imately equal to one half of the halfwidth of the spectrum produced $0.5\Delta\lambda_{1/2}/\lambda$. The aperture ratio of the apparatus should be regarded as being not less than 90%, since losses on reflection from a Fe-Co mirror do not exceed 3–5% if one takes into account the fact that the monochromator involves both a polarizer and an analyzer.

Among the shortcomings of the apparatus one should include the presence of a background due to the less than 100% polarization of the neutron beam (Fig. 3). For a line of halfwidth $\Delta\lambda_{1/2}/\lambda = 2.0\%$ picked out at the maximum of the given spectrum at a wavelength of $\lambda = 2.59$ Å, the signal to background ratio is equal to 2.5. This ratio, naturally, becomes worse if we go to longer wavelengths or better resolving power. One should note the presence in the spectrum produced with the aid of the monochromator of higher order maxima which spoil the shape of the line being selected.

The monochromator described above can find application in experiments in which it is necessary to retain a high aperture ratio for the apparatus together with a good resolving power. A not unimportant advantage is the possibility of a smooth adjustment of the line being produced with respect to the wavelength by means of a simple variation in the value of the direct current. The apparatus also enables us to produce with the aid of a number of contacts a rapid discrete change in the resolving power.

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Mesic atoms of light nuclei in the field of resonant electromagnetic radiation

I. S. Batkin, Yu. G. Smirnov, and T. A. Churakova

Voronezh State University
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We consider the influence of resonant electromagnetic radiation on the characteristics of light mesic atoms (^{1}H, ^{2}H, ^{3}He, and ^{6}Li). The conditions are investigated under which the populations of the hfs sublevel populations become equalized in the case of linearly polarized radiation. We calculate the dependence of the critical external field (at which saturation is reached) on the collision and Doppler widths of the level. The degree of muon polarization produced when the mesic atom interacts with circularly polarized resonant radiation is determined. It is noted that the most strongly polarized is the muon in the mesic atoms of hydrogen isotopes. The effect of resonant electromagnetic radiation on the rate of nuclear fusion in the $pd\mu$ molecule is also considered. It is shown that the yield of the fusion-reaction channel can be changed by 20% as a result of this action.

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1. It is known that a nucleus captures a muon from the K-shell of the mesic atom. If the nucleus has zero spin, then the 1s level is split by the hyperfine interaction of the magnetic moments of the meson and the nucleus. The spin of the muon + nucleus system takes on two values $F = j \pm \frac{1}{2}$ (j is the spin of the nucleus), and the probability of μ capture in light nuclei depends essentially on the hyperfine-structure (hfs) state from which the capture takes place. [1,2] The presence of the hyperfine interaction influences also the polarization properties of the muon, which in turn influences the asymmetry of the angular distribution of the electrons (neutrons) produced as a result of the decay (capture) of the bound muon. [3,4] The actual magnitude of these effects depends strongly on the combination of the weak-interaction constants and the sublevel population of the hyperfine structure. It is therefore of interest to study the decay (capture) properties of the bound muon by varying the hfs sublevel population in a definite manner. A convenient mechanism for varying the population is, for example, the interaction of the mesic atom with intense electromagnetic radiation from a laser, at a frequency that is resonant with the hyperfine-splitting frequency of the 1s level. High-power lasers that are tunable in a rather large frequency range have by now been developed. [5] This range includes the hyperfine-splitting frequencies of the mesic atoms of five light isotopes: ^{1}H, ^{2}H, ^{3}H, ^{3}He, and ^{6}Li. Some characteristics of the indicated isotopes are shown in Table I.

2. Consider at first the interaction of a system of mesic atoms with linearly polarized laser radiation whose frequency is close to the hyperfine-splitting frequency ω_{12} (we assume for the sake of argument that

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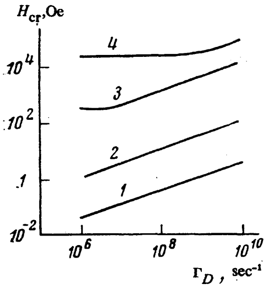

FIG. 1. Plot of H_{cr} against Γ_D at $\Delta\omega=0$. Curves; $1-\gamma=1$ sec^{-1}, $2-\gamma=10^3$ sec^{-1}, $3-\gamma=10^7$ sec^{-1}, $4-\gamma=10^9$ sec^{-1}.

the level 1 corresponds to a spin $F_-=j-\frac{1}{2}$, and level 2 to a spin $F_+=j+\frac{1}{2}$). The system can then be regarded as a two-level one, the transitions between the hfs levels being brought about by the operator $\hat{\mathbf{V}}=-\hat{\mu}\mathbf{H}_0\cos\omega t$ where $\hat{\mu}$ is the magnetic-moment operator and $\mathbf{H}_0$ is the amplitude of the magnetic component of the external electromagnetic radiation. Obviously, in this case the external field induces in the system magnetic-dipole transitions that are subject to the selection rules $\Delta F=1$ and $\Delta m_F=0$ (m_F is the projection of F on the quantization axis).

Assume that the considered system satisfies the condition that the sum of the populations of levels 1 and 2 is conserved during the time interval $\tau\gg\Lambda_\mu^{-1}$ (τ is the time of action of the external electromagnetic field on the system and Λ_μ is the muon decay probability), with relaxation from level 2 to level 1 possible. It is convenient to describe such a system by a statistical density matrix[9] whose components satisfy the following system of equations:

$$\begin{aligned}\dot\rho_{11}&=\gamma\rho_{22}-i(V\rho_{21}-V^*\rho_{12}),\\ \dot\rho_{22}&=-\gamma\rho_{22}+i(V\rho_{21}-V^*\rho_{12}),\\ \dot\rho_{12}&=-(\Gamma+i\omega_{12})\rho_{12}+iV(\rho_{11}-\rho_{22}),\\ \rho_{21}&=\rho_{12}^*,\end{aligned}\tag{1}$$

where

$$V=-{}^1/_2B(e^{i\omega t}+e^{-i\omega t}),\quad B=\langle 2|\hat{\mu}|1\rangle\mathbf{H}_0/\hbar.$$

The system is characterized by two relaxation constants, γ and Γ. The diagonal density-matrix elements determine the level populations, i.e., $\rho_{11}=n_1$ and $\rho_{22}=n_2$. The condition $n_1+n_2=1$ is satisfied. We put $\rho_{21}^*=\rho_{12}=n_{12}e^{-i\omega t}$, with $\dot n_{12}=0$. We transform the system of equations (1), taking into consideration only terms with the difference frequency $\Delta\omega=\omega-\omega_{12}$ and neglecting the rapidly oscillating terms, in analogy with[9]. Changing over to populations, we obtain

$$\begin{aligned}\dot n_1&=\gamma n_2+i(Bn_{21}-B^*n_{12})/2,\\ \dot n_2&=-\gamma n_2-i(Bn_{21}-B^*n_{12})/2,\\ n_{12}(\Gamma-i\Delta\omega)&=-iB(n_1-n_2)/2,\\ n_{21}(\Gamma+i\Delta\omega)&=iB^*(n_1-n_2)/2.\end{aligned}\tag{2}$$

It follows from the last two equations that

$$Bn_{21}-B^*n_{12}=i(n_1-n_2)\Gamma|B|^2/[\Gamma^2+(\Delta\omega)^2].\tag{3}$$

We introduce the notation

$$W=\Gamma|B|^2/2[\Gamma^2+(\Delta\omega)^2].\tag{4}$$

Substituting (3) and (4) in (2) we get

$$\dot n_1=\gamma n_2-W(n_1-n_2),\quad \dot n_2=-\gamma n_2+W(n_1-n_2).\tag{5}$$

The quantity W has the meaning of the probability with which the mesic atoms go over from one level to the other under the action of the external field. The stationary solution of interest to us can be easily obtained for the system (5):

$$n_1=\frac{\gamma+W}{\gamma+2W},\quad n_2=\frac{W}{\gamma+2W}.\tag{6}$$

It is seen from (6), when (4) is taken into account, that the population difference n_1-n_2 is given by the expression

$$n_1-n_2=\frac{(\Delta\omega)^2+\Gamma^2}{(\Delta\omega)^2+\Gamma^2+\Gamma|B|^2/\gamma}.\tag{7}$$

The width Γ consists of the Doppler width Γ_D, which depends substantially on the physical state of the substance, and the width Γ_{12} connected with the rate of relaxation of the upper hfs level to the lower one. The constant Γ_{12} is significantly different for the mesic atoms considered by us. Its largest value is for the hydrogen isotopes. Owing to the collisions of the muonic and ordinary atoms, the width Γ_{12} reaches values $\sim10^9$ sec^{-1} (^{1}H, ^{3}H) and $\sim10^7$ sec^{-1} (^{2}H), [10,11] which are noticeably larger than the decay widths of the other mesic atoms (for which $\Gamma_{12}<1$ sec^{-1}). We put for our estimates $\gamma=\Gamma_{12}$.

We introduce the concept of the critical-field amplitude H_{cr}, defined as the external field intensity at which the population of the i-th hfs level differs from the population reached when the transition is saturated by not more than 5%, i.e., $n_i-\frac{1}{2}(n_1+n_2)\leq0.05$ (here $i=1, 2$). Figure 1 shows a plot of H_{cr} against Γ_D in the case of $\Delta\omega=0$. As seen from the figure H_{cr} assumes the largest value for the mesic atoms of hydrogen isotopes. The reason is the relatively large value of Γ_{12}.

As already noted, the rate of capture of the bound muon depends on the state of the hfs of the mesic atom. The total capture probability is given by[1]

$$\Lambda_\mu=n_1\Lambda_1(j-{}^1/_2)+n_2\Lambda_2(j+{}^1/_2),\tag{8}$$

where $\Lambda_1(j-\frac{1}{2})$ and $\Lambda_2(j+\frac{1}{2})$ are the rates of capture from different hfs levels. These quantities are known for the mesic atoms of ^{1}H and ^{2}H. Figure 2 shows the capture probabilities for these isotopes as functions of the amplitude of the external resonant field for different Γ_D (see also [12]). It is seen that the probabilities change appreciably at laser fields intensities that are

TABLE I. *

Isotope	Nuclear spin, j	Nuclear magnetic moment μ_N	Hyperfine splitting $\Delta\nu/10^{10}$ Hz	$\Lambda_\mu(j-{}^1/_2)$, sec^{-1}	$\Lambda_\mu(j+{}^1/_2)$, sec^{-1}
^{1}H	$^1/_2$	2.793	4423	636	13
^{2}H	1	0.857	1184	334	15
^{3}H	$^1/_2$	2.979	5824	–	–
^{3}He	$^1/_2$	–2.127	16628	2000	1300
^{6}Li	1	0.822	10779	–	–

*The data for the table were taken from[2,6,7,8].

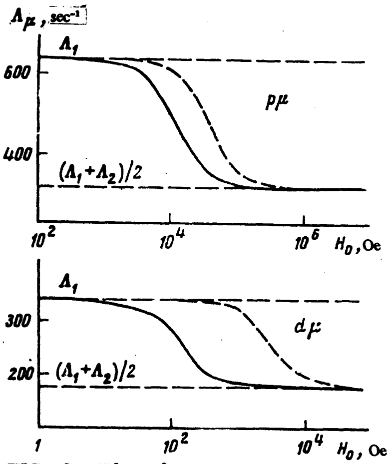

FIG. 2. Plot of μ-capture probability in the mesic atoms $p\mu$ and $d\mu$ vs H_0 at $\Delta\omega=0$ for $\Gamma_D=10^7$ sec (solid line) and $\Gamma_D=10^{10}$ sec^{-1} (dashed line).

not excessively large. They can apparently be obtained with modern lasers.

3. We consider now the interaction of a system of mesic atoms with a circularly polarized wave of resonant frequency.

The nuclei considered by us have angular momenta $j=\frac{1}{2}$ and $j=1$ (see Table I). For the mesic atoms whose nuclei have spin $\frac{1}{2}$, the state $F_+=1$ and $m_{F_+}=1$ consists of three substates that are degenerate in m_F. The substates with projections $m_{F_+}=1$ and $m_{F_+}=-1$ correspond to the spin functions $\chi^1_{1/2\,1/2}$ and $\chi^1_{-1/2-1/2}$, while the substate m_{F_+} is described by a superposition of the functions $\chi^1_{-1/2\,1/2}$ and $\chi^1_{1/2-1/2}$. The upper index of the spin function is equal here to the value of F, while the first and second lower indices denote the spin projections of the nucleus m_N and of the meson m_μ on the quantization axis. The state $F_-=0$ is also described in this case by a superposition of $\chi^0_{1/2-1/2}$ and $\chi^0_{-1/2\,1/2}$.

We assume that the resonant electromagnetic radiation has 100% left-hand circular polarization. In this case the matrix element of the transition between the hfs states takes the form

$$M=\mu H_0\delta(m_F, m_{F-+1}). \qquad (9)$$

We assume also that the electromagnetic transition is due only to the interaction of the nuon magnetic moment μ_μ with the magnetic field (since $\mu_N/\mu_\mu \ll 1$, where μ_N is the magnetic moment of the nucleus). It is easily seen that in this case the result of the action of the external field on the system will be the M1 transitions between the sublevels $\chi^0_{1/2-1/2}$ and $\chi^1_{1/2\,1/2}$ ($\Delta F=1$, $\Delta m_F=1$), for in this case the nuclear spin projection remains unchanged (transition with nuclear spin flip are much less probable, in view of the smallness of the nuclear magnetic moment). The redistribution of the sublevel populations under the influence of the external field leads to polarization of the muon on the levels F_+ and F_-. The muon polarization on a hfs level with total angular momentum F is given by

$$P_F=\sum_{m_\mu m_N} m_\mu n^F_{m_N m_\mu} \Big/ \sum_{m_\mu m_N} n^F_{m_N m_\mu}, \qquad (10)$$

where $n^F_{mNm\mu}$ is the population of the sublevel with total angular momentum F, on which the projections of the spins of the nucleus and the muon on the quantization axis are respectively equal to m_N and m_μ.

Estimates for the produced polarization upon saturation of the transition $\chi^0_{1/2-1/2}\to\chi^1_{1/2\,1/2}$, made under the assumption that in the absence of an external field the muon on the K shell is completely depolarized and that there are no transitions between degenerate sublevels in the system, are given in Table II.

In the case of a nuclear spin $j=1$, level $F_+=\frac{3}{2}$ consists of four sublevels degenerate in m_{F_+}. The substates with projections $m_{F_+}=\frac{3}{2}$ and $m_{F_+}=-\frac{3}{2}$ correspond to the spin functions $\chi^{3/2}_{1\,1/2}$ and $\chi^{3/2}_{-1-1/2}$, the substate with $m_{F_+}=\frac{1}{2}$ is described by a superposition of $\chi^{3/2}_{1-1/2}$ and $\chi^{3/2}_{0\,1/2}$, while the substate with $m_{F_+}=-\frac{1}{2}$ is described by a superposition of $\chi^{3/2}_{-1\,1/2}$ and $\chi^{3/2}_{0-1/2}$. The level $F_-=\frac{1}{2}$ consists of a sublevel with $m_{F_-}=-\frac{1}{2}$ (superposition of $\chi^{1/2}_{-1\,1/2}$ and $\chi^{1/2}_{0-1/2}$) and a sublevel with $m_{F_-}=\frac{1}{2}$ (superposition of $\chi^{1/2}_{1-1/2}$ and $\chi^{1/2}_{0\,1/2}$). In such a system, the interaction with an electromagnetic wave with 100% left-circular polarization leads to saturation of the transitions $\chi^{1/2}_{1-1/2}\to\chi^{3/2}_{1\,1/2}$ and $\chi^{1/2}_{0-1/2}\to\chi^{3/2}_{0\,1/2}$. Estimates from the resultant polarization, under the same assumptions as for nuclei with spin $j=\frac{1}{2}$ are also given in Table II.

4. We consider finally the influence of the resonant electromagnetic radiation on the rate of nuclear fusion in the $pd\mu$ molecule.

It is known that the fusion reaction proceeds in this mesic molecule along two channels

$$pd\mu \begin{array}{l} \nearrow {}^3He\mu+\gamma \\ \searrow {}^3He+\mu \end{array} \qquad (11)$$

with respective probabilities λ_1 and λ_2. The $pd\mu$ molecule has four hyperfine-structure sublevels with total angular momentum $J=0$ and $J=1$ (two levels), and $J=2$, the probability of the reaction being substantially dependent on the sublevel from which it proceeds. Such a reaction is realized in experiment in a mixture of hydrogen and deuterium gas.[13] It is known that the stopping of the meson is followed by the following processes:

$$\begin{array}{l} p+\mu\to p\mu, \\ d+\mu\to d\mu, \\ p\mu+d\to d\mu+p\ (\lambda_e), \\ d\mu+p\to pd\mu\ (\lambda_{pd}), \\ p\mu+p\to pp\mu\ (\lambda_{pp}), \\ d\mu+d\to dd\mu\ (\lambda_{dd}), \\ \mu\to e+\nu\ (\lambda_0). \end{array} \qquad (12)$$

TABLE II. Muon polarization on hfs levels under the influence of a resonant field for a saturated transition.

Population of hfs levels	$j=1/2$		$j=1$	
	P_+	P_-	P_+	P_-
$n_+=(J+1)/(2J+1)$, $n_-=J/(2J+1)$	$1/22$	$1/10$	$1/30$	$1/18$
$n_+=0$, $n_-=1$	$1/2$	$1/6$	$1/2$	$1/6$

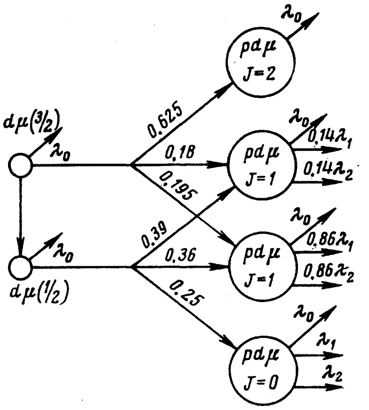

FIG. 3. Diagram of the process of formation of the $pd\mu$ molecule and of the subsequent fusion reaction.

The parentheses contain the probabilities of these processes, and their numerical values are given, e.g., in[14].

The yields of the γ photons (n_γ) and of the muons (n_μ) in the channels of reaction (11) per stopped meson can be written in the form[6,14]

$$n_\gamma=\frac{c_d\lambda_e}{c_d\lambda_e+\lambda_{pp}+\lambda_0}\,\frac{\lambda_{pd}}{\lambda_{pd}+\lambda_0}\sum_J \omega_J\frac{\lambda_1^J}{\lambda_0+\lambda_f^J},$$
$$n_\mu=\frac{c_d\lambda_e}{c_d\lambda_e+\lambda_{pp}+\lambda_0}\,\frac{\lambda_{pd}}{\lambda_{pd}+\lambda_0}\sum_J \omega_J\frac{\lambda_2^J}{\lambda_0+\lambda_f^J} \qquad (13)$$

where ω_j is the population of the molecular sublevel with angular momentum J; c_d is the deuterium concentration; λ_1^j and λ_2^j are the probabilities of the channels of reaction (11) from the level with angular momentum J, and $\lambda_f=\lambda_1+\lambda_2$. The populations of the molecular hfs sublevels depend on the populations of the hfs levels in the mesic atom $d\mu$, which takes part in the production of the mesic molecule. The process of the production of the $pd\mu$ molecule and the subsequent fusion reaction can be represented by the diagram shown in Fig. 3.[12] If we denote the sublevel populations of the atom $d\mu$ with total angular momentum $F_+=\frac{3}{2}$ and $F_-=\frac{1}{2}$ by $n_{3/2}$ and $n_{1/2}$, the population of molecular states with different J can be expressed in the form

$$\omega_2=0.625n_{3/2},\qquad \omega_1=0.18n_{3/2}+0.39n_{1/2},$$
$$\omega_1'=0.195n_{3/2}+0.36n_{1/2},\qquad \omega_0=0.25n_{1/2}. \qquad (14)$$

On the other hand, theoretical estimates are available for the rates of nuclear fusion from various substates.[10] It is assumed that the state with spin $J=2$ makes no contribution to the nuclear fusion, and from the states with $J=1$ and $J=0$ the fusion proceeds with the relative probabilities shown in the diagram (Fig. 3). Under the influence of an intense electromagnetic radiation of resonant frequency the populations $n_{1/2}$ and $n_{3/2}$ become equalized in accordance with formulas (6). As seen from (13) and (14), this leads, in turn, to a change in the yield of the γ quanta and the muons in the channels of the reaction (11). Simple estimates show that when the $n_{1/2}\to n_{3/2}$ transition saturates the yetld of the γ quanta and of the muons increases approximately 20% at a deuterium concentration in the H_2+D_2 mixture less than 10%.

5. The presented estimates have shown that the action of resonant electromagnetic radiation on a mesic molecule or a mesic atom leads to a number of effects whose experimental observation can be attempted. The characteristics of these effects are determined by the values of the weak-interaction constant, and therefore experimental investigations are of undisputed interest. In addition, these effects constitute a rare (if not the only) case when a relatively weak external field can alter the yield of a nuclear reaction.

We note that the present paper dealt only with one population-change mechanism. There are also other possibilities (the method of pulsed inversion, the method of inversion by adiabatic fast passage described, e.g., in[15]), the analysis of which is beyond the scope of the present paper. We note also that an examination of these mechanisms will not lead to fundamentally new results.

In conclusion, it is our pleasant duty to thank L. I. Ponomarev and B. A. Zon for a discussion of the results.

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