Nonstationary "intermediate" state of nonequilibrium superconductors

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The character of the phase transition of an optically pumped superconductor to the normal state is investigated. It is shown that a superconductor with nonequilibrium quasiparticle is unstable relative to a transition to the normal state if the gap becomes smaller than a certain value Δ_n that is small in comparison with the gap in the absence of pumping. The region of instability with respect to the optical pumping power β coincides with the interval where the gap is not a unique function of the pump; this region was obtained by the author earlier (Sov. Phys.-JETP 44, 780, 1976). The spatially inhomogeneous states that arise after the development of the instability are investigated using the Ginzburg-Landau equation generalized to include the nonequilibrium case. Depending on the pump power, stationary coexistence of the regions of normal and superconducting phases is possible (at $\beta=\beta_0$), as well as nonstationary coexistence when the interface moves with a definite velocity that depends on the difference $\beta-\beta_0$.

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INTRODUCTION

Experimental investigations of superconductors exposed to light have made it possible to study a number of characteristics of the superconducting nonequilibrium state.[1] It was observed in[2,3] that the sample acquires resistance to direct current not jumpwise when the gap Δ vanishes (as in an equilibrium superconductor), but smoothly, starting with a certain critical value of the pump power. The mechanism of the resistance-blurred transition has been discussed in a number of studies.[2–4] To explain the finite resistance, it was proposed that the superconductor can break up into regions of normal and superconducting phase, i.e., a certain analog of the mixed state in type-II superconductors. However, the causes of the transition of the system into the inhomogeneous state, the structure, and even the very possibility of the existence of the inhomogeneous state have remained unclear.

At the same time, the important role of the form of the distribution function of the nonequilibrium quasiparticles has been made clear[5,6] and it has been shown that the energy distribution of the quasiparticles deviates considerably from equilibrium.[5] In[7] I was able to obtain the distribution function of the particles n_p near the point of the phase transition and the dependence of the order parameter Δ on the pump power β. The dependence of Δ on β turned out to be multiply valued, namely, two solutions (not counting $\Delta=0$) exist for Δ starting at a critical power β_c. The reason for the ambiguity is quite general in character and is connected with the influence of the order parameters on the quasiparticle-energy relaxation processes. The ambiguous $\Delta(\beta)$ dependence means, generally speaking, that the transition to the normal state can be of first order (in agreement with the conclusion drawn in[8]), and a breakup into phases is possible.

It is shown in the present paper that a superconductor with nonequilibrium particles becomes unstable to a transition to the normal state if the gap becomes smaller than a certain value Δ_n (small in comparison with the gap Δ_0 in the presence of the pump). The instability is due to the decrease of the recombination rate of the quasiparticle with decreasing order parameter and arises in the following manner. The decrease of the order parameter decreases the recombination rate, and consequently increases the number of quasiparticles, leading in turn to a further decrease of the order parameter.

This paper deals with spatially inhomogeneous states that arise after the development of small perturbations of Δ and n with allowance for the nonlinear effects. These states are described by nonlinear Ginzburg-Landau equations generalized to include the nonequilibrium state. The main result is that a normal and a superconducting phase can coexist in a superconductor with nonequilibrium quasiparticles, the width of the separation boundary being equal to the larger of two lengths, the coherence length ξ_0 or the quasiparticle diffusion length L. A stationary coexistence of the phases, however, is possible only at one value of the pump power β_0 (or in a very narrow region near β_0). At $\beta\neq\beta_0$ the phase separation boundary moves with a velocity proportional to the difference $\beta-\beta_0$. The boundary moves towards the normal phase if $\beta_c<\beta<\beta_0$, and towards the superconducting phase at $\beta>\beta_0$.

Just as before,[7] we consider the case when the nonequilibrium phonons produced upon recombination of the quasiparticles can be neglected. This is valid for thin films if the time of departure of the phonons from the film is $\tau_{es}=4d/s_d$ (d is the film thickness and s_d is the speed of sound) is shorter than the time of reabsorption by the quasiparticles[6] (for aluminum, e.g., $d<4000$ Å).

1. EQUATIONS FOR THE ORDER PARAMETER AND FOR THE QUASIPARTICLES IN AN INHOMOGENEOUS NONEQUILIBRIUM SUPERCONDUCTOR

To investigate the inhomogeneous state of a superconductor in a nonequilibrium state it is necessary to gen-

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eralize the Eliashberg equations for the gap, i.e., to obtain equations of the Ginzburg-Landau type for a nonequilibrium superconductor in a region where the gap Δ is small in comparison with Δ_0 and varies weakly over the coherence length ξ_0. With the aid of the nonequilibrium Gor'kov equations[9,10] we can represent the expression for $\Delta(\mathbf{r})$ in the form of the infinite series

$$\Delta(\mathbf{r})=\sum_{\mathbf{r}_1} K_1(\mathbf{r},\mathbf{r}_1)\Delta(\mathbf{r}_1)+\sum_{\mathbf{r}_1,\mathbf{r}_2} K_2(\mathbf{r},\mathbf{r}_1,\mathbf{r}_2)\Delta(\mathbf{r}_1)\Delta(\mathbf{r}_2)+\ldots, \qquad (1)$$

where K_i are the coefficients of the expansions of the Green's functions and of the distribution function n of the nonequilibrium particles with respect to Δ.

It is known that if Δ is small and depends smoothly on $\mathbf{r}$ it suffices to retain the linear term of the expansion of Δ in $\mathbf{r}-\mathbf{r}_1$, so that the first term in (1) yields

$$\Delta(\mathbf{r})\int K_1(R)d^3R+\frac{1}{6}\frac{d^2\Delta}{d\mathbf{r}^2}\int K_1(R)R^2 d^3R.$$

In the remaining terms of the series (1), we take $\Delta(\mathbf{r}_i)$ outside the integral sign at the point $\mathbf{r}$. Summing the series and gathering the results, we obtain the following equation for the order parameter:

$$\xi_0^2\frac{d^2\Delta}{d\mathbf{r}^2}=\Delta(\mathbf{r})\left(\frac{1}{g}-\sum_p\frac{1-2n_p(\mathbf{r})}{(\xi^2+|\Delta(\mathbf{r})|^2)^{1/2}}\right),\quad \hbar=1,$$

$$\xi_0^2=\frac{p_0^3}{4\pi^2m}\int_0^\infty\frac{d\xi}{\xi}\frac{d}{d\xi}\frac{2n_0-1}{\xi},\quad \xi=\frac{p^2}{2m}-\mu,\quad \mu=\frac{p_0^2}{2m}, \qquad (2)$$

where $n_0(\xi)$ is the quasiparticle distribution function at $\Delta=0$.

Generally speaking, Eq. (2) should contain also a term with a linear derivative, since n_0 can depend on $\mathbf{r}$. In this paper we consider the case of spatially homogeneous pumping, and omit therefore the term with $d\Delta/d\mathbf{r}$.

The Ginzburg-Landau equation is obtained from (2) by replacing the function $n_p(\mathbf{r})$ by $n_T=(e^{\varepsilon/T}+1)^{-1}$ and expansion in powers of Δ^2. In the nonequilibrium case $n_p(\mathbf{r})$ satisfies the kinetic equation[11] (see below), and the expansion of n_p in terms of Δ contains not only quadratic terms. This is precisely why the sum is retained in (2), so as to carry out the expansion in Δ for concrete $n_p(\mathbf{r})$ (see[7]).

It is interesting to note that Eq. (2) can be obtained by varying a certain functional

$$\Phi=\int d^3r\left\{\frac{\xi_0^2}{2}\left(\frac{d\Delta}{d\mathbf{r}}\right)^2-U(\Delta)\right\}, \qquad (3)$$

$$U(\Delta)=-\int_0^\Delta\Delta'\,d\Delta'\left(\frac{1}{g}-\sum_p\frac{1-2n_p(\mathbf{r})}{(\xi^2+|\Delta'(\mathbf{r})|^2)^{1/2}}\right). \qquad (4)$$

The expression for the "potential energy" U can be obtained with the aid of the known statistical formula[12]

$$U(\Delta(g))=\int_0^g\frac{dg'}{g'}\langle H_{int}\rangle,$$

where $\langle H_{int}\rangle$ is the mean value of the BCS interaction Hamiltonian.

Including in the usual manner the vector potential $\mathbf{A}$ in (2), we obtain the following system of equations for Δ and n_p:

$$\xi_0^2\left(\frac{d}{d\mathbf{r}}-\frac{2ei\mathbf{A}}{c}\right)\Delta=\Delta\left[\frac{1}{g}-\sum_p\frac{1-2n_p(\mathbf{r})}{(\xi^2+|\Delta(\mathbf{r})|^2)^{1/2}}\right], \qquad (5)$$

$$\frac{\partial n_p}{\partial t}+\frac{\partial n_p}{\partial\mathbf{r}}\frac{\partial\varepsilon}{\partial\mathbf{p}}-\frac{\partial n_p}{\partial\mathbf{p}}\left(\frac{\partial\varepsilon}{\partial\mathbf{r}}+\mathbf{F}\right)=\left(\frac{\partial n}{\partial t}\right)_{st}+Q, \qquad (6)$$

$$Q=\frac{\pi\lambda\Delta_0^2}{2\omega}\left(\frac{\Delta_0}{\omega_D}\right)^{k+1}\beta\theta(\omega-\varepsilon),\quad \varepsilon=(\xi^2+\Delta^2)^{1/2},\quad \lambda=g^2\frac{mp_0}{2\pi^2}, \qquad (7)$$

$$n(\xi)=\frac{1}{4\pi}\int n_p\,d\Omega,$$

which together with Maxwell's equations and with the expression for the current constitute a complete system for the nonequilibrium state of the superconductor. Here $(\partial n/\partial t)_{st}$ are the integrals of the collisions with the impurities, phonons, and electrons; $n(\xi)$ is a symmetrical distribution function, and Q is the pump source. We shall assume below that a spatially homogeneous sources generates quasiparticles in a wide energy interval ω, and the potential $\mathbf{A}$ and the force $\mathbf{F}$ exerted on the quasiparticles by the external fields are equal to zero.

The collision integrals with the phonons, which plays the principal role in the energy relaxation, is of the form[11,7]

$$\left(\frac{\partial n}{\partial t}\right)_f=\frac{1}{\tau_f\Delta_0^{2+k}}\left\{(1-n)\int_\xi^{\omega_D}d\xi'\,n'\left(1-\frac{\Delta^2}{\varepsilon\varepsilon'}\right)(\varepsilon-\varepsilon')^{k+1}\right.$$
$$\left.-n\int_0^\xi d\xi'(1-n')\left(1-\frac{\Delta^2}{\varepsilon\varepsilon'}\right)(\varepsilon-\varepsilon')^{k+1}-n\int_0^{\omega_D}d\xi'\,n'\left(1+\frac{\Delta^2}{\varepsilon\varepsilon'}\right)(\varepsilon+\varepsilon')^{k+1}\right\}. \qquad (8)$$

Here T is set equal to zero, k is the exponent in the dependence of the square of the matrix element of the electron-phonon interaction on the wave vector, and ω_D is the Debye frequency.

The equation for the gap contains the distribution function $n(\xi)$ integrated over the angles. Proceeding in the usual manner, we obtain from (6) for this function the equation (omitting the terms quadratic in Δ)

$$\frac{\partial n}{\partial t}-L^2\frac{\partial^2 n}{\partial\mathbf{r}^2}+L^2\frac{\partial^2\Delta}{\partial\mathbf{r}^2}\frac{\Delta}{\varepsilon}\frac{\partial n}{\partial\varepsilon}=\left(\frac{\partial n}{\partial t}\right)_f+Q, \qquad (9)$$

where $L^2=v_0^2\tau\xi\tau_f/3\varepsilon$ is the quasiparticle diffusion length, which we assume for simplicity to be independent of ξ, while τ and τ_f are the relaxation times on the impurities and phonons, respectively.

The coherence length ξ_0, which enters in (5), depends on the form of the distribution function $n_0(\xi)$. For $k=-1$ we have $\xi_0=v_0/\Delta_0\sqrt{3}$, while for $k=1$ the value of ξ_0 was obtained numerically in[13] and turned out to be $0.52v_0/\Delta_0$.

The stationary spatially homogeneous solution of the system (5) and (6) was obtained earlier.[7] The quasiparticle distribution function is represented in the form $n=n_0+n_1$, where n_1 is a small correction proportional to $\Delta\ln(\Delta_0/\Delta)$ if $k=-1$ or to Δ if $k=1$. We present the equations for the gap in dimensionless variables

$$\Delta'=\Delta/\Delta_0,\quad k=-1\ \text{and}\ \Delta'=\pi a_1\Delta/2a_2,\quad k=1,$$

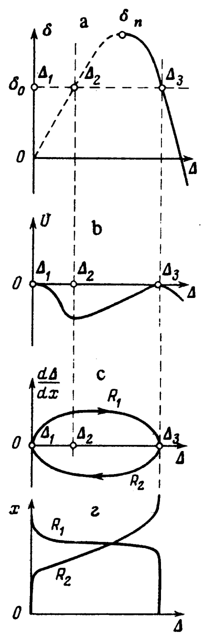

FIG. 1.

$$a_i = \int_0^\infty \xi^i n_0(\xi)\, d\xi,$$

henceforth $\Delta' \equiv \Delta''$

$$\Delta \frac{\beta^{1/2}-\beta_c^{1/2}}{\beta_c^{1/2}} \equiv \Delta\delta = \Delta^2\left(\frac{\pi}{2}\ln\frac{1}{\Delta} - 2G - 1\right), \quad k=-1, \tag{10}$$

$$\Delta\delta = \Delta^2(1-\alpha\Delta), \quad G=0.915, \quad k=1. \tag{11}$$

According to (10), the dependence of Δ on δ becomes ambiguous at $\delta > 0$ (see Figs. 1a and 2a). Indeed, starting with $\delta = 0$ there exist two solutions Δ_2 and Δ_3 (besides $\Delta_1 = 0$); Δ_3 corresponds to a solution that decreases with increasing δ, and Δ_2 to an increasing solution. The two solutions coalesce at Δ_n and δ_n satisfying the equation

$$\frac{\partial\delta}{\partial\Delta} = 0, \quad \frac{\pi}{2}\ln\frac{1}{\Delta_n} - 2G - 1 - \frac{\pi}{2} = 0, \quad \delta_n = \frac{\pi}{2}\Delta_n. \tag{12}$$

The equation for the gap in the case $k = 1$ was obtained earlier[7] in an approximation linear in Δ. It is obvious, however, that the increasing solution must merge with the decreasing one. Therefore a second term with an unknown coefficient α approximating the $\Delta(\delta)$ dependence was added to (11).

2. INSTABILITY OF NONEQUILIBRIUM SUPERCONDUCTOR RELATIVE TO SPATIALLY INHOMOGENEOUS PERTURBATIONS

We consider a superconductor with a pump power β such that $\delta > 0$. We investigate the stability of the system (5)–(8) relative to homogeneous perturbations. As usual, we seek the solution in the form

$$n(t) = n + \tilde{n}e^{\gamma t}, \quad \Delta(t) = \Delta + \tilde{\Delta}e^{\gamma t}. \tag{13}$$

The linearized system of equations for $\tilde{n}(\xi)$ and $\tilde{\Delta}$ is

$$\tilde{\gamma}_k\tilde{n} = -\tilde{n}\tilde{\Gamma}_k + \tilde{S}_k\{\tilde{n}\} + \tilde{\psi}_k, \tag{14}$$

$$\int \frac{2\tilde{n}}{\varepsilon}\, d\xi = -\Delta\tilde{\Delta}\int\frac{1-2n}{\varepsilon^3}\, d\xi, \tag{15}$$

where the following notation was used:

$$\Gamma_k = \int_0^{\omega_D} d\xi' n'\left[\left(1-\frac{\Delta^2}{\varepsilon\varepsilon'}\right)(\varepsilon-\varepsilon')^{k+1} + \left(1+\frac{\Delta^2}{\varepsilon\varepsilon'}\right)(\varepsilon+\varepsilon')^{k+1}\right]$$

$$+ \int_0^\xi d\xi'(1-2n')\left(1-\frac{\Delta^2}{\varepsilon\varepsilon'}\right)(\varepsilon-\varepsilon')^{k+1}, \quad \tilde{\gamma}_k = \gamma_k\tau_\varepsilon, \tag{16}$$

$$\tilde{\psi}_k \equiv \Delta\tilde{\Delta}\varphi_k = -\frac{2\Delta\tilde{\Delta}}{\varepsilon}\left[\int_\xi^{\omega_D}\frac{d\xi' n'}{\varepsilon'}(\varepsilon-\varepsilon')^{k+1}\right.$$

$$\times\left(1-\frac{\Delta^2}{2\varepsilon^2}-\frac{\Delta^2}{2\varepsilon'^2}-\frac{(k+1)}{2}\left(\frac{\Delta^2}{\varepsilon\varepsilon'}-1\right)\right) + 2n\varepsilon a_0(k+1) \tag{17}$$

$$\left. - n\int_0^\xi \frac{d\xi'(1-2n')}{\varepsilon'}(\varepsilon-\varepsilon')^{k+1}\left[1-\frac{\Delta^2}{2\varepsilon^2}-\frac{\Delta^2}{2\varepsilon'^2}-\frac{(k+1)}{2}\left(1-\frac{\Delta^2}{\varepsilon\varepsilon'}\right)\right]\right].$$

Here $\tilde{S}_k\{\tilde{n}\}$ is an integral linear operator acting on n. The equations for $\tilde{n}$ and $\tilde{\Delta}$ are analogous in form and in properties with the corresponding equations derived earlier for n_1 and Δ. [7]

Recognizing that the term $\tilde{S}_k\{\tilde{n}\}$ leads to corrections quadratic in Δ^2, we can represent $\tilde{n}$ in the linear approximation in the form

$$\tilde{n} = \tilde{\psi}_k/(\tilde{\gamma}_k + \tilde{\Gamma}_k).$$

Substituting $\tilde{n}$ in (15), we obtain an equation for $\tilde{\gamma}_k$:

$$\int_0^\infty \frac{2\varphi_k\, d\xi}{\varepsilon(\tilde{\gamma}_k+\Gamma_k)} + \int_0^\infty \frac{1-2n}{\varepsilon^3}\, d\xi = 0. \tag{18}$$

The main contribution to the integrals is made by small $\xi \sim \Delta$. Taking this circumstance into account, we get

$$\tilde{\gamma}_k = -\frac{J_0}{D_k}, \quad J_0 = \int_0^\infty \frac{2\varphi_k\, d\xi}{\varepsilon} + D_k\Gamma_k(0), \quad D_k = \Delta^2\int_0^\infty \frac{1-2n}{\varepsilon^3}\, d\xi. \tag{19}$$

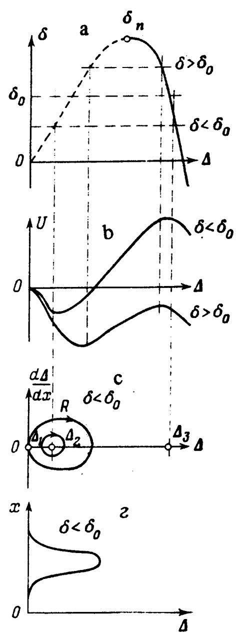

FIG. 2.

Substituting in (19) the previously obtained[7] functions $n=n_0+n_1$, we get

$$\tilde{\gamma}_{-1}=2\left(\frac{\pi}{2}\ln\frac{1}{\Delta}-2G-1-\frac{\pi}{2}\right)\left(\frac{\pi}{2}\ln\frac{1}{\Delta}-2G+3\right)^{-1}. \tag{20}$$

It is seen from (20) that if the gap becomes smaller than the value of Δ_n, which is a root of Eq. (12), then $\tilde{\gamma}_{-1}>0$ and the system is unstable to a transition to the normal state or to a state[1)] with $\Delta_3>\Delta_n$. The instability region coincide with the region where $\Delta(\delta)$ is a multiply valued function. The maximum decrement $\tilde{\gamma}_{-1}$ is reached at $\Delta=0$ and is equal to two.

The physical meaning of the instability consists in the following. Let the gap in the state with $\Delta<\Delta$ decrease somewhat as a result of the fluctuations. Then the recombination probability decreases, and consequently the number of quasiparticles increases, and this leads in turn to further decrease of the gap, etc. The instability is thus due to the influence of the order parameter on the processes of the energy relaxation of the quasiparticles. It is interesting to note that when the system goes over into the normal state, an additional number δn of quasiparticles is released because of the decreased rate of recombination of the quasiparticles at the same source power.

For the case $k=1$ we can obtain $\tilde{\gamma}_1$ only in the approximation linear in Δ, i.e., the maximum value, equal to $\tilde{\gamma}_1=1$. If we take the quadratic correction into account in accord with (11), we get

$$\tilde{\gamma}_1=1-2\alpha\Delta, \quad \Delta_n=1/2\alpha.$$

3. INSTABILITY OF NONEQUILIBRIUM SUPERCONDUCTOR RELATIVE TO INHOMOGENEOUS PERTURBATIONS

We consider the instability of a system relative to spatially inhomogeneous perturbations of the type

$$\tilde{n}(\mathbf{r},t)=\tilde{n}_{\mathbf{q}}e^{i\mathbf{q}\mathbf{r}+\gamma t}, \quad \tilde{\Delta}(\mathbf{r},t)=\tilde{\Delta}_{\mathbf{q}}e^{i\mathbf{q}\mathbf{r}+\gamma t},$$

In this case the system of equations for $\tilde{\Delta}_q$ and $\tilde{n}_q$ takes the form

$$\xi_0^2\mathbf{q}^2\tilde{\Delta}_{\mathbf{q}}=-\Delta\left[\int\frac{2\tilde{n}_{\mathbf{q}}\,d\xi}{\varepsilon}+\Delta\tilde{\Delta}_{\mathbf{q}}D_k\right], \tag{21}$$

$$\tilde{n}_{\mathbf{q}}(\tilde{\gamma}_k^{\mathbf{q}}+\Gamma_k+L^2\mathbf{q}^2)=\tilde{\psi}_k+\frac{\Delta\tilde{\Delta}_{\mathbf{q}}}{\varepsilon}L^2\mathbf{q}^2\frac{dn}{d\varepsilon}. \tag{22}$$

Substituting (22) in (21), we obtain $\tilde{\gamma}_k^q$

$$\tilde{\gamma}_k^{\mathbf{q}}=-\{J_0+\mathbf{q}^2L^2J_{\mathbf{q}}+\mathbf{q}^2\xi_0^2(\tilde{\Gamma}_k+\mathbf{q}^2L^2)\}[D_k+\mathbf{q}^2\xi_0^2]^{-1}, \tag{23}$$

$$J_{\mathbf{q}}=\Delta^2\int\left(\frac{1-2n}{\varepsilon^3}+\frac{2}{\varepsilon^2}\frac{dn}{d\varepsilon}\right)d\xi. \tag{24}$$

The first term in the numerator of the expression for $\tilde{\gamma}_k^q$ coincides with the decrement of the homogeneous state, the second term is connected with the motion of the quasiparticles under the influence of the concentration gradient of the quasiparticles and the gradient of the order parameter; the last term is due to diffusion of the order parameter. The diffusion of the order parameter makes a negative contribution to $\tilde{\gamma}_k^q$, leading to a dissolution of the instability. The contribution of the second term is determined by the sign of the expression J_q, which was calculated earlier[7,14] for different cases and it has been shown that $J_q>0$. We can therefore conclude that the decrement is maximal for homogeneous perturbations ($q=0$).

It is of interest to estimate those values of q_0 at which the instability becomes stabilized, i.e., $\tilde{\gamma}_k(\mathbf{q}_0)=0$. After simple calculations we obtain

$$\mathbf{q}_0^2L^2=\frac{\Gamma_k+\eta^2J_{\mathbf{q}}}{2}\left[\left(1+\frac{4\eta^2|J_0|}{(\Gamma_k+\eta^2J_{\mathbf{q}})^2}\right)^{1/2}-1\right], \quad \eta=\frac{L}{\xi_0}. \tag{25}$$

If $\eta\ll1$, then $\mathbf{q}_0^2L^2=|J_0|\eta^2\ll1$. In the opposite limiting case $\eta\gg1$ we have $\mathbf{q}_0^2L^2=|J_0|/J_q$. At $\Delta=0$ we have $(q_0L)_{max}=1$. It will be shown below that in the region of physical interest we have $|J_0|<1$, so that in stable inhomogeneous configurations we can expect small gradients of the order parameter and of the quasiparticle distribution function, i.e., $Ld\Delta/dr\ll\Delta$ and $Ldn/dr\ll n$.

4. "INTERMEDIATE" STATE OF NONEQUILIBRIUM SUPERCONDUCTORS IN THE CASE $L\ll\xi_0$

The growth of small perturbations of $\tilde{\Delta}$ and $\tilde{n}$ at $\Delta<\Delta_n$ is limited by nonlinear effects. One can expect the new state resulting from the instability development to be an inhomogeneous stationary state. This state is described by the nonlinear equations (5) and (9). The system is characterized by two lengths. We consider first the simpler case when $L\ll\xi_0$. Omitting in (9) the terms with L, we obtain n_1 accurate to Δ^2:

$$n_1(r)=\psi_k/\Gamma_k.$$

After substituting n_1 in (5) we obtain an equation for the gap

$$\xi_0^2\frac{d^2\Delta}{dr^2}=-\frac{\partial}{\partial\Delta}U_k(\Delta), \tag{26}$$

$$U_{-1}(\Delta)=-\Delta^2[\delta/2-{}^1/_3\Delta({}^1/_2\pi\ln(1/\Delta)-2G-1+\pi/6)],$$

$$U_1(\Delta)=-\Delta^2[\delta/2-\Delta/3+\alpha\Delta^2/4]. \tag{27}$$

It is simplest to ascertain the character of the possible distributions in the one-dimensional case (introducing the dimensionless variable $x'=x\xi_0$):

$$\frac{d^2\Delta}{dx'^2}=-\frac{\partial}{\partial\Delta}U_k(\Delta). \quad x'\equiv x \quad \text{hereafter}. \tag{28}$$

Equation (28) is the equation of motion of a "particle" in a field U that depends on the "coordinate" Δ. Equations of this type arise in plasma theory[15] and for semiconductors with negative differential conductivity.[16,17] In the analysis of Eqs. (26) and (28) we follow the paper of Volkov and Kogan.[17]

The potential $U_k(\Delta)$ at different values of the pump δ is shown in Figs. 1b and 2b. In the interval of δ from zero to δ_n, the potential U has three extrema, Δ_1, Δ_2, and Δ_3, corresponding to three possible homogeneous states at the given pump δ. We recall that the points Δ_1 and Δ_3 correspond to a stable equilibrium and Δ_2 to an unstable one. The potentials $U(\Delta_1)$ and $U(\Delta_3)$ coincide (Fig. 1b) at a single value of the pump $\delta=\delta_0$, defined by the equation

$$U(\Delta_1)-U(\Delta_3)=\int_0^{\Delta_3}\Delta'\,d\Delta'\left(\frac{1}{\lambda}-\int\frac{1-2n(\xi)}{\varepsilon}d\xi\right)=0, \tag{29}$$

$$\delta_0=2/9\alpha,\quad k=1.$$

The form of the phase trajectories in the $(d\Delta/dx,\Delta)$ plane is given by the first integral of Eq. (28):

$$\frac{1}{2}\left(\frac{d\Delta}{dx}\right)^2=-U(\Delta)+C,\quad C=U(\Delta_m), \tag{30}$$

where Δ_m is the maximal or the minimal value of the gap.

At $0<\delta<\delta_0$ the separatrix R represents a narrow layer with $\Delta\neq0$. Outside this layer we have $\Delta=0$. The layer width is of the order of the coherence length $\xi_0\delta^{1/2}$. The trajectories close to R show the distribution in the form of a series of such narrow layers (Figs. 2c and 2d), while those close to Δ_2 correspond to the oscillations of Δ near Δ_2 (singular point of the "center" type). At $\delta_0<\delta<\delta_n$ the separatrix, passing now through Δ_3, describes a narrow layer of the normal phase with a small value of Δ in the superconducting phase.

Two stable phases, normal $\Delta_1=0$ and superconducting Δ_3, can coexist only at $\delta=\delta_0$. This distribution corresponds to the trajectory R_1 or R_2 on Fig. 1c, passing through two singular points ("saddles") Δ_1 and Δ_3. The width of the transition layer between the phases is of the order of $\xi_0\delta^{1/2}$. If δ differs very little from δ_0, then a solution exists in the form of a single layer with $\Delta\approx0$ or $\Delta\approx\Delta_3$ and with a width greatly exceeding $\xi_0\delta_0^{1/2}$.

Of greatest interest to us are solutions in which the trajectory passes through the point $\Delta_1=0$. For such trajectories we have $C=U(0)=0$, and the $\Delta(x)$ dependence is given by

$$\pm\int^{\Delta}\frac{d\Delta'}{(-2U(\Delta'))^{1/2}}=x+\text{const}.$$

At $k=1$, the transition layer has a simple form, Fig. 1d (cf. [18]):

$$\Delta(x)=\frac{3\delta_0}{1+\exp(-x\delta_0^{1/2})}. \tag{31}$$

This distribution of Δ (layer solution) is stable (see [17]). The question of the stability of other possible distributions calls for an additional investigation.

What will happen at $\delta\neq\delta_0$? As can be expected from physical considerations, [19] and also from the analogy with the situation in semiconductors, [17] the phase boundary will move in this case with a velocity s that depends on the difference $\delta-\delta_0$. Indeed, at $\delta>\delta_0$ a stationary wave travels and transforms the system from superconducting to normal. At $\delta<\delta_0$, on the contrary, the wave transforms the normal phase into a superconducting one. It can be easily verified that in both cases the final homogeneous states correspond to an absolute minimum of the potential $\Phi=-U$.

In the considered limiting case $\eta\ll1$, the motion of the boundary is connected with the spatial and temporal diffusion of the order parameter. To estimate the velocity s it is therefore necessary to add to (28) a term $\tau_\Delta\partial\Delta/\partial t$, where τ_Δ is the relaxation time of the order parameter. In a stationary wave Δ depends on the coordinate x and on the time t like $\Delta(x-st)$. The equation for $\Delta(x-st)$ differs from (28) by a term

$$\frac{s\tau_\Delta}{\xi_0}\frac{\partial\Delta}{\partial x},$$

which the meaning of the friction force. Multiplying the modified equation by $d\Delta/dx$ and integrating over the trajectory of the motion, we obtain the velocity of the boundary for small $(\delta-\delta_0)/\delta_0$

$$s=\frac{\delta-\delta_0}{\delta_0}\frac{3\xi_0\delta_0^{1/2}}{\tau_\Delta}. \tag{32}$$

It should be noted that measurement of the velocity of the boundaries makes it possible to determine directly the relaxation time of the order parameter.

In the two- and three-dimensional cases, for distributions with axial $\Delta(\rho)$ and spherical $\Delta(R)$ symmetries, Eq. (26) takes the form

$$\frac{d^2\Delta}{d\rho^2}+\frac{1}{\rho}\frac{d\Delta}{d\rho}+\frac{\partial}{\partial\Delta}U_k=0, \tag{33}$$

$$\frac{d^2\Delta}{dR^2}+\frac{2}{R}\frac{d\Delta}{dR}+\frac{\partial}{\partial\Delta}U_k=0. \tag{34}$$

Equations (33) and (34) differ from (28) in the presence of a term that describes the friction. Obviously, if the radius ρ (or R) of the phase (say a normal phase in a superconducting environment) is large, then the boundary can be regarded as almost plane, and we arrive at the already considered one-dimensional case. At a relatively small radius of the normal phase, the growth of this region begins at a pump value somewhat larger than δ_0. It is easily seen also that there exists a certain critical radius ρ_{cr} such that the regions of the normal phase with $\rho<\rho_{cr}$ will be unstable at all values of δ. The critical radius is approximately equal to $\rho_{cr}\approx\xi_0\delta^{-1/2}\times\ln(1/\delta)$.

5. "INTERMEDIATE" STATE OF NONEQUILIBRIUM SUPERCONDUCTORS IN THE CASE L

As shown in Sec. 2, the parameter q_0L, which characterizes the scale of the inhomogeneity, is small also in this limiting case at $\delta\sim\delta_0$. We shall assume therefore that n and Δ vary slowly over a distance L, so that the solution of (9) can be sought in the form

$$n_1(\xi,x)=\frac{\psi_k}{\Gamma_k}+\eta^2\left\{\frac{d^2}{dx^2}\left(\frac{\psi_k}{\Gamma_k}\right)-\frac{\Delta}{\varepsilon}\frac{dn}{d\varepsilon}\frac{d^2\Delta}{dx^2}\right\}. \tag{35}$$

With the aid of (35) we obtain the equation for Δ

$$\frac{d^2\Delta}{dx^2}+\frac{1}{2}\left(\frac{d\Delta}{dx}\right)^2K_k(\Delta)+\frac{\partial}{\partial\Delta}\tilde{U}_k=0,$$

$$K_k=\frac{{}^5/_4\eta^2(\ln1/\Delta)^{(1-k)/2}}{1+\eta^2\Delta(\ln1/\Delta)^{(1-k)/2}}, \tag{36}$$

$$\tilde{U}_k=-\int^{\Delta}\frac{d\Delta'\,\partial U_k/\partial\Delta'}{1+\eta^2\Delta'(\ln1/\Delta')^{(1-k)/2}}.$$

The difference between (36) and (28) lies in the change of U_k and in the addition of the term $\sim (d\Delta/dx)^2$. Since this term is proportional to the square of $d\Delta/dx$, it does not lead to "genuine friction."

The first integral of (36) is

$$\frac{1}{2}\left(\frac{d\Delta}{dx}\right)^2=\exp\left(-\int^{\Delta}K_k(\Delta')d\Delta'\right)\left[C-\int^{\Delta}\exp\left(\int^{\Delta}K_k(\Delta'')d\Delta''\right)\frac{\partial\tilde{U}_k}{\partial\Delta'}d\Delta'\right]. \tag{37}$$

A layer solution passing through the singular points $\Delta_1=0$ and Δ_3 is realized under the following conditions:

$$C=0,\quad \int_0^{\Delta_3}\exp\left(\int^{\Delta}K_k(\Delta')d\Delta'\right)\frac{\partial\tilde{U}_k}{\partial\Delta}d\Delta=0, \tag{38}$$

which are analogous to (29). In particular, (38) yields at $\delta_0\approx 0.21/\alpha$ and $k=1$ a value that coincides with that for δ_0 in the case $\eta\ll 1$.

Thus, even in the case when the diffusion length of the quasiparticles is much larger than the coherence length, a stable layer solution exists only at $\delta=\delta_0$. The apparent reason is that in the approximation in question the characteristic lengths over which the distribution function of the quasiparticles and the order parameter vary are approximately the same and equal to the diffusion length. Therefore the quasiparticles cannot diffuse to a large distance from the transition region and cannot ensure coexistence of two phase in a wide pump region. In turn, the fact that the $\Delta(x)$ distribution almost coincides with the spatial distribution $n(x)$ follows from Eq. (5), which goes over at $\Delta\eta>1$ into an equation that describes the change of Δ in space (and time) only as a result of the quasiparticle distribution.

It should be noted that the situation considered differs substantially from that in type-II superconductors in the presence of a magnetic field. In the latter the order parameters changes over a coherence length ξ_0 much smaller than the penetration depth L of the magnetic field (the role of this depth is played in our case by the diffusion length).

The phase-boundary velocity at $\delta\neq\delta_0$ is determined by the quadiparticle diffusion velocity and is of the order of

$$s=\frac{L}{\tau_f}\frac{\delta-\delta_0}{\delta_0}\zeta, \tag{39}$$

where $\zeta\sim 1$.

The motion of the boundary at $\delta>\delta_0$ is formally analogous to the motion of the boundary of a combustion wave in gas mixtures.[19] In fact, on the phase separation boundary, owing to the instability of the state with $\Delta<\Delta_n$, a transition takes place to a normal phase, accompanied by a release of the excess quasiparticles (the analog of the heat of the reaction). The excess quasiparticles (which are due to the decreased recombination rate at $\Delta=0$) diffuse into the neighboring region, in which they decrease the gap to $\Delta<\Delta_n$. In this region, a transition takes place again to the normal phase, etc., i.e., a wave travels with velocity $\sim L/\tau_f$.

CONCLUSION

The process of the transition of a superconductor to a normal state with increasing pump power depends on additional conditions. If it is assumed that the fluctuations of the order parameter are small (or that there are no regions of normal phase in the sample), then the system remains in a superconducting state up to δ_n, and then goes jumpwise into the normal state. With decreasing pump, the system stays in the normal phase to $\delta=0$. We thus have hysteresis.

If we assume the existence of large fluctuations or regions of the normal phase (these can be inhomogeneities as well as regions of the normal phase near the illuminated surface of the sample), then at $\delta>\delta_0$ the seeds of the normal phase begin to grow and gradually fill the sample. Since the condition of the equilibrium between the phases corresponds to a single value of the pump power β_0, it is clear that the "intermediate" state of the nonequilibrium superconductors is nonstationary. Such a state can therefore be observed only under nonstationary conditions, by measuring the parameters during a time shorter than the time required to fill the sample with the normal phase.

The concepts developed above make it possible, generally speaking, to explain the presently existing experiments on the smooth growth of the resistance with increasing pump.[2,3] We recall that in these studies they measured the dc resistances of samples illuminated by short light pulses. Assuming that the pulse durations were of the order of the time required to fill the sample with the normal phase, then the observed resistance could be attributed to moving regions of the normal phase. Since the velocity of the boundary motion is proportional to the pump power, it follows that the volume occupied by the normal phase (and hence also the resistance) by the end of the pulse is proportional to the pump power, in agreement with experiment.

It is obvious that to verify the agreement of the interpretation with the experimental data it is necessary to increase the pulse duration to a value exceeding the filling time. Then the stationary state should correspond to the normal value of the resistance at any $\delta>\delta_0$.

We note in conclusion that the question of the stability of the other possible distributions, e.g., the oscillatory dependence of the order parameter on the coordinate, the distribution in the form of alternating regions of normal and superconducting phases, etc.

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[1)]It will be shown below (sec. 4) that at $\delta>\delta_0$ (δ_0 is defined by Eq. (20)) the system goes over to the normal state, i.e., $\tilde{n}>0$ and $\tilde{\Delta}<0$ in (13). If $\delta<\delta_0$, the transition is to a state with an order parameter $\Delta=\Delta_3>\Delta_n$ and accordingly $\tilde{n}<0$ and $\tilde{\Delta}>0$.

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