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On absorption of sound in ferrodielectrics

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Expressions are found for the sound attenuation due to interaction between the sound and magnons at temperatures $T_0 < T < \Theta_0^2/\Theta_c (T_0 = \Theta_c (\mu M_0/\Theta_c)^{4/7})$ and frequencies $\tau_{m\text{-}ph}^{-1}$, $\tau_{ph\text{-}m}^{-1} < \omega < \tau_m^{-1}$($\tau_m$, $\tau_{m\text{-}ph}$, and $\tau_{ph\text{-}m}$ are the magnon-magnon, magnon-phonon and phonon-magnon collision times). In a broad frequency range, the attenuation exceeds that of sound due to anharmonism. It is shown that the method proposed by Akhiezer for calculation of sound attenuation is valid over a broader frequency range than was previously assumed.

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1. INTRODUCTION

In an ideal ferrodielectric at temperatures $T \ll \Theta_c$ (Θ_c is the Curie temperature) sound absorption is due to its interaction with magnons and phonons and depends essentially on the relation between the frequency of the sound wave ω and the mean collision times: magnon-magon τ_m, phonon-phonon τ_{ph}, magnon-phonon τ_{m-ph} and phonon-magnon τ_{ph-m} (in a ferrodielectric, over a wide temperature ranges, $\tau_m \ll \tau_{ph-m}, \tau_{m-ph}$).[1] In the calculation of sound absorption, as a rule, two approaches are employed, the choice of which is also determined by the frequency interval under investigation.

At high frequencies, the sound absorption is usually represented as the result of the collisions of a sound quantum with phonons and magnons of the crystal. Such phonon-phonon damping at $\tau_{ph}^{-1} \ll \omega$, due to triple anharmonism, is determined for transverse sound in second order perturbation theory,[2] and for longitudinal sound, by the method of account of the anharmonism in all orders of perturbation theory[3] (the latter corresponds to account of the lifetime of the interacting phonons[4]). Sound absorption due to interaction with magnons at $\tau_m^{-1} \ll \omega$ is also considered in second order perturbation theory.[5,6] It was shown in Ref. 7 that at temperatures $T \ll \Theta_0^2/\Theta_c$ (Θ_0 is the Debye temperature), it is not sufficient to limit ourselves to second order for the calculation of phonon-magnon damping, rather it is necessary to take into account the contribution from fourth order perturbation theory. In this case, it turns out that the considered contribution can appreciably exceed the phonon-phonon damping.

At low frequencies, the sound absorption is usually calculated by the Akhiezer method.[8] In this case, the sound wave is considered as an external field, which produces a departure from equilibrium in the gas of magnons and phonons. Knowing the change in the particle distribution function under the action of the sound field, we can determine the change in the entropy of the gas and thus calculate the dissipation of energy of the sound wave. The phonon-phonon damping in dielectrics at $\omega \ll \tau_{ph}^{-1}$ was calculated by this method.[8] In what follows, the indicated method for phonon-phonon damping was developed in other researches.[9,10] Phonon-magnon damping of sound in ferrodielectrics was considered by the same method for the frequencies $\omega \ll \tau_m^{-1}$ at low temperatures $T \ll T_0 = \Theta_c(\mu M_0/\Theta_c)^{4/7}$ (μ is the Bohr magneton, M_0 the magnetization saturation), when the equilibrium in the magnon gas is established through dipole-dipole processes.[5,11]

In the present work, we consider the attenuation of sound in a ferrodielectric due to interaction with magnons at temperatures $T_0 \ll T$, when it is necessary to take exchange scattering into account, and at frequencies $\tau_{ph-m}^{-1}, \tau_{m-ph}^{-1} \ll \omega \ll \tau_m^{-1}$. The sound attenuation in the given frequency range was considered previously in the work of Kaganov and Chikvashvili,[5] who neglected the contribution to the damping from the anisotropic part of the phonon-magnon interaction and, in addition, in contrast to the case considered by us, assumed that $T_0 \sim \Theta_0^2/\Theta_c$.

As will be shown, account of the anisotropy has a significant effect on the results. It is also important here that the basic contribution to the damping is made by the interaction of the sound with subthermal intermediate magnons, while the phonon-magnon damping at $T \ll T_0$ and the phonon-phonon damping due to anharmo-

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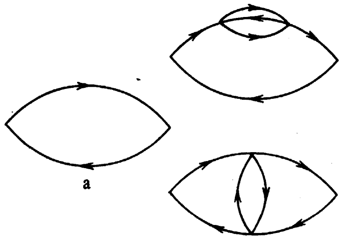

FIG. 1.

nism are determined by the interaction with thermal intermediate states. In this sense, the situation is similar to that considered earlier by two of the authors of the present work in the calculation of the damping of long-wave magnons, [12] and also to the situation considered by Gurevich and Shklovskiĭ [13] in an account of the effect of impurities on the phonon-phonon damping of the sound.

In the present work, the calculations are carried out by the diagram method.[1)] Such an approach allows us not only to obtain an expression for the damping decrement, but also to show that the method of Akhiezer (in the form put forward in Refs. 9 and 10) is applicable over a wider range of frequencies than has been assumed earlier. It is also proposed that the given method of calculation of the sound attenuation is applicable under two limitations: a) $\omega\tau_m \ll 1$, $\omega\tau_{ph} \ll 1$, and b) λ^*_m, $\lambda^*_{ph} \ll \lambda$, where λ is the sound wavelength, and λ^*_m, λ^*_{ph} are the wavelengths of the magnons and phonons which make a substantial contribution to the sound absorption. We shall show that the only necessary condition is the second one, which, for example at the temperatures $T \ll \Theta_0^2/\Theta_0$ considered by us and at the frequencies $\omega \ll T/\hbar$, is always satisfied.

2. EQUATION FOR THE VERTEX. DAMPING DECREMENT

We consider an isotropic ferrodielectric. The Hamiltonian of the phonon-magnon system has the following form[1]:

$$\mathscr{H}=\mathscr{H}_0+V^{(3)}_{m-ph}+V^{(3)}_d+V^{(4)}_{ex}. \tag{1}$$

Here

$$\mathscr{H}_0=\sum_{\mathbf{q}}\hbar\omega_{\mathbf{q}}b_{\mathbf{q}}^+b_{\mathbf{q}}+\sum_{\mathbf{k}}\varepsilon_{\mathbf{k}}a_{\mathbf{k}}^+a_{\mathbf{k}}$$

is the Hamiltonian of the noninteracting phonons and magnons; $\varepsilon_{\mathbf{k}}=\Theta_c(ak)^2$, $\hbar\omega_{\mathbf{q}}=\Theta_0 aq$ are the energies of the magnons and phonons (at the considered temperatures $T_0 \ll T$, we can neglect the gap in the magnon spectrum); $a^+_{\mathbf{k}}$, $b^+_{\mathbf{q}}$ are the creation operators of the magnons and phonons; the quantities

$$V^{(3)}_{m-ph}=\sum_{\mathbf{k}_1\mathbf{k}_2\mathbf{q}}\psi(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q})a_{\mathbf{k}1}^+a_{\mathbf{k}2}b_{\mathbf{q}}\Delta(\mathbf{k}_1-\mathbf{k}_2-\mathbf{q})+\text{herm. conj.} \tag{2a}$$

$$V^{(3)}_d=\sum_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}\Phi(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)a_{\mathbf{k}1}^+a_{\mathbf{k}2}^+a_{\mathbf{k}3}\Delta(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_3)+\text{herm. conj.} \tag{2b}$$

$$V^{(4)}_{ex}=\sum_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3\mathbf{k}_4}\Phi(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)a_{\mathbf{k}1}^+a_{\mathbf{k}2}^+a_{\mathbf{k}3}a_{\mathbf{k}4}\Delta(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_3-\mathbf{k}_4) \tag{2c}$$

describe the magnon-phonon interaction, and also the dipole and exchange interaction of the magnons.

$$\psi(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q})=i\Theta_c a^2(\hbar/2\rho V\omega_q)^{1/2}[\beta_1(\mathbf{ek}_1)(\mathbf{qk}_2)+\beta_1(\mathbf{ek}_2)(\mathbf{qk}_1)+2\beta_2(\mathbf{k}_1\mathbf{k}_2)(\mathbf{eq})], \tag{3a}$$

$$\Phi(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)=2\pi(2S)^{1/2}\mu^2a^{-3}[\sin 2\theta_{\mathbf{k}_1}\exp(-i\varphi_{\mathbf{k}_1})+\sin 2\theta_{\mathbf{k}_2}\exp(-i\varphi_{\mathbf{k}_2})], \tag{3b}$$

$$\Phi(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)=-\Theta_c a^2(\mathbf{k}_1\mathbf{k}_2+\mathbf{k}_3\mathbf{k}_4)/4S \tag{3c}$$

are the amplitudes of these processes. Here ρ is the density, a is the period of the lattice, V is the volume of the crystal, β_1 and β_2 are dimensionless constants of the order of unity, $\mathbf{e}$ is the unit polarization vector, S is the spin of the atoms of the magnetic lattice, $\theta_{\mathbf{k}}$ and $\varphi_{\mathbf{k}}$ are the angles which describe the position of the wave vector relative to the axis of quantization. We do not take into account the interaction of the phonons, since it gives a separate contribution to the sound absorption in the frequency range considered (phonon-phonon damping) and was considered previously.[2-4,8-10] We also neglect processes of phonon decay into two magnons, since they do not make a contribution in the frequency range considered.[5,6]

We shall calculate the sound attenuation by the diagram technique. In such an approach, the damping decrement of the sound is expressed in terms of the mass operator $\Sigma^R(\omega, q)$ of the equal-time retarded Green's function of the phonons[15]:

$$\gamma(\omega)=-\operatorname{Im}\Sigma^R(\omega, q).$$

The damping due to the interaction with magnons, calculated by second order perturbation theory, corresponds to a diagram of second order, shown in Fig. 1a. The law of conservation of energy and momentum in this case leads to the result that only those magnons with energy $\varepsilon_k \gtrsim \Theta_0^2/4\Theta_c$ can absorb or emit a phonon. At low temperatures $T \ll \Theta_0^2/\Theta_c$, the number of such magnons is exponentially small, and the damping calculated in this approximation turns out to be correspondingly small.[5] However, as shown by Shklovskiĭ,[7] we need to take into account higher order perturbation theory, the contribution of which to the damping at $T \ll \Theta_0^2/\Theta_c$, exceeds the contribution of second order perturbation theory. He considered the case of frequencies $\tau_m^{-1} \ll \omega$, where it sufficed to calculate the damping in fourth order perturbation theory (several diagrams of fourth order are shown in Fig. 1b), since the higher orders are small in the smallness parameter $(\omega\tau_m)^{-1} \ll 1$. In the opposite limiting case, $\omega\tau_m \ll 1$, which we shall consider, it is necessary to sum the contributions of all orders of perturbation theory in order to obtain the correct expression for the damping.

The total mass operator is shown in Fig. 2, where the heavy lines correspond to the renormalized Green's function of the magnon, and Γ denotes the vertex part. The principal contribution to the mass operator is made

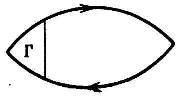

FIG. 2.

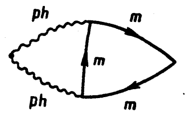

FIG. 3.

by the ladder diagrams. In what follows, we limit ourselves to not too low frequencies, $\tau_{m-ph}^{-1} \ll \omega$, when we can neglect the contribution to the mass operator from diagrams of the type shown in Fig. 3. The equation for the vertex Γ with account of the ladder diagrams only is shown in Fig. 4. The reason why we cannot always neglect the dipole processes at $T_0 \ll kT$, when the relaxation in the magnon system is determined by the exchange scattering, is that the sound absorption, as will be shown below, is due to the interaction with an isolated, relatively small group of subthermal magnons.

To begin with, we consider the case in which the dipole processes do not play a role. Here, the last two terms in the graphic equation for the vertex (see Fig. 4) are discarded. Using the method of analytic continuation, [15,16] we obtain the following expression for the damping decrement:

$$\gamma(\omega) = -\frac{\omega v_0}{4T\hbar}\,\mathrm{Im}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\,\frac{1}{\mathrm{sh}^2\,\varepsilon_k/2T}\,\frac{\Gamma(\mathbf{k},\mathbf{q})\,\psi^*(\mathbf{k},\mathbf{k},\mathbf{q})}{\omega-\mathbf{q}\mathbf{v}_k+2i\gamma_{ex}(\mathbf{k})}. \tag{4}$$

Here $\gamma_{ex}(\mathbf{k})$ is the damping of the magnon with momentum $\hbar\mathbf{k}$ due to the exchange scattering. The expression for $\lambda_{ex}(\mathbf{k})$ at $\varepsilon_k \ll T$ can be found, for example, in Ref. 17:

$$\gamma_{ex}(\mathbf{k}) = \frac{1}{96\pi^3 S^2}\,\frac{T^2}{\hbar\Theta_c}(ak)^4 \ln^2\frac{\varepsilon_k}{T}. \tag{5}$$

In the derivation of (4), the expression for the magnon mass operator $\Sigma_{ex}(z, k)$ in the renormalized Green's functions of the magnons

$$G(z,\mathbf{k}) = (z-\varepsilon_k-\Sigma_{ex}(z,\mathbf{k}))^{-1}$$

was taken in second order perturbation theory, since the perturbation theory works well in this region of intermediate wave vectors $\mathbf{k}$ of the magnons, which give the principal contribution to the damping. It has also been assumed that $\hbar\omega \ll T$ and $q \ll k$. In similar fashion, we find the equation for the vertex $\Gamma(\mathbf{k}, \mathbf{q})$:

$$\begin{gathered}\Gamma(\mathbf{k},\mathbf{q}) = \psi(\mathbf{k},\mathbf{k},\mathbf{q}) \\ -2i\iint d^3\mathbf{k}_2\,d^3\mathbf{k}_3\,\gamma_{ex}(\mathbf{k},\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)\,\{\nu(\mathbf{k}_2,\mathbf{q})-\nu(\mathbf{k}_3,\mathbf{q})-\nu(\mathbf{k}_4,\mathbf{q})\}, \\ \nu(\mathbf{k},\mathbf{q}) = \Gamma(\mathbf{k},\mathbf{q})/[\omega-\mathbf{q}\mathbf{v}_k+2i\gamma_{ex}(\mathbf{k})],\quad \mathbf{k}_4=\mathbf{k}+\mathbf{k}_2-\mathbf{k}_3.\end{gathered} \tag{6}$$

Here $\gamma_{ex}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)$ is the expression for the imaginary part of Σ in second order perturbation theory, not integrated over the intermediate momenta:

$$\gamma_{ex}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4) = \frac{v_0^2}{(2\pi)^5}\,\frac{\mathrm{sh}(\varepsilon_k/2T)\,|\Phi(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)|^2\,\delta(\varepsilon_{k_1}+\varepsilon_{k_2}-\varepsilon_{k_3}-\varepsilon_{k_4})}{\mathrm{sh}(\varepsilon_{k_2}/2T)\,\mathrm{sh}(\varepsilon_{k_3}/2T)\,\mathrm{sh}(\varepsilon_{k_4}/2T)} \tag{7}$$

We limit ourselves to consideration of the temperature range $T \ll \Theta_0^2/\Theta_c$, when the magnon energy difference $\varepsilon_{\mathbf{k}+\mathbf{q}} - \varepsilon_{\mathbf{k}} = \hbar\mathbf{q}\mathbf{v}_{\mathbf{k}}$ in expressions (4) and (6) can be neglected in comparison with the sound frequency.

3. CALCULATION OF THE DAMPING DECREMENT $\gamma(\omega)$

The situations are different for transverse and longitudinal sound.

A. Transverse sound

We shall show that in this case the renormalization of the vertex can be neglected. For this, we estimate the contribution to the bare vertex $\psi(\mathbf{k}, \mathbf{k}, \mathbf{q})$ for momenta $\hbar k \approx \hbar k^* \ll \hbar k_T = (\hbar/a)(T/\Theta_c)^{1/2}$ of the magnons, which make the principal contribution to the damping (an estimate of k^* is given below). We find the n-th iteration $\Delta\Gamma_n(\mathbf{k}, \mathbf{q})$ for the first few n by direct calculation from Eq. (6):

$$\Delta\Gamma_n \approx \frac{|\psi(\mathbf{k},\mathbf{k},\mathbf{q})|}{\ln^n(k_T/k)} \ll |\psi(\mathbf{k},\mathbf{k},\mathbf{q})|. \tag{8}$$

If the estimate (8) were valid at arbitrarily large numbers n, then the contribution to the vertex $\psi(\mathbf{k}, \mathbf{k}, \mathbf{q})$ would obviously be small. Such an assumption was made in Refs. 12 and 13, in which a similar problem of the renormalization of the vertex arose. A more critical consideration shows, however, that the estimate (8) is valid only at $n < n_0$. In our case, n_0 is determined from the condition

$$(\varepsilon_k/T)\,|\ln(\varepsilon_k/T)|^{n_0} \sim 1.$$

At $n > n_0$, all the iterations are of the same order:

$$\Delta\Gamma_n \sim |\psi(\mathbf{k},\mathbf{k},\mathbf{q})|/\ln^{n_0}(k_T/k).$$

For this reason, it is impossible to conclude from (8) that the contribution to the vertex is actually small. Moreover, the renormalization of the vertex is essential for longitudinal sound (see Sec. 3B). This circumstance is connected with the fact that the isotropic part is contained in the bare vertex for longitudinal sound. For transverse sound, the amplitude $\psi(\mathbf{k}, \mathbf{k}, \mathbf{q})$ is essentially anisotropic, and this leads to excellent convergence of the iteration series for the vertex $\Gamma(\mathbf{k}, \mathbf{q})$, which also allows us to neglect the renormalization of the vertex (for more details on this, see Sec. 3B).

For parametrically excited magnons in ferromagnets ($\theta_{\mathbf{k}} = \pi/2$) the bare amplitude (the dipole-dipole amplitude Φ_d) does not contain an istropic part, and therefore, the statement[12] that the renormalization of the vertex can be neglected is valid. Really, however, the renormalization of the vertex does not affect the order of the magnitude of the result. For this reason, the order-of-magnitude results of Gurevich and Shklovskiĭ[13] are also valid.

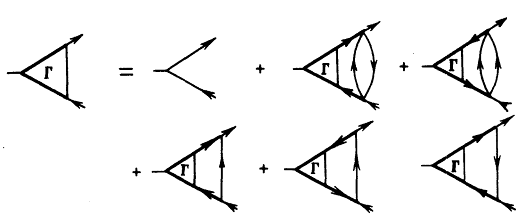

FIG. 4.

With account of what has been said above, we obtain the following expression for the damping of transverse sound:

$$\gamma(\omega)=\frac{\omega v_0}{4T\hbar}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{|\psi^{\perp}(\mathbf{k},\mathbf{k},\mathbf{q})|^2}{\mathrm{sh}^2(\varepsilon_k/2T)}\frac{2\gamma_{ex}(k)}{\omega^2+4\gamma_{ex}^2(k)}$$
$$=\frac{\pi}{15}\frac{1}{(2\pi)^3}\omega\frac{T}{Mv_{\perp}^2}\left(\frac{T}{\Theta_c}\right)^{1/2}\delta^2\int dx\,\mathrm{sh}^{-2}\frac{x}{2}\frac{x^{5/2}\ln^2 x}{\delta^4+x^4\ln^4 x}. \qquad (9)$$

Here

$$x=\varepsilon_k/T,\quad \delta^2=48\pi^3S^2(\hbar\omega/T)(\Theta_c/T)^3\approx\omega\tau_m\ll 1,$$

$\psi^{\perp}(\mathbf{k},\mathbf{k},\mathbf{q})$ is the amplitude of the magnon-phonon interaction for transverse sound (expression (3a) at $\mathbf{e}\perp\mathbf{q}$).

The expression (9) differs from that obtained in second order perturbation theory

$$\gamma_{\perp}(\omega)=\frac{\omega}{2T}\pi v_0\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{|\psi^{\perp}(\mathbf{k},\mathbf{k},\mathbf{q})|^2}{\mathrm{sh}^2(\varepsilon_k/2T)}\delta(\varepsilon_{\mathbf{k}}+\omega-\varepsilon_{\mathbf{k}+\mathbf{q}}) \qquad (10)$$

in that the δ function is replaced by a Lorentzian, a replacement sometimes simply postulated[4] on the basis of physical considerations of the finite lifetime of the particles. We see that such a procedure is correct if we can neglect the renormalization of the vertex.

It is easy to see that the principal contribution to the integral (9) is made by $x\sim\delta\ll 1$ and, correspondingly, by magnons with wave vectors $k^*\approx k_T(\omega\tau_m)^{1/4}\ll k_T$ (here $\hbar k_T\sim\hbar a^{-1}(T/\Theta_c)^{1/2}$ is the thermal momentum of the magnons), i.e., satisfying the condition $\omega\approx\gamma_{ex}(k^*)$.

Direct calculation of (9) gives:

$$\gamma_{\perp}(\omega)=A\frac{(\hbar\omega)^{1/4}\Theta_c^{3/4}}{Mv_{\perp}^2\hbar}\left(\frac{\Theta_c}{T}\right)^{1/2}|\ln\delta|^{-3/2}. \qquad (11)$$

Here $A=(\frac{8}{5}\sqrt{2})\,\pi(\pi/3)^{1/4}\cos(\pi/8)\beta_1^2S^{3/2}$ is a numerical factor, $v_{\perp}$ is the velocity of transverse sound, and M is the mass of the unit cell.

B. Longitudinal sound

For longitudinal sound, the amplitude of the interaction $\psi(\mathbf{k},\mathbf{k},\mathbf{q})$ at $q\ll k$ is of the form

$$\psi_{\parallel}(\mathbf{k},\mathbf{k},\mathbf{q})=2i\Theta_c a^2k^2q(\hbar/2\rho V\omega_q)^{1/2}[\beta_2+\beta_1\cos^2\widehat{\mathbf{qk}}]. \qquad (12)$$

We shall seek a solution of Eq. (12) in the form

$$\Gamma(\mathbf{k},\mathbf{q})=2i\Theta_c a^2\left(\frac{\hbar}{2\rho V\omega_q}\right)^{1/2}q$$
$$\times\left\{\left(\beta_2+\frac{\beta_1}{3}\right)f_1(k)+\beta_1\left(\cos^2\widehat{\mathbf{qk}}-\frac{1}{3}\right)f_2(k)\right\}, \qquad (13)$$

which corresponds to the division of the amplitude $\psi_{\parallel}(\mathbf{k},\mathbf{k},\mathbf{q})$ into an isotropic part independent of the angles and an essentially anisotropic part, the averaging of which over the solid angle in the space of the wave vectors $\mathbf{k}$ yields zero. We obtain the following equation for the functions $f_1(k)$ and $f_2(k)$:

$$f_1(k)=k^2-2i\iint d^3\mathbf{k}_2\,d^3\mathbf{k}_3\,\gamma_{ex}(\mathbf{k},\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)$$
$$\times\left\{\frac{f_1(k_2)}{\omega+2i\gamma_{ex}(k_2)}-\frac{f_1(k_3)}{\omega+2i\gamma_{ex}(k_3)}-\frac{f_1(k_4)}{\omega+2i\gamma_{ex}(k_4)}\right\}, \qquad (14)$$

$$f_2(k)=k^2-2i\iint d^3\mathbf{k}_2\,d^3\mathbf{k}_3\,\gamma_{ex}(\mathbf{k},\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)$$
$$\times\left\{\frac{f_2(k_2)}{\omega+2i\gamma_{ex}(k_2)}\frac{3\cos^2\widehat{\mathbf{k}_2\mathbf{k}}-1}{2}-\frac{f_2(k_3)}{\omega+2i\gamma_{ex}(k_3)}\frac{3\cos^2\widehat{\mathbf{k}_3\mathbf{k}}-1}{2}\right.$$
$$\left.-\frac{f_2(k_4)}{\omega+2i\gamma_{ex}(k_4)}\frac{3\cos^2\widehat{\mathbf{k}_4\mathbf{k}}-1}{2}\right\}. \qquad (15)$$

Direct substitution can establish the fact that the solution of Eq. (14), which corresponds to the isotropic part of the amplitude, is the following expression:

$$f_1(k)=k^2(1+2i\gamma_{ex}(k)/\omega). \qquad (16)$$

It is seen that the renormalization of the isotropic part of the bare amplitude is significant.

The anisotropic part of the amplitude $\psi_{\parallel}(\mathbf{k},\mathbf{k},\mathbf{q})$ corresponds to Eq. (15). An estimate of the first iterations for $f_2(k)$, similar to that given in the case of transverse sound (see Eq. (8)), shows that they are small. The angular factors $(3\cos^2\widehat{\mathbf{k}_i\mathbf{k}}-1)/2$ under the integral sign in (15) lead, in contrast to the case of the isotropic part of the amplitude, to excellent convergence of the iteration series. In this case, the contribution of the high-order iterations $n>n_0$ is small. For this reason, the zeroth approximation can be taken for $f_2(k)$:

$$f_2(k)=k^2,\quad k\ll k_T.$$

Thus, the renormalization of the vertex can be neglected if the bare amplitude is essentially anisotropic and the principal contribution to the damping is made by the subthermal momenta.

Direct substitution of the expression (13) with account of (16) in the expression (4) can easily establish the fact that the isotropic part of the amplitude $\psi_{\parallel}(\mathbf{k},\mathbf{k},\mathbf{q})$ does not make a contribution to the damping in our approximation, (we recall that we have neglected $\mathbf{q}\mathbf{v}_{\mathbf{k}}$ in comparison with the sound frequency ω in (4)).

When the anisotropy of the amplitude of the phonon-magnon interaction $\psi(\mathbf{k},\mathbf{k},\mathbf{q})$ is taken into account it turns out, as also in the case of transverse sound, that the principal contribution is made by the subthermal magnons: $\omega\approx\gamma_{ex}(k)$. The damping decrement of the longitudinal sound $\gamma_{\parallel}(\omega)$ can be expressed in terms of the damping of the transverse sound $\gamma_{\perp}(\omega)$:

$$\gamma_{\parallel}(\omega)={}^4/_3(v_{\perp}/v_{\parallel})^{1/4}\gamma_{\perp}(\omega). \qquad (17)$$

Here $v_{\parallel}$ is the velocity of longitudinal sound.

In the derivation of the formulas (11) and (17) for the damping of transverse and longitudinal sound, we have not taken the dipole-dipole interactions into account. It is physically clear that the dipole processes must be taken into account if the lifetime of the magnons, which make the principal contribution to the sound attenuation, is determined not only by the exchange scattering, but also by the triple dipole processes. Here, in the graphic equation at the vertex (Fig. 4), it is necessary to take into consideration the last three terms, which we have not previously considered.

The damping of the magnon states due to triple dipole processes is of the following form[1]:

$$\gamma_d(k) \approx \frac{(\mu M_0)^2}{\hbar\Theta_c}\frac{T}{\Theta_c}\frac{1}{ak}\ln\frac{\varepsilon_k}{2\mu H}, \quad \mu M_0 \ll \mu H \ll \varepsilon_k. \tag{18}$$

Comparing $\gamma_d(k)$ with $\gamma_{ex}(k)$ (see Eq. (5)) at the momenta $\hbar k \sim \hbar k^*$, we obtain the result that account of the dipole processes is necessary at frequencies

$$\omega < (T/\hbar)(\mu M_0/\Theta_c)^{4/3}(T/\Theta_c)^{1/3}. \tag{19}$$

In this case we estimate the sound attenuation in order of magnitude. For frequencies satisfying (19), the basic contribution to the sound attenuation is made by magnons with wave vectors k': $\gamma_d(k') \approx \gamma_{ex}(k')$. Here we get

$$\gamma_\perp(\omega) \approx \gamma_\parallel(\omega) \approx \frac{\hbar\omega^2}{Mv_\perp^2}\frac{\Theta_c}{T}\left(\frac{\Theta_c}{\mu M_0}\right)^{2/3}. \tag{20}$$

4. EQUIVALENCE OF THE AKHIEZER METHOD AND THE DIAGRAM METHOD IN THE CALCULATION OF SOUND ATTENUATION

We shall show the equivalence of both methods in the case considered here of the phonon-magnon sound attenuation at $\tau_{m-ph}^{-1} \ll \omega \ll \tau_m^{-1}$ for the case in which the dipole interaction of the magnons can be neglected. In the Akhiezer approach, [8] the sound wave is considered as an external field which modulates the energy of the magnons, leading to a departure of the distribution function from its equilibrium value. Knowing the change in the distribution function of the magnons under the action of the sound wave, we can determine the amount of energy absorbed by the crystal and, in the same way, the decrease in the sound energy.

Assuming the sound wavelength $\lambda = 2\pi/q$ ($\mathbf{q}$ is the wave vector of the sound) to be large in comparison with the magnon wavelength $\lambda_m = 2\pi/k$, we introduce the local density of number of magnons $n(\mathbf{k}, \mathbf{r}, t)$. In order to determine $n(\mathbf{k}, \mathbf{r}, t)$, we write down the kinetic equation with the collision integral in second order perturbation theory, for the case of exchange scattering:

$$\frac{dn}{dt} = \frac{\partial n}{\partial t} + \mathbf{v}_\mathbf{k}\frac{\partial n}{\partial \mathbf{r}} + \dot{\mathbf{k}}\frac{\partial n}{\partial \mathbf{k}} = \frac{8}{(2\pi)^5}\frac{v_0^2}{\hbar}\iint d^3\mathbf{k}_2\, d^3\mathbf{k}_3\, d^3\mathbf{k}_4 |\Phi(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)|^2$$
$$\times\{(n_1+1)(n_2+1)n_3n_4 - n_1n_2(n_3+1)(n_4+1)\}\delta(\tilde\varepsilon_1+\tilde\varepsilon_2-\tilde\varepsilon_3-\tilde\varepsilon_4)\Delta(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_3-\mathbf{k}_4). \tag{21}$$

Here $\tilde\varepsilon_\mathbf{k}$ is the energy of the magnon in the presence of the sound wave, [9,10] and $\mathbf{v}_\mathbf{k} = \partial\varepsilon_\mathbf{k}/\partial\mathbf{k}$ is the velocity of the magnon.

Assuming the deviation from equilibrium to be slight, we linearize the collision integral terms in the deviations $\delta n = n(\mathbf{k}, \mathbf{r}, t) - n_\mathbf{k}^0$, where $n_\mathbf{k}^0 = (\exp\{\tilde\varepsilon/T\} - 1)^{-1}$. Following Akhiezer, [8] we introduce the function $\varphi(\mathbf{k}, \mathbf{r}, t)$:

$$n(\mathbf{k}, \mathbf{r}, t) = n_k^0 + T^{-1}\varphi(\mathbf{k}, \mathbf{r}, t)n_k^0(n_k^0+1) \tag{22}$$

and obtain for it from (21) the following equation:

$$\frac{\partial\varphi}{\partial t} + \mathbf{v}_{\mathbf{k}_1}\frac{\partial\varphi}{\partial\mathbf{r}} = \dot{\tilde\varepsilon}(\mathbf{k}_1, \mathbf{r}, t) - 2\iint d^3\mathbf{k}_2\, d^3\mathbf{k}_3\, \gamma_{ex}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)\{\varphi_1+\varphi_2-\varphi_3-\varphi_4\}. \tag{23}$$

The magnon energy $\tilde\varepsilon(\mathbf{k}, \mathbf{r}, t)$ in the presence of the sound wave can be determined from the magnon-phonon Hamiltonian (1), retaining only the Hamiltonian of the free magnons and the interaction with the sound wave $V_{m-ph}^{(3)}$.[5] Here the annihilation operator $\hat b_\mathbf{q}$ of the sound quanta can be replaced by the classical quantity $b_\mathbf{q}e^{i(\mathbf{qr}-\omega t)}$ + c. c. corresponding to it, whence we obtain

$$\tilde\varepsilon(\mathbf{k}, \mathbf{r}, t) = \varepsilon_k + [\psi(\mathbf{k}, \mathbf{k}, \mathbf{q})b_\mathbf{q}e^{i(\mathbf{qr}-\omega t)} + \text{c.c.}] \tag{24}$$

Substituting $\varphi(\mathbf{k}, \mathbf{r}, t)$ in the form

$$\varphi(\mathbf{k}, \mathbf{r}, t) = \varphi(\mathbf{k}, \mathbf{q})e^{i(\mathbf{qr}-\omega t)} + \text{c.c.},$$

we get from (23) the following equation for $\varphi(\mathbf{k}, \mathbf{q})$:

$$(\omega - \mathbf{qv}_{\mathbf{k}_1})\varphi(\mathbf{k}_1\mathbf{q}) = \omega\psi(\mathbf{k}_1, \mathbf{k}_1, \mathbf{q})b_\mathbf{q}$$
$$-2i\iint d^3\mathbf{k}_2\, d^3\mathbf{k}_3\, \gamma_{ex}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)\{\varphi_1+\varphi_2-\varphi_3-\varphi_4\}. \tag{25}$$

Comparing (25) with the equation for the vertex (6), we find the relation between the vertex $\Gamma(\mathbf{k}, \mathbf{q})$ and the function $\varphi(\mathbf{k}, \mathbf{q})$ at which both equations are the same:

$$\varphi(\mathbf{k}, \mathbf{q}) = \Gamma(\mathbf{k}, \mathbf{q})\omega b_\mathbf{q}/[\omega - \mathbf{qv}_k + 2i\gamma_{ex}(k)]. \tag{26}$$

The energy dE/dt absorbed by the crystal per unit time can be determined from the dissipative function[8] or from the obvious relation[9]

$$\frac{dE}{dt} = \left\langle \frac{\partial}{\partial t}\sum_k \tilde\varepsilon(\mathbf{k}, \mathbf{r}, t)n(\mathbf{k}, \mathbf{r}, t)\right\rangle$$

(the angle brackets denote time averaging over the period of the wave). Recognizing that the energy of the sound wave is $E = \hbar\omega|b_\mathbf{q}|^2$, we find the sound damping decrement:

$$\gamma = \left|\frac{1}{2E}\frac{dE}{dt}\right| = \frac{1}{4T}\operatorname{Im}\int\frac{v_0 d^3\mathbf{k}}{(2\pi)^3}\frac{\varphi(\mathbf{k}, \mathbf{q})\psi^*(\mathbf{k}, \mathbf{k}, \mathbf{q})}{\operatorname{sh}^2(\varepsilon_k/2T)}, \tag{27}$$

which, with account taken of the expression (26), is identical with the diagram expression (4).

We note that in the derivation of Eqs. (25) and (27), only the condition of the longwave nature of the sound was used: $\lambda_m \ll \lambda$; it is sufficient to assume that this is satisfied for that group of magnons which make the principal contribution to the attenuation of the sound, that is, it is necessary that $\lambda_m^* \ll \lambda$ (or, correspondingly, $q \ll k^*$). This condition is considerably weaker than the conditions $\omega\tau_m \ll 1$ and $\omega\tau_{ph} \ll 1$, which were assumed to be necessary to be able to calculate the sound attenuation by the Akhiezer method.

Of course, the equivalence of the two methods also holds for the problem of the calculation of the phonon-phonon damping. It is therefore not surprising that the expression obtained by Woodruff and Ehrenreich[9] for the damping, which is valid, as was assumed, at $\omega\tau_{ph} \ll 1$, goes over at $1 \ll \omega\tau_{ph}$ into the Landau-Rumer formula.

It is interesting to note that if we solve the kinetic equation in the Akhiezer method in the $\tau_\mathbf{k}$ approximation, then this corresponds to neglect of the renormalization of the vertex in the diagram approach. Such an approach was used by Pomeranchuk[18] in the calculation of the sound attenuation in a dielectric with account of the fourth order anharmonism.

5. CONCLUSION

In conclusion, we compare the sound attenuation in a ferrodielectric, due to interaction with magnons at $T_0 \ll T \ll \Theta_0^2/\Theta_c$, τ_{m-ph}^{-1}, $\tau_{ph-m}^{-1} \ll \omega \ll \tau_m^{-1}$, with the phonon-phonon damping due to the anharmonism. At $\tau_{ph}^{-1} \ll \omega$, the damping of the transverse sound due to the anharmonism is described by the Landau-Rumer formula:

$$\gamma_{ph}(\omega) \approx \omega \frac{\Theta_0}{Mv_\perp^2}\left(\frac{T}{\Theta_0}\right)^4. \tag{28}$$

Comparing the damping of the transverse sound (11) with the phonon-magnon damping at $\omega \ll \tau_m^{-1}$, we find that it exceeds the phonon-phonon damping (28) at

$$\tau_m^{-1}\xi^{-2} \lesssim \omega, \quad \xi^{-1} = (\Theta_0^2/T\Theta_c)^{-1} \ll 1. \tag{29}$$

At $\omega\tau_{ph} \ll 1$, the phonon-phonon damping is proportional to the square of the frequency.[8] At the same time, the phonon-magnon damping changes either in proportion to $\omega^{7/4}$ (expression (11)), or in proportion to ω^2 (expression (20)). Therefore, at $(\tau_m\xi^2)^{-1} \lesssim \tau_{ph}^{-1}$, for frequencies $\omega \lesssim \tau_m^{-1}$ the phonon-magnon damping is the principal effect. This assertion is valid also for longitudinal sound. Taking into account the results of Shklovskiĭ,[7] we can conclude that at $(\tau_m\xi^2)^{-1} \lesssim \tau_{ph}^{-1}$ the sound attenuation is due to interaction with the magnons in the frequency range τ_{m-ph}^{-1}, $\tau_{ph-m}^{-1} \ll \omega \ll \tau_m^{-1}\xi^{3/2}$. At $\tau_{ph}^{-1} \lesssim (\tau_m\xi^2)^{-1}$ this region narrows: $(\tau_m\xi^2)^{-1} \lesssim \omega \lesssim \tau_m^{-1}\xi^{3/2}$.

We note that at $\omega\tau_m \ll 1$, the phonon-magnon damping falls off slowly with increase in temperature and at $1 \ll \omega\tau_m$ it increases ($\propto T^{13/2}$ at $\xi \gg 1$[7] and $\propto T\exp\{-\Theta_0^2/4T\Theta_c\}$ at $\xi \sim 1$).[5] Therefore, the temperature dependence of the phonon-magnon damping has a broad maximum at $\omega\tau_m \approx 1$. Just this behavior of the sound attenuation has been observed in yttrium iron garnet (YIG) at a frequency of $\nu = \omega/2\pi = 9$ MHz. The authors of Ref. 19 connected the broad maximum at $T = 210$ K with silicon impurities and possible inhomogeneities of the crystal. Although this region of temperatures corresponds to $\xi \approx 1$, which lies on the boundary of applicability of the results that we have obtained, it is interesting to observe that both the location of the maximum $T_{max}^{teor} \approx 200$ K, found under the condition $\omega\tau_m \sim 1$, and the order of magnitude of the quality factor $Q_{calc} = \omega/4\gamma(\omega) \approx 10^6$, obtained under the assumption that the sound attenuation is due to the interaction with magnons, are close to the experimental values. The experimental frequency dependence of the sound attenuation,[19] which is slower than ω^2, is also in agreement with this supposition.

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[1)]The EPR width in metals, with account of interaction with phonons, was considered by the diagram method.[14]

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