Influence of nonlocality effects on van der Waals interaction

E. I. Kats

L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences

(Submitted December 2, 1976)
Zh. Eksp. Teor. Fiz. 73, 212–220 (July 1977)

The van der Waals interaction between two macroscopic bodies (metals or dielectrics) is calculated. Nonlocality effects due to spatial dispersion of the permittivity or to inhomogeneity of the fluctuation field at characteristic distances of the order of the electron mean free path (anomalous skin effect) are taken into account. A similar analysis is carried out for cholesteric liquid crystals. A relation is found between the cholesteric-helix pitch and the molecular optical activity.

PACS numbers: 61.30.–v, 73.20.–r

I. INTRODUCTION

In recent years, numerous attempts have been made to generalize the macroscopic theory of van der Waals forces[1] to include a system with nonlocal dielectric properties (see, for example, the work of Chan and Richmond[2]; there is also a detailed bibliography on this subject in the review of Barash and Ginzburg[3]). However, in most cases, these generalizations were carried out with a patent exaggeration of the accuracy, or else the final formulas were used outside the range of applicability of the initial expansions of the permittivity tensor (for more detail, see below). In order to make this clear, we briefly recall the method of calculation of the van der Waals contribution to the free energy of a macroscopic body.[1]

The radiation Green's function in a medium satisfies the Dyson equation (see Fig. 1, where the shaded loop is the polarization operator Π_{ik} and the thin dashed line is the radiation Green's function in a vacuum). Upon complete neglect of the nonlocality, the polarization operator is expressed directly in terms of the permittivity ε_{ik}. In this approximation, all diagrams of the type shown in Fig. 2 are assumed to be unimportant. The smallness of these nonlocal contributions to the radiation Green's function is determined by the ratio of the characteristic interatomic distance a to the characteristic electromagnetic wavelength λ. However, account of nonlocality due to spatial dispersion corresponds to account of corrections $\sim a/\lambda$ (in a gyrotropic medium) or $(a/\lambda)^2$ (in a nongyrotropic one) in the permittivity tensor. Therefore the simple replacement of $\Pi_{ik}(\mathbf{r}_1, \mathbf{r}_2) \propto \varepsilon_{ik}(\mathbf{r}_1)\,\delta(\mathbf{r}_1 - \mathbf{r}_2)$ by the corresponding nonlocal function (with account of spatial dispersion) $\varepsilon_{ik}(\mathbf{r}_1, \mathbf{r}_2)$ is a patent exaggeration of the accuracy. This replacement does not take into account some of the terms of order $(a/\lambda)^2$. Thus, the problem of the calculation of the van der Waals contribution to the free energy of systems with nonlocal dielectric properties requires a concrete consideration of each given system. In certain cases, for example, collionless plasma,[3] the simple substitution mentioned above is legitimate. Another example – cholesteric liquid crystals – will be considered in the second section of the present work.

Much further progress can be made if we are interested not in the total van der Waals energy of an inhomogeneous system, but only in the force of interaction of two macroscopic bodies (the stress tensor). The basic physical characteristic by which the van der Waals force is determined in this case is the reflection coefficient of electromagnetic waves from the surface of the body. Knowing the reflection coefficient, for example from optical experiments, we can in principle calculate the van der Waals force of the interaction.

One more circumstance should be noted. The effects of spatial dispersion are always small. Therefore, the consideration of physical problems with account of nonlocality is valid only when we are talking about new phenomena, which would not exist without account, for example, of spatial dispersion, and not about the calculation of small corrections to the usual formulas. In the present work, the new effect, absent in local systems, is the possibility of a nonmonotonic dependence of the force on the characteristic parameters of the problem (the distance between the bodies or the magnetic field).

In the third part of the work, we consider the interaction between metallic plates located in a vacuum or an arbitrary nondispersive medium. The reflection coefficient is determined by the surface impedance of the plates. In the presence of an external magnetic field, the value of the impedance, together with the value of the van der Waals force, depends on the field in nonmonotonic fashion. These corrections are important only at distances that are large in comparison with the characteristic wavelength in the spectrum of the bodies, $d \gg \lambda_0$ (for more detail, see Ref. 1). At small distances, it is necessary to take into account the dependence of the surface impedance on the components of the wave vector in a direction q parallel to the surface. This range $d \ll \lambda_0$ corresponds to large wave vectors $q \gg \lambda_0^{-1}$. Here the expansion of the permittivity in powers of q, used, for example, by Chan and Richmond,[2] becomes invalid. At $q \gg \lambda_0^{-1}$, the problem can be analyzed only by using a model. We shall discuss this question very briefly.

In the fourth part, we consider the interaction between two dielectric plates separated by a vacuum. With al-

FIG. 1.

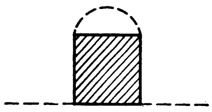

FIG. 2.

lowance for spatial dispersion, the reflection coefficient is determined with the help of additional boundary conditions.[4] We shall assume for concreteness that there is only a single dipole resonance, near which the spatial dispersion is significant (in the presence of several resonances, the corresponding contributions should be simply summed). The contribution to the force due to spatial dispersion exists only in a narrow range of distances (corresponding to the frequency width of the dipole resonance). The possibility of nonmonotonic dependence of the force on the distance is connected with this fact.

The final section of the work is devoted to estimates and means of experimental observation of the considered effects.

2. HELIX PITCH IN CHOLESTERIC LIQUID CRYSTALS

As is well known,[5] the appearance of a helix in the phenomenological theory of cholesteric liquid crystals, i.e., a component $Q_0\mathbf{n}$, curl $\mathbf{n}$ that is linear in the gradient of the director $\mathbf{n}$, is connected with the absence of a center of inversion, i.e., molecular optical activity. It is natural to attempt to obtain a relation between these quantities. Dzyaloshinskii, Dmitriev and the author[6] have shown that the van der Waals forces make a significant contribution to the elastic modulus of a nematic liquid crystal. It should therefore be expected that the van der Waals contribution will correctly determine also the value of the linear component Q_0.

It is therefore necessary to calculate the van der Waals energy with account of spatial dispersion. The difficulty mentioned in the Introduction does not allow us to carry out such calculations in general form. However, the case of cholesteric liquid crystals (and of gyrotropic media in general has certain singularities that facilitate the problem. Namely, if we calculate the correction to the radiation Green's function by perturbation theory in a/λ, then by virtue of symmetry conditions the first correction (of the type of the diagram shown in Fig. 2) will be of order $(a/\lambda)^2$. Therefore, if we are interested only in components of order a/λ, then we can make the already mentioned simple substitution in the formula for Π_{ik}.

Thus we have the following equation for the Green's function of the electric field with account of spatial dispersion:

$$\omega^2\int \varepsilon_{il}(\mathbf{r}.\mathbf{r}-\mathbf{r}'')D_{lk}(\mathbf{r}'',\mathbf{r}')\,d^3r''+\mathrm{rot}_{im}\,\mathrm{rot}_{ml}D_{lk}(\mathbf{r}.\mathbf{r}')=4\pi\omega^2\delta(\mathbf{r}-\mathbf{r}')\,\delta_{ik}. \tag{1}$$

The written-out argument of the permittivity takes into account both the inhomogeneity of the state and the spatial dispersion. It is convenient to separate the components that are inhomogeneous in the orientations of the director:

$$\varepsilon_{il}(\mathbf{r},\ \mathbf{r}-\mathbf{r}'')=\varepsilon_{il}{}^{0}(\mathbf{r}-\mathbf{r}'')+\varepsilon_{im}{}^{a}(\mathbf{r}-\mathbf{r}'')N_{ml}(\mathbf{r}), \tag{2}$$

where

$$N_{ml}(\mathbf{r})=n_m(\mathbf{r})n_l(\mathbf{r}). \tag{3}$$

Assuming that the anisotropic components ε^a_{im} are small in comparison with ε^0_{il}, we can calculate D_{ik} by perturbation theory. Then, as before,[6] we can find also the van der Waals energy in the necessary approximation. But, this calls for knowledge of the explicit forms of $\varepsilon^0_{il}(\mathbf{r}-\mathbf{r}'')$ and $\varepsilon^a_{im}(\mathbf{r}-\mathbf{r}'')$.

For a gyrotropic medium, as is known,[7] components with first derivatives of the field with respect to the coordinates are taken into account in the permittivity tensor. Transforming to Fourier components we can write

$$\varepsilon_{ik}=\varepsilon\delta_{ik}+i\gamma_{ikl}q_l. \tag{4}$$

The Onsager relations require only the antisymmetry of γ_{ikl} in the first two indices. To specify things more concretely, it is convenient to introduce the dual tensor of second rank:

$$\gamma_{ikl}=e_{ikm}g_{ml} \tag{5}$$

(e_{ikm} is a completely antisymmetric unit tensor). Both the characteristic optical activity, i.e., the interaction of the molecular electric and magnetic dipoles, and the interaction of the electric quadrupole and dipole make contributions to the value of g_{ik}.

We begin with consideration of the gas approximation: a weak solution of optically active molecules. The symmetry of the corresponding molecular quantities can easily be determined in each specific case (see Ref. 7). For example, for molecules of symmetry C_4,

$$g_{ik}=\begin{pmatrix} g_\perp & 0 & 0\\ 0 & g_\perp & 0\\ 0 & 0 & g_\parallel \end{pmatrix}, \tag{6}$$

where the subscripts refer to the molecular system of coordinates. We note also, for comparison, the symmetry S_4:

$$g_{ik}=\begin{pmatrix} g_1 & 0 & 0\\ 0 & -g_1 & 0\\ 0 & 0 & 0 \end{pmatrix}. \tag{7}$$

After this, the molecular tensors g_{ik} should be averaged over the orientation of the molecules. Thus, for a cholesteric liquid crystal, it is simply necessary to transform to the laboratory system of coordinates by means of the corresponding Euler angles (α,β,γ), such that $\langle\cos\alpha\rangle=0$ (absence of biaxiality), $\beta=q_0z$ (cholesteric order, q_0 = pitch of the helix), $\gamma=\pi/2$. Thus, we obtain in the symmetry C_4

$$g_{ik}=g_0\delta_{ik}+g_an_in_k, \tag{8}$$

where

$$g_0=g_\perp,\quad g_a=g_\parallel-g_\perp.$$

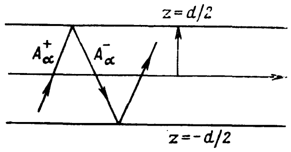

FIG. 3

Similar averaging for the symmetry S_4 gives $g_{ik}=0$. Of course, it should be kept in mind that all the conclusions reached above apply only to weak solutions of optically active molecules in a nongyrotropic medium. However, it is clear from symmetry considerations that formula (8) will be valid in any gyrotropic uniaxial medium (the quantities g_0 and g_a in the general case must be simply regarded as certain phenomenological parameters). So far as the assertion that $g_{ik}=0$ for the symmetry of the type S_4 is concerned, it is applicable only to dilute solutions and can change when account is taken of the interaction.

Thus, we shall use the relation (8). With account of (4) and (5), we have

$$\begin{aligned}\varepsilon_{ik}{}^0&=\varepsilon_0\delta_{ik}+ig_0e_{ikl}q_l,\\ \varepsilon_{ik}{}^a&=\varepsilon_a\delta_{ik}+ig_ae_{ikl}q_l.\end{aligned} \tag{9}$$

The subsequent analysis basically duplicates the calculations made previously[6]; therefore, we shall give only the fundamental results.

The Green's function of zeroth order in the inhomogeneity, accurate to a/λ, is of the form

$$D_{ik}{}^{01}=\frac{4\pi\omega^2}{\varepsilon_0\omega^2+q^2}\left[\delta_{ik}+\frac{q_iq_k}{\varepsilon_0\omega^2}\right]-\frac{4\pi g_0\omega^2}{(\varepsilon_0\omega^2+q^2)^2}e_{ikm}q_m. \tag{10}$$

The van der Waals energy in this approximation is

$$E=-\frac{T}{16\pi^2}\frac{1}{(2\pi)^6}\sum_n{}' \varepsilon_a{}^2\omega_n{}^4\int d^3\varkappa d^3q\, D_{ik}{}^{01}\left(\varkappa+\frac{\mathbf{q}}{2}\right)\times D_{ml}{}^{01}\left(\varkappa-\frac{\mathbf{q}}{2}\right)N_{im}(-\mathbf{q})N_{kl}(\mathbf{q}). \tag{11}$$

Substituting (10) in (11) and limiting ourselves only to the linear term, we obtain

$$Q_0=\frac{1}{4\pi^2}\int\limits_0^\infty\frac{\varepsilon_a{}^2g_0\omega^3}{\varepsilon_0{}^{3/2}}d\omega. \tag{12}$$

It should be noted that in obtaining (12), we discard quantities $\sim\varepsilon_a^2g_a$ which are known to be smaller, and also terms that reduce to the surface energy, for example,

$$\int D_{ik}{}^{00}g_aN_{ik}d^3r.$$

The pitch of the helix q_0 is obtained from (12) by division by the Frank modulus K_{22}. If we use the expression found previously[6] for the van der Waals contribution to K_{22}, we then have

$$q_0=6\int\limits_0^\infty\frac{\varepsilon_a{}^2g_0\omega^3}{\varepsilon_0{}^{3/2}}d\omega\Big/\int\limits_0^\infty\frac{\varepsilon_a{}^2}{\varepsilon_0{}^{3/2}}\omega d\omega. \tag{13}$$

We also note that, as usual, all the quantities in (12) and (13) are calculated at an imaginary value of the frequency.

3. VAN DER WAALS INTERACTION BETWEEN METALS

We shall consider in this section only the simplest case of two identical metallic plates, separated by a vacuum gap of thickness d. The geometry of the problem is shown in Fig. 3. As Barash and Ginzburg pointed out,[3] the problem reduces to the determination of the dispersion law for surface electromagnetic waves in such a layered system. Let $A_\alpha^\pm$ be the amplitudes of waves propagating in the positive and negative directions of the z axis. The index α denotes the polarization of the wave and takes on two values, corresponding to orientation of the electric field in the wave perpendicular ($\alpha=1$) and parallel ($\alpha=2$) to the plane of incidence (xz). Introducing the reflection coefficient R_α, we obviously have for plane waves into which the field is resolved (k is the component of the wave vector in the direction perpendicular to the surface, ω is the frequency of the wave. We have chosen the x axis along the wave vector q),

$$R_\alpha=(A_\alpha{}^-/A_\alpha{}^+)e^{-ikd}$$

for the upper plate and

$$R_\alpha=(A_\alpha{}^+/A_\alpha{}^-)e^{-ikd}$$

for the lower plate.

By virtue of the identity of the plates and the complete homogeneity of the surface in the horizontal direction, we obtain hence the dispersion law of the surface waves:

$$R_\alpha{}^2e^{2ikd}-1=0. \tag{14}$$

Using (14), we can determine (according to the argument (phase) principle of complex variable theory) the part of the energy which depends on the distance between the plates, and, along with this, the van der Waals force. The calculations are in complete analogy with those of Barash and Ginzburg,[3] and we therefore write out the result immediately:

$$F=-\frac{T}{4\pi^2}\sum_n\int qdq\,m_0\{[P_1e^{2m_0d}-1]^{-1}+[P_2e^{2m_0d}-1]^{-1}\}, \tag{15}$$

$$m_0{}^2=\omega_n{}^2+q^2,\quad P_1=R_1{}^{-2},\quad P_2=R_2{}^{-2}.$$

This formula can easily be generalized to the case in which the dielectric properties are anisotropic (the corresponding calculations can be found in Ref. 6).

As is well known, in the case of metallic plates, the electromagnetic wave outside the plates is entirely determined by the surface impedance $\zeta(\omega)$. Here,[7]

$$R_1=\left|\frac{m_0-\zeta\omega}{m_0+\zeta\omega}\right|,\quad R_2=\left|\frac{\omega-\zeta m_0}{\omega+\zeta m_0}\right|. \tag{16}$$

The region of large distances $d\gg\lambda_0$ corresponds in the integrals (15) to the relation $\omega\gg q$. Therefore, in this region, $R_1 \approx R_2 \approx (1-\zeta)(1+\zeta)^{-1}$. Since $\zeta \ll 1$ in good conductors, we can expand (16) in powers of the impedance and limit ourselves to the linear components. We thus obtain the additional van der Waals force relative to ideally conducting plates ($\zeta = 0$):

$$\Delta F = \frac{2T}{\pi^2} \sum_n \int q\,dq\,\omega_n \frac{e^{2\omega_n d}}{[e^{2\omega_n d}-1]^2} \zeta(i\omega_n). \tag{17}$$

The nonlocality manifests itself in the conditions of the anomalous skin effect. The expression for the surface impedance under these conditions is well known (see, for example, the book of Abrikosov[8]). In the isotropic case, for specular reflection of the electrons from the surface

$$\zeta = \frac{8}{9}\left(\frac{\pi\sqrt{3}\,\omega^2\Lambda}{\sigma_0}\right)^{1/3} (1 - i\sqrt{3}) \tag{18}$$

(Λ is the free path length of the electrons, σ_0 is the static conductivity; we use the system $\hbar = c = 1$ throughout). Substituting (18) in (17), we find

$$\Delta F = \mathrm{const}\,(\Lambda/\sigma_0)^{1/3} d^{-14/3}, \tag{19}$$

where the constant is determined by the integral over the frequencies and does not depend on any of the physical parameters of the system. The basic local contribution at $d \gg \lambda_0$ is equal to

$$F_0 = \mathrm{const}\, d^{-4}. \tag{20}$$

Therefore $\Delta F \ll F_0$ (at $d \sim 10^{-5}$ cm, we have $\Delta F/F_0 \sim 10^{-1}$). However the nonlocal contribution ΔF can be separated on the basis of its characteristic dependence on the applied external magnetic field H. The corresponding formulas are well known,[8] and we shall not give them here. We only make note of the possibility of nonmonotonic dependence on H and on the distance between the plates.

At $d \ll \lambda_0$, it is necessary to take into account the dependence of the impedance on the wave vector q, since $q \gg \omega$ in this region. A more general consideration is not possible here, since the nonlocal effects depend on the specific form of the permittivity. Moreover, at $q \gg \omega$, the expansion of the permittivity in powers of q is invalid. Therefore, the consideration of the region of small distances $d \ll \lambda_0$ must of necessity be based on a model. For a semi-infinite free electron gas it is easy to obtain

$$\zeta(\omega, q) = \frac{2i\omega}{\pi} \int_0^\infty \frac{dk}{Q^2} \left[\frac{q^2}{\omega^2\varepsilon_l(Q,\omega)} + \frac{k^2}{\omega^2\varepsilon_t(Q,\omega) - Q^2}\right], \tag{21}$$

where $Q^2 = q^2 + k^2$, ε_l and ε_t are the longitudinal and transverse components of the permittivity tensor:

$$\varepsilon_{ik}(\omega, Q) = \varepsilon_l(\omega, Q)\frac{Q_iQ_k}{Q^2} + \varepsilon_t(\omega, Q)\left[\delta_{ik} - \frac{Q_iQ_k}{Q^2}\right]. \tag{22}$$

In the absence of spatial dispersion, ε_l and ε_t do not depend on Q and we get the usual relation from (21) (at $\omega \gg q$) for a nonmagnetic medium $\zeta \sim \varepsilon^{-1/2}$. As has already been noted, for the calculation of the van der Waals force in this case it is necessary to specify concretely the dependence of ε_l and ε_t on the wave vector Q. However, such a model-based analysis is outside the scope of this paper.

4. VAN DER WAALS INTERACTION BETWEEN DIELECTRICS

In the case of a dielectric, the reflection coefficient cannot be expressed merely in terms of the surface impedance. Therefore, it is necessary to use the general formula for the van der Waals force (15) and determine the reflection coefficient separately.

There exist several approaches to the problem of the calculation of R_α. It is possible, for example, to determine the permittivity $\varepsilon(q, \omega, z, z')$ of a semiinfinite system from a simultaneous solution of the equation of motion for the polarization and the inhomogeneous Maxwell equations. Instead of this, we can use with the same accuracy, the permittivity of an unbounded system $\varepsilon(Q, \omega)$ but write additional boundary conditions corresponding to the possibility of the appearance of new waves in the vicinity of the resonance ω_r.

Far from the resonance, we obtain the usual Fresnel formulas and the usual expression for the van der Waals force at large distances $d \gg \lambda_0$: $F_0 = \mathrm{const} \cdot d^{-4}$. The general expression for F with account of spatial dispersion is very complicated (it is obtained by substitution of R_1 and R_2 in our formula (15)), and we shall not write it out. Actually, however, the spatial dispersion is important only in the immediate vicinity of resonance and therefore only those frequencies in the vicinity of resonance and therefore only those frequencies in the vicinity of resonance enter in the increment ΔF to the force. Corresponding to this circumstance, the increment connected with spatial dispersion differs from zero only in a narrow region of distances $d \sim 1/\omega_r \gg \lambda_0$. It turns out here that

$$\Delta R_1 \approx \Delta R_2 \approx \frac{\cos\theta - (n_1^2 - \sin^2\theta)^{1/2}}{\cos\theta + (n_1^2 - \sin^2\theta)^{1/2}} \frac{4\pi\Gamma}{n_2(1 - n_1^2)}. \tag{23}$$

ΔR_1, ΔR_2 are the increments to the Fresnel formulas due to spatial dispersion in the vicinity of resonance; n_1, n_2 are the indices of refraction of the two normal waves (see Ref. 4); Γ is a coefficient determining the additional boundary condition; $\cos\theta = (1 - q^2/\omega^2)^{1/2}$.

Substituting (23) in (15) and expanding in terms of ΔR_1 and ΔR_2, we obtain the formula

$$\Delta F = \mathrm{const}\,\Gamma d^{-3}, \tag{24}$$

which is valid only in the vicinity of resonance. Here we shall not touch at all on the region of small distances, since in this case, the basic contribution to the integrals (15) is made by large wave vectors, for which the expansion of ε_{ik} is invalid.

It is easy to carry out a similar analysis for gyrotropic crystals. We note that the gyrotropy coefficient enters in the answers in the second degree, which is in agreement with the general results[4] that the effect of gyrotropy appears in the linear approximation only in polarization characteristics.

5. CONCLUSION

The effects considered in the present work are small. Moreover, they appear at $d \gg \lambda_0$, when the basic local part of the van der Waals force is already small. Therefore, the only possibility of observation of the results are qualitative singularities associated with the nonmonotonic dependences. We note here that, in addition to the nonmonotonic sources mentioned (the external magnetic field in metals and the presence of an isolated resonance in dielectrics), other reasons are possible. For example, in multilayered coatings in the vicinity of resonance, an oscillating dependence of the reflection coefficient of electromagnetic waves on the coating thickness is possible.[4] Numerous oscillating size effects can occur in metals in a magnetic field.[8]

We now discuss the possibility of experimental observation of the effects that have been proposed. Contemporary precision measurements of the force of interaction between two plates[9] allow us to determine the van der Waals force at distances $\sim 5\times 10^3$ Å. On the other hand, effects associated with the anomalous skin effect appear at frequencies $\omega \gtrsim 10^{15}$ sec^{-1}. Correspondingly, at distances of 10^2-10^3 Å, the nonlocal correction (19) has a value within the limits of experimental accuracy. Of course, the complicated problem of the technical realization of the conditions of the anomalous skin effect (low temperatures) still remains. Perhaps, therefore, a more suitable object for observation of the considered effects would be helium films on a solid surface. As is well known, the size-dependent "van der Waals" part of the chemical potential of the film (per unit volume) is identical with the force of interaction between the surfaces. Equating the chemical potential of the film to the chemical potential of the vapor that is in equilibrium with the film at the given pressure, we can determine the equilibrium thickness of the film. The nonlocal effects will appear in the nonmonotonic dependence of the film thickness on the external parameters. Moreover, the nonmonotonic dependence $F(d)$ can lead[1] to instability of the film in a certain thickness range or to the presence of a finite wetting angle (but very small in contrast with the usual case of nonwettability). The corresponding effects appear, however, in the range of thicknesses $\geqslant 200$ Å and are at the borderline of modern experimental accuracy.

Formally, the case of dielectrics is very favorable for experimental test, since ΔF contains the distance raised to a lower power than F_0. However, for the presence of a nonmonotonic dependence, the coefficient of the additional boundary condition Γ should have a sufficient value $\Gamma/\omega_r \gtrsim 1$. Unfortunately, at the present time, there are no experimental data on the value of Γ. A more detailed theoretical discussion of the quantity Γ can be carried out only within the framework of a microscopic model, which takes us outside the scope of the present research. The problem of the value and possibilities of measurement of Γ is considered by Arganovich and Ginzburg.[4]

We also note that a general account of nonlocality due to the anomalous skin effect in a medium filling the intermediate space between the interacting bodies is possible. We consider, for example, the interaction of two identical metals, divided by liquid layers of another metal. As was shown,[1] in the local approximation, the van der Waals force changes from the law $F\sim d^{-3}$ at $d \ll \lambda_0$ to the law $F\sim d^{-5}$ (at $d \gg \lambda_0$), which is connected with the normal skin effect. However, in this same region $d \gg \lambda_0$, the anomalous skin effect can also be substantial. Using the expression (6) and the general theory of van der Waals forces,[1] we have in this region $F\sim d^{-6}$.

Thus, a general analysis is possible for the given case because the difficulty mentioned in the Introduction refers only to the nonlocality associated with the smallness of a/λ, while under the conditions of the anomalous skin effect the nonlocality is determined by the ratio of the characteristic inhomogeneity of the electric field and the free path length of the electrons. The transition from the law d^{-5} to the law d^{-6} takes place at distances determined only by the static conductivity of the liquid-metal layer.

So far as the formula for the pitch of the cholesteric helix is concerned, direct calculation of q_0 according to (12), for example, is difficult. However, the quantity g_0 can in principle be determined by optical methods. For example, g_0 shifts the frequencies at which selective reflection of light in cholesteric liquid crystals takes place. A rough estimate ($\varepsilon_a \sim 0.1$, $g_0 \sim 10^{-5}$, $K_{22} \sim 10^{-6}$ erg/cm) gives $q_0 \sim 10^5$ cm^{-1}, which agrees with the experimentally observed periods of the cholesteric liquid crystals.

The author thanks I. E. Dzyaloshinskiĭ for useful discussion of the research and also D. E. Khmel'nitskiĭ who called his attention to a number of questions considered in the second part.

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Translated by R. T. Beyer