Theory of the behavior of the magnetic moment of the μ^+ meson when muonium is produced in a normal metal

Yu. M. Belousov, V. N. Gorelkin, and V. P. Smilga

Moscow Physicotechnical Institute
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A theory is developed of the muonium mechanism of the relaxation of the spin of positive muons in normal metals. An equation is derived and solved for the spin density matrix; the equation takes consistent account of the hyperfine structure of the muonium atom and is valid in the entire range of realistic temperatures and external magnetic field. The depolarization and spin precession of the μ^+ meson are analyzed. It is shown that the longitudinal and transverse relaxation times have minima as functions of the temperature and of the external magnetic field, and the frequency shift of the muon spin precession is determined. A number of experimental ways of unequivocally answering the question of the existence of muonium atoms in normal methods are indicated.

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The question of the coexistence of muonium in metals, or the identical question of the charge state of monatomic hydrogen, has not been unequivocally answered to this day. It is not excluded, for examples, that in some metals muonium exists, but is not produced in others. It is therefore necessary to consider two variants. In this article we analyze the relaxation and precession of the μ^+-meson spin in metals, assuming that a hydrogenlike Mu atom has been produced.

It is obvious that the problem of the muon spin relaxation is analogous in principle to the calculation of the relaxation times of the nuclear spin of a paramagnetic impurity center. This question was considered relatively recently in[1,2], but the results are valid only in certain limiting cases, since no consistent account has been taken of the influence of the nuclear magnetic moment on the electron spin relaxation. In addition, in the analysis of the behavior of Mu the picture of muon spin precession, which was naturally not considered in[1,2], is important.

The muonium atom in polarizable media (metals, semiconductors) was considered in[3,4], where certain regularities in the behavior of muonium have been revealed. It was noted correctly in[4] that the electron moment becomes renormalized in a polarized medium, a fact not allowed for in[3]. In all other respects the initial equations obtained in[3] from phenomenological considerations, and in[4] on the basis of an analysis of the exchange scattering in a polarized gas[5] are practically identical. These studies, however, likewise disregarded the reaction of the muon magnetic moment on the electron spin. As a result, for example, the equilibrium state obtained in these studies do not satisfy the Gibbs distribution (see[3], formula (15)). It is therefore necessary to develop a consistent theory of the relaxation process in polarizable media, in order, on the one hand, to ascertain the region where the earlier results are valid, and to obtain on the other hand new qualitative laws.

1. We write the Hamiltonian of the system in the form

$$H=H_0+H_T+V, \tag{1}$$

where H_0 is the Hamiltonian of the dynamic subsystem, H_T is the Hamiltonian of the thermostat, V is the potential of the interaction of the dynamic subsystem with the thermostat. As usual, we assume the off-diagonal part of the thermostat interaction potential to be equal to zero (or to be included in H_0).

We choose as the basis the known equations of the NMR and ESR theories[6,7]:

$$\frac{\partial\rho_{mn}}{\partial t}+\frac{i}{\hbar}[H_0+\Gamma,\rho]_{mn}=\sum_{k,l}\{[\Gamma_{mkln}(\omega_{ln})+\Gamma_{lnmk}(\omega_{mk})e^{-\hbar\omega_{mk}/T}]\rho_{kl} \\ -\Gamma_{mkkl}(\omega_{kl})e^{-\hbar\omega_{kl}/T}\rho_{ln}-\Gamma_{knlk}(\omega_{lk})\rho_{ml}\}, \tag{2}$$

where $\hbar\omega_{kl}=E_k-E_l$, E_k are the eigenvalues of H_0, and the coefficients are given by

$$\Gamma_{mkln}(\omega_{ln})=\pi\hbar^{-1}\sum_{\alpha,\alpha'}V_{m\alpha k\alpha'}V_{l\alpha' n\alpha}\rho_{\alpha'\alpha'}\delta(\omega_{ln}+\omega_{\alpha'\alpha}), \tag{3}$$

$$\Gamma_{mk}=\mathscr{P}\sum_{\alpha,\alpha',l}V_{m\alpha l\alpha'}V_{l\alpha' k\alpha}\rho_{\alpha\alpha}(\omega_{lk}+\omega_{\alpha'\alpha})^{-1}, \tag{4}$$

where $\hbar\omega_{\alpha\alpha'}=\varepsilon_\alpha-\varepsilon_{\alpha'}$, ε_α are the eigenvalues of H_T, the symbol $\mathscr{P}$ means that the sum (4) must be understood in the sense of the principal value, and $\rho_{\alpha\beta}$ is the thermostat density matrix. It is assumed that the thermostat is in a state of thermodynamic equilibrium and $\rho_{\alpha\beta}=\delta_{\alpha\beta}\exp[(F-\varepsilon_\alpha)/T]$.

The actual form of the interaction potential is determined by the model of the thermostat. In metals, the relaxation of the muonium electron spin is due to exchange scattering by the electrons of the medium. As usual,[1,2,8,9] we use the model Hamiltonian

$$\tilde{V}(\mathbf{r})=(J/n)\sum_t\sigma_e\sigma_t\delta(\mathbf{r}-\mathbf{r}_t), \tag{5}$$

where J is the exchange interaction constant, n is the electron density, σ_e and σ_t are respectively the Pauli operators of the electrons of the muonium and of the medium. It is obvious that $\tilde{V}_{m\alpha n\alpha}\neq 0$, so that the diagonal part of the thermostat interaction will be incorporated in the Hamiltonian H_0, and it is necessary to substitute in (3) and (4) $V=\tilde{V}-\tilde{V}_d$, where $(\tilde{V}_d)_{mn}=\sum_\alpha\tilde{V}_{m\alpha n\alpha}\times\rho_{\alpha\alpha}$.

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In the calculation of the matrix elements we assume that the metal-electron wave functions are plane wave, and neglect their distortion near the muonium. We shall therefore not consider in this article any question connected with the Kondo effect. The muonium wave function is an S function of the hydrogen atom with an effective radius a. The coefficients (3) take the form

$$\Gamma_{mpln}=\sum_{s,s_1}\Gamma_{mpln;ss_1}(\sigma_{mp}\sigma_{ss_1})(\sigma_{ln}\sigma_{s_1s})$$
$$=\frac{1}{3\hbar}\left(\frac{J}{n}\right)^2\left(\frac{m}{2\pi\hbar^2}\right)^3\sum_{s,s_1}\int\limits_{\hbar\Omega/2}^{\infty}f(x,\Omega)\,n(x-\hbar\Omega^-/2)$$
$$\times[1-n(x-\hbar\Omega^+/2)]\,dx\,(\sigma_{mp}\sigma_{ss_1})(\sigma_{ln}\sigma_{s_1s}),\tag{6}$$

the summation is over the spin variables of the electrons of the medium, and

$$f(x,\Omega)=\frac{[x^2-(\hbar\Omega/2)^2]^{1/2}[(3+8x/\varepsilon_0)^2+3-4(\hbar\Omega/\varepsilon_0)^2]}{\varepsilon_0[1+4x/\varepsilon_0+(\hbar\Omega/\varepsilon_0)^2]^3},\tag{7}$$
$$\Omega=\omega_{ln}-\omega(s-s_1),\quad \Omega^{\pm}=\omega_{ln}\pm\omega(s+s_1),\tag{8}$$

$\hbar\omega=2\mu_eB$ is the electron-spin precession frequency, B is the external magnetic field, $n(x)$ is the Fermi distribution function, and $\varepsilon_0=\hbar^2/2ma^2$. As seen from (7), $f(\varepsilon_f,0)\approx 1$. For metals we have $\hbar\Omega/\varepsilon_f$, $T/\varepsilon_f\ll 1$ (ε_f is the Fermi level), the integral in (6) can be easily estimated, and we obtain

$$\Gamma_{mpln}=\Big[(1/2)\nu_\perp(\sigma_{mp}{}^+\sigma_{ln}{}^-+\sigma_{mp}{}^-\sigma_{ln}{}^+)+\nu_\parallel\sigma_{mp}{}^z\sigma_{ln}{}^z$$
$$+\frac{i}{2}r(\sigma_{mp}{}^-\sigma_{ln}{}^+-\sigma_{mp}{}^+\sigma_{ln}{}^-)\Big]\frac{\hbar\omega_{ln}/2T}{\operatorname{sh}(\hbar\omega_{ln}/2T)}\exp\left(\frac{\hbar\omega_{ln}}{2T}\right),\tag{9}$$

where

$$\nu_\perp=\nu[1-\hbar^2(\omega_{ln}{}^2+\omega^2/4)\varkappa_1],$$
$$\nu_\parallel=\nu[1-\hbar^2(\omega_{ln}{}^2+\omega^2/4)\varkappa_1+\hbar^2\omega^2\varkappa_2];\tag{10}$$
$$r=\nu\hbar^2\omega_{ln}\omega\varkappa_1,\quad \nu=gTf(\varepsilon_f,0)\hbar^{-1},$$

$$g=(3\pi/8)(J/2\varepsilon_f)^2.\tag{11}$$

Since $\varkappa_1$ and $\varkappa_2\sim\varepsilon_f^{-2}$, it follows that by neglecting terms of order $\hbar^2\omega^2\varepsilon_f^{-2}$ we obtain ultimately

$$\Gamma_{mpln}=\nu\frac{\hbar\omega_{ln}/2T}{\operatorname{sh}(\hbar\omega_{ln}/2T)}\exp\left(\frac{\hbar\omega_{ln}}{2T}\right)\left[\frac{1}{2}(\sigma_{mp}{}^+\sigma_{ln}{}^-+\sigma_{mp}{}^-\sigma_{ln}{}^+)+\sigma_{mp}{}^z\sigma_{ln}{}^z\right].\tag{12}$$

The coefficients in (4) are calculated analogously, and the energy-shift matrix is of the form

$$(\Gamma_{mn})=-(J\mu_e/\varepsilon_f)(\mathbf{B}\boldsymbol{\sigma}_e)-3g\Gamma\varepsilon_fI$$
$$+\hbar\Gamma_1g\omega_0(\boldsymbol{\sigma}_e\boldsymbol{\sigma}_\mu)4g[\Gamma_1(1+\zeta/2)+\Gamma_2]\mu_e(\mathbf{B}\boldsymbol{\sigma}_e)-2\Gamma_1g\mu_\mu(\mathbf{B}\boldsymbol{\sigma}_\mu),\tag{13}$$

where $\zeta=|\mu_\mu/\mu_e|$, I is a unit matrix, while Γ, Γ_1, and Γ_2 are positive constants of the order of unity. Thus, $H_0+\Gamma$ are replaced in (2) by the effective Hamiltonian

$$H_{eff}=\hbar\tilde{\omega}_0(\boldsymbol{\sigma}_e\boldsymbol{\sigma}_\mu)/4-\tilde{\mu}_e(\mathbf{B}\boldsymbol{\sigma}_e)-\tilde{\mu}_\mu(\mathbf{B}\boldsymbol{\sigma}_\mu),\tag{14}$$

where

$$\tilde{\omega}_0=\omega_0(1+4g\Gamma_1)\tag{15}$$

is the normalized frequency of the hyperfine interaction in the metal,

$$\tilde{\mu}_e=\mu_e[1+J/\varepsilon_f+4g(\Gamma_1(1+\zeta/2)+\Gamma_2)]\tag{16}$$

is the renormalized magnetic moment of the electron, and

$$\tilde{\mu}_\mu=\mu_\mu(1+2g\Gamma_1)\tag{17}$$

is the renormalized magnetic moment of the muon.

The first correction to formula (16) was obtained in[4]. In second order, the hyperfine-interaction frequency and the magnetic moments of the electron and the muon are renormalized. Estimates show that these corrections can reach 10%. It is easily seen that it is possible subsequently to substitute in the formula (2) for the relaxation coefficients the energy levels corresponding to the effective Hamiltonian (14). To simplify the notation, we shall therefore omit from now on the tilde, and let ω_0, ω, and ζ stand throughout for the renormalized quantities.

The eigenfunctions of the Hamiltonian (14), as is well known, are

$$\psi_1=|++\rangle,\quad \psi_2=(C_-|-+\rangle+C_+|+-\rangle)/\sqrt{2},$$
$$\psi_3=(C_+|-+\rangle-C_-|+-\rangle)/\sqrt{2},\quad \psi_4=|--\rangle.\tag{18}$$

The first symbol denotes here the state of the muonium electron, the second the state of the μ^+ meson, the magnetic field is directed along the z axis, and

$$C_\pm=[1\pm 2\omega(1+\zeta)/\alpha\omega_0]^{1/2}.\tag{19}$$

The eigenvalues of the Hamiltonian (14) are equal to

$$E_1=\hbar\omega_0/4+\hbar\omega(1-\zeta)/2,\quad E_2=\hbar\omega_0(\alpha-1)/4,$$
$$E_3=-\hbar\omega_0(\alpha+1)/4,\quad E_4=\hbar\omega_0/4-\hbar\omega(1-\zeta)/2,\tag{20}$$

where

$$\alpha=2[1+(2\omega(1+\zeta)/\omega_0)^2]^{1/2}.$$

We write down Eq. (2) for the spin density matrix of the muonium in operator form, substituting (12) in (2):

$$\frac{\partial\rho}{\partial t}+\frac{i}{\hbar}[H_{eff},\rho]=\frac{1}{2}[\sigma^+,\rho\bar{G}]+\frac{1}{2}[\sigma^-,\rho G]+[\sigma^z,\rho G_z]+\text{H.c.},\tag{21}$$

where G is a matrix with elements $g(\omega_{ln})\sigma_{ln}^+$, $\bar{G}$ is a matrix with elements $\sigma_{ln}^-\cdot g(\omega_{nl})$, G_z is a matrix with elements $g(\omega_{ln})\sigma_{ln}^z$, and

$$g(\omega_{ln})=\nu\frac{\hbar\omega_{ln}/2T}{\operatorname{sh}(\hbar\omega_{ln}/2T)}\exp\left(\frac{\hbar\omega_{ln}}{2T}\right).\tag{22}$$

The coefficients $g(\omega_{ln})$ determine the frequencies of the transitions between the different levels and can be interpreted as "effective collision frequencies." We note that at $\hbar\omega_{ln}\gg T$ the coefficients $g(\omega_{ln})$ increase linearly with ω_{ln}, meaning that under these conditions electrons with energies $\varepsilon\geqslant\varepsilon_f-\hbar\omega_{ln}$ can also take part in the scattering.

Equation (21) describes the behavior of the muonium spin density matrix at all realistic temperatures and external magnetic field. The stationary (asymptotic) solution is the Gibbs distribution.

We shall next be interested in the μ^+-meson polarization $\mathbf{P}_\mu(t) = \mathrm{Sp}(\sigma_\mu \rho)$, which takes in the chosen basis the form

$$
\begin{aligned}
P_z &= \rho_{11}-\rho_{44}+2(\rho_{33}-\rho_{22})\,\omega(1+\zeta)/\omega_0\alpha+2(\rho_{23}+\rho_{32})/\alpha,\\
P_+ &= (2+4\omega(1+\zeta)/\omega_0\alpha)^{1/2}(\rho_{21}+\rho_{43})\\
&\quad +(2-4\omega(1+\zeta)/\omega_0\alpha)^{1/2}(\rho_{42}-\rho_{31}),\\
P_- &= (2+4\omega(1+\zeta)/\omega_0\alpha)^{1/2}(\rho_{12}+\rho_{34})\\
&\quad +(2-4\omega(1+\zeta)/\omega_0\alpha)^{1/2}(\rho_{24}-\rho_{13}).
\end{aligned}
\tag{23}
$$

2. We present an expression for $\mathbf{P}_\mu$ in the thermodynamic equilibrium state

$$
\begin{aligned}
P_\mu(\infty) = \Big\{ & \frac{2\omega(1+\zeta)}{\omega_0\alpha}\exp\left(\frac{\hbar\omega_0}{4T}\right)\mathrm{sh}\left(\frac{\hbar\omega_0\alpha}{4T}\right)\\
& -\exp\left(-\frac{\hbar\omega_0}{4T}\right)\mathrm{sh}\left[\frac{\hbar\omega(1-\zeta)}{2T}\right]\Big\}\Big\{\exp\left(\frac{\hbar\omega_0}{4T}\right)\mathrm{ch}\left(\frac{\hbar\omega_0\alpha}{4T}\right)\\
& +\exp\left(-\frac{\hbar\omega_0}{4T}\right)\mathrm{ch}\left[\frac{\hbar\omega(1-\zeta)}{2T}\right]\Big\}^{-1},
\end{aligned}
\tag{24}
$$

and

$$P_x = P_y = 0.$$

At the present state of the experimental art, the asymptotic value of the muon polarization $P_\mu(\infty)$ can be observed if $\tau_{\mathrm{rel}} \lesssim \tau_\mu$, where τ_{rel} and τ_μ are the relaxation and muon lifetimes, respectively.

We shall show that $P_\mu(\infty)$ can differ noticeably from the equilibrium polarization of the free muon $P_\mu^{\mathrm{free}}(\infty) = \tanh(\zeta\omega/2T)$. At low temperature and in strong fields $\hbar\omega \gtrsim T$ (this corresponds to $B \gtrsim 10^4$ G at $T \sim 1$ K) the muon polarization in muonium is equal to

$$P_\mu(\infty) = \mathrm{th}\left(\frac{\hbar\omega}{2T}\right)\mathrm{th}\left(\frac{\hbar\omega_0}{4T}\right) + P_\mu^{\mathrm{cs}}(\infty) \tag{25}$$

and can differ noticeably from the polarization of the free μ^+ meson. For example, at $T = 0.5$ °K, $B = 10^4$ G, $\omega_0 = 0.1\,\omega_{0B}$ ($\omega_{0B} = 2\times10^{10}$ sec^{-1} is the frequency of the hyperfine interaction in vacuum) we have $P_\mu(\infty) = 0.025$ as against $P_\mu^{\mathrm{free}}(\infty) = 0.015$. In principle it is therefore possible to search for muonium via analysis of the equilibrium state. The most convenient experiment is probably in a field perpendicular to the polarization at the initial instant of time. We note that in such a scheme, by reversing the field direction, we can carry out a compensation experiment.

3. In the general case, the system (21) breaks up into a system of five equations, two Hermitian-conjugate systems of four equations, and two Hermitian-conjugate equations. The density-matrix components which determine P_z and $P_\pm$ will henceforth be called longitudinal and transverse, respectively.

We obtain the solution of the system (21) for strong external fields $\exp(\hbar\omega/2T) \gg 1$. In cases of practical interest, the obvious conditions $\hbar\omega_0 \ll T$, $\zeta\hbar\omega \ll T$ are satisfied. The longitudinal components have here two characteristic relaxation times, fast and slow. Physically this means to the fact that the population of the two higher triplet levels decreases with a short time $[g(\omega_{in})]^{-1}$, after which the system arrives relatively slowly at a thermal equilibrium.

Under the condition $\exp(\hbar\omega/2T) \gg 1$ we have for the longitudinal components

$$
\begin{aligned}
&\frac{d}{d(\nu t)}(\rho_{11}+\rho_{22}) = -4\frac{\hbar\omega}{T}(\rho_{11}+\rho_{22}) - 2\frac{\hbar\omega_0}{T}(\rho_{11}-\rho_{22}) - \frac{\omega_0}{\omega}\frac{\hbar\omega_0}{2T}(\rho_{23}+\rho_{32}),\\
&\frac{d}{d(\nu t)}(\rho_{11}-\rho_{22}) = -4\frac{\hbar\omega}{T}(\rho_{11}-\rho_{22}) - 2\frac{\hbar\omega_0}{T}(\rho_{11}+\rho_{22})\\
&\qquad + \frac{2\omega_0}{\omega}\left(1+\frac{\zeta\hbar\omega}{2T}\right)(\rho_{23}+\rho_{32}),\\
&\frac{d}{d(\nu t)}(\rho_{33}-\rho_{44}) = -2\left(\frac{\omega_0}{\omega}\right)^2(\rho_{33}-\rho_{44}) + 4\frac{\hbar\omega}{T}(\rho_{11}+\rho_{22})\\
&\qquad + 2\frac{\hbar\omega_0}{T}(\rho_{11}-\rho_{22}) + 2\frac{\omega_0}{\omega}\left(1+\frac{\hbar\omega}{T} - \hbar\frac{\omega_0+\zeta\omega}{2T}\right)(\rho_{23}+\rho_{32}),\\
&\frac{d}{d(\nu t)}\rho_{23} + i\frac{\omega(1+\zeta)}{2\nu}\rho_{23} = -2\left(1+\hbar\frac{2\omega-\omega_0}{2T}\right)\rho_{23}\\
&\qquad + \frac{\omega_0}{2\omega}(\rho_{11}-\rho_{22}+\rho_{33}-\rho_{44}) + \frac{\hbar\omega_0}{2T}(\rho_{11}-\rho_{22}),\\
&\rho_{23} = \rho_{32}^*.
\end{aligned}
\tag{26}
$$

As seen from the system (26), during the first stage $t \gtrsim T(4\hbar\omega\nu)^{-1}$ the levels 1 and 2 are completely freed (with exponential accuracy) and the components ρ_{23} are damped. Levels 3 and 4 are populated respectively at the expense of levels 1 and 2. Using (23), we can easily show that the muon polarization is conserved in this case: We thus have for the second relaxation stage the initial condition $P_z(t) \approx P_z(0)$ and the obvious equality $\rho_{33}(t) + \rho_{44}(t) = 1$. The system (26) reduces to a single equation for the polarization $P_z(t)$, and we obtain

$$P_z(t) = P_z(\infty)(1-e^{-\tau_1^{-1}t}) + P_z(0)e^{-\tau_1^{-1}t}, \tag{27}$$

where

$$\tau_1^{-1} = 2\nu(\omega_0/\omega)^2 \tag{28}$$

is the reciprocal of the longitudinal relaxation time.

A time of the same order was obtained in[1]. This is perfectly natural, since under the condition $\hbar\omega_0 \ll T$ we go over to the model assumed in[1]. If $\hbar\omega_0 \sim T$, then the analysis of the system (21) is perfectly similar to that presented above, and we obtain for the longitudinal relaxation time

$$\tau_1^{-1} = 2\nu\left(\frac{\omega_0}{\omega}\right)^2\frac{\hbar(\omega_0+2\zeta\omega)}{2T}\,\mathrm{cth}\left[\frac{\hbar(\omega_0+2\zeta\omega)}{4T}\right]. \tag{29}$$

For the transverse components of the density matrix, the system (21) is in the same approximation of the form

$$
\begin{aligned}
&\frac{d}{d(\nu t)}\rho_{12} + \frac{i}{2\nu}(\omega_0-2\zeta\omega)\rho_{12} = -4\frac{\hbar\omega}{T}\rho_{12} - \frac{\omega_0}{\omega}(\rho_{13}-\rho_{24}),\\
&\frac{d}{d(\nu t)}(\rho_{13}-\rho_{24}) + i\frac{\omega}{\nu}(\rho_{13}-\rho_{24}) + i\frac{\omega_0}{2\nu}(\rho_{13}+\rho_{24})\\
&\qquad = -4\left(1+\frac{\hbar\omega}{2T}\right)(\rho_{13}-\rho_{24}) - \frac{\hbar\omega_0}{T}(\rho_{13}+\rho_{24})\\
&\qquad\quad - \frac{2\omega_0}{\omega}\left(1+\hbar\frac{2\omega-\omega_0}{2T}\right)\rho_{12} - \frac{4\omega_0}{\omega}\rho_{34},\\
&\frac{d}{d(\nu t)}(\rho_{13}+\rho_{24}) + i\frac{\omega}{\nu}(\rho_{13}+\rho_{24}) + i\frac{\omega_0}{2\nu}(\rho_{13}-\rho_{24})\\
&\qquad = -4\left(1+\frac{\hbar\omega}{2T}\right)(\rho_{13}+\rho_{24}) - \frac{\hbar\omega_0}{T}(\rho_{13}-\rho_{24}),\\
&\frac{d}{d(\nu t)}\rho_{34} - \frac{i}{2\nu}(\omega_0+2\zeta\omega)\rho_{34} = -\left(\frac{\omega_0}{\omega}\right)^2\rho_{34} - \frac{\omega_0}{\omega}\left(1+\frac{\hbar\omega}{T}\right)(\rho_{13}-\rho_{24})\\
&\qquad - \frac{\omega_0}{\omega}\frac{\hbar\omega_0}{4T}(\rho_{13}+\rho_{24}).
\end{aligned}
\tag{30}
$$

The solution is obtained in exactly the same way as for the system (26):

$$P_-(t)=P_+^*(t)=P_-(0)\exp[-(\tau_2^{-1}+i\Omega_\perp)t], \tag{31}$$

where $\tau_2=2\tau_1$ is the transverse relaxation time, and the precession frequency

$$\Omega_\perp=\omega_0/2+\zeta\omega \tag{32}$$

is independent of temperature. These results are preserved also at $\hbar\omega_0\sim T$ and $\hbar\zeta\omega\sim T$.

As seen from (32), we can search for muonium in metals by measuring the shift $\Omega_\perp$ in comparison with $\zeta\omega$. At the present time we can resolve muon precession frequencies lower than 10^9 sec^{-1}, so that the proposed experiment is possible at $B\sim5\times10^3$ to 10^4 G and at $T\lesssim0.1$ K.

4. If there is no external magnetic field, then we obtain for the density-matrix components of interest to us the system

$$\frac{d}{dt}(\rho_{11}-\rho_{44})=-2\nu a(\rho_{11}-\rho_{44})+2\nu b(\rho_{23}+\rho_{32}),$$
$$\frac{d}{dt}(\rho_{23}+\rho_{32})+i\omega_0(\rho_{23}-\rho_{32})=-2\nu b(\rho_{23}+\rho_{32})+2\nu a(\rho_{11}-\rho_{44}), \tag{33}$$
$$\frac{d}{dt}(\rho_{23}-\rho_{32})+i\omega_0(\rho_{23}+\rho_{32})=-2\nu(a+2b-2)(\rho_{23}-\rho_{32}),$$

where

$$a=1+\frac{\hbar\omega_0/2T}{\operatorname{sh}(\hbar\omega_0/2T)}\exp\left(\frac{\hbar\omega_0}{2T}\right),\quad b=1+\frac{\hbar\omega_0/2T}{\operatorname{sh}(\hbar\omega_0/2T)}\exp\left(-\frac{\hbar\omega_0}{2T}\right). \tag{34}$$

The characteristic equation of the system (33) is of the form

$$\lambda^3-2\nu(2a+3b-2)\lambda^2+[\omega_0^2+4\nu^2(a+b)(a+2b-2)]\lambda-2a\omega_0^2\nu=0. \tag{35}$$

At high collision frequencies $\nu\gg\omega_0$, the solution is the particular case of high temperatures and weak fields, which will be considered below, while for low collision frequencies we have, accurate to terms of order $(\nu/\omega_0)^2$,

$$\lambda_1=2a\nu,$$
$$\lambda_2=\lambda_3^*=\nu(a+3b-2)+i\omega_0. \tag{36}$$

In this case we obtain for the polarization (23), with the same accuracy,

$$P(t)=\{\exp(-\lambda_1 t)+\exp(-\operatorname{Re}\lambda_2 t)[\cos\omega_0 t+(\nu/\omega_0)(3a+b-2)\sin\omega_0 t]\}P(0)/2. \tag{37}$$

If the times $\Delta t\sim\omega_0^{-1}$ are not resolved under the experimental conditions, then only an exponential decrease of the polarization from $P(0)/2$ is observed, at a rate λ_1. In particular, for infralow temperatures we have

$$\lambda_1=2\omega_0(1+T/\hbar\omega_0)gf(\varepsilon_f,0). \tag{38}$$

5. For high temperatures $T\gg\hbar\omega$, $\hbar\omega_0$ the matrices G, $\tilde G$, and G_z can be expanded in a complete set of 4×4 spin matrices. Retaining only the terms linear in $\hbar\omega/T$ and $\hbar\omega_0/T$, we write these matrices in the form

$$G=\nu[(1+\hbar\omega/2T)\sigma_e^++(\hbar\omega_0/4T)(\sigma_e^+\sigma_\mu^z-\sigma_e^z\sigma_\mu^+)], \tag{39}$$

$$\tilde G=\nu[(1-\hbar\omega/2T)\sigma_e^--(\hbar\omega_0/4T)(\sigma_e^-\sigma_\mu^z-\sigma_e^z\sigma_\mu^-)], \tag{40}$$

$$G_z=\nu[\sigma_e^z+(\hbar\omega_0/8T)(\sigma_e^-\sigma_\mu^+-\sigma_e^+\sigma_\mu^-)], \tag{41}$$

where σ_e^z, $\sigma_e^\pm$ and σ_μ^z, $\sigma_\mu^\pm$ are respectively the spin operators of the muonium electron and of the μ^+ meson.

Using the expansion (39)–(41), we linearize Eq. (21) and obtain

$$\dot\rho+i\hbar^{-1}[H_{eff},\rho]=2\nu\{\sigma_e\rho\sigma_e-3\rho+ie_{kln}(\hbar\omega_n/2T)\sigma_e^k\rho\sigma_e^l-[\rho(\hbar\omega\sigma_e)+(\hbar\omega\sigma_e)\rho]/2T+i(\hbar\omega_0/8T)e_{kln}(\sigma_e^n\rho\sigma_e^k\sigma_\mu^l-\sigma_e^k\sigma_\mu^l\rho\sigma_e^n)-(\hbar\omega_0/4T)[(\sigma_e\sigma_\mu)\rho+\rho(\sigma_e\sigma_\mu)]\}. \tag{42}$$

Equation (42) differs from that proposed in[3,4] for the description of the relaxation processes at infinite temperatures. The relaxation term is determined by the polarization of the muonium electron, and the stationary solution is a Gibbs distribution.

We obtain the solution of the system (42) in standard fashion.[10–12] For the longitudinal component of the muon polarization at $\nu\gg\omega_0$ we obtain, accurate to terms of order $(\omega_0/\nu)^2$ inclusive,

$$\tau_1^{-1}=\frac{4\nu\omega_0^2}{(8\nu)^2+\omega_0^2+\omega^2}\left(1+\frac{\hbar\omega_0}{4T}\right). \tag{43}$$

Formula is valid also at $\nu\ll\omega_0$, except that the formula for $P_z(t)$ contains rapidly oscillating terms, that are averaged out in the observation. Thus, formula (43) can be used for interpolation without restrictions. It is seen that τ_1^{-1} has a maximum. The decreasing branch has obtained already in[10].

The transverse component of the polarization at $\nu\gg\omega_0$ is determined by a single root $P_\pm(t)=P_\pm(0)\exp[-(\tau_2^{-1}\pm i\Omega_\perp)t]$, where we have, accurate to terms of order $(\omega_0/\nu)^2$,

$$\tau_2^{-1}=\frac{\omega_0^2}{32\nu}+\frac{2\nu\omega_0^2}{(8\nu)^2+[1+\zeta-\hbar\omega_0/2T]^2\omega^2}, \tag{44}$$

$$\Omega_\perp=\left[\zeta+\frac{\hbar\omega_0}{4T}+\frac{(1+\zeta)\omega_0^2}{(16\nu)^2+4(1+\zeta-\hbar\omega_0/2T)^2\omega^2}\right]\omega. \tag{45}$$

If the last term of (45) can be neglected, we can express the precession frequency in the form

$$\Omega_\perp=(\zeta+\hbar\omega_0/4T)\omega, \tag{46}$$

which agrees with the precession frequency given in[13,14]. At $\nu\ll\omega_0$ the solution practically coincides with the result obtained in[11].

As seen from (43), the maximum of τ_1^{-1} is reached at $(8\nu)^2=\omega_0^2+\omega^2$. Then τ_1^{-1} is equal to

$$\tau_{1\,max}^{-1}=\frac{\omega_0}{4(1+\omega^2/\omega_0^2)^{1/2}}. \tag{47}$$

Thus, a maximum can be observed even at $\omega_0\sim10^7$ sec^{-1}, which corresponds for a field $B\sim10^2$–10^3 G to $\tau_{1\,max}^{-1}\sim(10^4$–$10^5)$ sec^{-1}. The estimates yield $\nu=(10^8$–$10^{10})T^0$ sec^{-1}, and accordingly the maximum should be observed at a temperature $T\sim0.1$–10 K. As seen from (44), the quantity τ_2^{-1} has no maximum and decreases monotonically with increasing temperature. At the maximum of τ_1^{-1} the value of τ_2^{-1} (at $\omega \gg \omega_0$) is

$$\tau_2^{-1} = \tau_{1\,max}^{-1} + \frac{\omega_0}{4} \frac{(1+\omega^2/\omega_0^2)^{1/2}}{1+2\omega^2/\omega_0^2}. \qquad (48)$$

At the specified values of ω and ω_0 we always have $\tau_{max}^{-1} < \tau_2^{-1} < 2\tau_{1\,max}^{-1}$.

It follows from (28) that the relaxation rate $\tau_2^{-1} = \tau_1^{-1}/2$ in strong fields ($\exp(\hbar\omega/2T) \gg 1$) increases linearly with increasing temperature, and consequently τ_2^{-1} also has a maximum, but at temperatures lower than the maximum of τ_1^{-1}. For fields $B \sim 10^3$–10^5 G, the maximum of τ_2^{-1} is reached at a temperature $T \sim 0.1$–10 K. The maximum of τ_1^{-1} in such fields should occur at a temperature $T \sim 10$–100 K. Experiments in such fields make it possible to observe muonium at a level $\omega_0 \sim (10^8$–$10^9)$ sec^{-1}. In strong fields, the relaxation time given by formula (28) differs by a factor of two from the value of τ_1^{-1} calculated from formula (43) at $(8\nu)^2 \ll \omega^2$. In the case when $(8\nu)^2 \gg \omega_0^2 + \omega^2$, formulas (43) and (44) go over into the corresponding results of[1]. We note that at high temperatures, in contrast to[1], ω_1 and ω_2 are unequal and differ strongly at $8\nu \sim \omega_0$ and $8\nu \sim \omega$. It is precisely in this region that the experimental values of τ_1^{-1} given in[1] are smaller than $\omega_0^2/32\nu$. Although in the case analyzed in[1] (but not considered here) the electron shell had an angular momentum 7/2, the qualitative aspect of the effect remains unchanged and it can be assumed that the discrepancy between theory and experiment is due to unaccounted-for terms with ω_0 and ω.

We note in conclusion that the theory can be extended, with very slight elaboration, to cases when the total angular momentum of the electron shell of the impurity center is 1/2, and the spin of the nucleus is arbitrary. The theory can be generalized with practically no change to include an analysis of the behavior of muonium in semiconductors.

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