On spontaneously emitted photons and their multiplication by means of a quantum mechanical amplifier

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Analysis is made of the passage of photons through an amplifying medium. It is concluded that as a result of amplification of spontaneous radiation by a quantum mechanical amplifier no additional correlations arise and therefore no singularities appear in the intensity fluctuation spectrum.

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In the paper by Aleksandrov, Kozlov and Kulyasov[1] the thought was expressed concerning the possible presence of excess correlations in spontaneous radiation from a resonant medium. Preliminary calculations in which each of the photons was considered phenomenologically in the form of a train of electromagnetic waves seemed to support this. They showed that the spectrum of intensity fluctuations must have a special feature characteristic of the radiating medium (which would enable us to determine the width of the radiating stage). But this conclusion was refuted by experiment and by more exact calculations (a consistent theoretical analysis on the basis of quantum electrodynamics has been carried out in a recent paper by Smirnov and Sokolov[2]). It was shown sufficiently convincingly that if the excess correlations are not introduced in some manner from the outside (for example, by a system of excited atoms), then they can not be present in spontaneous radiation.

The circumstance that the phenomenological calculation predicted an incorrect result is in the opinion of the authors of the references[1,2] cited above explained by the fact that in being recorded by a photon receptor each photon in this calculation could participate in many sequential acts of absorption leading to a correlation between photoelectrons during times of the order of magnitude of the lifetime of the atom, and this contradicts the concept of the indivisibility of photons. Thus, it was shown that in an experiment on the investigation of the spectrum of intensity fluctuations of spontaneous radiation the model of a photon in the form of a train of electromagnetic waves is invalid. But from this it does not yet follow that in arbitrary other experimental situations the phenomenological model will turn out to be equally incorrect. For example, in passage though an amplifying medium it is possible that a photon should be treated in just this manner. Then the train of electromagnetic waves appearing at the output of the amplifier (corresponding to one photon at the input of the amplifier, but to a large number of photons at the output) even from the point of view of quantum electrodynamics can already participate in many sequential acts of absorption at the photocathode leading to characteristic correlations between photoelectrons. But also here experiment did not confirm this conclusion.[3] In what follows we show theoretically that it is incorrect.

We consider the problem with a single photon which on being radiated by an atom is multiplied into a large number of photons similar to itself by means of a quantum mechanical amplifier. We shall analyze the radiation at the output of the amplifier by means of a two-atom detector, i.e., we investigate its spectrum of intensity fluctuations. We shall obtain a continuous spectrum and this testifies to the fact that the photon recorder reacts to the "multiplied photon" in exactly the same manner as to a single photon, by a delta-pulse of photocurrent. Thus, the spectrum of the amplified spontaneous radiation must not have any special features compared to the unamplified radiation.

PASSAGE OF A PHOTON THROUGH AN AMPLIFYING MEDIUM

For our purposes the most convenient formalism is the one involving the space-time density matrix for the electromagnetic field.[4] In the neighborhood of each point in space (the linear dimensions of the neighborhood are much larger than a wavelength: $l \gg \lambda$) one can introduce a set of field oscillators. An oscillator with the propagation vector $\mathbf{k}$ at the point $\mathbf{r}$ at the time instant t (we have in mind neighborhoods of the points $\mathbf{r}$ and $\mathbf{k}$) can be described by the density matrix $\rho_\mathbf{k}(\mathbf{r}, t)$. In those cases when all the oscillators at a given point are independent of each other, the state of the field at that point is determined by the matrix

$$\rho(\mathbf{r}, t) = \prod_\mathbf{k} \rho_\mathbf{k}(\mathbf{r}, t)$$

($\mathbf{k}$ are the eigenvectors of the space cell of linear dimensions l, the distance between neighbors is $2\pi/l$). For $\rho_\mathbf{k}$ the following equation can be written

$$\frac{\partial \rho_\mathbf{k}}{\partial t} + \frac{\mathbf{k}}{k}\frac{\partial \rho_\mathbf{k}}{\partial \mathbf{r}} = \left(\frac{\partial \rho_\mathbf{k}}{\partial t}\right)_1. \quad (1)$$

(In this paper the system of units has been adopted for which $\hbar = c = 1$). Here the term second from the left determines the transfer of radiation. From an energy point of view the presence of this term is quite comprehensible and necessary. However, in equation (1) there is present not only a transfer of energy, determined by the diagonal elements of the density matrix, but also a transfer of phase determined by the non-diagonal elements of the density matrix. As a result of this a special justification of Eq. (1) was required.

The right hand side of equation (1) determines the variation of the field at the point $\mathbf{r}$ due to some other

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causes. For example, in the presence of an amplifying resonance medium it acquires the form

$$(\partial\rho_k/\partial t)_1 = -\tfrac{1}{2}\Gamma_k(a_k a_k^+\rho_k - 2a_k^+\rho_k a_k + \rho_k a_k a_k^+). \qquad (2)$$

This expression can be obtained in the same manner as was done, for example, in the paper by Lamb and Scully.[5] The coefficient Γ_k depends on the properties of the amplifying medium. In regions where there is no medium it is equal to zero. In order for (2) to be valid it is necessary to assume that the field varies sufficiently slowly, in particular one must neglect the variation of the field over times of the order of the lifetime of the amplifying atoms. Moreover, it is necessary to assume that the excitation of atoms in the amplifying medium has a regular character. This enables one to replace in the equation the summation over atoms by integration over the instant of excitation as a result of which Γ_k does not vary in time.

As the boundary condition for Eq. (1) with the right hand side given by (2) (in future we shall write: for the equation (1.2)) we take the density matrix describing the photon emitted by the source atom. Its form can be written based on most general considerations. It is apparently clear that it is a superposition of two states: the vacuum state $|\ldots\rangle\langle\ldots|$ (all the field oscillators are in the ground state) and the single-photon state $|\ldots 1_k\ldots\rangle\langle\ldots 1_k\ldots|$ (the k-th oscillator is in the first excited state, while all the others are in the ground state). Thus, for the field emitted by the atom we can write the following:

$$\rho_k(\mathbf{r},t) = \Big(1 - \sum_k \varphi_k(\mathbf{r},t)\Big)|\ldots\rangle\langle\ldots| + \sum_k \varphi_k(\mathbf{r},t)|\ldots 1_k\ldots\rangle\langle\ldots 1_k\ldots|. \qquad (3)$$

The absence in the superposition of terms of the type $|0\rangle\langle 1|$ is related to the fact that spontaneous radiation is of the nature of noise, and therefore the average intensity of the electric field for it must be equal to zero. This is possible only in the absence of terms of the type indicated above. Moreover, we do not take into account multi-quantum decays.

The characteristic time dependence of the function $\varphi_k(\mathbf{r},t)$ is determined by the lifetime of the atom. Thus, we must require in accordance with what has been said previously concerning equation (1.2) that the lifetime of the initial atom should be much greater than the lifetime of the amplifying atoms. Equation (1.2) with the boundary condition (3) is easily solved

$$\rho(\mathbf{r},t) \approx \prod_k \rho_k(\mathbf{r},t), \quad \rho_k(\mathbf{r},t) = \sum_{n=0}^{\infty} \rho_{nn}^{(k)}(\mathbf{r},t)|n\rangle\langle n|,$$

$$\rho_{nn}^{(k)} = R_n^{(k)} + R_n^{(k)}\varphi_k(0,\Omega_r,t-r)(n/\sigma_k - 1). \qquad (4)$$

Here $\varphi_k(\mathbf{r},t)$ is written in the form $\varphi_k(\mathbf{r},\Omega_r,t)$; the quantity $R_n^{(k)}$ is the Bose-Einstein equilibrium distribution

$$R_n^{(k)} = \sigma_k{}^n(1+\sigma_k)^{-n-1}, \quad \sigma_k = \exp(\Gamma_k r) - 1, \quad \Gamma_k = \mathbf{k}\Gamma_k/k.$$

The sign of approximate equality in (4) denotes that in multiplying different matrices ρ_k one must retain only terms independent of φ_k and those proportional to the first power of φ_k.

THE SPECTRUM OF INTENSITY FLUCTUATIONS OF RADIATION AT THE OUTPUT OF THE AMPLIFIER

The spectrum of intensity fluctuations is calculated by means of the formula[2,6]

$$I(\omega) = \int_{-\infty}^{\infty} dt \int_0^{\infty} d\tau \iint_{(S)} \frac{ds_1 ds_2}{S^2} e^{-i\omega\tau}.$$

$$\langle E^+(\mathbf{r}_1,t)E^+(\mathbf{r}_2,t+\tau)E(\mathbf{r}_2,t+\tau)E(\mathbf{r}_1,t)\rangle + \text{c.c.}$$

(This signal is observed against the background of shot noise.) Here S is the surface of the photocathode, $E(\mathbf{r},t)$ is the positive frequency operator for the intensity of the electromagnetic field in the Heisenberg representation. In our discussion it has the form[4]

$$E(\mathbf{r},t) \approx \sum_k i\left(\frac{k}{2l^3}\right)^{1/2} e^{i\mathbf{k}\mathbf{r}} a_k(\mathbf{r},t).$$

We can then write the following:

$$\langle E^+(\mathbf{r}_1,t)E^+(\mathbf{r}_2,t+\tau)E(\mathbf{r}_2,t+\tau)E(\mathbf{r}_1,t)\rangle$$
$$= \sum_{kk'} \frac{kk'}{(2l^3)^2}[\langle a_k^+(\mathbf{r}_1,t)a_{k'}^+(\mathbf{r}_2,t+\tau)a_{k'}(\mathbf{r}_2,t+\tau)a_k(\mathbf{r}_1,t)\rangle$$
$$+ \exp(i(\mathbf{k}'-\mathbf{k})(\mathbf{r}_1-\mathbf{r}_2))\langle a_k^+(\mathbf{r}_1,t)a_{k'}^+(\mathbf{r}_2,t+\tau)a_k(\mathbf{r}_2,t+\tau)a_{k'}(\mathbf{r}_1,t)\rangle].$$

All the other terms are omitted because spontaneous radiation is random with respect to phase, while the angular brackets, in particular, denote averaging over phase. Therefore only those terms differ from zero, in which the phase is completely cancelled out within the angular brackets. It is not difficult to verify that cancellation occurs only in the terms indicated above.

This conclusion can be confirmed by exact calculations. There exists one more term which we have not written down although the phase in it does cancel out:

$$\sum_k \left(\frac{k}{2l^3}\right)^2 \langle a_k^+ a_k^+ a_k a_k\rangle.$$

The point is that its order of smallness ($\sim(\lambda/l)^3$) does not allow us to take it into account. However, even if we did take it into account it would have given us a contribution of no interest to us at the zero frequency of the spectrum of intensity fluctuations.

The desired correlators can be most simply written down in the diagonal representation of the density matrix[6]:

$$\lambda_1 = \langle a_k^+(\mathbf{r}_1,t)a_{k'}^+(\mathbf{r}_2,t+\tau)a_k(\mathbf{r}_2,t+\tau)a_{k'}(\mathbf{r}_1,t)\rangle$$
$$= \int d^2\alpha_k\, d^2\alpha_k'\, d^2\alpha_{k'}\, d^2\alpha_{k'}'\, \alpha_k^*\alpha_{k'}'^*\alpha_k'\alpha_{k'}$$
$$\times W_{kk'}(\alpha_k,\alpha_{k'},\mathbf{r}_1,t;\alpha_k',\alpha_{k'}',\mathbf{r}_2,t+\tau)\exp[i(k'-k)\tau].$$

$$\lambda_2 = \langle a_k^+(\mathbf{r}_1,t)a_{k'}^+(\mathbf{r}_2,t+\tau)a_{k'}(\mathbf{r}_2,t+\tau)a_k(\mathbf{r}_1,t)\rangle$$
$$= \int d^2\alpha_k d^2\alpha_{k'} d^2\alpha_k' d^2\alpha_{k'}' |\alpha_k|^2|\alpha_{k'}'|^2 W_{kk'}(\alpha_k,\alpha_{k'},\mathbf{r}_1,t;\alpha_k',\alpha_{k'}',\mathbf{r}_2,t+\tau).$$

Here $W_{kk'}$ is the two-mode "two-point" density matrix in the diagonal representation.

Since we are dealing with a Markov process (we succeed in writing a closed equation for the field) and the points $\mathbf{r}_1$ and $\mathbf{r}_2$ are situated outside the medium, then the two-point probability $W_{kk'}$ can be written in terms of the one-point probability $P_{kk'}$:

$$W_{\mathbf{kk}'}(\alpha_\mathbf{k}, \alpha_{\mathbf{k}'}, \mathbf{r}_1, t; \alpha_\mathbf{k}', \alpha_{\mathbf{k}'}', \mathbf{r}_2, t+\tau)$$
$$=P_{\mathbf{kk}'}(\alpha_\mathbf{k}, \alpha_{\mathbf{k}'}, \mathbf{r}_1, t)\delta^{(2)}(\alpha_\mathbf{k}-\alpha_\mathbf{k}')\delta^{(2)}(\alpha_{\mathbf{k}'}-\alpha_{\mathbf{k}'}').$$

Then one can carry out a part of the integrations and obtain the following equation:

$$\lambda_1=\lambda_2\exp[i(k'-k)\tau]=\exp[i(k'-k)\tau]\,\mathrm{Sp}\,[a_\mathbf{k}^+a_\mathbf{k}a_{\mathbf{k}'}^+a_{\mathbf{k}'}\rho_{\mathbf{kk}'}(\mathbf{r}_1, t)].$$

The matrix $\rho_{\mathbf{kk}'}$ is related to its diagonal representation $P_{\mathbf{kk}'}$ by means of the integral relation

$$\rho_{\mathbf{kk}'}(\mathbf{r}_1, t)=\iint d^2\alpha_\mathbf{k}d^2\alpha_{\mathbf{k}'}P_{\mathbf{kk}'}(\alpha_\mathbf{k}, \alpha_{\mathbf{k}'}, \mathbf{r}_1, t)\,|\alpha_\mathbf{k}\alpha_{\mathbf{k}'}\rangle\langle\alpha_\mathbf{k}\alpha_{\mathbf{k}'}|,$$
$$a_\mathbf{k}|\alpha_\mathbf{k}\rangle=\alpha_\mathbf{k}|\alpha_\mathbf{k}\rangle,\quad |\alpha_\mathbf{k}\alpha_{\mathbf{k}'}\rangle=|\alpha_\mathbf{k}\rangle|\alpha_{\mathbf{k}'}\rangle,$$
$$d^2\alpha_\mathbf{k}=d\,\mathrm{Re}\,\alpha_\mathbf{k}\cdot d\,\mathrm{Im}\,\alpha_\mathbf{k}.$$

With the aid of formula (4) one can write

$$\langle E^+(\mathbf{r}_1, t)E^+(\mathbf{r}_2, t+\tau)E(\mathbf{r}_2, t+\tau)E(\mathbf{r}_1, t)\rangle=\sum_{\mathbf{kk}'}I_{\mathbf{kk}'}(\mathbf{r}_1, \mathbf{r}_1, t)$$
$$\times[1+\exp(i(\mathbf{k}'-\mathbf{k})(\mathbf{r}_1-\mathbf{r}_2)+i(k'-k)\tau)],$$

where the quantity $I_{\mathbf{kk}'}$ for large amplifications ($\Gamma_k\mathbf{r}\gg 1$) has the form

$$I_{\mathbf{kk}'}(\mathbf{r}_1, \mathbf{r}_2, t)=\frac{kk'}{4l^6}\exp(\Gamma_k\mathbf{r}_1+\Gamma_{\mathbf{k}'}\mathbf{r}_2)[1+\varphi_\mathbf{k}(0, \Omega_{r_1}, t-r)+\varphi_{\mathbf{k}'}(0, \Omega_{r_2}, t-r)],$$

and in the complete absence of amplification ($\Gamma_\mathbf{k}=0$) is equal to zero. The absence of a signal without amplification reflects only the fact that one photon can not ionize two atoms of the photocathode, as is required in the scheme of measurements with a two-atom detector.

In the absence of an external photon ($\varphi_\mathbf{k}=0$) we shall obtain the fluctuation spectrum of spontaneous radiation from the amplifier. Corresponding to our conditions this will be a broad Doppler background. The appearance of an external photon will not alter this qualitative behavior. The point is that the dependence of the coefficient Γ_k on k is much smoother than the dependence of the coefficient φ_k on k. This is associated with our initial requirement that the lifetime of the atom—the source of the initial photon—must be many times greater than the lifetime of the amplifying atoms. Thus, in summing over $\mathbf{k}$ and $\mathbf{k}'$ the spectral behavior of $\varphi_\mathbf{k}$ is simply integrated and does not give a qualitative contribution to the spectrum. This is already sufficient in order for us to draw the conclusion that the amplified radiation does not have any excess correlations.

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[1]E. B. Aleksandrov, V. P. Kozlov and V. N. Kulyasov, Zh. Eksp. Teor. Fiz. 66, 1269 (1974) [Sov. Phys. JETP 39, 620 (1974)].

[2]D. F. Smirnov and I. V. Sokolov, Zh. Eksp. Teor. Fiz. 70, 2098 (1976) [Sov. Phys. JETP 43, 1095 (1976)].

[3]D. B. Scare and Stephen R. Smith, Phys. Rev. A10, 709 (1974).

[4]Yu. M. Golubev, Zh. Eksp. Teor. Fiz. 65, 466 (1973)[Sov. Phys. JETP 38, 228 (1974)].

[5]W. E. Lamb and M. O. Scully, Phys. Rev. 69, 208 (1967).

[6]R. Glauber, translation in: Kvantovaya optika i kvantovaya radiofizika (Quantum Optics and Quantum Radiophysics), ed. by O. V. Bogdankevich and O. N. Krokhin, Mir, 1966, p. 93.

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