We note in conclusion that taking the inhomogeneous broadening into account can be done trivially in the linear theory of the stability of a 2π pulse; the answer is also given by Eq. (10).

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Note added in proof (*March* 31, 1977). Recently a paper by Gibbs *et al.* has been published (Phys. Rev. Lett. 37, 1743 (1976)) in which the observation of the instability predicted by us was reported for the case where a resonant light pulse traversed a cell with sodium vapor.

[1]S. L. McCall and E. L. Hahn, Phys. Rev. 183, 457 (1969).
[2]I. A. Poluéktov, Yu. M. Popov, and V. S. Roitberg, Kvantovaya Elektron, (Moscow) 4, 757 (1974) [Sov. J. Quantum Electron. 4, 423 (1974)]; Usp. Fiz. Nauk 114, 97 (1974) [Sov. Phys. Usp. 17, 673 (1975)].
[3]N. Wright and M. C. Newstein, Opt. Commun. 9, 8 (1973).
[4]F. P. Mattar and M. C. Newstein, Opt. Commun. 18, 70 (1976).
[5]G. A. Askaryan, Zh. Eksp. Teor. Fiz. 42, 1567 (1962) [Sov. Phys. JETP 15, 1088 (1962)].
[6]B. B. Kadomtsev and V. I. Petviashvili, Dokl. Akad. Nauk SSSR 192, 753 (1970) [Sov. Phys. Dokl. 15, 539 (1970)].
[7]V. I. Bespalov and V. I. Talanov, Pis'ma Zh. Eksp. Teor. Fiz. 3, 471 (1966) [JETP Lett. 3, 307 (1966)].
[8]L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika (Quantum Mechanics) Fizmatgiz, 1963 [English translation published by Pergamon, Oxford].

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Undisplaced resonant scattering line of a strong quasimonochromatic field

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The change in the shape of a narrow undisplaced resonant-scattering line of a strong quasimonochromatic field is investigated. At low field intensity it is known that the line duplicates the spectrum of the exciting radiation. The change in the shape and width of a narrow undisplaced line depends essentially on the statistical field of the exciting-radiation field. The dependence of the line width on the field intensity is investigated for typical radiation statistics. The physical phenomenon in question can, in particularly, serve as the basis of an investigation of the statistical properties of an electromagnetic field.

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1. INTRODUCTION

In view of the progress made in the development of tunable lasers, interest has greatly increased of late in investigation of resonant scattering (resonant fluorescence) of a strong laser field. In the last few years many theoretical papers[1–12] 1) have been devoted to the analysis of the spectrum of resonant scattering of intense laser radiation. The results of the first experimental investigations in this direction have also been published.[13–19] The physical concepts developed in[1–12] are based on simplified models, and it is therefore obvious that many more advances will be made in the nearest future in the study of the spectrum of resonant scattering, both theoretical and experimental.

The first theoretical analysis of the spectrum of the resonant scattering of a strong monochromatic field (analysis of the variation of the spectral density of spontaneous emission in the presence of a strong field) was carried out by Rautian and Sobel'man.[18] They have shown, in particular, that the resonant-scattering line splits into three components, corresponding to the splitting of the atomic levels into quasi-energy levels under the influence of a strong field.[19] The analysis in[18] pertained to scattering by atoms in excited states. In the "classical" formulation of the resonant-scattering problem one of the combining states is assumed to be the ground state.[20] Of course, in view of the fundamental character of the physical cause (level splitting), the multiplet structure of the spectrum (three components) and the position of the components on the frequency scale are valid also for the classical formulation. However, the participation of the ground state leads to a redistribution of the intensity between the components and to the development of a more complicated structure within the individual components, this being connected with the singularities of the relaxation processes under the given conditions. The problem of resonant scattering in which the ground state participates is therefore of independent interest.

The question of the spectrum of resonant scattering of a strong field in the classical formulation (the only one dealt with henceforth) was first raised by

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Apanasevich[21] and was subsequently developed in the already cited papers.[1–12]

In the general case, the resonant scattering takes the form of a triplet (see Fig. 1). The widths of the broad components of the triplet are determined by the atomic relaxation constants. In addition, there is also a narrow component at the frequency ω of the exciting radiation, which is represented in[1–12] by the delta function $\delta(\omega_\mu - \omega)$. The presence of a narrow undisplaced component is due to the fact that the atom in any of the states (excited or ground) does not cease to interact coherently with the field. The coherence can be violated only as a result of violation of the phase relations in the exciting radiation itself. It is clear therefore that the width of this component is connected with the width of the spectrum of the external field. When the intensity of the exciting radiation tends to zero, it is precisely this component which goes over into the resonant-scattering line investigated by Weisskopf,[20] so that its shape duplicates the spectrum of the external field.

The dependence of the shape of the narrow undisplaced component on the radiation intensity has not been studied before, and the present paper is devoted to this question. It will be shown below that the shape of the narrow component is sensitive not only to the intensity but also to the statistics of the radiation.

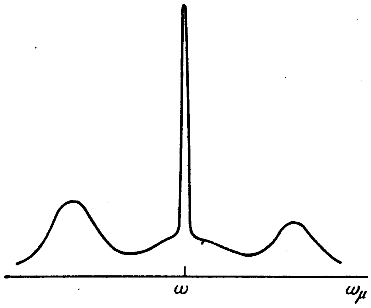

FIG. 1. Resonant-scattering triplet.

2. RESONANT-SCATTERING SPECTRUM

The problem of the spectrum of the resonant scattering reduces to finding the spectrum of the spontaneous emission on the transition $m-n$, where n is the ground state, in the presence of a strong quasi-monochromatic field of frequency ω close to the frequency ω_{mn} of the transition $m-n$. The spontaneous emission will be described, in accordance with the prescription of[18], by introducing an effective classical field. The density-matrix elements are described by the equations

$$
\begin{gathered}
(d/dt+\Gamma_m)\rho_{mm}=iV^*\rho_{mm}-iV\rho_{mn}^*,\\
\frac{d\rho_{nn}}{dt}=\Gamma_m\rho_{mm}-i(V^*+V_\mu^*)\rho_{mn}+i(V+V_\mu)\rho_{mn}^*,\\
(d/dt+\Gamma+i\omega_{mn})\rho_{mn}=iV(\rho_{mm}-\rho_{nn})+iV_\mu\rho_{mm},\\
V=G(t)e^{-i\omega t},\quad V_\mu=G_\mu e^{-i\omega_\mu t},\quad G(t)=\frac{d_{mn}}{2\hbar}E(t),\\
G_\mu=\frac{d_{mn}}{2\hbar}E_\mu.
\end{gathered}
\tag{1}
$$

Here Γ_m is the relaxation constant of the level m; Γ is the half-width of the luminescence line; d_{mn} is the dipole-moment matrix element; $E(t)$ is the amplitude of the electric field of the exciting radiation, which we assume to be a slow random function of the time, corresponding to smallness of the width γ of the emission spectrum in comparison with the relaxation constants Γ and Γ_m. The field $E(t)$ is assumed given, i.e., we do not consider the change in the radiation intensity in the course of propagation. The classical-field amplitude E_μ is the spectral density of the zero-point oscillations of the vacuum, and ω_μ is the frequency for which the spectral density of the spontaneous emission is calculated. The reason why V_μ and V in (1) are not equivalent is that the effective classical field V_μ has a bearing only on emission, whereas the genuine classical field leads to both emission and absorption.

We seek the solution of Eqs. (1) in the form

$$
\begin{gathered}
\rho_{jj}=R_j+2\mathrm{Re}[r_je^{-i\varepsilon t}],\qquad j=m,n,\\
\rho_{mn}=[R+re^{-i\varepsilon t}+\bar{r}e^{i\varepsilon t}]e^{-i\omega t},\qquad \varepsilon\equiv\omega_\mu-\omega.
\end{gathered}
\tag{2}
$$

The line contour of the spontaneous emission is determined by the value of $\mathrm{Im}(r)$ averaged over the time. The system (1) breaks up into the following two systems (cf. also[7,9,21]):

$$
\begin{gathered}
(d/dt+\Gamma_m)R_m=2\mathrm{Re}[iG^*R],\qquad R_m+R_n=N,\\
(d/dt+\Gamma-i\Omega)R=iG(R_m-R_n),\qquad \Omega\equiv\omega-\omega_{mn}\qquad G\equiv G(t)
\end{gathered}
\tag{3}
$$

and

$$
\begin{gathered}
(d/dt+\Gamma_m-i\varepsilon)r_m=iG^*r-iG\bar{r},\\
(d/dt-i\varepsilon)r_n=\Gamma_mr_m-iG^*r+iG\bar{r}+iG_\mu R^*,\\
(d/dt+\Gamma-i\Omega-i\varepsilon)r=iG(r_m-r_n)+iG_\mu R_m,\\
(d/dt+\Gamma+i\Omega-i\varepsilon)\bar{r}=-iG^*(r_m-r_n).
\end{gathered}
\tag{4}
$$

Equations (3) describe the interaction of the atom with a strong field. We have used here the condition that the particle number N be conserved; this makes it unnecessary to consider an equation for R_n. The solution of Eqs. (3) serves as the "right-hand side" for Eqs. (4).

We shall use next the condition that $G(t)$ be a slow function, or, equivalently,

$$
\gamma\ll\Gamma,\ \Gamma_m,
\tag{5}
$$

where γ is the width of the spectrum of the exciting radiation. Under this condition Eqs. (3) can be regarded as local with respect to time, i.e., their time derivatives can be discarded. As a result we obtain

$$
R_m=\frac{N}{2}\frac{a|G|^2}{1+a|G|^2},\quad R=\frac{iGN}{\Gamma-i\Omega}\frac{1}{1+a|G|^2}\quad a=\frac{4\Gamma}{\Gamma_m(\Gamma^2+\Omega^2)}.
\tag{6}
$$

An analogous procedure of discarding the time derivatives in Eqs. (4) cannot be employed directly, since the second equation does not contain the relaxation term for r_n, and dr_n/dt becomes the principal term at $\varepsilon=0$. We proceed in the following manner. As a result of adding the first two equations in (4), we obtain a solution for the sum r_m+r_n:

$$
r_m+r_n=iG_\mu\int_0^\infty e^{i\varepsilon\tau}R^*(t-\tau)\,d\tau\equiv f(t).
\tag{7}
$$

Identity transformations reduce (4) to the following equations:

$$(d/dt+\Gamma_m-i\varepsilon)r_m=iG^*r-iG\bar{r},$$
$$(d/dt+\Gamma-i\Omega-i\varepsilon)r=2iGr_m-iGf+iG_\mu R_m, \qquad (8)$$
$$(d/dt+\Gamma+i\Omega-i\varepsilon)\bar{r}=-2iG^*r_m+iG^*f.$$

It is obvious that $f(t)$, just as $G(t)$, is a slow function of the time, so that in Eqs. (8), in analogy with Eqs. (3), we can discard the time derivatives. As a result of simple transformations we arrive at the following expression for the quantity $\mathrm{Im}(r)$, which is of interest to us and in fact describes the spectrum of the resonant scattering

$$\mathrm{Im}(r)=G_\mu \,\mathrm{Re}\left\{\frac{R_m}{\Gamma-i\Omega-i\varepsilon}\left[1-\frac{\Gamma_m}{2\Gamma}\frac{\Gamma-i\Omega}{i\varepsilon}+\frac{2|G|_2}{\Gamma_m-i\varepsilon}\frac{1}{\Gamma+i\Omega-i\varepsilon}\right]\right.$$
$$\left.\times\,[1+a(\varepsilon)|G|^2]^{-1}+\frac{1}{N}\int_0^\infty R(t)R^*(t-\tau)e^{i\varepsilon\tau}\,d\tau\right\},$$
$$a(\varepsilon)=\frac{4(\Gamma-i\varepsilon)}{(\Gamma_m-i\varepsilon)[(\Gamma-i\varepsilon)^2+\Omega^2]}. \qquad (9)$$

The first part of Eq. (9) describes the broad components of the spectrum. It is precisely this part of the spectrum which was investigated in[1-12,21], and will not be analyzed here. We note only that our result coincides at this point, for example, with the result of[7], and after reduction to equivalent conditions also with the results of[4].2) In this paper we investigate the structure of the integral term in (9), which describes the narrow undisplaced line and which is proportional to $\delta(\varepsilon)=\delta(\omega_\mu-\omega)$ in the case of strictly monochromatic excitation ($R(t)$ independent of t), an assumption made before everywhere except in Weisskopf's classical paper,[20] where, however, the analysis was for the case of a weak field.

3. ANALYSIS OF THE SHAPE OF THE UNDISPLACED RESONANT-SCATTERING LINE

Thus, the shape of a narrow undisplaced line is determined by the expression

$$J(\varepsilon)=\frac{1}{N}\mathrm{Re}\int_0^\infty\langle R(t)R^*(t-\tau)\rangle e^{i\varepsilon\tau}\,d\tau$$
$$=\frac{N}{\Gamma^2+\Omega^2}\mathrm{Re}\int_0^\infty\left\langle\frac{G(t)}{1+a|G(t)|^2}\frac{G^*(t-\tau)}{1+a|G(t-\tau)|^2}\right\rangle e^{i\varepsilon\tau}\,d\tau, \qquad (10)$$

where the angle brackets denote averaging over the time t. The quantity $R(t)$ represents (apart from an inessential factor) the dipole moment induced in the atom by the external field and corresponding to the local (in time) field value. Consequently the shape of the narrow undisplaced resonant-absorption line is determined according to (10) by the Fourier transform of the correlation function for the induced dipole moment; this has a graphic classical analog. It might seem that this result is obvious and applies not only to (10) but to the entire spectrum (9). Analysis shows, however, that the complete resonant-scattering spectrum (9) is in no way connected with the correlation function of the induced dipole moment. The point is that a strong field leads, in particular, to a dynamic transformation of the spectrum, on account of the level-splitting phenomenon, which has no classical analog. The atomic correlation function corresponding to the resonant spectrum (see, e.g.,[4]) therefore does not coincide with the correlation function of the induced dipole moment. Only for a narrow undisplaced line does the classical analogy continue to hold, and furthermore at arbitrarily large radiation intensity. In order for (10) to be valid, it is important only that the time of establishment of the stationary value $R(t)$ be much shorter than the time of violation of the phase relations in the external field, and this is ensured by the condition (5).

If the intensity of the exciting radiation is not high ($a|G|^2\ll 1$), then we obtain from (10)

$$J(\varepsilon)=\frac{N}{\Gamma^2+\Omega^2}\mathrm{Re}\int_0^\infty\langle G(t)G^*(t-\tau)\rangle e^{i\varepsilon\tau}\,d\tau, \qquad (11)$$

i.e., $J(\varepsilon)$ duplicates the spectrum of the exciting radiation in accordance with[20]. With increasing intensity, the effect of saturation begins to assume importance for $R(t)$, and as a consequence the shape of the line $J(\varepsilon)$ changes. It is typical, however, that this change is sensitive also to the statistics of the external field; it is so sensitive that nothing can be stated concerning the character of the variation of $J(\varepsilon)$ beforehand (without a concrete description of the statistics). In the subsequent analysis of the effect of the field intensity on $J(\varepsilon)$ we shall therefore bear in mind its statistics, and shall analyze the cases that are most important in our opinion.

The simplest case corresponds to the statistics of the radiation of an "ideal" laser,[22] i.e., when the radiation is characterized by a random process with only phase and frequency modulation. With such statistics, $|G(t)|^2$ is practically independent of time. Consequently, the denominators in (10), which contain $|G|^2$, are affected neither by the averaging nor by the integration with respect to τ. Consequently $J(\varepsilon)$ is described by formula (11) to which there is added a factor that depends on the intensity but does not influence the line shape. Thus, if the exciting radiation is a process with only frequency-phase modulation, then the shape of the line $J(\varepsilon)$ duplicates the radiation spectrum at any value of the radiation intensity.

The change of the shape of $J(\varepsilon)$ can occur, thus, only in the presence of amplitude modulation in the radiation field. A typical example of an amplitude-modulated signal is random (Gaussian) radiation.[22] Let us consider this example. To carry out the averaging in (10) we need know the two-dimensional probability distribution $p(G_1, G_2)$ of the complex quantities G_1 and G_2, which in our case correspond to the values of G at the instants of time t and $t-\tau$, respectively. For a Gaussian radiation, $p(G_1, G_2)$ is given by[22]

$$p(G_1,G_2)=\frac{1}{\pi^2I^2(1-\varphi^2)}\exp\left[-\frac{|G_1-\varphi G_2|^2}{I(1-\varphi^2)}-\frac{|G_2|^2}{I}\right],$$
$$I=\langle|G(t)|^2\rangle,\qquad \varphi\equiv\varphi(\tau)=\langle G(t)G^*(t-\tau)\rangle/I. \qquad (12)$$

Here I characterizes the average field intensity, and $\varphi(\tau)$ is the normalized correlation function of the radiation field. Consider the correlation function for the quantity $R(t)$. With (12) taken into account it reduces to

$$J(\tau)=\frac{1}{N}\langle R(t)R^*(t-\tau)\rangle=N\frac{\Gamma_m}{8\Gamma}\frac{\varphi}{\varkappa}\int_0^\infty dx\int_0^\infty dy\frac{\exp[-(x+y)/\varkappa]}{[1+x+y+xy(1-\varphi^2)]^2},$$

$$\varkappa=4\Gamma I/\Gamma_m(\Gamma^2+\Omega^2). \tag{13}$$

We have introduced here a dimensionless parameter $\varkappa$ that determines the degree of saturation. The entire dependence of J on τ is contained in the function $\varphi(\tau)$, which enters, in particular, in the integrand of (13).

The exact calculation of $J(\tau)$ and consequently of the shape of the line $J(\varepsilon)$ entails mathematical difficulties; we confine ourselves here therefore to an analysis of the quantity

$$\Delta\varepsilon=J(\tau)|_{\tau=0}\Big/\int_0^\infty J(\tau)d\tau, \tag{14}$$

which characterizes the so-called effective half-width of the line, i.e., the ratio of the total area under the $J(\varepsilon)$ curve to the value of $J(\varepsilon)$ at the maximum ($\varepsilon=0$). For the sake of clarity we specify also the correlation function $\varphi(\tau)$:

$$\varphi(\tau)=\exp(-\gamma\tau), \tag{15}$$

where γ is the half-width of the emission spectrum.

If we retain in (13) only the first nonlinear corrections, then we obtain for $\Delta\varepsilon$ the relation

$$\Delta\varepsilon=\gamma(1+{}^4/_3\varkappa^2),\quad \varkappa\ll1. \tag{16}$$

With increasing field intensity, the narrow undisplaced scattering line broadens, and in relatively weak field the increment of the width is proportional to the square of the intensity.

We consider the inverse limiting case (strong field). An approximate integration in (13) yields

$$\Delta\varepsilon=\gamma\ln\varkappa/2\ln2,\quad \varkappa\gg1, \tag{17}$$

i.e., the line continues to broaden, but now as a logarithmic function of the intensity.

Thus, intense Gaussian radiation in resonant scattering produces an undisplaced line $J(\varepsilon)$ that is broader than the spectrum of the radiation itself, and its width increases with increasing radiation intensity.

One cannot exclude, however, the possibility of the inverse process, namely line narrowing with increasing radiation intensity. We shall demonstrate this possibility using the following model statistics of the radiation field: the radiation constitutes a random set of pulses, in which the slow amplitude $G(t)$ is of the form

$$G(t_0,\tau)=G(t_0)\exp(-\gamma\tau), \tag{18}$$

where t_0 is the instant of "turning on" an individual pulse. For this model of the spectrum, the radiation fields constitute a Lorentz contour with half-width γ. The contour of the narrow line of the resonant scattering is described by the formula

$$J(\varepsilon)=\frac{\gamma}{N}\mathrm{Re}\left\langle\int_0^\infty d\tau_1\int_0^\infty d\tau R^*(t_0,\tau)R(t_0,\tau_1+\tau)e^{i\varepsilon\tau}\right\rangle$$

$$=\frac{N\Gamma_m}{8\Gamma}\left\langle\int_0^\infty d\tau\, e^{i\varepsilon\tau}\frac{e^{-\gamma\tau}}{1-e^{-2\gamma\tau}}\ln\left[\frac{1+\varkappa(t_0)}{1+\varkappa(t_0)e^{-2\gamma\tau}}\right]\right\rangle, \tag{19}$$

$$\varkappa(t_0)=\frac{4\Gamma}{\Gamma_m(\Gamma^2+\Omega^2)}|G(t_0)|^2.$$

The averaging is carried out here over the instant of the "turning-on" t_0. The effective half-width of the line $J(\varepsilon)$, defined by formula (14), is easily obtained by integrating in (19):

$$\Delta\varepsilon=\left\langle\frac{\varkappa(t_0)}{1+\varkappa(t_0)}\right\rangle\frac{\gamma}{\langle[\mathrm{arctg}\,\varkappa^{1/2}(t_0)]^2\rangle}. \tag{20}$$

We assume that the distribution of $\varkappa(t_0)$ over the instants of time t_0 is narrow enough, i.e., the variance is much smaller than the mean value $\bar\varkappa$. Then the averaging over t_0 in (20) reduces to the substitution $\varkappa(t_0)\to\bar\varkappa$. Formula (20) then shows a monotonic decrease of $\Delta\varepsilon$ with increasing saturation parameter $\bar\varkappa$. At small values of $\bar\varkappa$ we have

$$\Delta\varepsilon=\gamma(1-\bar\varkappa/3);\quad \bar\varkappa\ll1, \tag{21}$$

i.e., the half-width of the line $J(\varepsilon)$ decreases linearly with increasing $\bar\varkappa$ in comparison with the width γ of the spectrum. At a large saturation parameter we have

$$\Delta\varepsilon=\gamma\frac{4}{\pi^2}\left(1+\frac{4}{\pi\bar\varkappa^{1/2}}\right),\quad \bar\varkappa\gg1. \tag{22}$$

The quantity $\Delta\varepsilon$ approaches monotonically the value $4\gamma/\pi^2$, i.e., for a given statistics of the radiation field the line $J(\varepsilon)$ can become narrower by approximately a factor of 2.5 in comparison with the line of the radiation itself.

The possibility of the narrowing of the $J(\varepsilon)$ line is due to the saturation effect for the induced dipole moment R (formulas (10) and (19)). When radiation of the type (18) acts on the atom, $R(t_0,\tau)$ saturates more strongly at small τ. The time of coherent interaction (the characteristic time of the variation of $R(T_0,\tau)$ as a function of τ) is therefore effectively lengthened, and the line $J(\varepsilon)$ is narrowed as a consequence.

We have thus shown that a narrow undisplaced resonant scattering line ceases in the general case to duplicate the spectrum of the radiation field if the exciting radiation is of high intensity. The changes in the shape and width of $J(\varepsilon)$ as functions of the field intensity can be quite appreciable. An analysis of the effective width has shown that these changes are extremely sensitive to the statistics of the radiation field. This uncovers the possibility of formulating the inverse problem, namely to investigate the radiation statistics on the basis of an analysis of the resonant scattering. Of course, the amount of information on the statistics increases if we are interested not only in the line width $J(\varepsilon)$ but also in the singularities of its shape.

In this paper, the external field $G(t)$ was assumed specified. This approximation is justified under conditions when the scattering is observed in a direction perpendicular to the direction of the incident radiation, or else when the optical thickness of the medium is small (in the case of collinear observation direction). There are, however, also other possible experimental conditions. If the distance between the active particles of the medium are shorter than the radiation wavelength, then interference causes the coherent part of the scattered radiation, corresponding precisely to the narrow undisplaced line, to propagate collinearly with the exciting radiation. At a large optical thickness the following will take place under these conditions: The external radiation will attenuate as it passes through the medium and will be converted into scattered radiation. The scattered radiation, on the other hand, begins to act as an excitation source with increase of its intensity, i.e., it experiences secondary scattering. Since, however, the spectrum of the scattered radiation differs from the spectrum of the exciting radiation, then a change of the spectrum will take place as a result of the propagation effect. We hope to deal with this interesting question elsewhere.

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1)See also the literature cited in[17].
2)The formulation of the problem in[4] pertains to microwave transitions, i.e., it is assumed that $\Gamma_m = \Gamma$ and the relaxation transition $n \rightarrow m$ is additionally taken into account.

1R. B. Mollow, Phys. Rev. **188**, 1969 (1969).
2P. L. Rubin and R. I. Sokolovskiĭ, Zh. Eksp. Teor. Fiz. **56**, 362 (1969) [Sov. Phys. JETP **29**, 200 (1969)].
3D. L. Huber, Phys. Rev. **178**, 93 (1969).
4R. B. Mollow, Phys. Rev. A **2**, 76 (1970).
5C. R. Stroud, Jr. Phys. Rev. A **3**, 1044 (1971).
6G. Oliver, E. Ressayre, and A. Tallet, Lett. Nuovo Cimento **2**, 777 (1971).
7E. V. Baklanov, Zh. Eksp. Teor. Fiz. **65**, 2203 (1973) [Sov. Phys. JETP **38**, 1100 (1974)].
8M. E. Smithers and H. S. Freedhoff, J. Phys. B **7**, L432 (1974).
9R. B. Mollow, Phys. Rev. A **12**, 1919 (1975).
10H. J. Carmichael and D. F. Walls, J. Phys. B **8**, L77 (1975).
11R. Sauders, R. K. Bullough, and F. Ahmad, J. Phys. A **8**, 759 (1975).
12P. A. Apanasevich and S. Ya. Kilin, Zh. Prikl. Spektrosk. **24**, 738 (1976).
13F. Schuda, C. R. Stroud, Jr., and M. Hercher, J. Phys. B **7**, L198 (1974).
14H. Walter, Proc. Second Laser Spectroscopy Conf., Mageve, France, 1975, publ. by Springer, Berlin, 1975.
15F. Y. Wu, R. E. Grove, and S. Ezekiel, Phys. Rev. Lett. **35**, 1426 (1975).
16R. E. Grove, F. Y. Wu, and S. Ezekiel, Opt. Commun. **18**, 61 (1976).
17J. L. Carlsten and A. Szöke, Phys. Rev. Lett. **36**, 667 (1976).
18S. G. Rautian and I. I. Sobelman, Zh. Eksp. Teor. Fiz. **41**, 456 (1961) [Sov. Phys. JETP **14**, 328 (1962)].
19S. H. Autler and C. H. Townes, Phys. Rev. **100**, 703 (1955).
20V. Weisskopf, Ann. Phys. (Leipzig) **9**, 23 (1931).
21P. A. Apanasevich, Opt. Spektrosk. **14**, 612 (1963); **16**, 709 (1964).
22J. Perina, Coherence of Light, Van Nostrand-Reinhold, 1972.

Stationary spectra of high-frequency oscillations of a plasma in a magnetic field

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We consider weak turbulence of Langmuir and electromagnetic waves, the source of which is a beam of relativistic electrons. We take into account the processes of induced scattering of the waves by the ions in the plasma and the damping of the waves due to Coulomb collisions. We find for the case of a weak magnetic field ($\omega_{He} < \omega_{pe}$) stationary turbulence spectra and we calculate the power transferred from the beam to the plasma. We show that the threshold heating power at which a condensation of energy into the long-wavelength part of the spectrum begins increases linearly with increasing magnetic field.

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1. INTRODUCTION

We consider in the present paper within the framework of weak turbulence theory[1-3] the non-linear stage of the instability of a beam of relativistic electrons in a plasma with a weak magnetic field ($\omega_{He} \ll \omega_{pe}$). We are dealing with the excitation by the beam of Langmuir oscillations (l) and of the restriction of their level due to induced scattering by ions. The role of the scattering consists in that it removes the oscillations from resonance with the beam electrons. This occurs, firstly, due to an increase in the phase velocity of the Langmuir oscillations when they are scattered by one another (ll process) and, secondly, due to the transformation of Langmuir into electromagnetic waves (lt process). Secondary waves arising in the scattering can be

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