Mobility of charges in crystalline helium in strong electric fields

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(Submitted December 8, 1976)
Zh. Eksp. Teor. Fiz. 72, 1953–1959 (May 1977)

The motion of carriers in a crystalline helium in strong electric fields is studied. The motion of the charges is determined by the inelastic scattering of vacancions by the charges with simultaneous spontaneous emission of phonons. The drift velocity of the charges is proportional to the cube root of the electric field intensity and decreases exponentially with decreasing temperature. At a certain critical value of the field intensity, a break, due to the formation of vacancion bound states in the charges, should occur in the dependence of the drift velocity on the field intensity. The experimental data confirm the main theoretical results.

PACS numbers: 67.80.Mg

There is great interest at the present time in the study of the quantum diffusion of impurity particles in crystalline helium. In helium, in addition to the impurity of the other isotope, only charge carriers can be dissolved in a sufficient quantity for observation. The determination of the mobility of the charges makes it possible to study the quantum features of diffusion simultaneously as functions of the temperature and of the external field.

Comparison of the activation energies of the transport of He^3 atoms and charges in crystalline He^4 indicates that the diffusion of these types of impurity particles is a vacancion diffusion. The vacancion mechanism of transport of the impurity particles was considered previously in Ref. 1 on the basis of the concepts of delocalization of point defects in quantum crystals.[2] It was shown that in the case of inelastic scattering of the delocalized vacancy—the vacancion—by an impurity particle, the latter can be displaced by the interatomic distance.

Upon displacement of a charge e by an interatomic distance a_n in an electric field $\mathbf{E}$, an energy $e\mathbf{E}\cdot\mathbf{a}_n$ is liberated. The energy of the vacancion should change by this amount as a result of the scattering process. The energy of the vacancion cannot change by an amount greater than the width Δ of its energy band. Therefore, in strong electric fields $eEa \gg \Delta$, pure vacancion processes of charge transfer are not possible. Charge displacement in inelastic scattering of vacancions should be accompanied by simultaneous spontaneous emission of phonons ($eEa \gg T$, T is the temperature) with a frequency of the order of $eEa/\hbar$; this emission is necessary for the energy balance. Expressions are obtained below which describe the dependence of the drift velocity of the charges on the temperature and on the field intensity in the case of such a mechanism of motion.

The probability of the spontaneous emission of phonons is proportional to the cube of the frequency, while the concentration of the vacancions falls off exponentially with decreasing temperature. Correspondingly, the drift velocity $\mathbf{v}$ of the charges in strong fields is proportional to the cube of the field intensity and depends exponentially on the temperature.[3] Such a dependence of the drift velocity on the temperature and on the electric field was observed experimentally for charges of both signs.[4,5] An example of a typical nonlinear dependence of the drift velocity on the field is shown in Fig. 1. The experimental method was described in detail in Ref. 5. The transition from the linear dependence of the drift velocity to the cubic one takes place in fields $eEa \sim \Delta$. This allows us to estimate the width of the energy band of the vacancions in He^4. It turns out that Δ is of the order of several degrees.[5]

The probability of charge displacement with simultaneous spontaneous emission of phonons is small and can be calculated with the help of perturbation theory. The unperturbed state corresponds to vacancy motion such that the charge remains localized at a definite site of the crystal lattice ($r=0$), i.e., the elastic scattering of the vacancions by the charge. It is convenient to describe the vacancions in terms of wave functions $\psi_{\mathbf{k}}^{+}(\mathbf{r})$ that contain at infinity the plane wave $\exp(i\mathbf{k}\cdot\mathbf{r})$ and a diverging spherical wave. In addition to the functions $\psi_{\mathbf{k}}^{+}$, we can also introduce the set of functions $\psi_{\mathbf{k}}^{-}=(\psi_{-\mathbf{k}}^{+})^{*}$,

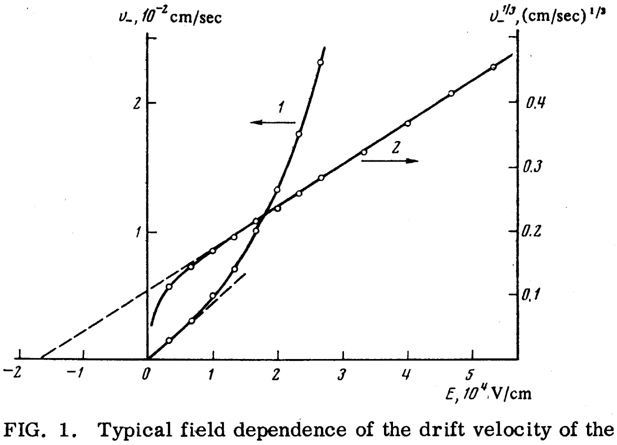

FIG. 1. Typical field dependence of the drift velocity of the charges. Negative charges. Curve 1 (left scale) is shown in a linear scale, curve 2 (right scale)—the dependence of the cube root of the drift velocity on the electric field for the same data; $p=36.6$ atm, $T=1.43$ K.

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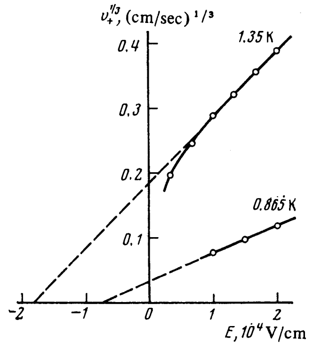

FIG. 2. Plot of $v^{1/3}(E)$ for different temperatures. The value of field $-E_0$ corresponds to points of intercept of the straight lines on the abscissa axis: $p = 32.4$ atm.

which describe the states in which there is at infinity a plane wave and a converging spherical wave.

For the calculation of the probability of the process in which the charge is displaced by an interatomic distance $\mathbf{a}_n (e\mathbf{E}\cdot\mathbf{a}_n > 0)$, a phonon of frequency ω is emitted, and the vacancy undergoes a transition from a state with quasimomentum $\hbar\mathbf{k}$ to a state $\hbar\mathbf{k}'$, we can use the well-known formula of perturbation theory for transitions to the continuous spectrum:

$$dw_n = |V|^2 \delta(\varepsilon - \varepsilon' + e\mathbf{E}\mathbf{a}_n - \hbar\omega)\, \omega^2 d\omega d^3k', \qquad (1)$$

where $\varepsilon \equiv \varepsilon(\mathbf{k}) = \varepsilon_0 + \varepsilon_1(\mathbf{k})$, $\varepsilon' = \varepsilon(\mathbf{k}')$, ε_0 is the activation energy of the vacancions, and the matrix element is taken over the wave functions of the phonon $|0_\omega\rangle$ and $\langle 1_\omega|$, the wave functions of the initial and final states of the vacancion, and also the wave functions of the charge localized at the sites $r = 0$ and $\mathbf{r} = \mathbf{a}_n$. As is well known, to determine the amplitude of the transition of a particle from the state with momentum $\hbar\mathbf{k}$ to the state $\hbar\mathbf{k}'$, we need to choose the function $\psi_\mathbf{k}^+(\mathbf{r})$ as the wave function of the initial state, and the wave function of the final state is chosen in the form $\psi_{\mathbf{k}'}^-(\mathbf{r} - \mathbf{a}_n)$.[6] Since in the final state the scattering center (ion) is located at the point $\mathbf{r} = \mathbf{a}_n$, the argument of the wave function of the final state of the vacancion is $\mathbf{r} - \mathbf{a}_n$.

The charge displacement can take place only under the condition that the vacancy turns out to be at the neighboring site of the crystal lattice; in other words, the excitation operator $\hat{V}$ acting on the vacancion variables, differs little from a δ-like operator, with matrix element

$$\langle \psi_{\mathbf{k}'}^-(\mathbf{r} - \mathbf{a}_n) | \hat{V} | \psi_\mathbf{k}^+(\mathbf{r}) \rangle = V \psi_{\mathbf{k}'}^-(0)^* \psi_\mathbf{k}^+(0).$$

Since the wave functions of the phonon are proportional to $\omega^{-1/2}$, and the matrix element of the emission of a phonon with a small wave vector is proportional to the wave vector, i.e., to the frequency ω, integration of (1) with respect to the phonon variable (1) leads to the appearance of a factor $(e\mathbf{E}\cdot\mathbf{a}_n + \varepsilon - \varepsilon')^3$.

The drift velocity $\mathbf{v} = \sum w_n \mathbf{a}_n$ of the charges is equal to

$$\mathbf{v} = \text{const} \sum_n \frac{\Delta \mathbf{a}_n}{\hbar} \int \frac{d^3k}{(2\pi/a)^3} \frac{d^3k'}{(2\pi/a)^3} \times \frac{(e\mathbf{E}\mathbf{a}_n + \varepsilon - \varepsilon')^3}{\Theta^3} n(\varepsilon) |\psi_\mathbf{k}^+(0)|^2 |\psi_{\mathbf{k}'}^-(0)|^2, \qquad (2)$$

where $n(\varepsilon)$ is the vacancion distribution function, Θ is the Debye temperature, and the summation is carried out over those of the neighboring sites of the crystalline lattice for which $e\mathbf{E}\cdot\mathbf{a}_n > 0$. Since $\varepsilon_0 \gg T$, the distribution function $n(\xi)$ is of the Boltzmann type.

In the following, we shall be interested only in the leading terms of the expansion $v(\mathbf{E})$ in terms of the electric field. It is conveneient to carry out the calculations separately for the case of wide bands $\Delta \gg T$ and narrow bands $\Delta \ll T$. Since the bandwidth of the vacancion decreases rapidly with increase in pressure, and the measurements of the drift velocity of the charges are made over a wide range of temperatures, both these cases can be achieved experimentally.

At $\Delta \gg T$, the principal role is played by particles with small $\mathbf{k}$, for which the spectrum $\varepsilon_1(\mathbf{k})$ is quadratic in $\mathbf{k}$ and the quantity $|\psi_\mathbf{k}^+(0)|^2$ does not depend on $\mathbf{k}$ (this corresponds to the well-known result for the inelastic-scattering cross section of slow particles $\sigma \propto 1/k$, used in Ref. 1). Finally, the drift velocity of the charges is determined to within terms of the order of $(T/eEa)^2$ and $(\Delta/eEa)^2$ by the expression

$$\mathbf{v} = \alpha e^{-\varepsilon_0/T} \left(\frac{T}{\Delta}\right)^{1/2} \frac{\Delta a}{\hbar} \left(\frac{ea}{\Theta}\right)^3 \sum_n \frac{\mathbf{a}_n}{a} \left\{ E^3 \xi_n^3 + 3E^2 \frac{3T - \Delta'}{2ea} \xi_n^2 \right\}, \qquad (3)$$

where α is a dimensionless constant, $\xi_n = \mathbf{E}\cdot\mathbf{a}_n / Ea$, and the quantity Δ' is of the order of Δ and is equal to

$$\Delta' = 2 \int_0^\Delta \varepsilon \varphi(\varepsilon) d\varepsilon \leqslant \Delta,$$

$$\varphi(\varepsilon) = \int |\psi_\mathbf{k}^-(0)|^2 dS_\varepsilon \Big/ \int_0^\Delta d\varepsilon \int |\psi_\mathbf{k}^-(0)|^2 dS_\varepsilon$$

(the integral with respect to dS_e is carried out over the constant-energy surface). The experimentally observed dependence of the drift velocity of the charges on the field is described empirically by the formula[5]

$$v \propto (E + E_0)^3 \approx E^3 + 3E^2 E_0.$$

The accuracy of the calculations (3) and of the experiment[5] permit a comparison of the leading terms of the expansion of the theoretical and experimental functions $\mathbf{v}(\mathbf{E})$ in powers of E. As follows from (3), the quantity

$$E_0 \approx \frac{3T - \Delta'}{2ea} \sum_n \xi_n^3 \Big/ \sum_n \xi_n^4$$

falls off with decrease in temperature and is determined by a quantity Δ' of the order of Δ as $T \to 0$. The tendency of E_0 to decrease with decrease in temperature was observed experimentally (Fig. 2); however, the measurements were not carried out at temperatures that were sufficiently low for the direct determination of Δ' (and consequently Δ) in the measurement of E_0.

In the opposite limiting case $eEa \gg T \gg \Delta$ the distribution function $n(\varepsilon)$ can be expanded in a series in the small parameter $\varepsilon_1(\mathbf{k})/T$. The expression (2), with accuracy to within terms of the order of $(\Delta/T)^2$, $(\Delta/eEa)^2$, and $(T/eEa)^2$, is of the form

$$\mathbf{v} = \alpha e^{-\varepsilon_0/T} \left(1 + \alpha_1 \frac{\Delta}{T}\right) \sum_n \frac{\Delta \mathbf{a}_n}{\hbar} \left(\frac{e\mathbf{E}\mathbf{a}_n}{\Theta}\right)^3 \left(1 + \alpha_2 \frac{\Delta}{T} \frac{\Delta}{e\mathbf{E}\mathbf{a}_n}\right), \qquad (4)$$

where α is a dimensionless constant and it is taken into account that $\varepsilon_1(\mathbf{k})=\varepsilon_1(-\mathbf{k})$,

$$\alpha_1=\frac{1}{\Delta}\int d^3k\,\varepsilon_1(\mathbf{k})\,|\psi_{\mathbf{k}}^+(0)|^2 \Big/ \int d^3k\,|\psi_{\mathbf{k}}^+(0)|^2,$$

$$\alpha_2=\frac{3}{\Delta^2}\Big\{\int d^3k\,\varepsilon_1^2(\mathbf{k})\,|\psi_{\mathbf{k}}^+(0)|^2 \Big/ \int d^3k\,|\psi_{\mathbf{k}}^+(0)|^2 - \Big[\int d^3k\,\varepsilon_1(\mathbf{k})\,|\psi_{\mathbf{k}}^+(0)|^2 \Big/ \int d^3k\,|\psi_{\mathbf{k}}^+(0)|^2\Big]^2\Big\}.$$

We also give the value of the drift velocity of the charges for weaker fields $T\gg eEa\gg\Delta$. In this case, the inelastic scattering of the vacancions is accompanied by the scattering of thermal phonons. Integration over $d\omega$ in the transition from formula (1) to (2) must be carried out with account of the phonon distribution function. As a result, the drift velocity of the charges turns out to be proportional to the field intensity:

$$\mathrm{v}=\alpha e^{-\varepsilon_0/T}\left(\frac{T}{\Theta}\right)^6\sum_n\frac{\Delta\mathbf{a}_n}{\hbar}\frac{(e\mathbf{E}\mathbf{a}_n)}{T}, \tag{5}$$

where the summation, just as in (3) and (4), is carried out over those of the neighboring sites for which $e\mathbf{E}\cdot\mathbf{a}_n>0$ and the constant

$$\alpha\sim\Big\{\int\frac{d^3k}{(2\pi/a)^3}|\psi_{\mathbf{k}}^+(0)|^2\Big\}^2.$$

It was understood above that the wave functions $\psi_{\mathbf{k}}^+$ and $\psi_{\mathbf{k}}^-$ of the vacancion and the elastic and inelastic cross sections do not depend on E. This statement is not valid for strong fields. The potential in which the vacancions move is determined by the strain tensor of the crystal. The presence of the electric field produces an additional deformation of the crystal in the vicinity of the charge, owing to the action of the force $e\mathbf{E}$ on the charge. As is shown by one of us, [7] up to fields $eEa\lesssim 10\Delta$, at not too low temperatures, allowance for such deformations leads to insignificant changes in the expressions for the drift velocity. The corresponding correction coefficients to formulas (3)–(5) are given in Ref. 7.

In fields stronger than critical: $eE_ca\sim 10\Delta$, [7]

$$\frac{1}{3\pi}\frac{eE_ca}{\Delta}\frac{\Omega}{a^3}\frac{1+\nu}{1-\nu}\approx 1.28 \tag{6}$$

(ν is the Poisson coefficient, Ω is the volume of the vacancy), the situation changes qualitatively. In the energy spectrum of the vacancions, below the states of the continuous spectrum, $\varepsilon=\varepsilon_0+\varepsilon_1(\mathbf{k})$, an infinite system of discrete energy levels appears, having a level condensation point $\varepsilon=\varepsilon_0$ and corresponding to the bound states of the vacancions on the charge. At low temperatures, the motion of the charge is determined by just these bound vacancions. Since $eEa\gg\Delta$, the tunneling of the charge to the location of the vacancion bound to it is accompanied by the emission of a phonon and a transition of the vacancion to another state of the discrete spectrum or to the continuous spectrum. The drift velocity of the charges in this case is also determined by the formula (2), in which it is necessary to include summation over the states of the discrete spectrum.

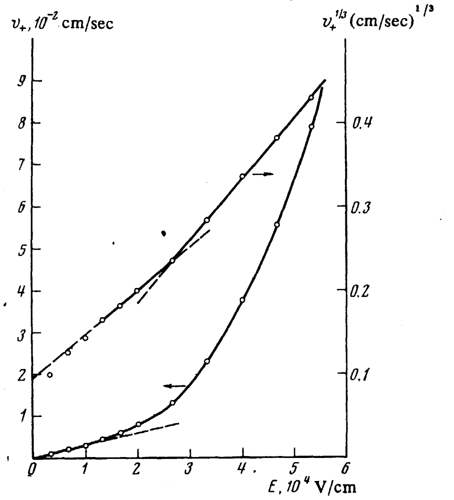

FIG. 3. The lower curve (left scale) gives the dependence of the drift velocity of the charges on the electric field intensity. The upper curve (right scale) gives, for the same data, the plot of $v^{1/3}(E)$. The break point corresponds to the electric field intensity $E_c\approx 2.7\times10^4$ V/cm; $p=40.5$ atm, $T=1.54$ k.

At a sufficiently low temperature, the principal contribution to (2) is made by the initial state, corresponding to the lowest level of the discrete spectrum $\varepsilon=\varepsilon_0-\Delta(\varkappa_0a)^2$, [7] Here the drift velocity of the charges is equal to

$$\mathrm{v}=\mathrm{const}\cdot\exp\left[\frac{\Delta(\varkappa_0a)^2-\varepsilon_0}{T}\right]\sum_n\frac{\Delta\mathbf{a}_n}{\hbar}\left(\frac{e\mathbf{E}\mathbf{a}_n}{\Theta}\right)^3 \times|\psi_{\varkappa_0}(0)|^2\Big\{\sum_i|\psi_i(0)|^2+\int\frac{d^3k}{(2\pi/a)^3}|\psi_{\mathbf{k}}^-(0)|^2\Big\}, \tag{7}$$

where the summation over i is carried out over the states of the discrete spectrum. Since the system of levels of the discrete spectrum arises in threshold fashion upon attainment of the critical field E_c (6), there should be a break corresponding to the transition from the regime (3), (4) to (7), on the straight line $v^{1/2}(E)$ at the point $E=E_c$. A similar effect was observed experimentally for the positive charges at a pressure of 40.5 atm and temperature $T=1.54$ K (Fig. 3). This problem calls for further investigations. Principal interest is attached here to the measurements of $v(T)$ in fields $E>E_c$, since at fields stronger than critical the activation energy of the transport process should change. The field $E_0\approx 2.7\times10^4$ V/cm (Fig. 3) corresponds to the value $\Delta\approx 1$ K.

The formulas (3)–(7), together with the results of Refs. 1, 3 and 7, determine the dependence of the drift velocity of the ions on the temperature and on the intensity of the electric field at any relation between the temperature, vacancion bandwidth, and the work needed to move the charge through the interatomic distance.

We express our gratitude to A. F. Andreev and A. I. Shal'nikov for constant attention and useful discussions in the course of the work.

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Translated by R. T. Beyer

Interaction of electron-hole drops in germanium with constant and alternating magnetic fields

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(Submitted December 8, 1976)
Zh. Eksp. Teor. Fiz. 72, 1960–1969 (May 1977)

The absorption of the energy of an alternating magnetic field as a result of excitation of eddy currents in electron-hole drops (EHD) in germanium is investigated. The mobility of the carriers in the EHD is determined from the dependence of the absorption on the intenstity of the constant magnetic field. It is shown that at impurity concentrations larger than 10^{14} cm^{-3} the mobility is determined by impurity scattering. Quantitative agreement is obtained between the absolute value of the absorption and the results of the calculation that takes into account the skin effect in EHD. It is observed that the ablation of large EHD in a constant magnetic field depends substantially on the mobility. It is shown that the ablation of EHD and the saturation of this effect in strong magnetic fields is a natural manifestation of the recombination magnetism of the EHD.

PACS numbers: 71.35.+z

1. INTRODUCTION

Even though electron-hole drops (EHD) in germanium have been investigated systematically since 1969, there were until recently no reliable experimental data whatever on the conductivity of EHD and on the mobility of the electrons and holes inside the drops. The reason is that the EHD are surrounded by a cloud of free carriers that determined both the dc and the alternating (microwave) conductivity of the crystal up to concentrations $\approx 10^{17}$ cm^{-3}, at which flow-through (percolation) conductivity via a chain of EHD appears. However, when such large carrier densities are produced, the crystal is bound to be overheated[1] and the results of measurements in this region can hardly describe the conductivity of the condensed phase. By way the example we can cite the results of[1,2], with which the results of earlier investigations[3,4] agree in the region of moderate excitation levels. It follows from[1,2] that at electron and hole densities in the samples 2×10^{17} cm^{-3}, which is the value of the density in EHD, the conductivity reaches only $2\times10^3\Omega^{-1}cm^{-1}$, while the mobility reaches 3×10^4 cm^2/V-sec. In[5] we have shown that the mobility of the carriers in the drops can be determined from the dependence of the absorption of the energy of the alternating magnetic fields, due to excitation of eddy currents in the EHD, on the intensity of the constant magnetic field. In the present paper we investigate in further detail the investigation of EHD with alternating and constant magnetic fields.

2. EXPERIMENTAL PROCEDURE

To investigate the losses in the frequency range 8–30 MHz, we used commercial apparatus of the Sh1-1 type to measure the induction of the constant magnetic field by means of nuclear magnetic resonance. The germanium samples were placed in the inductance coil, of the tank circuit of the apparatus, with quality vactor $Q_L = 50$, and were excited either by radiation pulses from a nitrogen laser at a wavelength 0.337 μm, duration 40 nsec, and energy 10^{-5} J, or else by radiation from an argon laser of power up to 200 mW, interrupted at a frequency of 320 Hz. Photoexcitation changed the Q of the tank circuit and produced a modulation ΔU of the RF voltage. The low-frequency component of the signal was detected, amplified, and fed to an oscilloscope. The modulation depth $\Delta U/U$ was determined from the ratio of the signal produced by the illumination to the signal U produced in the system when the lasing was stopped.

To produce large EHD, the germanium samples were subjected to inhomogeneous compression in liquid he-

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