Theory of magnetic domains

V. I. Marchenko

Physical Problems Institute, USSR Academy of Sciences
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The set of boundary conditions is solved and the surface tension is found for inclined boundaries in uniaxial ferromagnets. The behavior of the Landau-Lifshitz domain structure is determined in an external magnetic field directed along the anisotropy axis. A structure is proposed for the intermediate state of antiferromagnets in the case in which one of the relativistic constants is small.

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As was shown by Landau and Lifshitz,[1] ferromagnetic materials divide into domains with different directions of the magnetic moment. In uniaxial crystals, the magnetic moment within the domains is directed "up" or "down" along the anisotropy axis. In a plate with the anisotropy axis perpendicular to its surface, the domains are parallel layers. If the anisotropy is weak, then where the layers emerge at the surface there are formed closure domains that prevent the occurrence of a strong magnetic field: the Landau–Lifshitz structure (Fig. 1a). When the anisotropy is large, it is not advantageous for the moments to deviate from the anisotropy axis: the Kittel structure (Fig. 1b).

Privorotskiĭ[2] determined the thermodynamic condition for coexistence of phases in any ferromagnetic materials. It turned out that this condition was not satisfied at the boundaries of the closure domains. It will be shown below that the contradiction can be removed by qualitative consideration of the domain structure with an arbitrary anisotropy constant.

The paper investigates boundaries inclined to the anisotropy axis. For them it is possible to find an exact solution of the system of boundary conditions in the case in which the moments of both phases lie in a plane perpendicular to the boundary and passing through the anisotropy axis. The result makes it possible to introduce a "microscopic" definition of the position of the wall; such a definition is necessary, as is well known,[3] in order to separate out the surface part of any thermodynamic quantity, including the surface part of the thermodynamic potential, which plays the role of surface tension of the domain boundary. The equations that describe the structure of an inclined wall can be reduced to the form of the equations for walls directed along the anisotropy axis, which have been well studied. Also clarified is the dependence of the period of the Landau–Lifshitz structure on an external magnetic field directed along the anisotropy axis.

The second part of the paper considers the structure of the intermediate state of antiferromagnets, which is analogous to the Landau–Lifshitz structure in ferromagnets and which should occur when one of the relativistic constants is small. The Appendix gives a derivation of the Privorotskiĭ boundary condition.

I. FERROMAGNETIC DOMAINS

1. Domain boundaries

We shall clarify what phases (domains) can coexist in the case in which the moments of both phases lie in a single plane (the xz plane, where z is the anisotropy axis) perpendicular to the boundary. It is convenient to use the potential Φ' introduced by Privorotskiĭ[2]:

$$\Phi'=\tilde{\Phi}+H_nB_n/4\pi. \tag{1}$$

H_n and B_n are the components normal to the boundary of the magnetic field **H** and of the induction **B**; $\tilde{\Phi}$ is the usual thermodyanmic potential ([4], § 36),

$$\tilde{\Phi}={}^1/_2\beta\sin^2\theta-\mathbf{HM}-H^2/8\pi, \tag{2}$$

M is the magnetic-moment density, β is the anisotropy constant, θ is the angle between **M** and the anisotropy axis. In this section we shall suppose that $|\mathbf{M}|=1$. On substituting (2) in (1), we get

$$\Phi'={}^1/_2\beta\sin^2\theta-\mathbf{H}_t\mathbf{M}_t-B_nM_n+2\pi M_n^2, \tag{3}$$

$\mathbf{H}_t$ and $\mathbf{M}_t$ are the tangential components of the field **H** and of the magnetic moment. We have omitted terms that depend on $\mathbf{H}_t$ and B_n, which do not change during passage across the boundary. Because the moments lie in a single plane, $\mathbf{H}_t$ differs from zero only in this plane. Taking this into account, we introduce the following notation:

$$d\sin 2\theta_0=-4\pi\sin 2\psi,\quad d\cos 2\theta_0=-\beta-4\pi\cos 2\psi,$$
$$H_t=cd\sin(A+\psi),\quad B_n=cd\cos(A+\psi), \tag{4}$$

ψ is the angle between the z axis and the boundary. In this notation, Φ' has the simple form

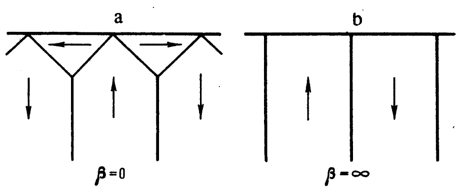

FIG. 1. Ferromagnetic domain structures: a, the Landau-Lifshitz when $\beta\to 0$; b, the Kittel when $\beta\to\infty$.

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$$\Phi'={}^1/_4 d\cos 2(\theta-\theta_0)-cd\sin(\theta+A). \quad (5)$$

The boundary conditions are expressed by the following equations:

$$d\Phi_1'/d\theta_1=0, \quad d\Phi_2'/d\theta_2=0, \quad \Phi_1'=\Phi_2'. \quad (6)$$

The first two are the stability conditions of phases 1 and 2; the third is Privorotskiĭ's thermodynamic condition[2] for the coexistence of magnetic phases. Thus by use of expression (5) for the potential Φ' we get the system of equations

$$\sin 2(\theta_1-\theta_0)+2c\cos(\theta_1+A)=0,$$
$$\sin 2(\theta_2-\theta_0)+2c\cos(\theta_2+A)=0,$$
$$\cos 2(\theta_1-\theta_0)-4c\sin(\theta_1+A)=\cos 2(\theta_2-\theta_0)-4c\sin(\theta_2+A).$$

By transformation to the variables $\theta_1+\theta_2$ and $\theta_1-\theta_2$, the solution is easily found:

$$\theta_1+\theta_2=2\theta_0=\operatorname{arctg}\frac{\sin 2\psi}{\cos 2\psi+\beta/4\pi}+\pi,$$

$$\cos\frac{\theta_1-\theta_2}{2}=\pm\left(\frac{H_t^2+B_n^2}{\beta^2+8\pi\beta\cos 2\psi+16\pi^2}\right)^{1/2} \quad (7)$$

Here H_t and B_n are related by the condition

$$H_t[4\pi+\beta\cos 2\psi+(\beta^2+8\pi\cos 2\psi+16\pi^2)^{1/2}]=\beta B_n\sin 2\psi.$$

The answers obtained must be investigated with respect to stability. In both phases we must have $d^2\tilde{\Phi}/d\theta^2>0$ and $H_x\sin\theta>0$. The latter inequality is the condition that $\tilde{\Phi}$ be a minimum with respect to departure of the moments from the xz plane. For arbitrary β too complicated expressions are obtained. For $\beta\to\infty$ all phases defined by the solution (7) are stable. For $\beta\to 0$ we obtain the simple condition

$$(1-c^2)^{1/2}\cos 2\psi\pm c\sin 2\psi>0.$$

Thus formation of boundaries inclined to the anisotropy axis at an angle greater than 45° is impossible, and the difference $\theta_1-\theta_2$ can take values within the limits 4ψ to $2\pi-4\psi$. We note that in the limiting cases $\beta\gg 4\pi$ and $\beta\ll 1$, the solution (7) coincides with the results obtained by Privorotskiĭ.[2]

Thus the moments in the coexisting domains are located symmetrically with respect to a straight line directed at angle θ_0 to the anisotropy axis. Hence it is clear that for the case being considered, it is natural to define the position of the mean boundary in such a way that the component of the magnetic moment of the boundary perpendicular to this straight line, and lying in the xz plane, shall be zero.

Since the potential Φ' is the same on both sides of the boundary, its surface part Δ' is simply the integral

$$\Delta'=\int_{-\infty}^{+\infty}(\Phi'-\Phi_\infty')\,d\xi, \quad (8)$$

the ξ axis is perpendicular to the boundary. By use of the relation (1) of the potential $\tilde{\Phi}$ to Φ', the surface part Δ of the potential $\tilde{\Phi}$ (that is, the surface tension) can be expressed in terms of Δ' and the normal component M_n^s of the magnetic moment of the boundary, which must be found on the basis of the definition of the mean boundary:

$$\Delta=\Delta'+B_nM_n^s. \quad (9)$$

The equations that describe the structure of the boundary can be found by variation of the potential (8). Here it is necessary to take into account the nonuniformity energy due to exchange interaction. It is convenient to introduce a spherical system of coordinates γ, η, whose polar axis is directed at angle θ_0 to the anisotropy axis and lies in the xz plane. In these variables, the potential (8) has the following form:

$$\int_{-\infty}^{+\infty}\left\{\frac{g}{2}\sin^2\gamma\sin^2\eta-\frac{d}{2}\sin^2\gamma+cd\sin(A+\theta_0)\cos\gamma+\frac{\alpha}{2}[(\gamma')^2+\sin^2\gamma(\eta')^2]\right\}d\xi, \quad (10)$$

α is the nonuniformity constant, and $2g=d+\beta-4\pi$. We see that in the variables γ and η, the potential differs from the well investigated case $\psi=0$ only in the value of the constants. Therefore we shall merely present several solutions. When $c=0$, the value of γ is constant and equal to $\pi/2$, and

$$\cos\eta=-\operatorname{th}(\xi/\delta), \quad \delta=(\alpha/g)^{1/2}. \quad (11)$$

If $\beta\gg 4\pi$, then $\delta=\delta_0=(\alpha/\beta)^{1/2}$; if $\beta\ll 4\pi$, then $\delta=\delta_0\cos^{-1}\psi$. When $c\neq 0$ and $\beta\ll 4\pi$, then the variation of the angle η, in the zeroth approximation with respect to $\beta/4\pi$, is again described by the expression (11), while the angle γ varies as follows:

$$\gamma=\arccos c-\frac{\beta}{2\pi}c(1-c^2)^{-1/2}\cos^2\psi\sin^2\eta. \quad (12)$$

We record the value of the surface tension Δ for this boundary:

$$\Delta=\Delta_0(1+\cos^2\gamma_\infty)\cos\psi, \quad (13)$$

$\Delta_0=2\beta\delta_0$ is the surface tension of the usual Bloch boundary, investigated macroscopically by Landau and Lifshitz[1]; γ_∞ is the value of the angle γ at infinity.

2. The Landau-Lifshitz domain structure

It would appear that the impossibility of existence of interdomain boundaries directed at angle 45° to the anisotropy axis, between phases $\theta_1=0$ and $\theta_2=\pi/2$, for $\beta\to 0$ contradicts the Landau–Lifshitz structure. But actually a transition from the state $\theta=0$ to the state $\theta=\pi/2$ can occur over macroscopic distances, dependent on the plate thickness L and small in comparison with the period a of the structure, to the degree that β is small. It is clear that for $\beta\sim 4\pi$, in the unbranched domain structure the walls emerge at the surface just as in the Kittel structure, but the moment is inclined at an appreciable angle to the anisotropy axis at distances of the order of a. For $\beta\to\infty$, the angle of inclination is small,

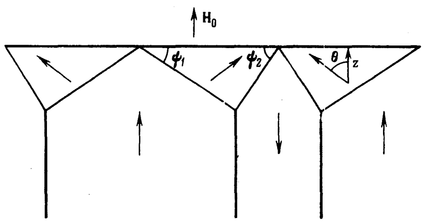

FIG. 2. The Landau-Lifshitz structure in an external magnetic field $H_0 \parallel z$.

and only in a small neighborhood of the point of emergence of the interdomain boundary is the moment inclined significantly, since in this place there develop strong fields ~ βM (at the point of emergence, $\theta = \pi/2$ for arbitrary β). For $\beta \to 0$, the condition of absence of fields larger than βM leads naturally, in the zeroth approximation in β, to the Landau-Lifshitz structure. In the next approximation, slight departures from Fig. 1 occur, and the transition between the primary and the closure domains occurs over macroscopic distances. These departures lead to the appearance of a magnetic field ~ βM, which stabilizes the structure.

We shall now consider the behavior of the Landau-Lifshitz domain structure in an external field H_0 directed along the anisotropy axis. It is clear that the structure must change as is shown in Fig. 2. The angles ψ_1, ψ_2, and θ, which describe the closure domains, are determined by the condition of continuity of M_n:

$$H_0 = 4\pi M \cos\theta, \quad \psi_1 = \theta/2, \quad \psi_2 = \pi/2 - \theta/2.$$

The concentration c^+ of domains with moment directed along the field is found from the condition that H_0 must equal the component normal to the plate surface of the mean induction **B** inside the plate; that is

$$H_0 = 4\pi c^+ M - 4\pi(1-c^+)M,$$

whence

$$c^+ = {}^1/_2 (1 + H_0/4\pi).$$

The volume of a closure domain is $\frac{1}{2}a^2 \sin\theta$. Thus the energy density in the plate is

$${}^1/_4\, \beta M^2 a \sin^3\theta + 2\beta M^2 \delta_0 L/a.$$

The first term is the energy of emergence of domains at the surface, the second the energy of the domain boundaries. On minimizing, we obtain the period

$$a = a_0[1 - (H_0/4\pi M)^2]^{-3/4}, \quad a_0 = 2(2\delta_0 L)^{1/2}. \tag{14}$$

II. THE INTERMEDIATE STATE OF ANTIFERROMAGNETS

A theory of the intermediate state of antiferromagnets, which occurs in a certain field interval near the sublattice-flip field H_c, has been developed by Bar'yakhtar, Borovik, and Popov.[5,6] They determined the surface tension at a boundary between phases in one of which the antiferromagnetic vector **l** is directed along the anisotropy axis, in the other perpendicular to this axis; they investigated a structure analogous to the Kittel structure in ferromagnets[6] and a branched structure.[5]

We shall write the energy density of an antiferromagnet in the form

$$\frac{1}{2\chi_\perp}[\mathbf{lM}]^2 + \frac{1}{2\chi_\parallel}(\mathbf{lM})^2 + \frac{\beta}{2}(M_x^2 + M_y^2) + \frac{\rho}{2}(l_x^2 + l_y^2) - \mathbf{HM} - \frac{H^2}{8\pi}. \tag{15}$$

The first two terms correspond to the exchange energy; $\chi_\perp$ and $\chi_\parallel$ are the transverse and longitudinal susceptibilities in the exchange approximation; β and ρ are relativistic constants. When $\beta > 0$, then during a change of the external magnetic field H_0 from H_{c1} to H_{c2}, where

$$H_{c1} = H_c(1 + 4\pi\chi_\parallel), \quad H_{c2} = H_c(1 + 4\pi\chi_\perp), \tag{16}$$

antiferromagnets divide into domains with different directions of the vector **l**: $\mathbf{l} \parallel z$ and $\mathbf{l} \perp z$. The field **H** inside the domains is directed along the z axis and is equal to the flip field H_c:

$$H_c^2 = \rho l^2/(\chi_\perp - \chi_\parallel). \tag{17}$$

The concentrations $c_\perp$ and $c_\parallel$ of the phases are determined by the condition of continuity of the magnetic flux and are

$$c_\perp = (H_0 - H_{c1})/(H_{c2} - H_{c1}),$$
$$c_\parallel = (H_{c2} - H_0)/(H_{c2} - H_{c1}). \tag{18}$$

When $\beta \ll 4\pi$, in perfect analogy to the case of ferromagnets, the structure should have the form shown in Fig. 3. In the closure domains, the z component of the magnetic moment is determined by the condition

$$H_0 = H_c(1 + 4\pi\chi_{zz}), \tag{19}$$

χ_{zz} is the zz component of the magnetic susceptibility tensor, which depends on the angle θ of inclination of the vector **l** to the z axis as follows:

$$\chi_{zz} = \chi_\parallel \cos^2\theta + \chi_\perp \sin^2\theta.$$

On substituting this value of χ_{zz} in (19), we find the angle θ:

$$\cos 2\theta = c_\parallel - c_\perp.$$

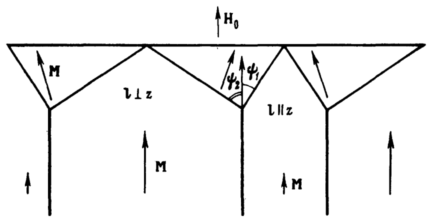

FIG. 3. Structure of the intermediate state of antiferromagnets when $\beta \to 0$.

The component M_x of the magnetic moment is determined by the xz component of the susceptibility tensor, $M_x = \chi_{xz} H_c$, and

$$\chi_{xz} = -(\chi_\perp - \chi_\parallel) \sin\theta \cos\theta.$$

We find the angles ψ_1 and ψ_2 from the condition of continuity of the normal component of the magnetic moment:

$$\operatorname{ctg}\psi_1 = \operatorname{tg}\psi_2 = (c_\perp/c_\parallel)^{1/2}.$$

We now have everything that is needed in order to calculate the energy density of the plate as a function of the period of the structure:

$$\tfrac{1}{8}\rho\beta l^2(\chi_\perp - \chi_\parallel)(c_\perp c_\parallel)^{1/2} a + \Delta L/a. \qquad (20)$$

The quantity $2\Delta = (\chi_\perp - \chi_\parallel)\rho\beta l^2 \delta$ is the surface tension of a boundary between phases $\mathbf{l} \parallel z$ and $\mathbf{l} \perp z$; $\delta^2 = \alpha/\rho\beta(\chi_\perp - \chi_\parallel)$ is the "thickness" of this boundary[5]; α is the nonuniformity constant. On minimizing the energy, we find the period

$$a = 2(\delta L)^{1/2}(c_\perp c_\parallel)^{-1/4}.$$

It is interesting that this expression agrees exactly with formula (14) for ferromagnets, if in the latter we introduce, instead of the field H_0, the concentrations of the phases with magnetic moment directed along the field and opposite to the field.

Here we shall present, for comparison, the formula for the period of the structure considered in Ref. 6, when $\beta \gg 4\pi$,

$$a = \frac{\pi}{2}\left(\frac{\beta\delta L}{f(c_\perp)}\right)^{1/2}, \quad f(c_\perp) = \sum_{n=1}^{\infty} n^{-3} \sin^2 \pi c_\perp n,$$

and its limiting expressions

$$c \to 0, \quad a = \frac{1}{2c}\left(\frac{\beta\delta L}{-\ln c}\right)^{1/2},$$

here $c = c_\perp$ or $c_\parallel$. These expressions are correct as long as $a \ll L$. The problem can be solved also, quite analogously, when $a \sim L$. We shall present only the value of the period when $a \gg L$,

$$a = \frac{1}{c}\left(\frac{\beta\delta L}{2\ln(L/\beta\delta)}\right)^{1/2}.$$

The formula is correct with logarithmic accuracy in $L/\beta\delta$. When the plate thickness exceeds a certain critical L_{cr}, branching becomes advantageous. The value of L_{cr} can be found by comparison of the energy density of a structure of the Kittel type[6] with the energy of the branched structure treated in Ref. 5; we get

$$L_{cr} = \beta\delta\chi_\perp^{-2}.$$

Since $\delta \sim 10^{-5}$ cm, $\chi_\perp \sim 10^{-3}$, and $\beta \gg 1$, L_{cr} considerably exceeds one centimeter; that is, the unbranched structure should usually occur.

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APPENDIX

We consider the change of the thermodynamic potential $\tilde{\Phi}$ during an infinitely small change of the boundary between two magnetic phases:

$$\delta\tilde{\Phi} = \int (\tilde{\Phi} - \tilde{\Phi}_0)\, dV.$$

Here the integration extends over all space; $\tilde{\Phi}_0$ and $\tilde{\Phi}$ are the densities of the thermodynamic potential before and after, respectively, the variation. We separate out from this expression the integral over the volume δV where a change of phase occurred:

$$\delta\tilde{\Phi} = \int_{-\delta V} (\tilde{\Phi} - \tilde{\Phi}_0)\, dV + \int_{\delta V} (\tilde{\Phi} - \tilde{\Phi}_0)\, dV. \qquad (A.1)$$

The first integral extends over all of space except δV. Since there small changes occur, and in particular the magnetic field $\mathbf{H}$ changes slightly, the difference $\tilde{\Phi} - \tilde{\Phi}_0$ can be expressed as follows (see Ref. 4):

$$\tilde{\Phi} - \tilde{\Phi}_0 = -\mathbf{B}\delta\mathbf{H}/4\pi.$$

By virtue of the equations $\operatorname{div}\mathbf{B} = 0$ and $\operatorname{curl}\delta\mathbf{H} = 0$, the integral of $\mathbf{B}\delta\mathbf{H}$ over all space is zero; therefore the first integral in (A.1) can be transformed to an integral over the volume δV:

$$\delta\tilde{\Phi} = \int_{\delta V} \left(\tilde{\Phi} - \tilde{\Phi}_0 + \frac{\mathbf{B}\delta\mathbf{H}}{4\pi}\right) dV.$$

Finally, retaining in this expression terms linear in the boundary deviation f, we find

$$\delta\tilde{\Phi} = \int \left(\tilde{\Phi}_1^0 - \tilde{\Phi}_2^0 + \frac{B_n^0 H_{n1}^0}{4\pi} - \frac{B_n^0 H_{n2}^0}{4\pi}\right) f\, dS. \qquad (A.2)$$

We have replaced integration over the volume δV by integration over the area of the boundary; the indices 1 and 2 denote quantities related to the different phases. In a state of thermodynamic equilibrium, the change of potential (A.2) must vanish for an arbitrary variation f; therefore

$$\tilde{\Phi}_1^0 + \frac{B_n^0 H_{n1}^0}{4\pi} = \tilde{\Phi}_2^0 + \frac{B_n^0 H_{n2}^0}{4\pi}.$$

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