# Pion condensation and the region of stability of abnormal nuclei

A. B. Migdal, O. A. Markin,[1)] I. N. Mishustin,[1)] and G. A. Sorokin

*L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences*
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The possibility of the existence of superdense nuclei with various $Z/A$ ratios is demonstrated. The equilibrium density and binding energy of abnormal nuclei are computed within the framework of simple models. It is shown that the accuracy of the theory at the present time is not sufficient to conclude definitely that there are such nuclei, but the possibility exists when reasonable nuclear constants are chosen.

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**1.** The possibility, due to pion condensation, of the existence of superdense nuclei was first discussed in[1]. Subsequently, this question was considered in greater detail in[2], where the possibility of the existence of neutron nuclei was also discussed. In the present communication, using the latest results in the theory of pion condensation, we carry out a more detailed analysis of the properties of abnormal nuclei.

Nuclear matter, starting from some density, becomes unstable against pion production[1,3–10]. This instability leads to a phase transition with the formation of a $\pi$-condensate. The appearance of the condensate leads to substantial softening of the equation of state, as a result of which the compressibility of the nuclear matter can become negative at densities close to the critical density $n_c$[2,11,12]. The energy gain connected with the condensate may turn out to be sufficient for the appearance of a new (abnormal) bound state of the finite nucleon system.

Recently, there have been published model calculations of the energy of the pion condensate for both weak[6] and strong[8–10] condensate fields. On the basis of these results, below we derive interpolation formulas giving the energy of the condensate and the energy of the baryon subsystem for an arbitrary density and going over, for $n \approx n_0$ and $n \gg n_0$, into the well-known expressions. Conditions are formulated for the stability of abnormal nuclei against particle evaporation, fission, and $\beta$ decay. It is demonstrated that two stability regions are possible: in the region of small $A$ with $Z/A \approx \frac{1}{2}$ (superdense nuclei) and in the region of large $A$ with $Z/A \ll 1$ (neutron nuclei). The energy versus density curves for such nuclei have been constructed. For some values of the parameters of the theory, the appearance in these curves of minima corresponding to stable or $\beta$-active superdense, or neutron, nuclei is possible. At the same time it will be shown that the accuracy of the available computations of the condensate energy, as well as of the computations of the nucleon energy is not sufficient for unequivocal conclusions to be drawn about the possibility of the existence of abnormal nuclei. Possible ways of detecting superdense and neutron nuclei are discussed.

**2.** The energy of a system of $A$ baryons of charge $Z$ and density $n$, measured from the nucleon-mass sum $\Sigma m_N = mA$, can be written in the form

$$E(n, A, \nu) = \varepsilon(n, \nu)A + a_S(n, \nu)A^{2/3} + a_Q(n)\nu^2 A^{5/3}, \qquad (1)$$

where $\nu = Z/A$, $Z = Z_B + Z_\pi$ is the sum of the baryon and pion charges. The terms proportional to $A^{2/3}$ and $A^{5/3}$ are respectively the surface and Coulomb energies. The volume energy per particle, $\varepsilon(n, \nu)$, is a sum of two terms

$$\varepsilon(n, \nu) = \varepsilon_B(n, \nu) + \varepsilon_\pi(n, \nu), \qquad (2)$$

where $\varepsilon_B(n, \nu)$ is the energy of the baryon subsystem, while $\varepsilon_\pi(n, \nu)$ is the energy connected with the appearance of the pion condensate. We neglect in (1) the corrections from pairing and deformation, as well as the shell effects.

The computation of the condensate energy in various models is the subject of[5,6–12]. For our purposes, it is necessary to know the energy of both the weak ($(n - n_c)/n_c \lesssim 1$) and the developed ($(n - n_c)/n_c > 1$) condensate.

In the region of weak condensate fields, it is possible to derive an analytic expression for $\varepsilon_\pi(n, \nu)$ by taking the perturbation series in the amplitude of the condensate field to fourth order[6]:

$$\varepsilon_\pi(n, \nu) = -\beta(\nu)[n - n_c(\nu)]^2/2n. \qquad (3)$$

In Table I we present $n_c$ and $\beta$ values computed within the framework of the model assumptions of[10] for vari-

TABLE I. The parameters determining the condensate energy and the values of the coefficients entering into the interpolation formula (8).

| $\nu = Z/A$ | $f'/f$ | $\gamma$ | $n_c$ | $\beta$ | $A$ | $B$ | $C$ |
|---|---|---|---|---|---|---|---|
| 0.5 | 0.9 | 0.45 | 0.54 | 0.69 | 1.44 | 0.28 | −1.03 |
| 0.5 | 0.9 | 0.5 | 0.65 | 0.81 | 1.31 | 0.46 | −0.96 |
| 0.5 | 0.9 | 0.55 | 0.79 | 0.89 | 1.18 | 0.58 | −0.87 |
| 0 | 0.9 | 0.45 | 0.69 | 0.80 | 0.90 | −0.23 | 0.13 |
| 0 | 0.9 | 0.5 | 0.79 | 0.63 | 0.77 | −0.23 | 0.09 |
| 0 | 1 | 0.4 | 0.48 | 1.19 | 1.40 | −0.11 | −0.10 |
| 0 | 1 | 0.45 | 0.54 | 1,11 | 1.24 | −0.11 | −0.02 |

*The quantities $n_c$ and $\beta$ were computed for neutron matter ($\nu = 0$) for $k = 3$ without allowance for nucleon recoil.

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ous isotopic compositions of the medium and for different values of the constant $g^{-1}$, by whose introduction the nucleon correlations are taken into account. [13,3–5]

Calculations of the energy of a developed condensate in neutron matter are contained in[9], where an approach was developed which enabled the authors to derive an analytic expression for the energy of a charged-pion condensate within the framework of a fairly realistic model, including $S$- and $P$-wave $\pi$-$N$ interaction, $\pi$-$\pi$ interaction, the $N^*$ (1236) resonance, and nucleon correlations. [10] Below we shall use the results of the papers, [9,10] but in our case, in contrast to[9,10], where an infinite system was considered, we should discard the electrical-neutrality condition. At high densities (in a strong condensate field), the baryon subsystem is substantially reconstructed: for any $\nu$, instead of two proton and neutron Fermi spheres, one Fermi sphere of baryon quasiparticles, which are a superposition of six baryons—$N^{*++}$, $N^{*+}$, $p$, $n$, $N^{*0}$, $N^{*-}$—and which have the least energy, fills up. In the limiting-condensate-field model, [10] which describes the system at high densities, the charge of the baryon subsystem is equal to $A/2$, while the energy per particle has the form (2), where

$$\varepsilon_B(n)=\frac{3(3\pi^2n)^{2/3}}{10m}+U(n) \tag{4}$$

is the sum of the kinetic and potential energies of the baryon quasiparticles,

$$\varepsilon_\pi(n,\nu)=\varepsilon_\pi(n)+\alpha_\pi(n)(1-2\nu)^2, \tag{5}$$

$$\varepsilon_\pi(n)=-[{}^{81}/_{50}f'^2(1-\gamma)n-\Delta/3], \tag{5a}$$

$$\alpha_\pi(n)=n/2F^2, \tag{5b}$$

$\Delta=m_{N^*}-m_N=294$ MeV $=2.1$. In these expressions $\hbar=c=m_\pi=1$; $F=1.35\,m_\pi$ is the pion-decay constant.

The $\pi$-$N$ coupling constant $f$ is related to the constant $F$ and the axial constant $g_A=1.36$ by the relation $f=g_A/F$. As comparison of the theory with experiment shows, the slight renormalization $f\to f'=(0.8-1)f$ [13] occurs in ordinary nuclei. Below, for the calculation of the energy of superdense and neutron nuclei, we shall use the value $f'=0.9f$. The quantity $\gamma$, with the aid of which the contribution of the nucleon correlations is taken into account, is connected with the local amplitude, $g^{-1}$, of the spin-spin interaction[13] by the relation

$$g^{-}=f'^2\frac{2mp_0}{\pi^2}\gamma, \tag{6}$$

where $p_0=1.92$ is the Fermi momentum in the nuclear matter at the normal nuclear density $n_0=0.17$ F$^{-3}$ $=0.48$. In the formula (5a) we have discarded the term $F^2/4n$, since in the region of densities of interest to us it is virtually completely compensated by the second term of the expansion of the condensate energy in the parameter $\Delta/g_Ak$, where $k$ is the wave number of the condensate field.

It was assumed in the derivation of the expression (5a) in[10] that, up to Clebsch–Gordan coefficients, the local amplitudes of the $N$-$N$, $N$-$N^*$, and $N^*$-$N^*$ interactions in a nucleon medium are equal, but this assumption is theoretically unjustified. No direct experimental information about the $N$-$N^*$ and $N^*$-$N^*$ interactions exists at present. Apparently, the localized $N$-$N^*$ interaction is significantly weaker than the $N$-$N$ interaction, as follows from experiments on $pp\to nN^*$ scattering involving large momentum transfers. [14] This should lead to an increase in the gain in condensate energy in comparison with (5a). On the other hand, in[10] the suppression of the $\pi NN^*$ vertex at large pion momenta, which leads to a decrease in the condensate energy, [5] was not taken into account. In view of the fact that it is quite difficult to take these effects into account in the presence of other unknown parameters, we shall use the expressions (5a) and (5b) as reasonable estimates.

Below we shall use in the entire density range of interest to us the single interpolation formula

$$\varepsilon_\pi(n,\nu)=-\beta(n)(n-n_c)^2/2n, \tag{7}$$

where

$$\beta(n)=A+Bn_c/n+C(n_c/n)^2. \tag{7'}$$

The $\nu$-dependent coefficients $A$, $B$, and $C$ are chosen such that the values of $\beta(n)$ for $n=n_c$ coincide with the results presented in the table ($\beta=\beta(n_c)$), while for $n/n_c\to\infty$ the condensate energy $\varepsilon_\pi(n,\nu)$ coincides together with $\partial\varepsilon_\pi/\partial n$ with the expressions (5). The values of $A$, $B$, and $C$ for various cases are given in the table. The formula (7) for the condensate energy allows us to manage without tedious numerical computations. The baryon energy $\varepsilon_B(n,\nu)$ has been computed in numerous papers on the theory of nuclear and neutron matter. There exist calculations for the two limiting cases: $(1-2\nu)\ll 1$ and $\nu\ll 1$. As we shall see, it is precisely these cases that are of greatest interest.

For $\nu\approx\frac{1}{2}$ we can, in the region of densities $(n-n_0)\ll n_0$, express the volume part of the baryon energy in terms of the compressibility, $K$, of nuclear matter:

$$\varepsilon_B(n,{}^1/_2)=-\varepsilon_0+{}^1/_2K(1-n/n_0)^2, \tag{8}$$

where $\varepsilon_0=15.7$ MeV $=0.11$. For $K$ we use the expression obtained in the theory of the Fermi liquid[13]:

$$K={}^2/_3\varepsilon_F(1+2f_0)$$

($f_0$ is the local amplitude of the scalar interaction between nucleons). As a comparison of the theory with experiment shows, [15] $f_0=0.25\pm0.1$. For making estimates we shall take $f_0=0.25$, which corresponds to $K=40$ MeV $=0.29$. At high densities ($n\gg n_0$), as has already been noted, the nucleon subsystem is substantially reconstructed: the baryon quasiparticles fill one Fermi sphere. The energy $\varepsilon_B$ corresponding to this distribution includes the kinetic energy and the interaction energy of the baryon quasiparticles (see the formula (4)), which, at high densities, is determined by baryon repulsion at small distances. If we assume that this repulsion is the same for all the participating baryons, then the potential energy $U(n)$ should, at fairly high densities, coincide with the energy of neutron matter without allowance for condensation (see, for example, [16]). Under this assumption, the volume part of the baryon energy can be written in the form of a simple interpolation formula:

$$\varepsilon_B\left(x=\frac{n-n_0}{n_0}\right)=-0.11+\frac{0.14x^2}{1+0.37x}. \quad (9)$$

The coefficients in this expression have been chosen such that, for $x \ll 1$, the expression goes over into (8), while for $n=7.35\,n_0=1.25\ F^{-3}$ the values of $\varepsilon_B(n)$ and $\partial\varepsilon_B/\partial n$ coincide with the results of the calculations for neutron matter. At high densities $\varepsilon_B(n)$ is taken from the paper.[16]

For the baryon energy in the case when $\nu \ll 1$, a case which we shall need for the elucidation of the question of stability of neutron nuclei, we shall use in the entire density range the results of the calculations for neutron matter.[16]

No reliable surface-energy calculations for high densities exist at present. For the coefficient $a_s$ in the formula (1), let us use the estimate obtained under the assumption that the thickness of the surface layer coincides with the range of nuclear forces, i.e., that it does not depend on the density. Then the surface energy is proportional to the energy per unit volume. We have

$$a_S(n,\nu)=0.13\left(\frac{n}{n_0}\right)^{1/3}\frac{\varepsilon(n,\nu)}{\varepsilon(n_0,{}^1/_2)}. \quad (10)$$

For $n=n_0$ and $\nu=\frac{1}{2}$, this expression coincides with the corresponding term in the Weizsäcker formula.

Under the assumption of uniform charge distribution, the factor $a_Q(n)$, which determines the Coulomb energy, is equal in pion units to

$$a_Q(n)=5\cdot10^{-3}(n/n_0)^{1/3}. \quad (11)$$

3. Let us formulate the equilibrium conditions which should be satisfied by a finite particle system at zero pressure.

1) Positive mass defect,

$$-E>0. \quad (12)$$

The condition for the nucleons to be bound particles is then automatically fulfilled, i.e., the chemical potentials of the neutrons and protons are negative:

$$\mu_n=(\partial E/\partial N)_Z<0,\quad \mu_p=(\partial E/\partial Z)_N<0. \quad (13)$$

It is not difficult to verify that, for $E>0$, the chemical potentials of the nucleons are positive, and, consequently, the system is unstable against particle evaporation.

2) $\beta$-equilibrium (the electrons escape freely):

$$(\partial E/\partial Z)_A=\mu_p-\mu_n=0. \quad (14)$$

3) Stability against fission, as determined by the well-known relation

$$\frac{Z^2}{A}<2\frac{a_S(n,\nu)}{a_Q(n)}. \quad (15)$$

From (10), (11), and (15) we obtain

$$Z^2/A<50f(n,\nu), \quad (16)$$

where $f(n,\nu)=\varepsilon(n,\nu)/\varepsilon(n_0,\frac{1}{2})$. For $n=n_0$, (16) goes over into the well-known criterion for stability of normal nuclei against fission.

Let us now assume that the condition (1) is fulfilled.

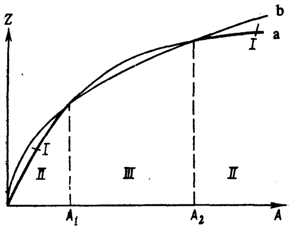


FIG. 1. The regions of existence of abnormal nuclei. a) $\beta$-equilibrium curve, b) boundary of stability against fission. The heavy curve separates out the sections of the $\beta$-equilibrium curve that correspond to stable abnormal nuclei. I) Regions of $\beta^+$ activity; II) regions of $\beta^-$ activity that ends in the formation of a stable nucleus; III) region of $\beta^-$ activity that ends in fission.

Then from (1), (5), and (14) we find the equilibrium value of $\nu$ at high densities:

$$\nu=\frac{1}{2}\left[1+\frac{a_Q(n)}{4\alpha_\pi(n)}A^{2/3}\right]^{-1}. \quad (17)$$

It follows from (16) and (17) that there can be two stability regions:

a) $A<A_1\approx 200f(n,\frac{1}{2})$ and $\nu\approx\frac{1}{2}$ (superdense nuclei);

b) $A>A_2\approx 2\times10^5\,(n/n_0)^4f^{-3}(n,0)$ and $\nu\approx 50\,(n/n_0)^{2/3}A^{-2/3}\ll 1$ (neutron nuclei). Such nuclei, in spite of the small $Z/A$ ratio, have a charge that is large enough for the Coulomb energy to inhibit $\beta$ decay and, at the same time, a $Z^2/A$ ratio that is small enough for fission to be impossible.

Nuclei with a $Z/A$ ratio that is different from the equilibrium value will be $\beta$-active. The regions of existence of the stable and $\beta$-active abnormal nuclei are shown in Fig. 1. Notice that in the region, I, of $\beta^+$-activity and in the region, II, of $\beta^-$- activity, the evolution of a nucleus terminates at the stability line, whereas from the region III a nucleus reaches the fission curve. It can be seen from the expressions given that the quantities $A_1$ and $A_2$ essentially depend on the parameters of the model.

Thus, if the bound-state condition, (12), for the system is fulfilled, then the formation of $\beta$-active nuclei with small $\nu$ is possible. From (5) and (3) follows the estimate for the $\beta$-decay energy at the beginning of the cascade:

$$\varepsilon_\beta=-(\partial\varepsilon/\partial\nu)_A\approx-\partial\varepsilon_\pi/\partial\nu=4\alpha_\pi(1-2\nu). \quad (18)$$

For $n=5\,n_0=2.4$ and $\nu=0$, $\varepsilon_\beta=2.6\approx 400$ MeV. Such nuclei acquire over a short period of time ($10^{-8}-10^{-10}$ sec) a positive charge and go over either into superdense nuclei with $Z\approx A/2$, or, in the case of sufficient dimensions, into stable nuclei with $Z\ll A$.

4. Let us now use the above-obtained interpolation formulas to verify the bound-state condition, (12), for the system. Let us note to begin with that at high densities the total energy of the system is the difference between two large numbers—the energy of the baryon subsystem and the energy of the condensate—which to a large extent cancel each other. The computational accuracy of each of the terms is not high at present (at best they can be regarded as order-of-magnitude esti-

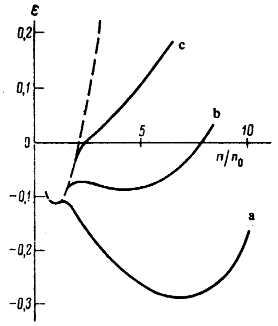


FIG. 2. The energy of nuclear matter ($\nu=\frac{1}{2}$). The dashed curve was computed without allowance for condensation )the formula (9)). The curves $a$, $b$, and $c$ were computed respectively for $\gamma=0.45$, $0.5$, and $0.55$.

mates), and, therefore, the results of the total-energy calculations for the system should be regarded only as an illustration of the various possible cases.

The results of the $\epsilon(n)$ calculations for superdense nuclei ($\nu=\frac{1}{2}$) are shown in Fig. 2. The curve $a$, which was computed for $\gamma=0.45$, demonstrates the case when the binding energy of the superdense nuclei exceeds the binding energy of ordinary nuclei. If such a situation actually existed, ordinary nuclei would be metastable with respect to transitions into the superdense state. The minimum, corresponding to superdense nuclei, in the curve $b$, which was computed for $\gamma=0.5$, lies above the minimum corresponding to ordinary nuclei. In this case superdense nuclei would be metastable. The curve $c$, which was computed for $\gamma=0.55$, corresponds to the case when no abnormal bound state exists at all.

Let us note another important circumstance. In computing the curves $a$–$c$ in Fig. 2, we used for the nuclear constants values for which $n_c>n_0$. It is possible that $n_c<n_0$. Then the pion condensate exists in ordinary nuclei,[3] and all the quantities characterizing ordinary nuclei now contain contributions from the condensate. It is most probable that superdense nuclei do not exist in this case.

The results of the energy calculation for neutron matter ($\nu=0$) with allowance for the condensate are presented in Fig. 3. The dashed curve represents the energy of neutron matter without allowance for condensation.[16] The curves $a$ and $b$ were computed with the same nuclear-constant values as the corresponding curves in Fig. 2. Evidently, in this case the energy gain on account of the $\pi$-condensation is insufficient for the formation of a bound state. The possibility of some change occurring in the constants as a result of a change in the isotopic composition of the medium is, however, not to be excluded. The curves $c$ and $d$ in Fig. 3, which were computed with $f'=f=1.0$ and respectively with $\gamma=0.4$ and $0.45$, illustrate the case of the appearance of a bound state in a system with a nearly zero charge-to-mass ratio (neutron nuclei). It should be noted that such a change in the constants can significantly alter the $\beta$-decay energy (18), as well as the pattern of stability regions.

Thus, the analysis performed shows that the parameters of abnormal nuclei essentially depend on the pion-nucleon and nucleon-nucleon coupling constants. Furthermore, the model[9,10] used by us allows us to take account of only the energy connected with the condensate of only the charged pions, a condensate which has the simple spatial structure of a running wave. As shown in[3–5], a neutral-pion condensate should also appear in a nucleonic medium at a density close to $n_c$. Furthermore, the minimum energy of the system should correspond to a spatial structure for the condensate fields that is more complex than that of a running wave.[6,17,18] All these effects lead to a gain in energy and are additional factors in favor of the existence of abnormal nuclei. On the other hand, the choice of an equation of state that is more rigorous than the one used in the estimation of $\epsilon_B$, as well as allowance for the suppression of the pion-nucleon vertices in the case of large momentum transfers, would lead to an increase in the total energy of the system. It is impossible at present to take all these effects into account with the required accuracy, and the primary conclusion that can be drawn on the basis of our analysis is that the possibility of the existence of abnormal nuclei is theoretically not to be excluded. The definitive resolution of the question can be given only by an experiment.

5. Let us now make a few remarks concerning possible experiments for the detection of abnormal nuclei.

If by chance superdense nuclei exist, then it is not clear to which nuclei—the normal or the superdense—corresponds the higher binding energy. In principle, it is possible that the superdense nuclei have the higher binding energy. Therefore, the experimental delimitation of the probability of spontaneous transitions of normal nuclei into the superdense state[19] is of interest. Let us note that so far the search for nuclei with anomalously high binding energies has not yielded results.[20–22]

Of interest is the search for stable or $\beta$-active abnor-

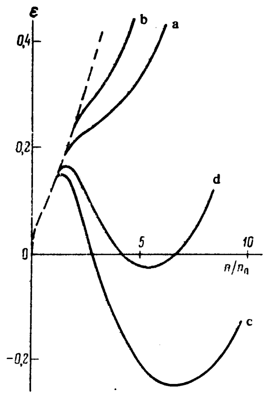


FIG. 3. The energy of neutron matter ($\nu=0$). The dashed curve was computed without allowance for condensation.[16] The curves $a$ and $b$ were computed with the same parameter values used to compute the corresponding curves in Fig. 2. The curves $c$ and $d$, which were computed with $f'=f=1.0$ for $\gamma=0.4$ and $\gamma=0.45$, correspond to the appearance of a bound state.

mal nuclei of small dimensions ($A \approx 100$) in the fission products of ordinary nuclei.

Possibly, superdense nuclei can be produced in collisions of heavy ions with energies of the order of several hundred MeV per nucleon. The shock wave that arises in such a collision may then lead to substantial compaction of the nuclear matter. At a sufficiently high value of $\beta$, the compressibility of the system becomes negative even at $n=n_c$. Therefore, it is sufficient to compress the system to the density $n_c$ for the formation of the superdense phase to begin. Irrespective of whether or not stable superdense nuclei exist, pion condensation should have appreciable influence on the collision dynamics.

And, finally, as was noted back in 1971 in[1], it is to be hoped that abnormal nuclei will be detected in cosmic rays. The possibility of observing in cosmic rays stable abnormal nuclei or their $\beta$-active fragments with anomalous $Z/A$, produced in the interaction with the nuclei of the atmosphere, should be taken into account when setting up, and when analyzing, experiments. Thus, for example, we cannot exclude the possibility that the unusual track that was originally ascribed to a magnetic monopole[23] is the track of an abnormal ("neutron") nucleus with $Z \approx 10^2$ and $A \approx 10^4$.

Also of interest is the search for superdense nuclei of cosmic origin which have accumulated over cosmic intervals of time in the surface layers of the lunar soil and in meteorites.

1) I. V. Kurchatov Institute of Atomic Energy.

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