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Charge exchange in plasmas

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Charge exchange induced by collisions with electrons is considered. It is shown that in a plasma under certain conditions, three-body collisions resulting in charge exchange become significant at electron densities n_e of $\sim 10^{12}$ cm^{-3}. As an example, it is shown that for an Rb-Cs plasma with atomic velocities v_n of $\sim 10^6$ cm/sec, electron velocities v_e of $\sim 7\cdot 10^7$ cm/sec, and a resonance defect $\varkappa_0$ of $\sim$0.28 eV, the electron-induced charge exchange cross section is $3\cdot 10^{-32} n_e$ cm^2; this is a thousand times greater than the charge exchange cross section calculated with allowance for Coulomb detuning (R. Z. Vitlina and A. V. Chaplik, Zh. Eksp. Teor. Fiz. 70, 543 (1976)/Sov. Phys.-JETP 43, 280 (1976)/). It is also shown that in a cesium plasma the decrease in the resonance charge exchange cross section due to Coulomb detuning is compensated by charge exchange induced by electron collisions.

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Charge exchange is usually treated as a binary (two-body) process since under laboratory conditions the particle density n and the interaction range r are usually such as to satisfy the condition $nr^3 \ll 1$ for binary interactions.[1)] The effect of the potential field of the medium on charge exchange under conditions of binary interactions between the particles of the medium has been discussed in recent papers[2,3] by Vitlina, Dychne, and Chaplik.

Actually, in treating charge exchange in a plasma, i.e., in a medium containing free electrons, one must make allowance for ternary collisions—specifically, for collisions of the charge-exchanging pair with a plasma electron. As will be shown below, under certain conditions such collisions may considerably increase the nonresonance charge-exchange cross section and compensate for the decrease in the resonance charge-exchange cross section due to Coulomb detuning.[3]

1. The probability for nonresonance charge exchange is known to be exponentially small at low relative velocities of the nuclei.[4] However, a collision with a plasma electron taking place while the nuclei are close together can considerably alter this probability. A transition from the initial term $U_i(R)$ of the quasimolecule to the term $U_f(R)$ as a result of electron-impact excitation or deexcitation will lead to a final result, after separation of the nuclei, that must be recognized as charge exchange, provided the final state corresponds to transfer of an electron from the atom to the ion. Such a process we shall call electron-induced charge exchange.

Let $\sigma_{if}(R)$ be the cross section for transition from term i to term f at a fixed distance R between the nuclei. The probability per unit time for electron-induced charge exchange is $n_e v_e \sigma_{if}(R)$, where n_e and v_e are the density and velocity of the plasma electrons. Integrating along the path of the nuclei, we obtain the following expression for the charge-exchange probability for a given impact parameter (we are using atomic units):

$$P(\rho)=\int_{R_{min}}^{\infty}\frac{2n_e v_e \sigma_{if}(R)\,dR}{v_n[1-2U_i(R)/Mv_n^2-\rho^2/R^2]^{1/2}}. \qquad (1)$$

Here R_{min} is the distance of closest approach of the nuclei, v_n their initial velocity, and M their reduced mass, while $U_i(R)$ is the potential energy for the interaction of

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the atom and the ion in the initial state. It is assumed that the probability for both ordinary and electron-induced charge exchange is much smaller than unity.

A molecular wave function forms in the region in which the exchange potential $\Delta(R)$ becomes of the order of the initial resonance defect $\varkappa_0$. The internuclear distance R_1 at which the change from atomic to adiabatic wave functions takes place is accordingly determined by the condition $\Delta(R_1) = \varkappa_0$. The most interesting case is that in which the resonance defects are not large (fractions of an electron volt), since in that case R_1 is large and the ternary-collision process makes its most appreciable contribution to the charge exchange cross section.

When the internuclear distance R is larger than R_1 the wave functions are centered on different nuclei and the excitation probability is low; hence we may assume the excitation cross section to vanish when $R > R_1$. When $R < R_1$ the adiabatic wave functions ψ_i and ψ_f of the initial and final states correspond to symmetric and antisymmetric combinations of atomic wave functions: $\psi_g = (\varphi_1 + \varphi_2)/\sqrt{2}$ and $\psi_u = (\varphi_1 - \varphi_2)/\sqrt{2}$. The Coulomb field of the molecular ion distorts the incident-electron wave function, so it is desirable to calculate σ_{if} in the distorted wave approximation:

$$\sigma_{if} = \frac{v_2}{4\pi^2 v_1} \int \left| \langle F_{v_1}(r_2)\psi_i(r_1) | \frac{1}{r_{12}} | F_{v_2}(r_2)\psi_f(r_1)\rangle \right|^2 d\Omega,$$

where v_1 and v_2 are the velocities of the electron before and after scattering, $d\Omega$ is an element of solid angle in the direction of the scattered electron, and $F_{v_{1,2}}$ are the wave functions of the incident and scattered electron in the Coulomb field of the molecular ion. We write the interaction between the electrons in the form $1/r_{12} = 1/r_2 + \mathbf{r}_1 \cdot \mathbf{r}_2 / r_2^3$ for the case $r_2 \gg r_1$. After averaging over the direction of the line joining the nuclei, we obtain

$$\sigma_{if} = \frac{v_2 D_{12}^2}{12\pi^2 v_1} \int \left| \left\langle F_{v_2}(r_2) \left| \frac{\mathbf{r}_2}{r_2^3} \right| F_{v_1}(r_2) \right\rangle \right|^2 d\Omega. \quad (2)$$

Making use of the relation $D_{12} = |\langle \psi_u(r_1) | \mathbf{r} | \psi_g(r_1)\rangle| = R/2$, we rewrite Eq. (2) in the form

$$\sigma_{if} = \frac{2\pi^2 R^2}{3^{3/2} v_1^2} g(v_1, v_2). \quad (3)$$

Below we shall use the empirical value $g(v_1, v_2) \approx 0.2$ proposed by Seaton[5] for the Gaunt factor, which gives a satisfactory account of the behavior of optically allowed transition cross sections for ions near the threshold. Assuming straight-line trajectories to perform the integration in Eq. (1), we obtain the following result for the probability ($v_1 \equiv v_e$):

$$P(\rho) = \frac{0.8\pi^2 n_e}{3^{3/2} v_n v_e} \left[\frac{1}{3} (R_1^2 - \rho^2)^{3/2} + \rho^2 (R_1^2 - \rho^2)^{1/2} \right] \quad (4)$$

for $\rho < R_1$, and $P(\rho) = 0$ for $\rho > R_1$. For the electron induced charge exchange cross section, we have

$$Q_{ei} = 1.6\pi^3 n_e R_1^5 / 15\sqrt{3} v_e v_n. \quad (5)$$

The energy difference between the quasimolecular levels falls off exponentially with increasing R in the region $R < R_1$: $|U_f(R) - U_i(R)| \sim \gamma^2 e^{-\gamma R}$; hence in the case in which the initial state is the more tightly bound and the electron energy $v_e^2/2$ is fixed there will be an energy threshold for the excitation reaction at the point $R_2 \approx \gamma^{-1} \ln(2\gamma^2/v_e^2)$ and we shall have $\sigma_{if}(R) = 0$ for $R < R_2$. The presence of this reaction threshold complicates formula (5) somewhat, but the result is not substantially changed even when $v_e^2/2 \gtrsim 5\varkappa_0$. We also note that for large values of R_1 of the order of 10–15 atomic units and electron energies of $\sim (1-10)\varkappa_0$, it is hardly reasonable, within the limitations of the present problem, to calculate the excitation or deexcitation cross section of the molecular ion in the region $R_2 < R < R_1$ more accurately than has already been done.

As an example let us estimate the charge exchange cross section in an Rb-Cs plasma for the following parameter values (given in atomic units): $v_n \sim 5 \cdot 10^{-3}$, $v_e \sim 0.3$, $\varkappa_0 = 0.01$, and $R_1 = 15.3$. In this case the cross section calculated with formula (5) is $Q_{ei} = 1.6 \cdot 10^{-5} n_e$ (Q_{ei} is in a.u., and n_e, in cm^{-3}); this is about a thousand times larger than the nonresonance charge exchange cross section in the plasma as calculated with allowance for the effect of the surrounding medium by the method developed in Ref. 3. It should be noted that the cross section for electron induced charge exchange in an Rb-Cs plasma with the parameter values given above becomes comparable with the nonresonance two-body charge exchange cross section *in vacuo* at electron densities n_e of the order of 10^{12} cm^{-3} and increases further with further increase in n_e.

2. As was shown by Vitlina and Dykhne,[2] a nonzero resonance defect arises in the case of resonance charge exchange in a medium. The deviation from resonance is due either to polarization or to the Coulomb interaction of the medium with the charge-exchanging pair. In the case of a plasma with a fairly high charged-particle density the Coulomb detuning will be predominant: $\varkappa_0 \sim R_1 n_e^{2/3}$. When $\varkappa_0/\gamma v_n \ll 1$, the cross section is of the order of the resonance charge exchange cross section $Q_r \approx \pi\rho_0^2/2$ ($\rho_0 = R_1$). As n_e, and consequently $\varkappa_0$, increases the cross section begins to fall, and when $\varkappa_0/\gamma v_n \gg 1$ we have $Q_r = 0.03\gamma v_n / n_e^{2/3}$.

This approach, which is valid for a dense, weakly ionized medium (i.e., for the case of polarization detuning) turns out not to be adequate for a plasma having a fairly high electron density n_e because of the electron-

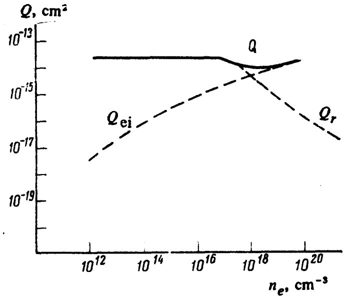

FIG. 1.

induced charge exchange effect discussed above. It is evident from formula (5) that Q_{ei} must increase with increasing n_e (though not in direct proportion) since $R_1 \approx \gamma^{-1}\ln(\gamma^2/\varkappa_0)$ while $\varkappa_0 \sim R_1 n_e^{2/3}$. In this case several iterations will be needed to determine the true value of R_1.

The rise of the cross section Q_{ei} essentially compensates the fall of the resonance charge exchange cross section due to Coulomb detuning. Figure 1 shows the charge exchange cross sections for a cesium plasma as functions of the electron density: Q_r is the cross section for two-body charge exchange with allowance for deviation from resonance,[2] Q_{ei} is the electron induced charge exchange cross section, and $Q = Q_r + Q_{ei}$. The cross sections were calculated for the parameter values $v_n = 5\cdot 10^{-3}$ and $v_e = 0.3$. Q_{ei} will be 15 times larger if the atoms are at room temperature. The behavior of the total cross section Q shows that the charge exchange cross section for a one-component plasma varies little over a wide range of electron densities and is roughly equal to the resonance charge exchange cross section.

The effect of the plasma electrons on charge exchange discussed above must be taken into account in measuring charge exchange cross sections for plasmas in the adiabatic velocity region.

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1) Several years ago, Lisitsa [1] considered the effect of multiparticle interactions but did not discuss charge exchange.

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A new example of a quantum mechanical problem with a hidden symmetry

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The quantum mechanical problem of the behavior of a neutron in the magnetic field of a linear current is considered. An exact solution of this problem is found. It is found that the system possesses the hidden symmetry $O(3)$. The generators of the symmetry group are constructed. The Schrödinger equation is reduced to an explicitly invariant form.

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1. INTRODUCTION

Several quantum mechanical problems are known in which the degeneracy of the energy levels is stronger than anticipated by starting from the usual spatial symmetry of the system.[1)] The following pertain to such problems: the oscillator, the Kepler problem, the rotator, and several other problems which do not have an immediate physical interpretation. The additional or "accidental" degeneracy which appears in these problems is associated with the existence of a so-called dynamical symmetry. For example, in addition to the usual rotational symmetry the hydrogen atom possesses the symmetry $O(4)$ (for the discrete spectrum) due to the existence of the conserved Runge-Lenz vector.[2]

Investigating problems in which the internal degrees of freedom of a particle interact with an inhomogeneous field, the authors discovered an example of such a system which possesses a dynamical symmetry: a neutron in the magnetic field of a linear current forms bound states whose spectrum is determined by the dynamical symmetry group $O(3)$. The energy levels are determined by the quantum number n:

$$E_n = -\frac{1}{n^2}\frac{(c\mu I)^2 M}{2\hbar^2}, \qquad (1)$$

where M and μ denote the neutron's mass and magnetic moment, I is the current, and the constant c depends on the system of units ($c = 0.2$ in the practical system of units).

2. SOLUTION OF THE SCHRÖDINGER EQUATION

A neutral nonrelativistic particle with a magnetic moment is described by the Hamiltonian

$$\mathscr{H} = \mathbf{p}^2/2M - \mu\mathbf{H}, \qquad (2)$$

where $\mathbf{H}$ denotes the external magnetic field. Let us consider the field created by a linear current I directed along the z axis:

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