function $\rho(x)$ describing the inhomogeneities); D is the dispersion of $\rho(x)$; in our case $D=1$ by definition.

The spectral density that we have chosen has the form characteristic for white noise for $k\ll k_0$ and is cut off sufficiently sharply when $k>k_0$. This form of $S(k)$ describes a sufficiently wide class of inhomogeneities. If the nonuniformity has a macroscopic character, the analysis is valid for wave numbers both smaller and larger than k_0. If the inhomogeneities have a size of the order of the lattice constant a, our treatment is valid only for $k\ll k_0$, as the long-wavelength approximation is limited by the relationship $ka\ll 1$.

We substitute $S(k)$ into (1.6) and consider the integral that is formed. It has poles on the real axis, which give the damping, and poles in the complex plane, which lead to modification of the dispersion relation. The integral (like all subsequent integrals in this paper), is taken exactly, and (1.6) takes the form

$$q^2-k^2=(\gamma q)^2\frac{k_0^2+q^2-k^2+ik_0(k_0^2+q^2+k^2)/q}{[k_0^2+(k-q)^2][k_0^2+(k+q)^2]}, \tag{1.8}$$

where $q=\omega/s$ is the frequency normalized to the dimensions of wave number.

Because of the approximation (1.4) we can treat only small departures from the unperturbed dispersion relation (1.3). Solving Eq. (1.8) for q, in first order in γ^2 we obtain the final form of the dispersion relation for a one-dimensional medium with a fluctuating density:

$$\omega=sk\left\{1+\frac{\gamma^2}{2}\frac{k^2}{k_0^2+4k^2}+\frac{i\gamma^2 k}{2k_0}\frac{k_0^2+2k^2}{k_0^2+4k^2}\right\}. \tag{1.9}$$

By the same method, for waves in a one-dimensional medium with a fluctuating interaction constant A we ob-

tain an expression corresponding to the results of[10] [1]):

$$\omega=sk\left\{1-\frac{\gamma_1^2}{2}\frac{k_0^2+3k^2}{k_0^2+4k^2}+\frac{i\gamma_1^2 k}{2k_0}\frac{k_0^2+2k^2}{k_0^2+4k^2}\right\}, \qquad (1.10)$$

where $\gamma_1=\Delta A/A$.

For real waves in three-dimensional space the wave number k in the expressions (1.2)–(1.6) is replaced by a three-dimensional wave vector. In the spherical system of coordinates the correlation function and the corresponding spectral density will have the form

$$K(\mathbf{r})=De^{-k_0 r}, \quad S(\mathbf{k})=\frac{D}{\pi^2}\frac{k_0}{[k_0^2+k^2]^2}. \qquad (1.11)$$

For a plane wave propagating in a three-dimensional region with fluctuating density G we obtain the following dispersion relation:

$$\omega=sk\left\{1-\frac{\gamma^2}{2}\frac{k^2}{k_0^2+4k^2}+i\gamma^2\frac{k^3}{k_0(k_0^2+4k^2)}\right\}. \qquad (1.12)$$

For a wave of the same type in the presence of fluctuation of the interaction constant A the dispersion relation has the form

$$\omega=sk\left\{1-\frac{\gamma_1^2}{2}\frac{k_0^2+5k^2}{k_0^2+4k^2}+i\gamma_1^2\frac{k^3}{k_0(k_0^2+4k^2)}\right\}. \qquad (1.13)$$

Comparison of these expressions with the corresponding one-dimensional expressions (1.9) and (1.10) shows that both the modification of the dispersion relation and the damping change sharply (even to the extent of a change of sign of the correction) when we go over to three-dimensional space. Consequently, the one-dimensional model is a very poor approximation to the real situation. This is connected with the fact that waves with different $\mathbf{k}$ make a contribution both to the modification and to the damping (this is expressed by the integral term in (1.6)); with change of the dimensionality of space the number of such waves and the character of the contribution made by them change sharply.

We now consider the spin waves in a medium with a fluctuating exchange-interaction constant α with neglect of the dipole–dipole interaction. The spin system is described by the Landau–Lifshitz equation:

$$\dot{\mathbf{M}}=-g[\mathbf{M},\mathbf{H}'], \quad \mathbf{H}'=-\frac{\partial\mathcal{H}}{\partial\mathbf{M}}+\frac{\partial}{\partial x_i}\frac{\partial\mathcal{H}}{\partial(\partial\mathbf{M}/\partial x_i)}. \qquad (1.14)$$

The Hamiltonian of the system and the effective magnetic field corresponding to it have, in our case, the form

$$\begin{aligned}\mathcal{H}&=\tfrac{1}{2}[\alpha+\Delta\alpha\rho(\mathbf{r})](\nabla\mathbf{M})^2-\mathbf{MH},\\ \mathbf{H}'&=\mathbf{H}+\alpha\nabla^2\mathbf{M}+\Delta\alpha\{\rho\nabla^2\mathbf{M}+(\nabla\rho)(\nabla\mathbf{M})\}.\end{aligned} \qquad (1.15)$$

Putting

$$\mathbf{M}(\mathbf{r},t)=\mathbf{M}_0+\mathbf{m}(\mathbf{r},t) \qquad (1.16)$$

and orienting the z axis along $\mathbf{M}_0\parallel\mathbf{H}$, we obtain for the circular projections $m^{\pm}=m_x\pm im_y$ the following system of linearized equations of motion:

$$\left[H-M_0(\alpha+\Delta\alpha\rho)\nabla^2-M_0\Delta\alpha(\nabla\rho)\nabla\pm\frac{i}{g}\frac{\partial}{\partial t}\right]m^{\pm}=0. \qquad (1.17)$$

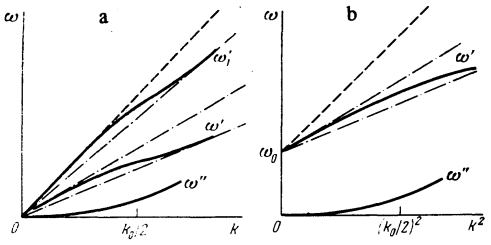

FIG. 1. Dispersion relations (the curves ω' and ω_1') and damping (the curves ω'') for isotropic inhomogeneities: a) the dependence $\omega(k)$ for phonons; b) the dependence $\omega(k^2)$ for spin waves. The dashed curves correspond to the unperturbed dispersion relations, the solid curves ω' to fluctuation of the interaction constant, and the curve ω_1' to fluctuation of the density.

Next, by the method described above, we obtain the following dispersion relation for m^+:

$$\omega=gH+g\alpha M_0k^2\left[1-\gamma_2^2\frac{k_0^2+5k^2}{k_0^2+4k^2}+i\gamma_2^2\frac{2k^3}{k_0(k_0^2+4k^2)}\right], \qquad (1.18)$$

where $\gamma_2=\Delta\alpha/\alpha$.

Comparison of this expression with the corresponding expression (1.13) for elastic waves shows that the corrections have the same form if we put $\gamma_2^2=2\gamma_1^2$.

Graphs of the expressions (1.12), (1.13) and (1.18) are shown in Fig. 1. The size $b=2/k_0$ of an inhomogeneity is clearly manifested in the shape of the dispersion curves: a change in the character of the dispersion law occurs in the vicinity of the point $k=k_0/2$. For $k\ll k_0/2$ formula (1.18) goes over into the corresponding formulas of[4,5].

The dispersion relation for fluctuation of the exchange constants of an antiferromagnet can be obtained analogously. It is similar in form to the dispersion relation (1.13) for phonons and for $k\ll k_0/2$ it goes over into the corresponding expression of[6].

2. ANISOTROPIC INHOMOGENEITIES. THE STOCHASTIC MAGNETIC STRUCTURE

Spatial nonuniformity of the anisotropy can consist both in fluctuation of the direction of the anisotropy axis and in fluctuation of the magnitude of the anisotropy. In this section we consider only the fluctuation of the direction of the anisotropy axis.

We choose the Hamiltonian in the form (as in the first section, we neglect the dipole–dipole interaction)

$$\mathcal{H}=\frac{1}{2}\alpha\left(\frac{\partial\mathbf{M}}{\partial x_i}\right)^2-\mathbf{MH}_0-\frac{1}{2}\beta_0(\mathbf{Mn})^2-\frac{1}{2}\beta(\mathbf{Ml})^2 \qquad (2.1)$$

and the effective magnetic field is, correspondingly,

$$\mathbf{H}'=\alpha\nabla^2\mathbf{M}+\mathbf{H}_0+\beta_0\mathbf{n}(\mathbf{nM})+\beta\mathbf{l}(\mathbf{lM}). \qquad (2.2)$$

Here β_0 is the constant of the uniaxial anisotropy that is uniform over the whole material, $\mathbf{n}$ is the unit vector along the easy axis of this anisotropy, β is the nonuniform-anisotropy constant and $\mathbf{l}=\mathbf{l}(\mathbf{r})$ is the easy-axis unit vector, which has a different direction in different

crystallites.

This model of the two anisotropies—the uniform and nonuniform—is very convenient for taking texture into account. Indeed, in this model we can assume that the unit vector **l** is oriented completely randomly in space, so that the probability density that the orientation of **l** is in the solid angle $d\Omega = \sin\theta\, d\varphi\, d\theta$ is a constant:

$$f(\mathbf{l}) = f(\theta, \varphi) = 1/4\pi. \tag{2.3}$$

The degree of texturing can be taken into account not by change of the distribution function but by the relationship between the constants β_0 and β; in this way, all cases are encompassed: the perfect monocrystal ($\beta_0 \neq 0$, $\beta = 0$), polycrystals textured to different extents ($\beta_0 \neq 0$, $\beta \neq 0$), and the "ideal" (untextured) amorphous state ($\beta_0 = 0$, $\beta \neq 0$).

A Hamiltonian similar to (2.1) was considered earlier in[14], in which the effect of nonuniformity of the orientation of the anisotropy axis on the uniform FMR in thin films was studied. Therefore, the results of this section will partly repeat the results of[14]; however, the concrete expressions differ from those of[14] on account of the difference in the models: there we had a thin film and the dipole–dipole interaction was taken into account; here we have an unbounded medium and the dipole–dipole interaction is neglected.

Let the external constant magnetic field $\mathbf{H}_0$ be directed along the easy axis **n**, parallel to the z axis. Substituting (2.2) into (1.14) we convince ourselves that the linearization (with respect to the dynamical variable) used in the spin-wave theory cannot be carried out by the usual representation (1.16) of the magnetization, which is valid both in a perfect crystal and in a randomly inhomogeneous isotropic medium. The spatial fluctuations of the anisotropy axis lead to the result that the ground state of the magnetization becomes directionally nonuniform and the linearization with respect to the dynamical variable should be carried out by a representation of the magnetization in which the ground state is a function of the space coordinate **r**:

$$\mathbf{M}(\mathbf{r}, t) = \mathbf{M}(\mathbf{r}) + \mathbf{m}(\mathbf{r}, t). \tag{2.4}$$

Substituting this expression into the Landau-Lifshitz equation we obtain two systems of differential equations: a static system for the ground state $\mathbf{M}(\mathbf{r})$ and a dynamical system for the elementary excitations $\mathbf{m}(\mathbf{r}, t)$ propagating against the background of the nonuniform ground state. The dynamical system will be considered in the next section; here we study the static system.

For $\beta = 0$ and the chosen orientations of the field $\mathbf{H}_0$ and the unit vector **n** of the uniform anisotropy the static system has a uniform solution: $M_z = M$, $M_x = M_y = 0$. We shall assume that for $\beta \neq 0$ the deviations from the uniform solution are not large, and linearize the system ($M_z \approx M$; M_x, $M_y \ll M$); the limits of applicability of this approximation will be considered below.

Then the static system will take the form

$$\begin{aligned} \alpha\nabla^2 M_x - (H_0 + \beta_0 M + \beta M\rho_x) M_x + \beta M\rho_{xy} M_y &= -\beta M^2 \rho_{xz}, \\ \alpha\nabla^2 M_y - (H_0 + \beta_0 M + \beta M\rho_y) M_y + \beta M\rho_{xy} M_x &= -\beta M^2 \rho_{yz}, \end{aligned} \tag{2.5}$$

where $\rho_{ij} = l_i l_j$ $(i \neq j)$, $\rho_i = l_z^2 - l_i^2$, and all the $\rho = \rho(\mathbf{r})$ are stationary random functions of the coordinates. Some of their probability characteristics were determined from the distribution function (2.3) in[14].

In the system (2.5) we shall neglect all products of the functions with the transverse magnetization components M_x and M_y; the limits of applicability of this approximation will also be considered below. Then this system breaks down into two independent equations, from which we can determine the Fourier transforms $M_x(\mathbf{k})$ and $M_y(\mathbf{k})$:

$$M_x(\mathbf{k}) = \frac{\beta M^2 \rho_{xz}(\mathbf{k})}{\alpha M k^2 + H}, \quad M_y(\mathbf{k}) = \frac{\beta M^2 \rho_{yz}(\mathbf{k})}{\alpha M k^2 + H}, \tag{2.6}$$

where $H = H_0 + \beta_0 M$.

We shall determine the probability characteristics of the solution for $M_i(\mathbf{k})$. The mathematical expectation $\langle M_i(\mathbf{k})\rangle \sim \langle \rho_{ij}(\mathbf{k})\rangle = 0$. The spectral densities of the magnetizations M_x and M_y are the same and will be denoted by S_M:

$$S_M(\mathbf{k}) = \frac{\beta^2 M^2}{(\alpha M k^2 + H)^2} S(\mathbf{k}), \tag{2.7}$$

where $S(\mathbf{k})$—the spectral density of the functions ρ_{xz} and ρ_{yz}—is determined by the expression (1.11).

Thus, the spectral density of the spatial fluctuations of the magnetization has the form

$$S_M = \left(\frac{\beta M}{\alpha\pi}\right)^2 \frac{k_0 D}{(k_\alpha^2 + k^2)^2 (k_0^2 + k^2)^2}, \tag{2.8}$$

where $k_\alpha = (H/\alpha M)^{1/2}$ is the characteristic wave number of the exchange correlations and $r_\alpha = 1/k_\alpha$ is the range of the exchange correlations. It follows from this expression that the properties of the stochastic magnetic structure depend on the relative sizes of the correlations lengths r_α and r_0.

The case $r_\alpha \ll r_0$ corresponds to the approximation of noninteracting (by exchange) crystallites; in this case S_M takes the form

$$S_M \approx \left(\frac{\beta M^2}{H\pi}\right)^2 \frac{k_0 D}{(k_0^2 + k^2)^2}, \tag{2.9}$$

i.e., it does not contain the exchange constant α and is analytically the same as the spectral density $S(\mathbf{k})$ of the function $\rho_{ij}(\mathbf{r})$. Physically, this signifies that in each crystallite the orientation of the magnetization is established in accordance with the local minimum of the energy associated with the anisotropy and magnetic field of the given crystallite.

The case $r_\alpha \gg r_0$ corresponds to the approximation of strongly interacting crystallites; in this case S_M takes the form

$$S_M \approx \left(\frac{\beta M}{\pi\alpha k_0^2}\right)^2 \frac{k_0 D}{(k_\alpha^2 + k^2)^2}. \tag{2.10}$$

In this case the properties of the stochastic magnetic structure differ sharply from those of the random function $\rho(\mathbf{r})$ which generates this structure. The correlation length of the random function $\mathbf{M}(\mathbf{r})$ is now determined not by the size of the crystallites but by the interaction constant and other parameters of the magnetic system. The local minimum of the energy associated with the anisotropy and magnetic field no longer has decisive significance for the orientation of the magnetization in a given crystallite. The exchange correlations defeat the disordering action of the nonuniform anisotropy, making the magnetizations of neighboring crystallites almost parallel to each other. A magnetic structure arises in which small spatial fluctuations of the magnetization vector relative to the mean position have a characteristic wavelength $2\pi/k_\alpha$ that depends on the magnitude of the magnetic field $\mathbf{H}$.

The case $r_\alpha \gg r_0$ is characteristic for, e.g., thin magnetic films ($r_0 \lesssim 10^{-6}$ cm) in sufficiently low magnetic fields. In films the stochastic magnetic structure (often called the "fine magnetic structure" or "magnetization ripple") is observed by means of an electron microscope.[16] A theoretical calculation of the characteristic wavelength of such a structure and of the dispersion for films was first carried out by Hoffmann[17]; the form of its spectral density was obtained theoretically in[14] and investigated experimentally in[18,19]; the effect of the stochastic structure on the uniform FMR was studied theoretically in[15].

Here we wish to draw attention to the fact that the stochastic magnetic structure is not connected with thin-film size effects and should arise in all finely dispersed magnetic systems and in all magnetic materials with microscopic inhomogeneities, if these inhomogeneities have an anisotropic character; in particular, the latter situation is evidently characteristic for amorphous magnets.

The correlation function of the magnetic structure and its dispersion are determined by the expressions

$$K_M(\mathbf{r}) = \int_{-\infty}^{\infty} S_M(\mathbf{k}) e^{i\mathbf{k}\mathbf{r}}\, d\mathbf{k}. \qquad D_M = K_M(0). \tag{2.11}$$

In the calculation of the dispersion the integral is taken exactly and we have

$$D_M = \frac{\beta^2 M^2 D}{\alpha^2 k_\alpha (k_0 + k_\alpha)^3} \tag{2.12}$$

or, in the limiting cases,

$$\begin{aligned} &D_M \approx D(\beta M^2/H)^2, \quad r_\alpha \ll r_0, \\ &D_M \approx D\frac{(\beta M)^2}{\alpha^2 k_0^3}\left(\frac{M}{H}\right)^{1/2}, \quad r_\alpha \gg r_0. \end{aligned} \tag{2.13}$$

We shall discuss the conditions of applicability of the expressions obtained. The linearization of the static system of equations corresponds to the inequality $D_M \ll M^2$. However, we did not confine ourselves to the linearization: even in the linearized system (2.5) we neglected the products ρM_i; as shown in[14], this is possible only when the stronger inequality $8D_M \ll M^2$ is fulfilled. Using the latter inequality we obtain the following conditions in the limiting cases:

$$\begin{aligned} &H \gg \beta M/\sqrt{2}, \quad r_\alpha \ll r_0, \\ &H \gg \frac{1}{4}\left(\frac{\beta}{\alpha k_0^2}\right)^3 \beta M, \quad r_\alpha \gg r_0. \end{aligned} \tag{2.14}$$

Thus, for noninteracting crystallites we obtain the well-known result: for the declination of the magnetization from the external field to be small the latter should be very much greater than the effective anisotropy field $H_a = \beta M$, the axis of which does not coincide with the direction of the field. For interacting crystallites the effective anisotropy field is multiplied by the coefficient $(\beta/\alpha k_0^2)^3$, which can be several orders smaller than unity; in this case our treatment remains valid down to very low magnetic fields.

3. SPIN WAVES IN AN ANISOTROPICALLY INHOMOGENEOUS MEDIUM

We turn now to the dynamical system of equations for $\mathbf{m}(\mathbf{r}, t)$ that is formed as a result of substituting the expression (2.4) into the Landau–Lifshitz equation. After the change to Fourier components the linearized dynamical system of equations takes the form

$$\begin{aligned} &\frac{i\omega}{g} m_x(\mathbf{k}) + (\alpha M k^2 + H) m_y(\mathbf{k}) - \beta[F_1(\mathbf{k}) - G_1(\mathbf{k})] = 0, \\ &\frac{i\omega}{\sigma} m_y(\mathbf{k}) - (\alpha M k^2 + H) m_x(\mathbf{k}) + \beta[F_2(\mathbf{k}) - G_2(\mathbf{k})] = 0. \end{aligned} \tag{3.1}$$

where the terms proportional to the nonuniform anisotropy are determined by the expressions

$$\begin{aligned} F_1(\mathbf{k}) &= M\int d\mathbf{k}_1 [m_x(\mathbf{k}_1)\rho_{xy}(\mathbf{k}-\mathbf{k}_1) - m_y(\mathbf{k}_1)\rho_y(\mathbf{k}-\mathbf{k}_1)], \\ G_1(\mathbf{k}) &= \iint d\mathbf{k}_1\, d\mathbf{k}_2\, m_y(\mathbf{k}_2)[M_x(\mathbf{k}_1)\rho_{xz}(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2) + 4M_y(\mathbf{k}_1)\rho_{yz}(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)]. \end{aligned} \tag{3.2}$$

F_2 and G_2 are obtained from F_1 and G_1 by replacing x by y and y by x.

Thus, anisotropic inhomogeneities lead to the appearance of terms of two types in the equation of motion. In coordinate space the terms of the type F have the form $\rho(\mathbf{r})m_i(\mathbf{r}, t)$ and correspond to the direct interaction of the dynamical part of the magnetization $\mathbf{m}$ with the defining random function ρ; only terms of this form appeared in the equations of motion of the first section of this paper, in which isotropic inhomogeneities were considered. In coordinate space the terms of the type G have the form $\rho(\mathbf{r})M_i(\mathbf{r})m_j(\mathbf{r}, t)$ and describe the interaction of the spin waves $\mathbf{m}$ with the stochastic magnetic structure $\mathbf{M}(\mathbf{r})$ due to $\rho(\mathbf{r})$. The need to take nonlinear terms of the type G into account is justified in[15], where it is shown that they make the same contribution to the shift in the uniform FMR frequency as the terms of the type F; as will be seen from the following, this also remains valid for $k \neq 0$.[2)]

We substitute (2.6) into (3.2), change to the circular variables $m^\pm = m_x \pm i m_y$ in the dynamical system (3.1), and average the resulting equations over the random realizations, taking into account the relations that follow from (2.3) and the stationarity of the random functions $\rho(\mathbf{r})$:

$$\begin{aligned}&\langle\rho_{ij}(\mathbf{k})\rho_{ij}^*(\mathbf{k}_1)\rangle=S(\mathbf{k})\delta(\mathbf{k}-\mathbf{k}_1),\\&\langle\rho_i(\mathbf{k})\rho_j^*(\mathbf{k}_1)\rangle_{i\neq j}=2S(\mathbf{k})\delta(\mathbf{k}-\mathbf{k}_1),\\&\langle\rho_i(\mathbf{k})\rho_i^*(\mathbf{k}_1)\rangle=4S(\mathbf{k})\delta(\mathbf{k}-\mathbf{k}_1),\\&\langle\rho_l(\mathbf{k})\rho_{ij}^*(\mathbf{k}_1)\rangle=0,\end{aligned}\qquad(3.3)$$

where $\rho(\mathbf{k})=\rho^*(-\mathbf{k})$ by virtue of the fact that the function $\rho(\mathbf{r})$ is real.

Decoupling the correlations $\langle m\rho\rangle$ exactly to terms $\sim\beta^2$, we obtain the following dispersion relation for $\langle m^+\rangle$:

$$\frac{\omega}{g}=\alpha Mk^2+H+(\beta M^2)\int dk_1\left[\frac{3S(k-k_1)}{L^+(k_1)}-\frac{2S(k-k_1)}{L^-(k_1)}+\frac{5S(k_1)}{B(k_1)}\right],\qquad(3.4)$$

where

$$L^{\pm}(k_1)=\omega/g\mp B(k_1),\ B(k_1)=\alpha Mk_1^2+H.$$

The first two terms under the integral in this expression arise from the inhomogeneous terms of the type F and the last term in the integrand arises from the inhomogeneous terms of the type G. We substitute $S(\mathbf{k})$, determined by the expression (1.11), into (3.4) and consider the integrals that are formed. The first integral has poles on the real axis, which give the damping, and poles in the complex plane, which lead to modification of the dispersion relation. The second and third integrals lead only to modification of the dispersion relation, since they do not have poles on the real axis. The integrals are taken exactly; when the equation that is obtained after the integration is solved for ω in the same approximations as in Sec. 1, we obtain the final form of the dispersion relation:

$$\omega/g=H+\alpha Mk^2+(\beta^2MD/\alpha k_0^2)[\delta'(\varkappa,\varkappa_\alpha)+i\delta''(\varkappa)].\qquad(3.5)$$

where the corrections δ' and δ'' are determined by the following expressions:

$$\begin{aligned}\delta'(\varkappa,\varkappa_\alpha)&=-\frac{3}{1+4\varkappa^2}-2\frac{(1-\sqrt{\varkappa^2+2\varkappa_\alpha^2})^2+\varkappa^2}{(1-2\varkappa_\alpha^2)^2+4\varkappa^2(\varkappa^2+2\varkappa_\alpha^2)}+\frac{5}{(\varkappa_\alpha+1)^2},\\\delta''(\varkappa)&=\frac{6\varkappa}{1+4\varkappa^2},\quad \varkappa=\frac{k}{k_0},\quad \varkappa_\alpha=\frac{k_\alpha}{k_0}.\end{aligned}\qquad(3.6)$$

The real and imaginary parts of (3.5) are depicted in Fig. 2a for the case $k_\alpha\ll k_0$. The damping ω'' reaches a maximum at $k=k_0/2$, going to zero as $k\to0$ and at $k\gg k_0$. The modification of the dispersion relation depends not only on the parameter k_0, as was the case in an isotropically inhomogeneous medium, but also on the second parameter k_α—the characteristic wave number of the stochastic magnetic structure: at $k\approx2k_\alpha$ the correction δ' changes sign.

The frequency shift also remains nonzero for the uniform oscillation ($k=0$):

$$\delta'(0,\varkappa_\alpha)=-\varkappa_\alpha\frac{4+18\varkappa_\alpha+20\varkappa_\alpha^2+6\varkappa_\alpha^3}{1+5\varkappa_\alpha+9\varkappa_\alpha^2+7\varkappa_\alpha^3+2\varkappa_\alpha^4}.\qquad(3.7)$$

This expression is always negative, and tends to zero as $\varkappa_\alpha\to0$ (the point 0 is unphysical; cf. (2.14)) and to -3 for $\varkappa_\alpha\gg1$.

We now consider the spin waves in a medium in which the anisotropy constant fluctuates while the direction of the easy axis of the anisotropy is the same over the whole material (and coincides, for simplicity, with the direction of the external magnetic field $\mathbf{H}_0$). The Hamiltonian in this case takes the form

$$\mathcal{H}=\frac{1}{2}\alpha\left(\frac{\partial\mathbf{M}}{\partial x_i}\right)^2-\mathbf{MH}_0-\frac{1}{2}[\beta+\Delta\beta\rho(\mathbf{r})](\mathbf{Mn})^2,\qquad(3.8)$$

where β is the mathematical expectation and $\Delta\beta$ the mean-square fluctuation of the anisotropy constant; $\rho(\mathbf{r})$ is a dimensionless stationary random function with mathematical expectation equal to zero and dispersion $D=1$.

By writing the equations of motion for this Hamiltonian we see that the ground state in this case remains uniform, as in the case of isotropic inhomogeneities, and a stochastic magnetic structure does not arise. For the dynamical variables we obtain the following system of linearized equations in the circular projections:

$$\left[H_0+\beta M-\alpha M\nabla^2+\Delta\beta M\rho(\mathbf{r})\pm\frac{i}{g}\frac{\partial}{\partial t}\right]m^{\pm}=0.\qquad(3.9)$$

The dispersion relation for m^+ has the form

$$\begin{aligned}\frac{\omega}{g}&=H_0+\beta M+\alpha Mk^2+[(\Delta\beta)^2M/\alpha k_0^2][\delta_1'(\varkappa)+i\delta_1''(\varkappa)],\\\delta_1'(\varkappa)&=-\frac{1}{1+4\varkappa^2},\quad \delta_1''(\varkappa)=\frac{2\varkappa}{1+4\varkappa^2}.\end{aligned}\qquad(3.10)$$

The real and imaginary parts of (3.10) are depicted in Fig. 2b. The damping ω'' is functionally similar to the damping determined by the dispersion relation (3.5).

The modification of the dispersion relation depends only on the parameter k_0, as was the case in an isotropically inhomogeneous medium; it is always negative, is maximum for the uniform oscillation $k=0$, and tends to zero for $\varkappa\gg1$.

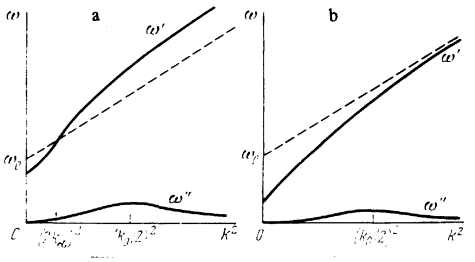

FIG. 2. Dispersion relations ω' and damping ω'' for anisotropic inhomogeneities: a) fluctuation of the direction of the anisotropy axis; b) fluctuation of the magnitude of the anisotropy. The dashed curves correspond to the unperturbed dispersion relations.

CONCLUSION

The presence of random magnetic anisotropy in a medium leads to physical effects that are essentially new in comparison with the effects induced by isotropic inhomogeneities (fluctuations of the density of the material, the interaction constant, etc.).

1. The static equilibrium equations (2.5) for the magnetization become inhomogeneous: in the right-hand side there is an external force—the random function $\rho(\mathbf{r})$,

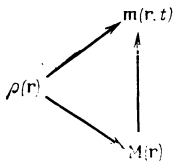

FIG. 3. Scheme of the interactions of the function $\rho(\mathbf{r})$ of the inhomogeneities with the dynamical variable $\mathbf{m}(\mathbf{r}, t)$ for fluctuation of the direction of the anisotropy axis.

which "excites" a static stochastic magnetic structure. (Unlike domain structure, which is excited by surface nonuniformity and is, therefore, described by homogeneous equations, the stochastic magnetic structure is excited by the bulk nonuniformity $\rho(\mathbf{r})$.) For finely dispersed and amorphous systems, in which the size of the inhomogeneities is much smaller than the range of the exchange correlations, only the dispersion D_M of the magnetic structure $\mathbf{M}(\mathbf{r})$ is proportional to the dispersion D of the "external" force $\rho(\mathbf{r})$; the correlation properties of the random function $\mathbf{M}(\mathbf{r})$ can differ sharply from the correlation properties of $\rho(\mathbf{r})$.

2. In the dynamical equation for the spin waves, besides the interaction terms of the usual type ρm_i new terms of the type $\rho M_i m_j$ arise; the corresponding scheme of the interactions is depicted in Fig. 3. The random function $\rho(\mathbf{r})$ of the inhomogeneities gives rise to two effects: a complex modification of the dispersion relation for $\mathbf{m}(\mathbf{r}, t)$, and the "generation" of a stochastic magnetic structure $\mathbf{M}(\mathbf{r})$; in its turn, the latter, interacting with $\mathbf{m}(\mathbf{r}, t)$, leads to an additional modification of the dispersion relation. The two channels of the interaction of ρ with $\mathbf{m}$—the direct one and through the stochastic magnetic structure—give contributions of the same order to the modification of the dispersion relation. In the dispersion law thus modified, two characteristic values of the wave number arise: one corresponds, as usual, to the correlation length of the random function $\rho(\mathbf{r})$, and the other to the correlation length of the random function $\mathbf{M}(\mathbf{r})$.

These effects should be observed, however, only under the condition that the direction of the anisotropy axis fluctuates. If it is only the magnitude of the anisotropy that fluctuates, while the direction remains constant, a stochastic magnetic structure does not arise, and in the modified dispersion relation there is only one characteristic wave-number value, corresponding to the correlation length of the random function $\rho(\mathbf{r})$. At the same time, properties appear (a shift in the frequency of the uniform oscillation ($k = 0$) and a decrease of the damping for $k \gg k_0$) that are general for all anisotropic inhomogeneities and absent for fluctuation of the density of the material or of the interaction constant.

[1)] The calculation in[10] was carried out for a fluctuating density of the material; however, the neglect of the quadratic term in the expansion of the denominator led in reality to an expression corresponding to a fluctuating interaction constant.

[2)] It is necessary to note the following: here we have not taken into account nonlinear terms of the type $M_i(\mathbf{k})\, M_i(\mathbf{k}_1) m_j(\mathbf{k}_2, t)$, although, since $M_i \sim \rho(\mathbf{k})$, they are very close in structure to the terms of the type G. However, the symmetry of these terms is such that their nonvanishing contribution to the dispersion relation begins at β^4, and in the following we shall confine ourselves to terms giving a contribution no higher than β^2.

[1]A. S. Maradudin, E. W. Montroll and G. H. Weiss, Theory of Lattice Dynamics in the Harmonic Approximation (Solid State Physics. Supplement 3), Academic Press, N. Y., 1963 (Russ. transl. Mir, M., 1965).
[2]A. A. Maradudin, Solid State Physics (ed. F. Seitz and D. Turnbull), Vol. 18, p. 273; Vol. 19, p. 1, Academic Press, N. Y., 1966 (Russ. Transl. Mir, M., 1968).
[3]Yu. A. Izyumov and M. V. Medvedev, Teoriya magnitouporyadochennykh kristallov s primesyami (Theory of Magnetically Ordered Crystals with Impurities) "Nauka," M., 1970).
[4]T. Kaneyoshi, Progr. Theor. Phys. **42**, 477 (1969); **43**, 560 (1970); **44**, 328 (1970).
[5]S. F. Edwards and R. C. Jones, J. Phys. C **4**, 2109 (1971).
[6]R. C. Jones and S. F. Edwards, J. Phys. C **4**, L194 (1971).
[7]C. G. Montgomery, J. I. Krugler and R. M. Stubbs, Phys. Rev. Lett. 25, 669 (1970).
[8]J. E. Gubernatis and P. L. Taylor, Phys. Rev. **B9**, 3828 (1974).
[9]Yu. N. Barabanenkov, Yu. A. Kravtsov, S. M. Rytov and V. I. Tatarskiĭ, Usp. Fiz. Nauk **102**, 3 (1970) [Sov. Phys. Usp. **13**, 551 (1971)].
[10]M. S. Howe, J. Fluid Mech. **45**, 769 (1971).
[11]R. G. Henderson and A. M. de Graaf, p. 331 in Amorphous Magnetism, ed. H. O. Hooper and A. M. de Graaf, Plenum, N.Y., 1973.
[12]R. Harris, M. Plischke and M. J. Zuckermann, Phys. Rev. Lett. **31**, 160 (1973).
[13]E. Schlömann, J. Phys. Chem. Solids **6**, 257 (1958); Phys. Rev. **182**, 632 (1969).
[14]V. A. Ignatchenko, Zh. Eksp. Teor. Fiz. **54**, 303 (1968) [Sov. Phys. JETP **27**, 162 (1968)].
[15]V. A. Ignatchenko and G. V. Degtarev, Zh. Eksp. Teor. Fiz. **60**, 724 (1971) [Sov. Phys. JETP **33**, 393 (1971)]; Fiz. Metal. Metalloved. **32**, 549 (1971) [Phys. Metals Metallog. (USSR) **32**, No. 3, 99 (1971)].
[16]H. W. Fuller and M. E. Hale, J. Appl. Phys. **31**, 238 (1960).
[17]H. Hoffman, J. Appl. Phys. **35** 1790 (1964).
[18]I. S. Édel'man and L. I. Chernysheva, Fiz. Metal. Metalloved. **28**, 440 (1969) [Phys. Metals Metallog. (USSR) **28**, No. 3, 57 (1969)].
[19]L. A. Chebotkevich, L. A. Yudina, L. G. Kashina and V. V. Veter, Fiz. Metal. Metalloved. **39**, 1297 (1975) [Phys. Metals Metallog. (USSR) **39**, No. 6, 158 (1975)].

Translated by P. J. Shepherd