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Theory of hysteresis reflection and refraction of light by a boundary of a nonlinear medium

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A theory is constructed for the reflection and refraction of light from the boundary of a semi-infinite nonlinear medium whose refractive index depends on the light intensity. It is shown that when the incidence angle or the intensity of the incident light is varied, hysteresis jumps should be observed from the transmission regime to the regime of total internal reflection (TIR) and back. At small nonlinearity, the necessary condition for the existence of the hysteresis effect is closeness of the linear refractive indices of both media (linear and nonlinear) and smallness of the glancing angles; all the observed effects should in this case be independent of the polarization of the incident field. At a negative nonlinearity, the phenomenon is due to the ambiguity of the transmission regime, while at positive nonlinearity it is due to the ambiguity of the TIR. At a definite light intensity, complete transparentization of the boundary can take place for all the incident angles in the region of stability of the transmission regime; in this case jumps take place from total reflection to total transmission and vice versa.

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INTRODUCTION

1. In a preceding paper[1] I reported the possibility of observing new effects in the case of almost glancing incidence of light from a linear medium onto the boundary of a nonlinear medium whose permittivity ε_1 depends on the field intensity and is close to the permittivity ε_0 of the linear medium. The principal effect was that when the glancing angle or the incident-field intensity were varied, strong hysteresis jumps should be observed in the refractive index and in the reflection coefficient, from the nonlinear reflection regime ("transmission regime") to the regime of nonlinear total internal reflection (TIR), and back. A theory of this phenomenon is constructed in the present paper.

The Snell formulas that follow from the generalized boundary conditions (§ 1) for the refraction angle and the Fresnel formulas for the amplitudes of the fields become coupled to one another by virtue of the nonlinearity of the refracting medium, and this leads to a single self-consistent equation for the nonlinear refraction (§ 2). In the case of TIR, a surface wave propagates inside the refractive medium along the boundary, and is likewise described by a nonlinear wave equation (§ 3). The self-consistency of these equations leads, under definite conditions, to the appearance of severally physically realizable solutions (states), some of which are unstable and this in fact is the reason for the hysteresis. The transmission regime becomes ambiguous at negative nonlinearity ($\Delta\varepsilon_{nl} < 0$, § 4), and the TIR regime at psoitive nonlinearity ($\Delta\varepsilon_{nl} > 0$, § 5). At a certain field intensity, total nonlinear transparentization of the boundary takes place at all incidence angles (§ 2), and the hysteresis jumps take place between the states of the total transmission and the total reflection, while the boundary operates as an ideal "optical flip-flop" (§§ 4, 5).

2. One of the main premises for the observation of the indicated effect is the matching of the optical densi-

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ties of both media. The difference between their linear permittivities $|\Delta\varepsilon_\lambda|$ should be smaller than or of the same order as the possible nonlinear increment $|\Delta\varepsilon_{nl}|$, for only then can the nonlinearity cause a strong self-consistency of the problem. This is why strong effects appear only at almost grazing incidence of the wave on the boundary. To observe the hysteresis it is necessary to have glancing angles ψ of the order of $\sim(|\Delta\varepsilon_{nl}|/\varepsilon_0)^{1/2}$, which amounts to $\psi\sim 0.5°$ at $|\Delta\varepsilon_{nl}|\sim 10^{-4}-10^{-5}$; this pertains also to the width of the hysteresis loop relative to the glancing angle $\Delta\psi_h$. The smallness of ψ and $\Delta\varepsilon$ simplifies the theory and makes the phenomenon independent of the polarization of the incident field. When the requirement that $|\Delta\varepsilon_l|$ be small is not satisfied, only "weak" nonlinear effects proportional to $\Delta\varepsilon_{nl}$ remain (changes in the refraction angle, in the reflection coefficient, and in the shift of the TIR angle, § 2), while the hysteresis becomes small. This may be the reason why hysteresis was not observed either theoretically or experimentally in studies of the type,[2-5] devoted to harmonic generation in the reflection of light from the boundary of a nonlinear dielectric (see also[6,7]).

3. Hard excitation and hysteresis are possible in certain types of lasers with nonlinear absorbers, ([8,9] see also the references in[10,11]). A similar mechanism of hysteresis formation is possessed also by a passive system[12-15] comprising a Fabry-Perot resonator filled with a resonant nonlinear absorbed with saturation. The bistable regimes in these systems are due to the presence of a resonance and a strong dissipative nonlinearity, and are in principle analogous to hard excitation in Thomson generators (or tuned feedback amplifiers) with "hard" nonlinearity.[16] The hysteresis effect proposed in[17] is also based on the use of a Fabry-Perot resonator filled with a transparent medium with Kerr (i.e., reactive) nonlinearity,[1)] and is in essence the analog of the hysteresis in a tank circuit with nonlinear capacitance or inductance[18] (inasmuch as the nonlinear polarization causes the natural frequency of the resonator to deviate from the field frequency when the intensity is changed). Thus, in all these systems the hysteresis is due to the presence of a resonator that provides the feedback (and in[8-15] also to the resonant character of the medium), and this, in particular, causes them to be strongly selective with respect to the frequency of the incident field.

In contrast to the foregoing, the nature of the effects considered here is connected with a very simple wave phenomenon—Fresnel reflection and absorption by a nonlinear boundary—and is determined by the strong sensitivity of this phenomenon to nonlinear variation of the permittivity in the case of almost grazing incidence of the waves. Therefore, in contrast to[8-15,17], these effects are nonresonant, can cause intensity jumps up to 100% (in contrast to systems with dissipative nonlinearity, where the jumps constitute a small fraction of the input intensity, for example ~1%[13]). In addition, they make it possible to obtain nonlinear refraction, and in particular, angular scanning and jumps of the refracted beam, and to observe and employ the phenomenon of nonlinear TIR; neither are possible in resonators.

4. The self-action effect known from nonlinear optics, such as self-focusing and self-channeling of light beams in substances with $\Delta\varepsilon_{nl}>0$, which were predicted in[19] and observed in[20,21] (see the reviews[22,23]), and the self-bending of trajectories of asymmetrical beams in substances with an arbitrary sign of $\Delta\varepsilon_{nl}$, which was predicted by us in[24], observed in[25,26], and investigated in[27], are the consequence of the transverse inhomogeneity of the incident wave. Wave beams for these effects should have a bound cross section and a definite intensity profile (bell-shaped for self-focusing[22,23] and wedge-shaped for self-bending[24,27]). In contrast to these, the phenomenon considered here is in principle not due to the inhomogeneity of the incident wave (although it should naturally be observed in bounded beams, where threshold conditions obtain, § 6). Therefore the principal results pertain here to the case of a homogeneous plane incident wave.

§ 1. BOUNDARY CONDITIONS

Let a plane wave with amplitude E_0 be incident from a linear medium with permittivity ε_0 at an angle φ (Fig. 1) on the boundary of a nonlinear medium whose permittivity ε_1 depends on the field amplitude E_1 in the medium:

$$\varepsilon_1=\varepsilon_0+\Delta\varepsilon_l+\Delta\varepsilon_{nl},$$

where

$$\Delta\varepsilon_{nl}=\varepsilon_2|E_1|^2,$$

$\Delta\varepsilon_l$ does not depend on the field (and $\Delta\varepsilon_l$ and $\Delta\varepsilon_{nl}$ can have arbitrary signs). Neglecting here, as is customary in the investigation of self-action,[22-24] the generation of new frequencies, and assuming that the field depends on the time like $e^{-i\omega t}$, we write down the expression for the wave in the linear medium in the form

$$\vec{\mathscr{E}}={}^1/_2 e^{ik_x x}(\mathbf{E}_0 e^{ik_z z}+r\mathbf{E}_{\rm ref}e^{-ik_z z}),$$

where

$$k_x=k_0\sin\varphi,\quad k_z=k_0\cos\varphi,\quad k_0=\omega\varepsilon_0^{1/2}/c,$$

r is the reflection coefficient, $\mathbf{E}_{\rm ref}$ is the image of the vector $\mathbf{E}_0$ relative to the yz plane. The expression for the wave in a nonlinear medium is

$$\vec{\mathscr{E}}_1={}^1/_2\mathbf{E}_1(z)\exp\left[i\left(k_{1x}x+\int k_{1z}(z)\,dz\right)\right].$$

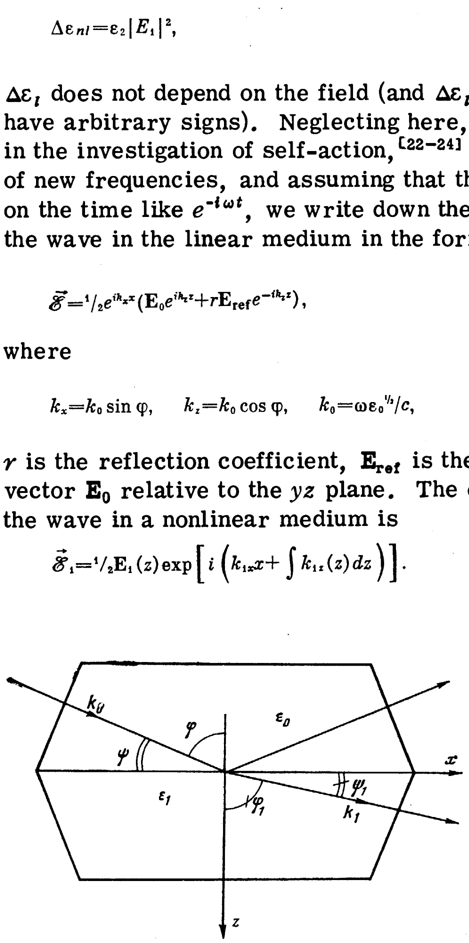

FIG. 1. Wave diagram of the phenomenon. $\varepsilon_1=\varepsilon_0+\Delta\varepsilon_l+\varepsilon_2|E_1|^2$.

Calculating the magnetic field $\mathbf{H}$ from Maxwell's equations

$$\operatorname{rot}\mathbf{H}=-i\varepsilon\omega\vec{\mathscr{E}}/c, \quad \operatorname{rot}\vec{\mathscr{E}}=i\omega\mathbf{H}/c,$$

and equating, as usual, the tangential values of the fields on the boundary, we find that $k_x = k_{1x}$ regardless of the direction of the plane of polarization of the incident wave, and in addition, if the plane of polarization is normal to the incidence plane ($\perp$), we have

$$1+r_\perp=\frac{E_1(0)}{E_0}, \quad E_0k_z(1-r_\perp)=E_1(0)k_{1z}(0)-i\frac{dE_1(0)}{dz}, \tag{1}$$

while at a polarization direction parallel to the incidence plane ($\parallel$) we have

$$1+r_\parallel=\frac{E_1(0)k_1(0)}{E_0k_0}, \quad E_0(1-r_\parallel)\frac{k_z}{k_0}=\frac{1}{k_1(0)}\left[E_1(0)k_{1z}(0)-i\frac{dE_1(0)}{dz}\right], \tag{2}$$

where $k_1(z)=\omega(\varepsilon_1(z))^{1/2}/c$. The boundary conditions in this form, including the derivatives of the field with respect to the coordinates, make it possible to describe both refraction and TIR, and remain valid for a nonlinear refracting medium.

§ 2. TRANSMISSION REGIME; EQUATIONS AND GENERAL PROPERTIES

1. If in a nonlinear medium the field is also a plane homogeneous wave that goes off from the boundary at the refraction angle φ_1 (Fig. 1):

$$dE_1/dz=0, \quad k_1(z)=\text{const}, \quad k_{1x}=k_1\sin\varphi_1, \quad k_{1z}=k_1\cos\varphi_1,$$

then the boundary condition $k_x = k_{1x}$ leads to the usual Snell's formula, generalized to include the nonlinear case:

$$\frac{\sin\varphi}{\sin\varphi_1}=\left[\frac{\varepsilon_1(|E_1|^2)}{\varepsilon_0}\right]^{1/2}=\left(1+\frac{\Delta\varepsilon_l}{\varepsilon_0}+\frac{\varepsilon_2|E_1|^2}{\varepsilon_0}\right)^{1/2}. \tag{3}$$

The boundary conditions (1) and (2), with allowance for the fact that $dE_1/dz=0$, lead to the usual Fresnel formulas,[28] which are also formally valid in the nonlinear case:

$$(E_1)_\perp=E_0\frac{2\sin\varphi_1\cos\varphi}{\sin(\varphi_1+\varphi)}, \quad (E_1)_\parallel=E_0\frac{2\sin\varphi_1\cos\varphi}{\sin(\varphi_1+\varphi)\cos(\varphi_1-\varphi)},$$
$$r_\perp=\frac{\sin(\varphi_1-\varphi)}{\sin(\varphi_1+\varphi)}, \quad r_\parallel=\frac{\operatorname{tg}(\varphi-\varphi_1)}{\operatorname{tg}(\varphi+\varphi_1)}. \tag{4}$$

Substituting expression (4) for the field E_1 in (3), we obtain an exact self-consistent equation that relates the angles φ_1 and φ at a given polarization and incident-field amplitude E_0.

2. Usually, especially in optics, the nonlinearity is weak, $|\Delta\varepsilon_{nl}|\ll\varepsilon_0$. It is therefore clear that substantial deviations of the nonlinear-regime parameters from those of the linear regime appear likewise only if $|\Delta\varepsilon_l|\ll\varepsilon_0$, and consequently in the case of small glancing angles $\psi=\pi/2-\varphi$ and transmission angles $\psi_1=\pi/2-\varphi_1$ (Fig. 1):

$$|\Delta\varepsilon_l|,\ |\Delta\varepsilon_{nl}|\ll\varepsilon_0, \quad \psi,\psi_1\ll\pi/2. \tag{5}$$

But then formulas (4) become the same for any polarization

$$\left(\frac{E_1}{E_0}\right)_\perp=\left(\frac{E_1}{E_0}\right)_\parallel=\frac{2\psi}{\psi+\psi_1}, \quad r_\perp=r_\parallel=\frac{E_1}{E_0}-1=\frac{\psi-\psi_1}{\psi+\psi_1}. \tag{6}$$

Substituting formula (6) for the field E_1 in (3), introducing the relative intensity of the incidence field Δ_E^2 and the linear permittivity difference Δ_l^2:

$$\Delta_E^2=4\varepsilon_2|E_0|^2/\varepsilon_0, \quad \Delta_l^2=\Delta\varepsilon_l/\varepsilon_0 \tag{7}$$

(where Δ_E^2 and Δ_l^2 can have arbitrary signs), using the condition (5), and retaining in (3) only terms of the same order of smallness, we obtain the "nonlinear Snell's formula" for the transmission angle ψ_1:

$$(\psi+\psi_1)^2(\psi^2+\Delta_l^2-\psi_1^2)+\psi^2\Delta_E^2=0 \quad (\psi,\psi_1\geqslant0) \tag{8}$$

or the "nonlinear Fresnel formula" for the reflection coefficient r:

$$16r\psi^2+4\Delta_l^2(1+r)^2+\Delta_E^2(1+r)^4=0 \quad (|r|\leqslant1). \tag{9}$$

3. The fourth-order equations (8) and (9) can be solved in terms of radicals. Since, however, Eq. (9) is of first order relative to all the given parameters of the problem (ψ^2, Δ_l^2, and Δ_E^2), the dependence of r on these parameters can be easily obtained and investigated as an inverse function.

4. It is easy to show that $\varepsilon_2>0$ the transmission regime is always single-valued, and at $\varepsilon_2<0$ it is single-valued under the condition $|\Delta_E^2|\leqslant\Delta_l^2$ and doubly-valued under the condition $|\Delta_E^2|>\Delta_l^2$. In particular, for negative or zero linear permittivity difference ($\Delta_l^2\leqslant0$) and at $\varepsilon_2<0$, the transmission is doubly-valued for any field inside a definite range of angles. It is seen from (8) and (9) that at an arbitrary sign of the nonlinearity the plot of the transmission regime approaches the TIR (i.e., $\psi_1\to0$, $r\to1$) at the following relation between the parameters of the problem

$$(\psi^2+\Delta_l^2+\Delta_E^2)_0=0, \tag{10}$$

which determines the critical value of any of them if the two others are fixed. At $\varepsilon_2<0$ the angle $\psi=\psi_0$ is also the maximal TIR angle (§ 4, Sec. 2), where a jump from TIR to transmission takes place.

5. In a "weak" field, i.e., when the change of the transmission regime is small in comparison with the linear case, we have for the stable branch of the regime

$$\psi_1-\psi_{1l}\approx\Delta_E^2\psi^2/2\psi_{1l}(\psi+\psi_{1l})^2, \quad r-r_l\approx-\Delta_E^2\psi^3/\psi_{1l}(\psi+\psi_{1l})^4, \tag{11}$$

where

$$\psi_{1l}=(\psi^2+\Delta_l^2)^{1/2}, \quad r_l=(\psi-\psi_{1l})/(\psi+\psi_{1l})$$

are the values of ψ_1 and r in the linear case. In particular, at large glancing angles, $\psi^2\gg|\Delta_l^2|+|\Delta_E^2|$, we have

$$\psi_t-\psi_{tl}\approx\Delta_E^2/8\psi,\quad r-r_l\approx-\Delta_E^2/16\psi^2, \tag{12}$$

and at small angles, $\psi \ll |\Delta_l|$ (in the case of $\Delta_l^2>0$), we have

$$\psi_t-\psi_{tl}\approx\Delta_E^2\psi^2/2\Delta_l^2,\quad r-r_l\approx-\Delta_E^2\psi^3/\Delta_l^5. \tag{13}$$

From (11) follows the criterion of the "weakness" of the field:

$$E_0(|\varepsilon_2|/\varepsilon_0)^{1/2}={}^1/_2|\Delta_E|\ll\psi_{tl}(\psi+\psi_{tl})/\psi. \tag{14}$$

The criterion (14), just as the expression (10), confirms the necessity of satisfying the previously assumed condition (5) in order to observe "strong" effects.

6. Let us examine the effective total nonlinear transparentization. From expression (9) it is seen that there exists an infinite-field intensity $E^2=E_{bl}^2$, such that $4\Delta_l^2+\Delta_{E_{bl}}^2=0$, i.e.,

$$E_{bl}^2=-\Delta\varepsilon_l/\varepsilon_2, \tag{15}$$

at which there is no reflection at all ($r=0$) for any glancing angle ψ. This phenomenon is not the nonlinear analog of the Brewster transparentization (since it does not depend on either the incidence angle or the polarization), and is due to the fact that when (15) is satisfied the field equalizes the permittivities of the two media ($\Delta\varepsilon_l=-\Delta\varepsilon_{nl}$), and by the same token makes the boundary completely transparent. This is possible at any sign of ε_2, the only requirement being that ε_2 and $\Delta\varepsilon_l$ be of opposite sign. It should be noted, however, that at $\varepsilon_2<0$ part of the branch of the nonlinear transparentization regime ($0<\psi<|\Delta_l|/2^{1/2}$ is stable, and a part ($2^{-1/2}<\psi/|\Delta_l|<3^{1/2}$) is one of the two stable regimes (see curve 4 of Fig. 4a below, and also § 4); the latter pertains also to the section ($0<\psi<|\Delta_l|/2^{3/2}$) at $\varepsilon_2>0$ (see curve 6 of Fig. 3a below and also § 5).

§ 3. NONLINEAR TIR; EQUATION AND BOUNDARY CONDITIONS

1. In the TIR regime, a refracting medium, only a transverse inhomogeneous surface wave propagates along the interface ($k_{1z}\equiv0$), and under the condition $k_x=k_{1x}$ (§ 1) it is expressed in the form $\mathscr{E}_1=\frac{1}{2}\mathbf{E}_1(z)\exp(ik_xx)$. The equation for $E_1(z)$ follows from Maxwell's equations (§ 1) and under the condition (5) its form is independent of the direction of the incidentfield polarization plane

$$\frac{d^2E_1}{dz^2}+E_1[k_1^2(|E_1|^2)-k_x^2]=0,\quad k_1^2=k_0^2\varepsilon_1(|E_1|^2)/\varepsilon_0. \tag{16}$$

Equation (16) is exact in the case when the direction of the polarization plane is normal to the incidence plane; then $E_1=E_{1y}$ and the magnetic component H_{1x} is longitudinal. When the polarization is parallel to the plane of incidence, it is the electric field E_{1x} which is longitudinal; under the condition (5) we have $E_{1x}\ll E_{1z}\approx E_1$. Introducing the dimensionless amplitude of the field in the linear medium u and the dimensionless coordinate ζ:

$$u=E_1(|\varepsilon_2|/\varepsilon_0)^{1/2},\quad \zeta=k_0z, \tag{17}$$

we reduce (16), with (5) taken into account, to the form

$$\frac{d^2u}{d\zeta^2}+u\left(\psi^2+\Delta_z^2{}_l+\frac{\varepsilon_2}{|\varepsilon_2|}|u|^2\right)=0. \tag{18}$$

2. At infinity ($\zeta\to\infty$), by virtue of the absence of sources at $\zeta>0$, the solutions of (18) should satisfy the conditions

$$u(\infty)=0\quad\text{and (or)}\quad du(\infty)/d\zeta=0. \tag{19}$$

The boundary conditions (1) and (2) for E_1 also coincide if (5) is satisfied. Recognizing that $k_{1z}\equiv0$ in the case of TIR, they reduce, in the notation of (7) and (17), to the form

$$i\frac{du(0)}{d\zeta}+\psi[|\Delta_E|-u(0)]=0,\quad r=\frac{E_1(0)}{E_0}-1=\frac{2u(0)}{|\Delta_E|}-1. \tag{20}$$

It is seen from this, in particular, that $|r|=1$, as expected. Writing now u and r in the form $u=e^{i\delta}|u(\zeta)|$ and $r=e^{i\delta_r}$, we obtain the boundary condition for the real amplitude of the wave $|u(\zeta)|$:

$$[|u|_\zeta'(0)]^2+\psi^2(|u(0)|^2-|\Delta_E^2|)=0 \tag{21}$$

and a formula for the phases of the surface (δ) and reflected (δ_r) waves:

$$\delta=\delta_r/2=\mp\arccos|u(0)/\Delta_E|=\mp\arccos|2E_1(0)/E_0|, \tag{22}$$

where the upper sign corresponds to $|u|_\zeta'(0)<0$ and the lower to $|u|_\zeta'(0)>0$.

3. In § 2 and here we have obtained equations for only two types of waves in the nonlinear medium: a) a plane homogeneous wave (transition regime) and b) inhomogeneous surface waves (TIR regime). It can be shown, however, that at $\varepsilon_2>0$ there can be no other wave regimes. At $\varepsilon_2<0$, in addition to these regimes, solutions of (18) can exist also in the form of nonlinear traveling waves of complicated structure in plasma electrodynamics (see, e.g., [29]). It follows from the exact solutions that the region of existence of these waves is bounded precisely by the zone of the purely nonlinear TIR. They require a special analysis, in which account is taken, in particular, of the transient behavior in time. However, owing to the localization of these solutions in the hysteresis zone only, and by virtue of the definite limitations on their amplitude, the region where they exist is such that the hysteresis is preserved even for solutions that lie on the boundary of this region.

§ 4. HYSTERESES AT NEGATIVE NONLINEARITY ($\varepsilon_2<0$)

1. We investigate first the transmission regime. We consider refraction and reflection (8) and (9) in the

case $\Delta_E^2 < 0$ for different signs of the linear mismatch Δ_l^2(7).

We note beforehand that whereas in the case of ambiguity of the transmission regime (which takes place only at $\varepsilon_2 < 0$) the points where they jump from the TIR to stable transmission takes place are determined by relation (10), the backward jump (from transmission to TIR) occurs at those points where the dependence of ψ_1 or of r on ψ or on Δ_E^2 has a vertical tangent. It follows from (8) that the geometric locus of these points is

$$\psi_1 = {}^1/_4[(9\psi^2+8\Delta_l^2)^{1/2}-\psi]. \tag{23}$$

Thus, the points $\psi = \psi_{cr}$, where $d\psi_1/d\psi = \infty$, are determined by eliminating ψ_1 from (8) and (23). For $r(\psi)$ the corresponding curve is obtained by substituting (23) in (6), and for $r(\Delta_E^2)$ it is determined from (9) by the formula

$$\Delta_E^2/\Delta_l^2 = 4(1-r)/(1+r)^2(3r-1). \tag{24}$$

The geometric loci of the point at which $d\psi_1/d\psi = \infty$ or $d\psi_1/dE_0^2 = \infty$ are shown in all the diagrams as thick dashed lines.

1A. If the linear optical densities of both media are equal ($\Delta_l^2 = 0$), i.e., if the boundary is completely transparent to the weak wave (Fig. 2a, curve 1), then in a strong field (curve 2) the functions $\psi_1(\psi)$ or $r(\psi)$ become doubly valued at $\psi_{cr} < \psi < \psi_0$, where $\psi_0 = |\Delta_E| = 2E_0(|\varepsilon_2|/\varepsilon_0)^{1/2}$ and $\psi_{cr} = 4\psi_0/3^{3/2}$. The width of the hysteresis loop in terms of the glancing angle is here

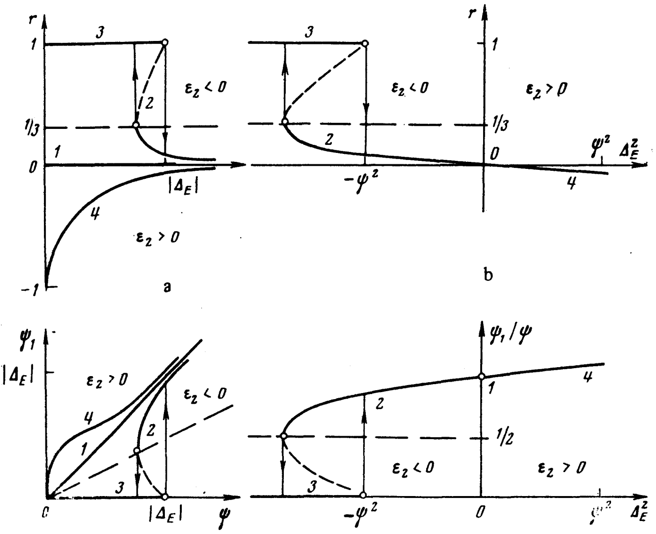

FIG. 2. Dependence of the reflection coefficient r and of the transmission angle ψ_1 on the glancing angle ψ on a fixed value of the relative intensity of the incident light Δ_E^2 (a) and on Δ_E^2 at a fixed value of ψ (b), in the case of exact equality of the linear permittivities of both media ($\Delta_l^2 = 0$). a) line 1—linear case, and b) curves: 2—transmission regime at negative nonlinearity ($\varepsilon_2 < 0$); 3—TIR regime; 4—transmission at positive nonlinearity ($\varepsilon_2 > 0$). Here and in the figures that follow the thin dashed lines represent the unstable branches of the regime and the arrows indicate the directions of the jumps.

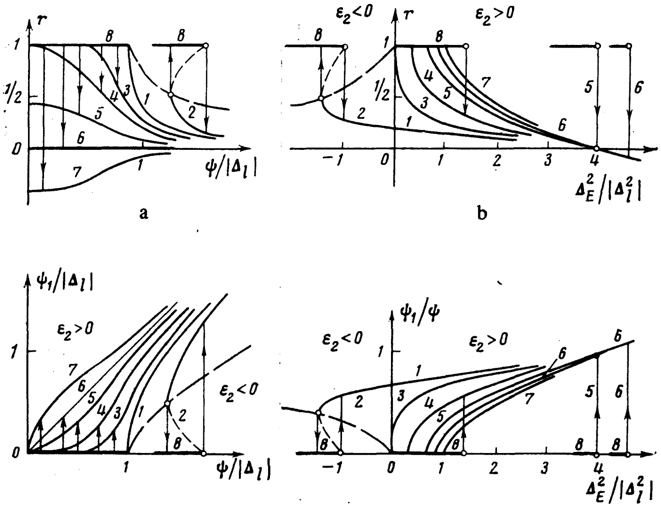

FIG. 3. Dependence of the reflection r and of the transmission angle ψ_1 on the glancing angle ψ at different intensities Δ_E^2(a) and on Δ_E^2 at different ψ (b) in the case of a negative linear mismatch of the permittivities ($\Delta_l^2 < 0$). Curves: 1—linear case; 2—transmission at $\varepsilon_2 < 0$; 3–7—transmission at $\varepsilon_2 > 0$ (3—$0 < \Delta_E^2 < |\Delta_l^2|$, 4—$\Delta_E^2 = |\Delta_l^2|$, 5—$1 < \Delta_E^2/|\Delta_l^2| < 4$, 6—$\Delta_E^2 = 4|\Delta_l^2|$, 7—$\Delta_E^2 > 4|\Delta_l^2|$); 8—TIR regime, b) Curve 2—transmission at $\varepsilon_2 < 0$; curves 1 and 3 to 7—transmission at $\varepsilon_2 > 0$ ($1-\psi > |\Delta_l|$, 3—$\psi = |\Delta_l|$, 4—$1 > \psi/|\Delta_l| > 1/2\sqrt{2}$, 5—$\psi = |\Delta_l|/2\sqrt{2}$, 6—$\psi < |\Delta_l|/2\sqrt{2}$, 7—$\psi \ll |\Delta_l|$); 8—TIR regime.

$$\Delta\psi_h \equiv \psi_0 - \psi_{cr} = (1-4/3^{3/2})|\Delta_E| \approx 0{,}46E_0(|\varepsilon_2|/\varepsilon_0)^{1/2}. \tag{25}$$

In particular, for the CdS crystal, where $n_2 (= 2^{-1}\varepsilon_2\varepsilon_0^{-1/2}) = -10^{-10}$ cgs esu, [27] in a field $E_0 \sim 10^6$ V/cm at $\varepsilon_0^{1/2} \sim 2.5$, we obtain $\psi_0 \sim 3.6°$; $\psi_{cr} \sim 2.75°$; $\Delta\psi_h \sim 0.85°$. The intensity hysteresis zone (Fig. 2b) is given by $\psi^2 < |\Delta_E^2| < \frac{27}{16}\psi^2$. We note that at the point of breakaway to the TIR we have $\psi_1 = \psi_{cr}/2$ and $r = 1/3$.

It is easy to conclude from qualitative considerations that the smallest of all the possible values of the field amplitude in the nonlinear medium E_1 is always stable, as is consequently also the maximum value of the transmission angle ψ_1 (Figs. 2–4, where the thin dashed lines represent the unstable regimes).

1B. If the refractive medium in the linear case is optically less dense than the medium bordering on it (upper medium in Fig. 1), i.e., $\Delta_l^2 < 0$, then hysteresis exists also at any intensity of the incident field (Fig. 3). Here, however, in a relatively weak field ($|\Delta_E^2| \ll |\Delta_l^2|$), the width of the hysteresis loop is very small:

$$\Delta\psi_h = \psi_0 - \psi_{cr} \approx {}^1/_2\Delta_E^4/|\Delta_l^3| \tag{26}$$

(ψ_0 is determined from (10), and ψ_{cr} from the simultaneous solutions of (8) and (23)), i.e., $\Delta\psi_h$ is proportional to E_0^4. In a sufficiently strong field ($|\Delta_E^2| \gg |\Delta_l^2|$) the value of $\Delta\psi_h$ is proportional to E_0 and approaches (25).

1C. If the refractive medium in the linear case is optically denser than the upper medium, i.e., $\Delta_l^2 > 0$, then hysteresis appears only in a sufficiently strong field, $|\Delta_E^2| > \Delta_l^2$ (Fig. 4). At $\Delta_l^2 < |\Delta_E^2| < 4\Delta_l^2$ a mini-

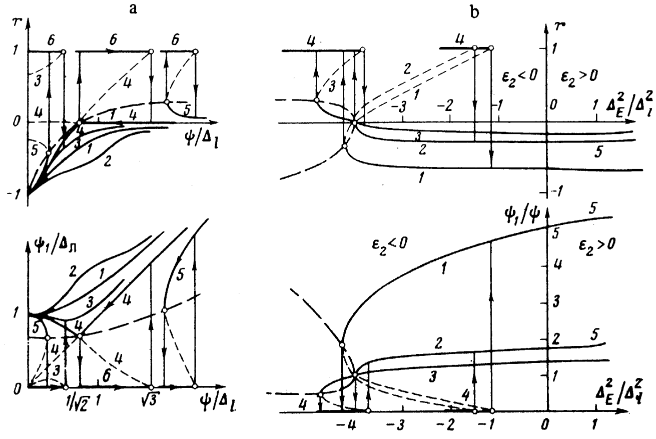

FIG. 4. Dependence of the reflection r and of the transmission angle ψ_1 on the glancing angle ψ at different intensities Δ_E^2 (a) and on Δ_E^2 at different ψ (b) in the case of positive linear mismatch of the permittivities ($\Delta_l^2>0$). a) Curves: 1—linear case, 2—tranmission at $\varepsilon_2>0$; curves 3 to 5—transmission at $\varepsilon_2<0$ (3—$|\Delta_E^2|<4\Delta_l^2$, 4—$|\Delta_E^2|>4\Delta_l^2$); 6—TIR regime. b) Curves: 1 to 3—transmission at $\varepsilon_2<0$ (1—$\psi<\Delta_l/\sqrt{2}$, 2—$\psi=\Delta_l/\sqrt{2}$, 3—$\psi>\Delta_l/\sqrt{2}$); 4—TIR regime; 5—transmission at $\varepsilon_2>0$.

mum appears on the stable branch of the transmission regime $\psi_1(\psi)$ (Fig. 4a, solid curve 3), and simultaneously there appears a nonstable branch of this regime (dashed curve 3), as well as an isolated stable TIR section (curve 6). It is possible to land on the latter only in the following manner: the glancing angle is fixed at $\psi<\Delta_l 3^{1/2}$, the intensity is increased until a jump into the TIR regime takes place (curve 1 or 2 of Fig. 4b), and the intensity is then decreased to a value somewhat higher than $|\Delta_E^2|_{\mathrm{TIR}}$ for the given angle ψ. The extrema of the $\psi_1(\psi)$ curves lie in this on the $\psi^3=\psi_1(\Delta_l^2-\psi_1^2)$ curve, and the jump from the TIR to refraction occurs at $\psi_0(10)$.

Let us examine the optical flip-flop. When the equality (15) is satisfied, total linear transparentization of the boundary takes place in the transmission regime for all angles $\psi>\psi_{\mathrm{tr}}=\Delta_l/2^{1/2}$ (line 4 of Fig. 4a). Therefore at the instant of the hysteresis jumps a breakaway is observed from total transmission of the wave to total reflection (curve 6 of Fig. 4a) and back. The "blocking" of the boundary (the jump from $r=0$ to $r=1$) occurs if the glancing angle goes through the value $\psi=\psi_{\mathrm{tr}}$ as it decreases (at a fixed intensity $|\Delta_E^2|_{\mathrm{tr}}=4\Delta_l^2$), or else if the intensity goes through the value $4\Delta_l^2$ as it increases (at a fixed angle $\psi=\psi_{\mathrm{tr}}$)—Fig. 4a, curve 2. (The course of the second branch of the transmission regime $\psi_1(\psi)$ and $r(\psi)$, which contains also stable and unstable sections (curve 4 of Fig. 4a) is determined by a cubic equation that follows from (8) and (9) and is written for ψ_1 in the form $(\psi_1+\psi)^3=\Delta_l^2(\psi_1+3\psi)$, and for r in the form $\Delta_l^2(1+r)^2(2+r)=4\psi^2$.) The return jump, which leads to the total "opening" of the boundary (from $r=1$ to $r=0$), occurs at $\psi=3^{1/2}\Delta_l$ (Fig. 4a) or at $|\Delta_E^2|=\frac{3}{2}\Delta_l^2$ (Fig. 4b). At $n_2\sim-10^{-10}$ cgs esu, $n_0\sim 2.5$, and $\Delta\varepsilon_l\sim 10^{-3}$ we have $E_{\mathrm{tr}}\sim 4\times 10^5$ V/cm.

Thus, at this setting the system constitutes an ideal optical "flip-flop" of sorts, which affords in optics a rare opportunity of an abrupt reversal of a high-power energy flux with small losses.[2)] At other settings the jumps of the reflection are also large. For example, in the case $\Delta_l^2=0$ (Sec. 1A) the jump from transmission ($r=1/3$) to TIR ($|r|=1$) produces a change of power reflection $\Delta R=1-\frac{1}{9}\sim 89\%$, and the return jump (from $|r|=1$ to $r\sim\frac{1}{16}$) produces a change $\Delta R\sim 99.6\%$. With further increase of the intensity ($|\Delta_E^2|>4\Delta_l^2$), the functions $\psi_1(\psi)$ and $r(\psi)$ have already two hysteresis loops (curve 5 of Fig. 4b), while the intensity dependences of Δ_E^2 always have one loop (Fig. 4b). At $|\Delta_E^2|=4\Delta_l^2$ the width of the loop in terms of the glancing angle is $\Delta\psi_h\approx 0.5\,|\Delta_E|$ (approximately twice as large than the value obtained in (25)) and tends to the value obtained in (25) with increasing $|\Delta_E^2|$.

2. We consider the TIR regime. Equation (18) at $\varepsilon_2<0$ has a solution that satisfies the conditions (19) and (21):

$$|u|=\begin{cases} 2^{1/2}\gamma/\mathrm{sh}\left[k_0\gamma z+\mathrm{arsh}\left(\dfrac{2^{1/2}\gamma}{u_0}\right)\right] & \text{if}\quad \Delta_l^2<0,\quad \psi^2<|\Delta_l^2|,\\ 2^{1/2}/(k_0 z+2^{1/2}/u_0)\quad \text{if}\quad \Delta_l^2<0, & \text{if}\quad \psi^2=|\Delta_l^2|,\\ |\gamma|\,\mathrm{cth}\left[\dfrac{k_0 z}{2^{1/2}}+\mathrm{arcth}\left|\dfrac{u_0}{\gamma}\right|\right] & \text{if}\quad 0<\psi^2+\Delta_l^2<|\Delta_E^2|,\\ \mathrm{const}=|\Delta_E|=u_0 & \text{if}\quad \psi^2=\psi_{\mathrm{TIR}}^2\equiv|\Delta_E^2|-\Delta_l^2, \end{cases} \tag{27}$$

where $\gamma=(-\psi^2-\Delta_l^2)^{1/2}$, $u_0\equiv|u(0)|$ is the amplitude of the field at the boundary:

$$u_0\equiv|u(0)|=\begin{cases} [(\Delta_l^4+2\psi^2|\Delta_E^2|)^{1/2}-|\Delta_l^2|]^{1/2} & \text{for}\quad \Delta_l^2<0,\quad \psi^2\leqslant|\Delta_l^2|,\\ [\psi(2|\Delta_E^2|-2\Delta_l^2-\psi^2)^{1/2}+\Delta_l^2]^{1/2} & \text{for}\quad 0\leqslant\psi^2+\Delta_l^2\leqslant|\Delta_E^2|, \end{cases} \tag{28}$$

where $\psi=\psi_{\mathrm{TIR}}\equiv(|\Delta_E^2|-\Delta_l^2)^{1/2}$ is the maximum angle at which nonlinear TIR exists (jump to the transmission regime). Comparison with relation (10) yields $\psi_0=\psi_{\mathrm{TIR}}$.

The first of the solutions (27) is the nonlinear analog of the surface wave of the linear TIR, which is realized under the same conditions ($\Delta_l^2<0$, $\psi^2\leqslant|\Delta_l^2|$). As $z\to\infty$, it attenuated exponentially $\sim e^{-k_0\gamma z}$, just as in the linear case, but the depth of penetration of the field in the medium (at the half-intensity level), L_{nl}, decreases in comparison with the linear penetration, $L_\lambda=\ln 2^{1/2}/k_0\gamma$:

$$L_{nl}=L_l\left[1+\frac{1}{\ln 2^{1/2}}\ln\frac{1+(1+u_0^2/4\gamma^2)^{1/2}}{1+(1+u_0^2/2\gamma^2)^{1/2}}\right]. \tag{29}$$

In a weak field we have $L_{nl}\to L_l$, and in a strong field ($u_0^2\gg\gamma^2$), which is equivalent to the condition $|\Delta_E^2|\gg|\Delta_l^2|(|\Delta_l^2|/\psi^2-1)$, we have $L_{nl}\approx 0.6\gamma L_l/u_0\ll L_l$.

The second, transition, solution (27) corresponds to the boundary of the linear TIR (when $E_1=\mathrm{const}=2E_0$, which is valid also for the boundary of the nonlinear TIR, the fourth of the solutions (27)). The third of the solutions (27) corresponds already to those glancing angles ψ, at which the linear TIR does not exist at all, i.e., at $|\Delta_l|<\psi<\psi_{\mathrm{TIR}}$, if $\Delta_l^2<0$, and at $0<\psi<\psi_{\mathrm{TIR}}$, if $\Delta_l^2>0$. As $z\to\infty$ it tends to a constant $|\gamma|=(\psi^2+\Delta_l^2)^{1/2}$ that is independent of the intensity Δ_E^2 (in particular, at $\Delta_l^2=0$ we have $|\gamma|=\psi$).

The phases of the waves are obtained by substituting (28) in (22), where it is necessary to take only the upper sign, inasmuch as at $\varepsilon_2 < 0$ we always have $|u|'(0) < 0$ (17). At the points where the TIR breaks away, i.e., at $\psi = \psi_0 = \psi_{\mathrm{TIR}}$, we have $\delta = \delta_r = 0$.

§ 5. HYSTERESES AT POSITIVE NONLINEARITY ($\varepsilon_2 < 0$)

1. The transmission regime described by Eqs. (8) and (9) is always single-valued at $\varepsilon_2 > 0$ (Fig, 2, curves 4; Fig. 3, curves 3–7; Fig. 4a, curve 2). With increasing Δ_E^2, the reflection increases, in such a way that at $r \to -1$ we have

$$r \approx -1 + 2^{1/2}[(\Delta_l^4 + 4\psi^2\Delta_E^2)^{1/2} - \Delta_l^2]^{1/2}/\Delta_E, \tag{30}$$

as follows from (9) for any field at $\psi^2 \ll \Delta_l^2$, if $\Delta_l^2 > 0$, and in a sufficiently strong field we have $\Delta_E^2 \gg 4\psi^2 + |\Delta_l^2|$), if $\Delta_l^2 \leqslant 0$. If $\Delta_l^2 = 0$, then $r + 1 \approx 2(\psi/\Delta_E)^{1/2}$ (at $\Delta_E^2 \gg 4\psi^2$). In a weak field (see (14)) the transmission is determined by formulas (11)–(13). At $\Delta_l^2 < 0$ the angle limit of the transmission regime shifts towards zero (see (10) and Fig. 3a, curve 3); at $\Delta_E^2 = |\Delta_l^2|$ we have already from (10) $\psi_0 = 0$ (curve 4). At $\Delta_E^2 > |\Delta_l^2|$ the reflection coefficient r in the transmission regime no longer reaches the value +1 at all (curves 5–7). The extremum ($r_m = -1 + 2|\Delta_l|/\Delta_E$) is reached here at $\psi = 0$. On the other hand, if the condition (15) is satisfied, then total nonlinear transparentization takes place (line 6) for all glancing angles ψ (see also curves 1 and 3 to 7 of Fig. 3b).

2. The TIR regime in the case $\varepsilon_2 > 0$, just as in the linear case, exists only at $\Delta_l^2 < 0$. Jumps of the reflection coefficient appear in this case on going from the TIR to transmission; in this case they are due to the ambiguity of the TIR regime (and not of the transmission as in the case $\varepsilon_2 < 0$). The return transition (from transmission to TIR) at $\psi = \psi_0$ (see (10)) is accompanied in the case when $|\Delta_E^2| < |\Delta_l^2|$ only by a jump in the phase of the reflection. At any intensity Δ_E^2, the maximum angle at which TIR exists, $\psi = \psi_{\mathrm{TIR}}$, is always larger than the minimal glancing angle for the transmission regime, $\psi = \psi_0$ (see Fig. 3a, curves 3–7).

2A. Equation (18) has a first integral satisfying the condition (19), namely

$$(|u|_\zeta')^2 = |u|^2(\gamma^2 - {}^1/_2|u|^2), \quad \gamma^2 = |\Delta_l^2| - \psi^2, \tag{31}$$

(and in addition, $|u|^2 = \mathrm{const} = \gamma^2$, which by virtue of (21) is realized only at $\psi^2 = \psi_0^2 = |\Delta_l^2| - \Delta_E^2$; this solution is unstable, see Sec. 2E below). The solution of Eq. (31) is (see Fig. 5a)

$$|u| = 2^{1/2}\gamma/\mathrm{ch}\,[k_0\gamma z \pm \mathrm{arch}\,(2^{1/2}\gamma/u_0)], \quad u_0 \equiv |u(0)|. \tag{32}$$

Eliminating $|u|'(0)$ from Eq. (31) taken at $\zeta = 0$ and from the boundary conditions (21), we obtain an equation for u_0, namely

$$u_0 = [|\Delta_l^2| \pm (\Delta_l^4 - 2\psi^2\Delta_E^2)^{1/2}]^{1/2}. \tag{33}$$

2B. Stipulating that the amplitude (33) and the phases of the waves (22) be real quantities, we obtain the conditions for the realization of the TIR

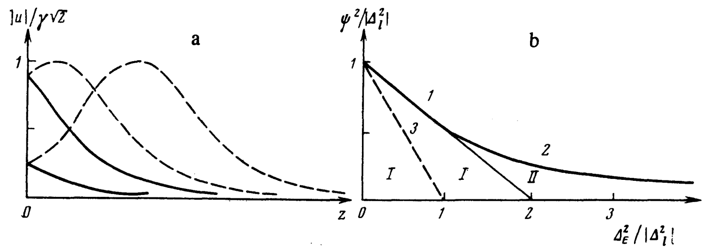

FIG. 5. Amplitude profiles $u(z)$ of the nonlinear surface wave in TIR in the case $\varepsilon_2 > 0$, $\Delta_l^2 < 0$ (a) and the regions of the existence of TIR regimes in the plane of the parameters of the incident wave (b).

$$2\psi^2\Delta_E^2 \geqslant \Delta_l^4, \quad 0 \leqslant u_0^2 \leqslant \Delta_E^2, \tag{34}$$

from which follows the maximum possible glancing angle at TIR

$$\psi_{\mathrm{TIR}} = \begin{cases} (|\Delta_l^2| - {}^1/_2\Delta_E^2)^{1/2} & \Delta_E^2 \leqslant |\Delta_l^2|, \\ |\Delta_l^2|/2^{1/2}\Delta_E & \Delta_E^2 \geqslant |\Delta_l^2|, \end{cases} \tag{35}$$

(Fig. 5b, curves 1 and 2), i.e., we always have $|\Delta_l| > \psi_{\mathrm{TIR}} > \psi_0 = (|\Delta_l^2| - \Delta_E^2)^{1/2}$ (curve 3).

2C. It follows from (32) and (33) that at $\varepsilon_2 > 0$ the number of possible states of TIR ranges from one (at $\psi = \psi_{\mathrm{TIR}}$, $\Delta_R^2 \leqslant |\Delta_l^2|$, when $|u| = \Delta_E/\cosh(k_0\gamma z)$), to four (their amplitude profiles are indicated in Fig. 5a). If we disregard the critical values of the parameters, then the number of regimes can be either two (region I on Fig. 5b) or four (region II). The two states of the TIR in the region I, which is defined by the inequality

$$\psi^2 < |\Delta_l^2| - {}^1/_2\Delta_E^2, \quad \Delta_E^2 < 2|\Delta_l^2|, \tag{36}$$

correspond to one value of u_0 (the lower sign in (33)). These states differ in the signs of the derivatives $|u|'(0)$ (32), and consequently in the signs of the phases (22), and the form of their profiles is given, say, by any of the pairs of curves on Fig. 5a with coinciding u_0. In region II on Fig. 5b

$$\Delta_l^4/2\Delta_E^2 > \psi^2 > \max\{0, (|\Delta_l^2| - {}^1/_2\Delta_E^2)\}, \tag{37}$$

the presence of two values of u_0 (see (33)) and of two signs in (32) for each of them yields already four states (both pairs of curves on Fig. 5a).

2D. The power transported in the surface wave along the interface of the media (per unit length of the y axis) is

$$W(E_0^2, \Delta\varepsilon_l, \psi) = \frac{c\varepsilon_0^{1/2}}{4\pi}\int_0^\infty E_t^2(z)\,dz = \frac{c\varepsilon_0|\Delta\varepsilon_l|^{1/2}}{\pi k_0\varepsilon_2} I\left(\frac{\varepsilon_2 E_0^2}{|\Delta\varepsilon_l|}, \frac{\psi^2\varepsilon_0}{|\Delta\varepsilon_l|}\right), \tag{38}$$

where, as follows from (32) and (33) (see Fig. 6),

$$I = \frac{1}{2}\left(1 - \frac{\psi^2}{|\Delta_l^2|}\right)^{1/2}\left\{1 \mp \left[1 - \frac{1 \pm (1 - 2\psi^2\Delta_E^2/\Delta_l^4)^{1/2}}{2(1 - \psi^2/|\Delta_l^2|)}\right]^{1/2}\right\}. \tag{39}$$

The signs in front of the square bracket correspond here to the signs in (32), and the signs in front of the

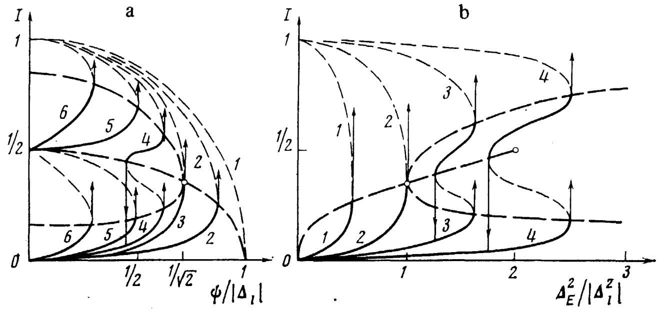

FIG. 6. Dependence of the total relative power I transported by the surface waves in TIR in the case $\varepsilon_2>0$; $\Delta_l^2<0$, on the glancing angle at different intensities Δ_E^2 (a) and on Δ_E^2 at different ψ (b). a) Curves: 1—linear case; 2—$\Delta_E^2<|\Delta_l^2|$; 3—$\Delta_E^2=|\Delta_l^2|$; 4—$1<\Delta_E^2/|\Delta_l^2|<4$; 5—$\Delta_E^2=4|\Delta_l^2|$; 6—$\Delta_E^2>4|\Delta_l^2|$. b) Curves: $1-1/\sqrt{2}<\psi/|\Delta_l|<1$; 2—$\psi=|\Delta_l|/\sqrt{2}$; 3, 4—$\psi<|\Delta_l|\sqrt{2}$.

round parentheses correspond to the signs in (33). Calculating the derivatives $\partial I/\partial(\Delta_E^2)\propto\partial W/\partial(E_0^2)$ (Fig. 6b) and assuming their positiveness to be the criterion of the stability, we find from (39) that the solutions in (32) which are stable are those with the upper sign (for which $|u(0)|'_\zeta<0$; they consequently have no maxima inside the nonlinear medium—solid curves of Fig. 5a). The same solutions, which have a maximum inside the refracting medium (dashed curve of Fig. 5a) and are sort of two-dimensional self-focusing channels excited not through the end face but "from the side," are unstable.

The relative power I (39) in the principal stable state of the TIR reaches a maximum $I_{\max}=1/2^{3/2}$ at $\Delta_E^2=|\Delta_l^2|$ and $\psi=|\Delta_l|/2^{1/2}$.

2E. The phases of the waves are determined by substituting (33) in (22), where the stable states correspond to the upper sign. On going from the TIR to transmission (i.e., at $\psi=\psi_{\mathrm{TIR}}$ (35)) we have

$$\delta=\delta_r=0 \quad \text{if} \quad \Delta_E^2\leqslant|\Delta_l^2|,$$
$$\delta=\frac{\delta_r}{2}=-\arccos\frac{|\Delta_l|}{\Delta_E} \quad \text{if} \quad \Delta_E^2\geqslant|\Delta_l^2|.$$

In the return transition—from transmission to TIR ($\Delta_E^2\leqslant|\Delta_l^2|$) the phase of the reflection jumps from $\delta_r=0$ to

$$\delta_r=-2\arccos\{1-[(|\Delta_l^2|/\Delta_E^2-1)^2+1]^{1/2}\}^{1/2}.$$

The fifth TIR regime which is possible at this point $|u|=\mathrm{const}=\Delta_E$ (Sec. 2A) is unstable, since any infinitesimal decrease of intensity Δ_E^2 changes the solution jumpwise by a finite amount in virtue of (32).

The width of the hysteresis loop relative to the glancing angle $\Delta\psi_h=\psi_{\mathrm{TIR}}-\psi_0$ in weak fields ($\Delta_E^2\ll|\Delta_l^2|$) first increases like $\Delta\psi_h\approx\Delta_E^2/4|\Delta_l^2|/2^{1/2}\Delta_E$, reaching a maximum $\Delta\psi_h=|\Delta_l|/2$ at $\Delta_E^2=|\Delta_l^2|$, and then decreases like $|\Delta_l^2|/2^{1/2}\Delta_E$. The depth of penetration of the field into the medium (L_{nl}) is of the same order as in the linear TIR (L_l) independently of the field.

2F. We present numerical estimates for CS_2, where $n_2\sim9\times10^{-12}$ cgs esu and $n_0\sim1.5$. Here, at $\Delta\varepsilon_l\sim-10^{-3}$ the transparency inducing field is $E_{\mathrm{tr}}\sim1.8\times10^6$ V/cm (see (15)), in which case $\psi_{\mathrm{TIR}}\sim0.52°$, and in the TIR regime at $\psi\lesssim\psi_{\mathrm{TIR}}$ and $k_0\sim10^5$ cm^{-1} we have $W\sim1.7\times10^7$ W/cm (see (38) and (39)), and $L_{nl}\sim1.4$ μm.

§ 6. THRESHOLD CHARACTERISTICS FOR BOUNDED BEAMS

Under real conditions, the transverse dimension a of the light beam, as well as its total power P, is limited, and this leads to additional requirements. By virtue of the diffraction of the narrow beam, the glancing angles of the wave at the different points of the spot of the boundary are different; for the case $\Delta_l^0=0$ and $\varepsilon_2<0$ (Sec. 4), if we require that the diffraction-induced angular smearing of the Gaussian beam over the length of its interaction with the boundary be not less than the width of the hysteresis loop (25), we obtain an estimate for its threshold power:

$$P_{\mathrm{thr}}\sim{}^{27}/_{11}c\varepsilon_0^{1/2}/4|\varepsilon_2|k_0^2=0.3c/k_0^2|n_2|. \tag{40}$$

Here P_{thr} does not depend on the beam radius. At $n_2\sim-10^{-10}$ cgs esu and $k_0\sim10^5$ cm^{-1} we have $P_{\mathrm{thr}}\sim1$ kW. On the other hand, if $\Delta_l^2\neq0$, then P_{thr} depends on the radius of the beam, tending to the value (40) if the field at the center exceeds the value

$$E_{\mathrm{thr}}\sim{}^{1}/_{2}|\Delta\varepsilon_l/\varepsilon_2|^{1/2}, \tag{41}$$

i.e., $|\Delta_E^2|\sim|\Delta_l^2|$. In turn, relation (41) imposes the following condition on the beam radius a:

$$a<a_{max}\sim3(P/P_{\mathrm{thr}})^{1/2}/k_0|\Delta\varepsilon_l|^{1/2}. \tag{42}$$

At $k_0\sim10^5$ cm^{-1}, $\Delta\varepsilon_l\sim10^{-3}$, and $P\sim100P_{\mathrm{thr}}$ we have $a_{\max}\sim0.1$ mm.

CONCLUSION

1. It appears that the simplest possibility of experimentally verifying the phenomenon is to use thermal nonlinearity, which makes it possible to work with low powers and in the cw regimes of the laser. The linear medium can be glass, and the nonlinear medium a liquid such as ether, alcohol, benzene, etc. The matching of ε_0 and ε_1 can be easily attained here by dyeing the liquid. This makes the liquid a convenient component also in the investigation of other nonlinearity mechanisms (the Kerr effect, striction, etc.[22]), which are also large for organic liquids.[30] We note that the thermal nonlinearity due to the heating of the thin near-surface layer of the refractive medium may turn out to be significant for arbitrary media and in sufficiently short pulses. The time of establishment of $\Delta\varepsilon_{nl}$ at the penetration depth L_{nl} of the surface wave in the case of TIR is $T_{nl}\sim L_{nl}/v$ (where v is the speed of sound in the medium), and amounts to $\sim10^{-9}$–10^{-10} sec at typical parameters.

2. The main results obtained here pertain to an unbounded stationary plane wave incident on a semi-infinite nonlinear medium. If the beam cross section, the pulse duration, the curvature of the wave front, and the thickness of the nonlinear medium are all bounded,

then additional aspects of the interaction should appear (besides the threshold conditions, Sec. 6), which call for further investigation.

Limiting the transverse dimension of the beam, even in the linear case in TIR, leads to a shift of the reflected beam along the boundary,[31, 32] and the limiting of the duration of the pulse leads to a change in the frequency spectrum of the reflected wave.[32] Understandably, there should exist also nonlinear analogs of these phenomena, which are of particular interest here because different sections of the boundary can be in different states. In addition, at $\varepsilon_2 > 0$ in a nonlinear medium, under certain conditions, self-focusing channels can be produced even in the transmission regime (in contrast to the near-surface "channel" in TIR, § 5, Sec. 2), which can also influence the reflection. The presence of very strong fields can lead in some cases to breakdown, which in the first experiments only facilitates the observation of the effect, just as it occurred when self-focusing[2] and self-bending[25] were observed. If the incident wave is not plane (owing to focusing, defocusing, or diffraction), then by virtue of the difference in the reflection of the "near" and "far" edges of the beam from the boundary, the wave in the linear medium can have an asymmetrical amplitude profile, and this can lead to self-bending.[24]

It appears that a finite thickness of the nonlinear layer leads, under certain conditions, to formation of several reflection-hysteresis loops corresponding to the edges of the regions in which different modes of the nonlinear waveguide made up by the layer exist. After a beam with a bounded cross section passes through the layer, depending on the amplitude and the front of the wave, one can observe both self-bending and "external" self-focusing or self-defocusing.[33]

We note, finally, that when higher harmonics are generated on the nonlinear boundary,[2-5] if the conditions described here are satisfied, hysteresis can likewise be observed (both for the fundamental and for the higher harmonics).

3. Let us point some possible applications of the hysteresis reflection and refraction of light on a boundary of a nonlinear medium, for the purpose of investigating the nonlinear properties of a medium and in laser technology.

1) With the aid of the phenomena considered above we can measure ε_2 with high accuracy, due to the accuracy with which the instants of the jumps are determined (for example, on oscillograms of the pulses).

2) In contrast to other self-action effects in nonlinear reflection (in the TIR regime), the field penetrates into the medium to a very small depth (L_{nl}). This could be quite useful as applied to nonlinear substances with strong absorption, since it makes it possible, despite the absorption, to investigate and to use their nonlinearity in the case when $L_{nl} \ll L_d$, where L_d is the length of the dissipative damping of the wave. We note that for many semiconductors the nonlinearity increases near the edge of the absorption band by several orders of magnitude,[34] i.e., the threshold power is accordingly decreased.

3) In a high-power nonstationary field at resonance with the quantum transition of the medium, oscillations of the populations of the resonance levels can occur, and are strongest when the pulse duration becomes of the order of or less than the relaxation times of the system.[35] Under the conditions described here, one could observe and investigate oscillations of reflection by the boundary of such a system from $|r| \ll 1$ to $|r| = 1$, i.e., so to speak strongly enhanced amplitude quantum oscillations.

4) In the resonance lines produced by several transitions, some of which are weak and unresolvable, the nonlinear properties of each differ strongly.[36] This would make it possible to realize with the aid of nonlinear TIR a modification of nonlinear spectroscopy for the resolution of such lines, which would be analogous to linear spectroscopy of internal reflection[37] and would have its main advantages.

5) At a definite setting, the nonlinear boundary changes at the instant of the jump from $r = 0$ to $|r| = 1$, and therefore can serve as an ideal threshold element—shutter—in lasers for the generation of giant pulses; this shutter would have the following advantages in comparison with a saturable resonant absorber: a) it absorbs practically no energy during the time of the lasing itself, when we already have $|r| = 1$ (the TIR regime); b) it is resonant when ordinary nonlinear media are used and has a high operating speed, limited only by the relaxation of the nonlinearity; c) the intensity of the light at which total reflection is turned on jumpwise can be easily regulated by choosing the linear mismatch $\Delta\varepsilon_l$ and the glancing angle ψ. In addition, the intensity of the "turning-off" field is always smaller than that of the "turning-on" field, thus increasing the efficiency in the generation of a giant pulse. All this makes it possible to use the considered system also for the formation of short pulse fronts, and also to shorten pulses.

6) The bistability of the regimes which is present in the hysteresis-loop zone makes it possible to realize an optical flip-flop in all those applications where this bistability is significant,[8-15, 17] and in particular to realize a binary logic element for optical computers.

7) Nonlinear refraction can be used for an angular switching and scanning of the refracted beam (when the intensity of the incident light is varied), resulting in angular deflections of the beam and in a deflection time of the same order as in self-bending,[24, 27] and not requiring a mandatory asymmetry of its amplitude profile.

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[1)]The reactive component of the nonlinearity can be significant also in the case of resonant absorption.[14,15]

[2)]It was noted in the Introduction that resonator systems with dissipative nonlinearity[12-15] have large losses.

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