Angular, spectral, and polarization properties of radiation emitted by high-energy electrons passing through a layer of matter

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The angular, spectral, and polarization characteristics of photons generated by a particle passing through a layer of matter are determined. It is shown that in the high-energy limit the contribution made to the radiation intensity by multiple scattering is proportional to the mean squared scattering angle. A kinetic equation is obtained for the electron with allowance for the energy lost in the radiation process.

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The influence of multiple scattering on the emission of protons by charged particles passing through the interface between matter and vacuum has been widely discussed in the literature (see, e.g., the review of Ter-Mikaelyan[1] and Pafomov[2] and the bibliography cited therein). However, principal attention has been paid so far to radiation in the optical and x-ray regions of the spectrum. The properties of transition radiation and bremsstrahlung of a particle moving through a layer of matter (emerging from the medium), in the presence of multiple scattering, were analyzed without allowance for the absorption of the photons in the medium. For this reason, the results cannot be used, for example, to study the generation of resonant photons (optical, x-ray, or Mössbauer). On the other hand, the singularities of the emission of high-energy γ photons (on the order of giga electron volts and higher) by ultrarelativistic electrons (positrons) were investigated only for the bremsstrahlung mechanism of radiation.[3–6] At the same time, it was shown by us[7] that at electron energies $E \geqslant 10^4$ eV the intensity of the transition radiation of high-energy γ photons ($E \sim 10^9$ eV) becomes comparable with and begins to exceed the intensity of the bremsstrahlung γ photons. In addition, for high-energy γ photons we have $\mathrm{Re}(\varepsilon-1) < \mathrm{Im}\,\varepsilon$ (ε is the dielectric constant of the medium). As a result, the theory of radiation in a medium can likewise not be applied to this case.

It should be noted that in connection with the discussion of the possible influence of the energy losses on the bremsstrahlung, an investigation of the angular and spectral distributions of the photons produced by an ultrarelativistic electron emitted from a medium into a vacuum was carried out by Varfolomeev and Zhevago.[8] Unfortunately, we cannot agree with their approach. As indicated by Varfolomeev and Zhevago,[8] their general formula (13) was obtained by Gol'dman's method[9]; in the absence of γ-photon absorption and without allowance for the electron energy losses, it agrees with the results of[9]; when absorption is taken into account, it agrees with the results of Ternovskiĭ.[10] However, the papers by Gol'dman and Ternovskiĭ[9,10] are devoted to the boundary radiation produced when the particle enters into the medium perpendicularly. As a result, the results of[8–10] cannot be used to describe the angular distribution of the radiation intensity of a particle emitted from a substance into a vacuum, inasmuch as the results of the multiple scattering in the medium the vacuum will contain, besides the normal trajectory, also a "fan" of trajectories. Furthermore, in the derivation of (13) by Varfolomeev and Zhevago,[8] following Gol'dman,[9] used the operation of the so-called "macroscopic renormalization of the charge." Therefore formula (13) likewise does not describe the spectral distribution of the total radiation intensity even in the case of normal entry, because, as specially emphasized by Gol'dman,[9] in this operation the contribution to the total intensity of the ordinary bremsstrahlung (not connected with the presence of the boundary) is discarded.

In addition, the earlier analysis[8,11,12] of the influence of the energy losses on the radiation process also needs to be reviewed, since a Fokker-Planck equation with a time dependent multiple-scattering constant can no longer be used to describe the process of bremsstrahlung losses. In fact, according to Varfolomeev, Zhevago, and Bazylev,[8,11,12] all that the losses introduce is a variation of the multiple-scattering angle, which enters as a parameter in the kinetic equations, per unit length q of the path traversed by the particle in the medium. Without allowance for the losses we have $q \sim 1/E^2$. In the presence of radiation losses, in the Bethe-Heitler region, the average electron is $\langle E \rangle = E_0 e^{-t/L}$ (L is the radiation length). This does not mean, however, as assumed in the cited papers,[8,11,12] that $q \equiv q(t) = q_0 e^{2t/L}$, for as a result of the large energy fluctuations, which occur when bremsstrahlung photons are emitted[13] in the Bethe-Heitler region, we have for the mean value $\langle E^{-2} \rangle \neq 1/\langle E \rangle^2$.

We obtain in this paper expressions that describe the angular, spectral, and polarization characteristics of the protons generated by a particle emitted from a substance into a vacuum (or else passing through a layer of matter), with allowance for the absorption of the photons in the medium and with allowance for the multiple scattering and energy losses of the particle in the course of the emission of the photon. The description of the multiple scattering and of the energy losses of the electron in the medium is carried out with the aid of a kinetic equation derived on the basis of the density-matrix formalism and free of the shortcomings indicated above.

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1. To obtain the indicated expressions it is necessary to find first the field produced in the vacuum by the particle. To this end it is necessary to solve Maxwell's equations, which for an arbitrary medium has in the Fourier representation with respect to time the following form[14]:

$$\left[-\mathrm{rot}\,\mathrm{rot}\,\mathbf{E}(\mathbf{r},\omega)+\frac{\omega^2}{c^2}\mathbf{E}(\mathbf{r},\omega)\right]_i+\frac{4\pi i\omega}{c^2}\hat{\sigma}_{ij}E_j=-\frac{4\pi i\omega}{c^2}J_i(\mathbf{r},\omega), \quad (1)$$

where $\hat{\sigma}_{ij}$ is the conductivity tensor of the medium and is in the general case, in the presence of boundaries, an integral operator with respect to the coordinate; $J_i(\mathbf{r}, \omega)$ is the Fourier transform of the i-th component of the current generated by the moving particle. In the quantum-mechanical case, $J_i(\mathbf{r}, \omega)$ must be taken to mean the current of the transition of the particles from one quantum state to the other.

The transverse solution of (1) outside the medium, which is of interest to us in the radiation processes, is easiest to obtain with the aid of the Green's function G of Eqs. (1), which satisfies an equality of the type

$$G=G_0+G_0 i\omega\hat{\sigma}G/c^2, \quad (2)$$

where G_0 is the transverse Green's function (1) at $\hat{\sigma}=0$. For its explicit form see, e.g., the book of Morse and Feshbach.[15] Using G, we have

$$E_i(\mathbf{r},\omega)=\int G_{il}(\mathbf{r},\mathbf{r}',\omega)\frac{i\omega}{c^2}J_l(\mathbf{r}',\omega)\,d^3r'. \quad (3)$$

As $r\to\infty$, the Green's function (2) is expressed in terms of the solution of the homogeneous Maxwell's equation $\mathbf{E}^{(-)}(\mathbf{r}, \omega)$, which contains at infinity a convergent spherical wave[6]:

$$\lim_{r\to\infty} G_{il}(\mathbf{r},\mathbf{r}',\omega)=\frac{e^{ikr}}{r}\sum_s e_i^s E_{\mathbf{k}l}^{(-)s*}(\mathbf{r}',\omega), \quad (4)$$

$$\left[-\mathrm{rot}\,\mathrm{rot}\,\mathbf{E}^{(-)}(\mathbf{r},\omega)+\frac{\omega^2}{c^2}\mathbf{E}^{(-)}(\mathbf{r},\omega)\right]_i-\frac{4\pi i\omega}{c^2}\hat{\sigma}_{ij}^* E_j^{(-)}=0, \quad (5)$$

where $\mathbf{e}^s$ is a unit transverse polarization vector.

If the wave is incident on an object with finite dimensions, then

$$\mathbf{E}_{\mathbf{k}s}^{(-)}(\mathbf{r},\omega)=\mathbf{e}^s e^{i\mathbf{k}\mathbf{r}}+\mathrm{const}\frac{e^{-ikr}}{r}, \quad (6)$$

where $k=|\mathbf{k}|=\omega/c$. With the aid of (3) and (4) we get

$$\mathbf{E}(\mathbf{r},\omega)=\frac{e^{ikr}}{r}\frac{i\omega}{c^2}\sum_s \mathbf{e}^s\int(\mathbf{E}_{\mathbf{k}}^{(-)s*}(\mathbf{r}',\omega)\mathbf{J}(\mathbf{r}',\omega))\,d^3r'. \quad (7)$$

Using (7) we easily obtain the intensity of the radiation in a unit solid angle and a unit interval of the frequencies of the photons having a polarization vector $\mathbf{e}^s$:

$$W_{s\mathbf{n}\omega}=\frac{cr^2}{4\pi^2}\langle|\mathbf{e}^{s*}\mathbf{E}(\mathbf{r},\omega)|^2\rangle=\frac{\omega^2}{4\pi^2c^3}\left\langle\left|\int(\mathbf{E}_{\mathbf{k}}^{(-)s*}(\mathbf{r}',\omega)\mathbf{J}(\mathbf{r}',\omega))\,d^3r'\right|^2\right\rangle, \quad (8)$$

where the vector $\mathbf{n}=\mathbf{k}/k$ is directed towards the observation point, and $\langle\ldots\rangle$ denotes averaging over the state of motion of the particle in the medium. Thus, to obtain $W_{s\mathbf{n}\omega}$ it suffices to find only the solution of the homogeneous Maxwell equation (5), which describes the refraction and the specular reflection of the waves by an arbitrary target.

We investigate next the emission of photons with energy much lower than the energy of the moving particles. In this case, the classical description is applicable. We take into account the fact that for a pointlike particle the current is

$$\mathbf{J}(\mathbf{r}',\omega)=\int e\mathbf{v}(t)\delta[\mathbf{r}'-\mathbf{r}(t)]e^{i\omega t}\,dt. \quad (9)$$

Substituting (9) in (8), we have

$$W_{s\mathbf{n}\omega}=\frac{e^2\omega^2}{4\pi^2c^3}\left\langle\int_{t_1}^{t_2}\int(\mathbf{E}_{\mathbf{k}}^{(-)s*}(\mathbf{r}(t))\mathbf{v}(t))e^{i\omega t}(\mathbf{E}_{\mathbf{k}}^{(-)s}(\mathbf{r}(t'))\mathbf{v}(t'))e^{-i\omega t'}\,dt\,dt'\right\rangle, \quad (10)$$

where t_1 and t_2 are respectively the starting and stopping instants of the charge motion.

To proceed further, it is necessary to average in (10). Usually this averaging is carried out with the density of the joint probability $w(\mathbf{r}, \mathbf{v}, t; \mathbf{r}', \mathbf{v}', t')$ of finding that the particle has coordinates and velocities $\mathbf{r}$ and velocity $\mathbf{v}$ at the instant t, and $\mathbf{r}'$ and $\mathbf{v}'$ at the instant t'. In the investigation of the influence of the energy losses, however, it is more convenient use for the averaging with a similar function, but dependent on the variables $\mathbf{r}$ and $\mathbf{p}$, where $\mathbf{p}$ is the particle momentum. As a result we have $(c=1)$

$$W_{s\mathbf{n}\omega}=\frac{e^2\omega^2}{4\pi^2}\iiint_{t_1}^{t_2}\left(\mathbf{E}_{\mathbf{k}}^{(-)s*}(\mathbf{r})\frac{\mathbf{p}}{E(p)}\right)\left(\mathbf{E}_{\mathbf{k}}^{(-)s}(\mathbf{r}')\frac{\mathbf{p}'}{E(p')}\right) \times w(\mathbf{r},\mathbf{p},t;\mathbf{r}',\mathbf{p}',t')\exp[i\omega(t-t')]\,d^3r\,d^3r'\,d^3p\,d^3p'\,dt\,dt'. \quad (11)$$

We choose the coordinate system such that the xy plane coincides with the interface of the medium and the vacuum. We direct the z axis from the medium into the vacuum. We assume further that a particle with momentum $\mathbf{p}_0$ begins to move along the z axis at the instant of time $(-T)$ from the point $(0, 0, -z_0)$ located inside the medium. Let then this particle cross the medium—vacuum interface at the instant of time $t=0$.

In the case of high γ-photon energies of interest to us, we can neglect in the expressions for the fields $\mathbf{E}_{\mathbf{k}}^{(-)s}$ the specularly reflected waves. As a result we have[6]

$$\mathbf{E}_{\mathbf{k}}^{(-)s}=\begin{cases}\mathbf{e}^s e^{i\mathbf{k}\mathbf{r}} & \text{when } z>0\\ \mathbf{e}^s e^{i\mathbf{k}'\mathbf{r}} & \text{when } z<0\end{cases}, \quad (12)$$

where $\mathbf{k}'$ is the wave vector of the photon in the medium, and has components

$$\mathbf{k}_\perp'=\omega\mathbf{n}_\perp,\quad k_z'=\omega\varepsilon^{1/2}n_z.$$

Using (12) and changing over as usual[2,9,16] from the variables p, θ, and φ to the variables E and $\boldsymbol{\theta}$ (E is the energy and $\boldsymbol{\theta}=\theta_x\mathbf{i}+\theta_y\mathbf{j}$ is the transverse angle vector), we obtain the following expression for the intensity distribution $W_{\parallel\mathbf{n}\omega}$ of the photons polarized in the plane of emergence from the medium:

$$W_{\parallel n\omega}=\frac{e^2\omega^2\vartheta^2}{4\pi^2}\Big\{\int_0^\infty dt\int_0^\infty dt'\int d\xi\, d\xi' F(\theta)F(\theta')$$

$$\times\exp[-i\mathbf{k}(\mathbf{r}-\mathbf{r}')+i\omega(t-t')]w_1(\mathbf{r},\theta,E,t+T)w_2(\mathbf{r},\theta,E,t|\mathbf{r}',\theta',E',t')$$

$$+2\mathrm{Re}\int_{-T}^{0}dt\int_0^\infty dt'\int d\xi\, d\xi' F(\theta)F(\theta')\exp[-i(\mathbf{k}'\mathbf{r}-\omega t)$$

$$+i(\mathbf{k}\mathbf{r}'-\omega t')]w_1(\mathbf{r},\theta,E,t+T)w_2(\mathbf{r},\theta,E,t|\mathbf{r}',\theta',E'\, t')$$

$$+2\mathrm{Re}\int_{-T}^{0}dt\int_0^{-t}d\tau\int d\xi\, d\xi' F(\theta)F(\theta')\exp[-i\omega\tau+i\mathbf{k}'^*(\mathbf{r}'-\mathbf{r})+\omega z\,\mathrm{Im}\,\varepsilon]$$

$$\times w_1(\mathbf{r},\theta,E,t+T)w_2(\mathbf{r},\theta,E,0|\mathbf{r}',\theta',E',\tau)\Big\}, \qquad (13)$$

where ξ denotes the aggregate of coordinates $\mathbf{r},\theta,E$; $w_1(\mathbf{r},\theta,E,t)$ is the probability of observing $\mathbf{r}$, θ, and E at the instant t; $w_2(\mathbf{r},\theta,E,t\,|\,\mathbf{r}',\theta',E',t')$ is the conditional probability of observing $\mathbf{r}',\theta',E'$ at the instant t' if the variables were $\mathbf{r}$, θ, and E at the instant t:

$$F(\theta)=\beta[1-(\theta_x\cos\vartheta_x+\theta_y\cos\vartheta_y)\vartheta^{-2}];\ \beta=v/c\ (c=1);$$

$\vartheta_x,\vartheta_y,\vartheta_z\equiv\vartheta$ are the direction angles of the vector $\mathbf{k}(\mathbf{n})$.

An expression for the spectral-angular distribution of photons having a polarization vector perpendicular to the emission plane is obtained from (13) by replacing the quantity ϑ^2 in front of the curly brackets by ϑ^{-2} and replacing $F(\theta)$ by

$$B(\theta)=\beta(\theta_x\cos\vartheta_y-\theta_y\cos\vartheta_x).$$

We call attention to the fact that some of the integrals of (13) contain probability densities that depend on the instants of time corresponding to the particle motion both in the medium and outside the medium. It is convenient, however, to deal with probabilities that depend on the instants of time pertaining only to motion of the particle in the medium or outside the medium. To attain this we use the following general properties of distribution functions[17]:

$$w_2(\xi,t|\xi',t')=\int w_2(\xi,t|\xi'',t'')w_2(\xi'',t''|\xi',t')d\xi''. \qquad (14)$$

Substituting (14) in (13) and choosing an instant of time t'' corresponding to the instant of the emission of the particle from the medium, i.e., $t''=0$, we obtain for $W_{sn\omega}$ an expression that depends only on the distribution functions describing the motion of the particle either in the medium or outside the medium.

2. The probabilities w_1 and w_2 satisfy a kinetic equation of general form[18]

$$\frac{\partial w}{\partial t}+\frac{\mathbf{p}}{E}\frac{\partial w}{\partial \mathbf{r}}=\left(\frac{\partial w}{\partial t}\right)_{\mathrm{coll}}. \qquad (15)$$

The time variation of the collision term $(\partial w/\partial t)_{\mathrm{coll}}$ is in our case due to the processes of scattering and emission, and can be described by an equation with the following structure:

$$\left(\frac{\partial w^{(n)}}{\partial t}\right)_{\mathrm{coll}}=-\sum_{n'}g_{n'n}w^{(n)}+\sum_{n'}g_{nn'}w^{(n')}, \qquad (16)$$

where $g_{nn'}$ is the probability, per unit time, that a system in a state n (in our case, an electron in a state with momentum $\mathbf{p}$) will go over into a state n (into a state with momentum $\mathbf{p}'$ as a result of scattering or into a state with momentum $\mathbf{p}'$ plus secondary particles as a result of the radiation process).

The probabilities $g_{nn'}$ can be obtained in accordance with the usual rules.[19] As a result, if we take into account the change of $w^{(n)}$ only as a result of multiple scattering and bremsstrahlung, we have

$$\left(\frac{\partial w(\mathbf{p},t)}{\partial t}\right)_{\mathrm{coll}}=-N\sigma_{\mathrm{tot}}w(\mathbf{p},t)+N\int\frac{d^3p'}{(2\pi)^2}\delta(E_{\mathbf{p}}-E_{\mathbf{p}'})$$

$$\times\frac{|M_{\mathbf{p}'\mathbf{p}}|^2}{4E_p^2}w(\mathbf{p}',t)+N\int\frac{d^3p'd^3k}{(2\pi)^5}\delta(E_{\mathbf{p}}-E_{\mathbf{p}'}-k)\frac{|M_{\mathbf{p}'\mathbf{k},\mathbf{p}}|^2}{8E_pE_{p'}k}w(\mathbf{p}',t), \qquad (17)$$

where N is the number of scatterers (nuclei) per unit volume, the amplitude $M_{\mathbf{p}'\mathbf{p}}$ describes the scattering of an electron in the Coulomb field of the nucleus, the amplitude $M_{\mathbf{p}'\mathbf{k},\mathbf{p}}$ describes the emission of γ photons by the electron in the field of the nucleus, and σ_{tot} is the total cross section of these processes.

We now investigate in greater detail the case when the process of the emission of the γ quanta can be described by the Bethe-Heitler expression in the case of complete screening of the field of the nucleus.[13,19] 1) Changing from the probabilities that describe the electron momentum distribution to probabilities that describe the energy and scattering-angle distributions of the particles, we can obtain from (17) after suitable transformations the following equation:

$$\left(\frac{\partial w(\theta,E,t)}{\partial t}\right)_{\mathrm{coll}}=q(E)\,\Delta_\theta w(\theta,E,t)+K(E)\,w(\theta,E,t),$$
$$\Delta_\theta=\frac{\partial^2}{\partial\theta_x^2}+\frac{\partial}{\partial\theta_y^2},\quad q=\frac{\delta}{E^2};\quad \delta=\frac{E_s^2}{4}; \qquad (18)$$

$E_s=21$ MeV, and $K(E)$ is an integral operator given, if t is measured in radiation units, by

$$K(E)w(\theta,E,t)=\int_E^\infty\frac{u^2+E^2-{}^2/{}_3uE}{u^2(u-E)}w(\theta,u,t)\,du$$
$$-\int_0^E\frac{u^2+E^2-{}^2/{}_3uE}{E^2(E-u)}du\,w(\theta,E,t). \qquad (19)$$

It is interesting to note that this equation can be obtained from the well known equations of the cascade theory of showers,[16,21] if we discard in the latter the terms connected with pair production.

The initial conditions for the distribution functions w_1 and w_2 are

$$w_1(t=-T)=\delta(\mathbf{r}-\mathbf{r}_0)\delta(\theta)\delta(E-E_0),$$
$$w_2(t=t')=\delta(\mathbf{r}-\mathbf{r}')\delta(\theta-\theta')\delta(E-E');$$

E_0 is the initial energy of the particles; $\mathbf{p}/E=\mathbf{v}$; the vector v has components $v(\theta_x,\theta_y,1-\frac{1}{2}(\theta_x^2+\theta_y^2))$.

With the aid of the kinetic equation (15) we can obtain the time dependence of the mean squared multiple-scattering angle, $\langle\theta^2(E,t)\rangle$, of an electron having an energy E in the interval dE. To this end we multiply (15) by

$\theta^2 = (\theta_x^2 + \theta_y^2)$ and integrate with respect to **r** and $\boldsymbol{\theta}$. As a result we have

$$\frac{\partial\langle\theta^2(E,t)\rangle}{\partial t} = 4q(E)w(E,t) + K(E)\langle\theta^2(E,t)\rangle, \tag{20}$$

where

$$w(E,t) = \int w(\mathbf{r},\theta,E,t)\,d^3r\,d^2\theta$$

is the probability of observing an electron with energy E at the instant of time t if it had an energy E_0 at the instant $t = 0$.

Solving (20) with the aid of the Mellin transformation, we obtain

$$\langle\theta^2(E,t)\rangle = 4\delta\int_0^t dt'\int_E^{E_0}\frac{dE'}{E'}w(E_0|E',t')w(E'|E,t'-t). \tag{21}$$

For the actual calculation of (21) we use an approximate expression obtained by Bethe and Heitler[13]:

$$w(E_0|E,t) = \left(\ln\frac{E_0}{E}\right)^{t/\ln 2-1} \Big/ E_0\Gamma\left(\frac{t}{\ln 2}\right), \tag{22}$$

where t is measured in radiation units; $\Gamma(t/\ln 2)$ is the gamma function.

Substituting (22) in (21), we obtain

$$\langle\theta^2(E,t)\rangle = \frac{4q(E)}{E_0\Gamma(t/\ln 2)}\left(\ln\frac{E_0}{E}\right)^{t/\ln 2-1}\int_0^t \Phi\left(\frac{\tau}{\ln 2},\frac{t}{\ln 2}, 2\ln\frac{E_0}{E}\right)d\tau, \tag{23}$$

where Φ is a confluent hypergeometric function. Expression (23) differs substantially from the simple exponential dependence obtained by substituting the equality $\langle E\rangle = E_0 e^{-t/L}$ in the expression $\langle\theta^2(E,t)\rangle = 4qt$.

We assume now that the electron passes through a plate of thickness z_0. In these cases the formulas that describe the angular and spectral distributions of photons polarized perpendicular to the emission plane remain unchanged, but it is necessary to add to the expression for $W_{\parallel\mathbf{n}\omega}$ a term in the form

$$\begin{aligned} W'_{\parallel\mathbf{n}\omega} = \frac{e^2\omega^2\vartheta^2}{4\pi^2}\Big\{ & \frac{\beta_0^2\exp(-z_0\omega\,\mathrm{Im}\,\varepsilon)}{\omega^2(1-\beta_0+\vartheta^2/2)^2} + 2\,\mathrm{Im}\frac{\beta_0\exp(iz_0k_z'-i\omega T)}{\omega(1-\beta_0+\vartheta^2/2)} \\ & \times\int_{-T}^{0}dt\int d\xi\,F(\theta)\exp[-i(\omega t-\mathbf{k}'\cdot\mathbf{r})]w_1(\mathbf{r},\theta,E,t+T) \\ & + 2\,\mathrm{Im}\frac{\beta_0\exp(iz_0k_z'-i\omega T)}{\omega(1-\beta_0+\vartheta^2/2)}\int_0^\infty dt\int d\xi\,F(\theta) \\ & \times\exp[-i(\omega t-\mathbf{k}\mathbf{r})]w_2(-z_0,\theta_0=0,E_0,-T|\mathbf{r},\theta,E,t)\Big\}, \end{aligned} \tag{24}$$

where

$$\beta_0 = [1-(m/E_0)]^{1/2}.$$

Equations (13) and (24) assume a particularly simple form for a thin plate. In this case the right-hand side of the kinetic equation (18) can be regarded as a perturbation. As a result, in the limit of small T we obtain the expressions:

$$\begin{aligned} W_{\parallel\mathbf{n}\omega} = {} & \frac{e^2\vartheta^2}{16\pi^2}\frac{|\varepsilon-1|^2(\omega T)^2}{(1-\beta_0+\vartheta^2/2)^2} + \frac{e^2qT}{2\pi^2}\frac{(1-\beta_0-\vartheta^2/2)^2}{(1-\beta_0+\vartheta^2/2)^4} \\ & + \frac{e^2\vartheta^2}{4\pi^2}\frac{T}{(1-\beta_0-\vartheta^2/2)^2}\int_{E_1}^{E_0}\frac{(\beta_0-\beta)^2}{(1-\beta+\vartheta^2/2)^2}\sigma(E_0|E)\,dE, \end{aligned} \tag{25}$$

$$W_{\perp\mathbf{n}\omega} = \frac{e^2qT}{2\pi^2(1-\beta_0+\vartheta^2/2)^2}, \tag{26}$$

where $\sigma(E_0|E) = N\sigma_\gamma(E_0|E)$, $\sigma_\gamma(E_0|E)$ is the bremsstrahlung cross sections per unit energy interval E of an electron having an initial energy E_0, and E_1 is the end-point energy (see[20,21] concerning its choice).

According to (25) and (26), allowance for the radiative losses leads to the appearance of one more term besides the usual transition and bremsstrahlung terms (the first and second terms of (25)) even if a particle passes through a thin plate.

We assume now that the energy losses can be disregarded. This case can in principle be realized when x-ray and resonant γ photons are generated by the electron, when the absorption depth of the γ photons is much less than the radiation length, and also at very high electron energies, when the Landau-Pomeranchuk effect causes the energy losses to decrease so that their influence on the radiation process can be neglected also for plates of thickness on the order of a radiation length.[6]

When the radiative energy losses are neglected, we can carry out the succeeding transformation of formulas (13) and (24) by following the procedure described by Pafomov.[2] As a result, integrating over the electron scattering angles, we obtain for the radiation intensity $W_{s\mathbf{n}\omega}$ very cumbersome expressions containing double integrals with respect to time (their explicit form will be published separately). If the γ-ray absorption is disregarded, then a direct comparison shows that the indicated expressions go over into the expressions obtained by Pafomov in an analysis of the radiation process in a plate (see[2], Sec. 26, formulas (26.13)–(26.22)). We note, however, that the second and third terms of formula (26.15) and the fifth and sixth terms of (26.16) contain misprints. The authors thank V. E. Pafomov for a discussion of the results of the comparison.

We note that simplification of the expressions for $W_{s\mathbf{n}\omega}$ in the absence of losses occurs when the conditions $q \ll \omega(\mathrm{Im}\,\varepsilon)$ and $qT \ll 1-\beta^2+\vartheta^2$, are satisfied. It turns out that as $q \to 0$ the radiation intensity contains a term that does not depend on q (and coincides with the expression for the transition radiation of a particle that moves uniformly perpendicular to the interface), as well as terms proportional to the first power of q. The resultant expressions are still too cumbersome to be presented here.

In the general case, the formulas for $W_{s\mathbf{n}\omega}$, even in the absence of losses, can be analyzed only by numerical methods. We present below the results of such an analysis in the case of generation of resonant photons by an electron moving through a plate containing W^{183} nuclei ($\omega = 46.5$ keV). This process is of great interest in

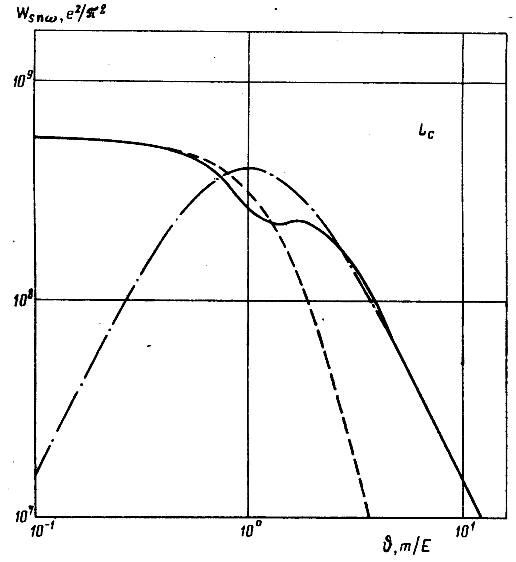

FIG. 1.

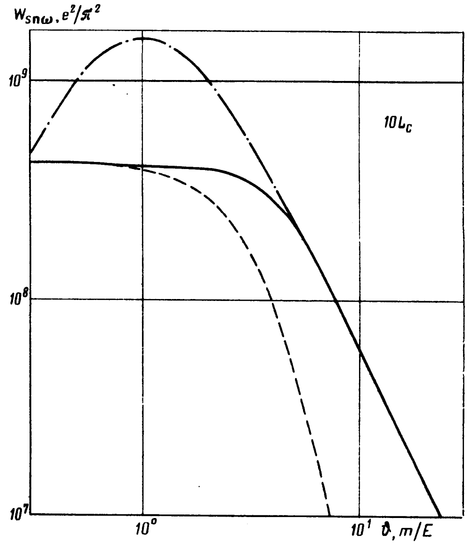

FIG. 2.

connection with the possibility, discussed in the literature, of producing sources of resonant radiation with the aid of beams of relativistic electrons.[22,7]

Figures 1 and 2 show the angular distributions of $W_{sn\omega}$ for γ photons produced by an electron of 40 GeV energy, and for plates with thicknesses L_c and $10L_c$, respectively (the value of the γ-photon absorption depth L_c, of $\mathrm{Re}(\varepsilon-1)$, and of $\mathrm{Im}\,\varepsilon$ were taken from[22]). The solid lines in the figures correspond to the radiation energy density $W_{\parallel n\omega}$; the dash-dot lines describe the transition radiation produced by the electron passing through the plate with constant velocity directed normal to its surface; the dashed lines describe the angular distribution of $W_{\perp n\omega}$.

We see that scattering of the particle in the medium greatly influences $W_{\parallel n\omega}$, so that at $\vartheta \lesssim m/E$ the radiation intensity of waves with polarization parallel to the γ-photon emission plane is not equal to the transition-radiation intensity. We call attention also to the fact that at angles $\vartheta \lesssim m/E$ the main contribution to $W_{\parallel n\omega}$ is made by a term equal to $W_{\perp n\omega}$ and connected with the scattering of the electrons in the medium. We note that a numerical analysis of the formulas shows that in the considered electron energy region ($E \geqslant 1$ GeV), up to γ-photon emission angles $\vartheta \sim (\mathrm{Im}\,\varepsilon)^{1/2}$, the predominant contribution to $W_{\parallel n\omega}$ is made by the term due to the appearance of a fan of electron-motion trajectories behind the plate as a result of multiple scattering in the medium. This contribution increases with increasing electron energy, at a fixed γ-photon frequency. It is seen from the plot that for angles $\vartheta > m/E$ the radiation intensity $W_{\parallel n\omega}$ coincides with the intensity of the transition radiation in the case of normal passage through the plate. It follows from them, in addition, that increasing the plate thickness leads to a broadening of the angular distribution of the γ-photon radiation energy density.

We have also calculated the irradiation intensity $W_{sn\omega}$ for electrons of energy 1 GeV and for a plate of thickness L_c. The obtained angular distributions are similar to the distributions shown in Fig. 1 for 40-GeV electrons. The formulas for nonresonant γ photons of energy 40 and 200 MeV and for electrons of energy 40 and 200 GeV, respectively, were also integrated numerically. The calculations were performed for tungsten plates $0.05L$ and $0.1L$ thick. In this case, too, a substantial contribution is made by the vacuum fan of the trajectories to the radiation intensity $W_{\perp n\omega}$, which is proportional to q.

Thus, allowance for the fan of vacuum trajectories changes significantly the picture of the angular spectral distributions of the intensity of the radiation from a particle passing through a medium–vacuum interface.

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1)The influence of multiple scattering (the Landau-Pomeranchuk effect) and the effect of the polarization of the medium on the bremsstrahlung cross section can be taken into account by the method described by Ter-Mikaelyan[1] (see also [20]).

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Charge exchange in plasmas

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Charge exchange induced by collisions with electrons is considered. It is shown that in a plasma under certain conditions, three-body collisions resulting in charge exchange become significant at electron densities n_e of $\sim 10^{12}$ cm^{-3}. As an example, it is shown that for an Rb-Cs plasma with atomic velocities v_n of $\sim 10^6$ cm/sec, electron velocities v_e of $\sim 7\cdot 10^7$ cm/sec, and a resonance defect $\varkappa_0$ of ~ 0.28 eV, the electron-induced charge exchange cross section is $3\cdot 10^{-32} n_e$ cm^2; this is a thousand times greater than the charge exchange cross section calculated with allowance for Coulomb detuning (R. Z. Vitlina and A. V. Chaplik, Zh. Eksp. Teor. Fiz. 70, 543 (1976)/Sov. Phys.-JETP 43, 280 (1976)/). It is also shown that in a cesium plasma the decrease in the resonance charge exchange cross section due to Coulomb detuning is compensated by charge exchange induced by electron collisions.

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Charge exchange is usually treated as a binary (two-body) process since under laboratory conditions the particle density n and the interaction range r are usually such as to satisfy the condition $nr^3 \ll 1$ for binary interactions.[1] The effect of the potential field of the medium on charge exchange under conditions of binary interactions between the particles of the medium has been discussed in recent papers[2,3] by Vitlina, Dychne, and Chaplik.

Actually, in treating charge exchange in a plasma, i.e., in a medium containing free electrons, one must make allowance for ternary collisions—specifically, for collisions of the charge-exchanging pair with a plasma electron. As will be shown below, under certain conditions such collisions may considerably increase the nonresonance charge-exchange cross section and compensate for the decrease in the resonance charge-exchange cross section due to Coulomb detuning.[3]

1. The probability for nonresonance charge exchange is known to be exponentially small at low relative velocities of the nuclei.[4] However, a collision with a plasma electron taking place while the nuclei are close together can considerably alter this probability. A transition from the initial term $U_i(R)$ of the quasimolecule to the term $U_f(R)$ as a result of electron-impact excitation or deexcitation will lead to a final result, after separation of the nuclei, that must be recognized as charge exchange, provided the final state corresponds to transfer of an electron from the atom to the ion. Such a process we shall call electron-induced charge exchange.

Let $\sigma_{if}(R)$ be the cross section for transition from term i to term f at a fixed distance R between the nuclei. The probability per unit time for electron-induced charge exchange is $n_e v_e \sigma_{if}(R)$, where n_e and v_e are the density and velocity of the plasma electrons. Integrating along the path of the nuclei, we obtain the following expression for the charge-exchange probability for a given impact parameter (we are using atomic units):

$$P(\rho)=\int_{R_{min}}^{\infty}\frac{2n_e v_e \sigma_{if}(R)\,dR}{v_n[1-2U_i(R)/Mv_n^2-\rho^2/R^2]^{1/2}}. \tag{1}$$

Here R_{min} is the distance of closest approach of the nuclei, v_n their initial velocity, and M their reduced mass, while $U_i(R)$ is the potential energy for the interaction of

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