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Translated by J. G. Adashko

Two-mode locking in a standing-wave gas laser

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(Submitted March 17, 1977)
Zh. Eksp. Teor. Fiz. 73, 462–469 (August 1977)

An experimental and theoretical investigation was made of the locking of two orthogonally polarized modes of an He-Ne laser on the neon transitions $3s_2$–$3p_4$, $3s_2$–$3p_2$, and $2s_2$–$2p_1$ in which the angular momenta of the upper and lower levels were 1 and 2, 1 and 1, and 1 and 0 respectively. It was observed that the locking can take place in the region when the modes are symmetrically tuned relative to the center of the gain line. It is shown that in the case of weak degeneracy of the levels of the working transition (with angular moment 1 and 1 or 1 and 0) the mode locking is due to the influence of atomic collisions. Results of investigations of the spectral and polarization characteristics of the radiation in the two-mode locking region are reported.

PACS numbers: 42.55.Fn

INTRODUCTION

The mode locking or self-synchronization phenomenon is typical of a laser working in the multitude regime and is due to the presence of combination tones. Despite the rather large number of studies of this question, the locking of two modes in a standing-wave laser has not yet been considered in detail, apparently because of the difficulties involved in producing lasing of two closely-located modes. Of particular interest is the case of two orthogonally polarized modes, since lasers with internal phase anisotropy, which can generate modes with such polarizations and in which the intermode distance can be continuously varied in a wide range, offer promise of being of practical use. From this point of view, in particular, it is important to know the minimum attainable intermode distances in a laser, which are limited by mode locking.

On the other hand, the very existence of locking of two orthogonally polarized modes on transitions with weak level degeneracy (angular momenta 1 and 0 or 1 and 1) remains open. As shown by Dienes,[1] when two orthogonally polarized modes on such transitions are generated there are no combination tones and there should be no locking.

From the experiment results of Doyle and White[2] who dealt with this problem, it follows more readily that the combination tones on transitions with weak degeneracy are more readily of decreased intensity than completely absent. We note that the experimental investigations devoted to direct study of the locking of two orthogonally polarized modes on such transitions have not been carried out.

We present here the results of a theoretical and experimental investigation of the locking of two orthogonally polarized modes in a gas laser with different degrees of degeneracy of the working-transition levels. The theoretical analysis is based on the paper of Vdovin *et al*,[3] in which account is taken of level degeneracy, elastic collisions of atoms, and dragging of the resonant radiation.

THEORETICAL ANALYSIS

It was noted earlier[4] that on the 1–0 transition a stable two-mode regime is realized at all values ω_{12} of the intermode distance in that frequency region where the gain exceeds the losses. At ω_{12} smaller than the resonator band width $\omega/4\pi Q$ it is necessary to take the possibility of locking into account. This phenomenon manifests itself in the fact that at values $\Omega_{12}<\Omega_{12\,\mathrm{cr}}$, where $\Omega_{12}=\Omega_1-\Omega_2$ is the distance between the modes of the empty resonator, the relative phase shift

$$\psi=[(\omega_2-\omega_1)t+\varphi_2-\varphi_1] \tag{1}$$

of the generated modes becomes constant, meaning lasing on a single frequency $\omega_1=\omega_2=\omega$ with a radiation polarization that is in general elliptic.

In the case of locking, when $\omega_{12}\leqslant\omega/4\pi Q$, it is necessary to take into account the combination tones that are produced at the frequencies $2\omega_1-\omega_2=\omega_1+\omega_{12}$ and $2\omega_2-\omega_1=\omega_2-\omega_{12}$. We denote by E_x and E_y the amplitudes of the generated modes with different polarizations, by Q_x and Q_y their quality factors, and assume in the general case that the Q values are different. At locking, when lasing takes place at a single frequency, the quantities E_x and E_y no longer mean mode amplitudes and determine, together with the relative phase shift ψ (see (1)), the magnitude and the position of the principal axes of the polarization ellipse of the radiation.

Assuming that we are in the locking region, we have, taking the combination tones into account, a coupled system of equations for the quantities E_x, E_y, and ψ:

$$\begin{aligned}\beta E_x^2+(\theta+\chi\cos 2\psi+\xi\sin 2\psi)E_y^2&=\alpha_x,\\ \beta E_y^2+(\theta+\chi\cos 2\psi-\xi\sin 2\psi)E_x^2&=\alpha_y,\\ \chi(E_x^2+E_y^2)\sin 2\psi+(\rho-\tau+\xi\cos 2\psi)(E_x^2-E_y^2)&=\Omega_{12},\end{aligned} \tag{2}$$

where α_x and α_y are the usual linear gains with allowance for the losses. The meaning of the coefficients β, θ, ρ, τ is the same as in the paper of Vdovin *al.*,[3] except that account is taken of the fact that in the locking regime at $\omega_1=\omega_2$ we have $\beta_1=\beta_2=\beta$, $\theta_{12}=\theta_{21}=\theta$, etc. The coefficients χ and ξ take into account the influence of the combination tones. To investigate the system (2) we calculated the expressions for α, β, θ, ρ, and τ for different transitions with allowance for the depolarizing atomic collisions. The influence of the dragging of the resonant radiation was neglected in this study, since it does not influence qualitatively the results of[3]. The coefficients χ and ξ were calculated by the same procedure.

Allowance for the depolarizing collisions leads to the existence of combination tones on transitions with level angular momenta 1 and 0, or 1 and 1, in contrast to the results of Dienes.[1] In transitions with other values of the angular momenta of the levels, the combination tones take place also without allowance for the collision of the atoms, in agreement with his results.[1] For example, for a transition with level momenta 1 and 0 we have

$$\chi(\omega_{10})=\delta\beta(\omega_{10}),\qquad \xi(\omega_{10})=-\delta\rho(\omega_{10}),$$

where $\omega_{10}=\omega_1-\omega_0$ is the detuning of the lasing frequency from the central frequency ω_0 of the transition,

$$\delta=\frac{1}{2}\left(\frac{1}{\gamma_b^{(2)}}-\frac{1}{\gamma_b^{(1)}}\right)\bigg/\left(\frac{1}{\gamma_a^{(0)}}+\frac{1}{3\gamma_b^{(0)}}+\frac{2}{3\gamma_b^{(2)}}\right)$$

is a parameter that depends on the spectroscopic characteristics of the levels, $\gamma_a^{(0)}$ and $\gamma_b^{(0)}$ are the radiative widths of the levels, and $\gamma_b^{(\varkappa)}=\gamma_b^{(0)}+\Gamma_b^{(\varkappa)}$, $\varkappa=1, 2$. The quantity $\Gamma_b^{(\varkappa)}$ characterizes the influence of the depolarizing collisions $\Gamma_b^{(\varkappa)}=A_b^{(\varkappa)}p$, where p is the pressure of the mixture and $A_b^{(\varkappa)}$ is determined by the interaction of the colliding atoms. It is important that $A_b^{(1)}\neq A_b^{(2)}$. For example, in a van der Waals interaction we have for the level angular momentum 1[3]

$$\Gamma_b\ /\Gamma_b^{(2)}\approx 1.1. \tag{3}$$

Thus, in the absence of collisions $\delta=0$ and there are no combination tones, in agreement with the Dienes results.[1]

The meaning of this result becomes understandable with the aid of a very simple model, in which one mode is connected with a transition between the lower level and the Zeeman sublevel of the upper level with $m=1$, and the other with $m=-1$. The appearance of a field at the frequency $2\omega_1-\omega_2$ of the combination tone means the feasibility of a process in which a quantum with ω_1 is emitted, a quantum with ω_2 is absorbed, and then a quantum with ω_1 is again emitted. It is clear that in this model, after absorption of the quantum with ω_2, the system cannot radiate a quantum with ω_1, since this process has different selection rules with respect to m. The depolarizing collisions lead to a partial mixing of the states with different Zeeman sublevels, and it is this which ensures the possibility of such processes. An analogous picture takes place also for a transition with level angular momenta 1 and 1. In the transition with angular momenta 1 and 2, the modes with different polarizations have common Zeeman sublevels both on the upper and the lower level, and this leads to the presence of combination tones even without allowance for the collisions.

We now estimate the distance $\Omega_{12\,\mathrm{cr}}$ between the resonator modes, below which mode locking takes place. Assume that the laser is tuned to the line center $\omega_{10}=0$ and the adjustment is ideal, $Q_x=Q_y$. Recognizing that $\xi=0$ and $\alpha_x=\alpha_y=\alpha$, we get from (2)

$$\begin{aligned}&E_x^2=E_y^2=\alpha(\beta+\theta+\chi\cos 2\psi)^{-1},\\ &2\chi\alpha(\beta+\theta+\chi\cos 2\psi)^{-1}\sin 2\psi=\Omega_{12}.\end{aligned} \tag{4}$$

The condition for the existence of a solution of (4) requires

$$\Omega_{12}^2\leqslant 4\chi^2\alpha^2[(\beta+\theta)^2-\chi^2]^{-1}. \tag{5}$$

The value of $\Omega_{12\,\mathrm{cr}}$ corresponds to the equality in (5), and therefore in the limit of inhomogeneously and homogeneously broadened lines we obtain the same results for the transition with level angular momenta 1 and 0:

$$\Omega_{12\mathrm{cr}}=\frac{\omega}{4\pi Q}(\eta-1)\left(\frac{1}{\gamma_b^{(2)}}-\frac{1}{\gamma_b^{(1)}}\right)\bigg/\left(\frac{2}{\gamma_a^{(0)}}+\frac{2}{3\gamma_b^{(0)}}+\frac{5}{6\gamma_b^{(2)}}+\frac{1}{2\gamma_b^{(1)}}\right), \tag{6}$$

where η is the relative excitation of the medium.

Let us estimate the order of magnitude of $\Omega_{12\,\mathrm{cr}}$. For typical values of the spectroscopic constants $\gamma_a^{(0)}=\gamma_b^{(0)}=10$ MHz and $A_b^{(2)}=3$ MHz, taking (3) into account we obtain from (6) at $p=3$ Torr and $\eta=2$

$$\Omega_{12\,\mathrm{cr}}\approx 10^{-2}\ \omega/4\pi Q, \tag{7}$$

i.e., locking takes place when the intermode distance becomes much less than the resonator band widths.

At $\Omega_{12}<\Omega_{12\,\mathrm{cr}}$, locking takes place in a certain region $0\leqslant|\omega_{10}|\leqslant\omega_{10\,\max}$. It is difficult to obtain an accurate expression for the locking region from the system of nonlinear equations (2). We confine ourselves therefore to a certain estimate.

At a tuning $\omega_{10}=0$ and at ideal adjustment, the second term in the left-hand side of the last equation of the system (2) is equal to zero. Neglecting this term also at $\omega_{10}\neq 0$, we obtain the following equation for the definition of $\omega_{10\,\max}$:

$$\frac{\alpha(\omega_{10max})}{\alpha(\omega_{10}=0)}=\frac{\Omega_{12}}{\Omega_{12\,cr}}. \tag{8}$$

From (8), for example, for an inhomogeneously broadened line, we obtain

$$\omega_{10max}=ku\left[\ln\frac{\eta}{(1+(\eta-1)\Omega_{12}/\Omega_{12\,cr})}\right]^{1/2}, \tag{9}$$

from which it follows that the locking region can reach appreciable values.

The connection of the quantities E_x, E_y, and ψ with the radiation characteristics at locking, such as the lengths a and b of the polarization ellipse semiaxes and their rotation angle φ relative to the selected axes x and and y, is given by the well known expressions[5]

$$a^2+b^2=E_x^2+E_y^2,\quad ab=E_xE_y\sin\psi, \tag{10}$$

$$\operatorname{tg}2\varphi=2E_xE_y(E_y^2-E_x^2)^{-1}\cos\psi. \tag{11}$$

Thus, after obtaining E_x, E_y, and ψ from the system (2), we can get a, b, and φ with the aid of (10) and (11). We note that it follows from (11) that the directions of the principal axes of the ellipse do not coincide with the axes x and y.

EXPERIMENTAL INVESTIGATIONS

For the experimental investigations we used two-mode He^3–Ne^{20} lasers with controllable internal phase anisotropy on the neon transitions $2s_2-2p_1$, $3s-3p_2$, and $3s_2-3p_4$ (the respective angular momenta are 1, 1, and 1 for the upper working level and 0, 1, and 2 for the lower level. The phase-shift elements were quarter-wavelength plates of crystalline quartz. The distance between two orthogonally polarized modes was varied in a wide range by relative rotation of two $\lambda/4$ plates located inside the resonator.[3] A selecting methane cell placed inside the resonator made it possible to work both on the transition $3s_2-3p_4$ and on the transition $3s_2-3p_2$.[6]

The output radiation of the lasers passed through a polarization light filter and was incident on photodiodes. The polarization light filters were used to determine the character of the polarization of the radiation and to separate the signals due to the beats between the modes. The optical length of the laser resonator was varied by vibrating one of the mirrors with the aid of piezoceramic. All the measurements were performed on the fundamental TEM_{00} mode, which was separated with the aid of a diaphragm inside the resonator.

The registration system was essentially the same for the different wavelengths, differing only in the types of photoreceivers and in the frequency characteristics of the amplification channels, of the filters, of the RF mixers, and the heterodynes. The registration system made it possible to observe the intermode beat signals from the photodiodes with the aid of an oscilloscope and a spectrum analyzer. Heterodyning and amplitude limitation made it possible to observe, with the aid of a frequency discriminator, the law governing the variation of the beat frequency as a function as a function of the laser tuning. Thus, the experimental setup made it possible to measure the internode beats, the locking region, the relative phase, and the radiation polarization as functions of the laser parameters.

The measurements have shown that locking is realized in the region of symmetrical tuning of the modes relative to the line center at all the considered transitions. The mode locking process is most clearly illustrated by the behavior of the frequency ω_{12} as a function of the detuning ω_{10}. Typical oscillograms of this behavior, obtained with the aid of a frequency discriminator are shown in Fig. 1 for three values of the laser discharge current on the transition $3s_2-3p_2$ at a constant value of Ω_{12}. In Fig. 1a the value of ω_{12} varies little with detuning, since the excitation in the laser is weak the combination tones have no influence. With increasing excitation, more perturbations by the combination tones come into play, as manifest by the attraction of one mode by the other (Fig. 1b). Further increase of the excitation leads to a collapse of the modes and a combination of tones and to the formation of a locking region $2\omega_{10\,\max}$ (Fig. 1c). The above-described onset of the locking was analogous for all the considered transitions.

The minimum value of the intermode distances $\omega_{12\,cr}$ prior to locking, at optimal laser parameters, was approximately 10 kHz for the $2s_2-2p_1$ transition and approximately 500 kHz for the transitions $3s_2-3p_2$ and $3s_2-3p_4$. Owing to the smallness of $\omega_{12\,cr}$ in comparison with the widths of the lines and of the transition levels, the linear and nonlinear mode shifts should not lead to

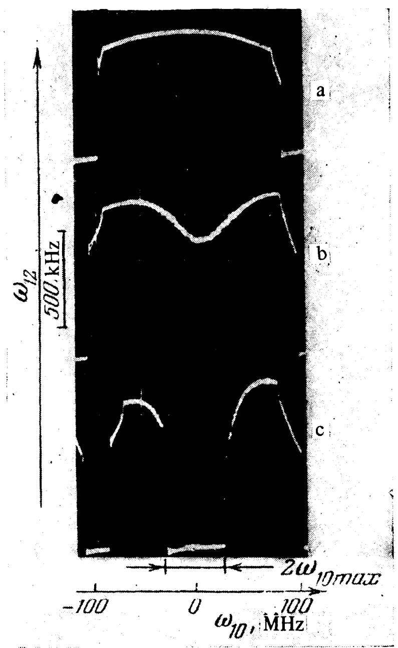

FIG. 1. Oscillograms of the behavior of the frequency ω_{12} as a function of the detuning ω_{10} on the neon transition $3s_2-3p_2$ at constant Ω_{12} for three different laser excitations corresponding to: a) $\Omega_{12}>\Omega_{12\,cr}$, b) $\Omega_{12}\gtrsim\Omega_{12\,cr}$, c) $\Omega_{12}<\Omega_{12\,cr}$.

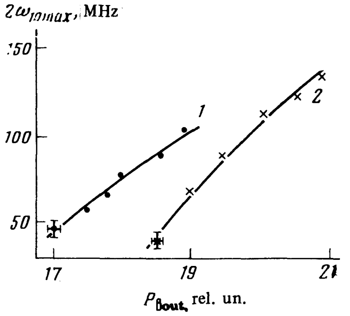

FIG. 2. Dependence of the locking region $2\omega_{10\,\max}$ on the radiation output power P_{out} for the neon transition $3s_2$–$3p_2$ at constant Ω_{12} and pressures $p = 2.4$ Torr (curve 1) and $p = 1.7$ Torr (curve 2).

an appreciable difference between $\omega_{12\,\text{cr}}$ and $\Omega_{12\,\text{cr}}$, and expressions (6) and (7) can be used to explain the results. Estimates have shown that the substantial difference between the values of $\omega_{12\,\text{cr}}$ for the transitions $2s_2$–$2p_1$ and $3s_2 - 3p_2$, $3s_2 - 3p_4$ is due primarily to the bandwidths of the laser resonators for the considered transitions. For all the transitions, a growth of $\Omega_{12\,\text{cr}}$ is observed with increasing mixture pressure. This indicates that atomic collisions influence the mode locking not only in transitions with weak level degeneracy, but also in a transition with level angular momenta 1 and 2.

It was observed that the locking region is extremely sensitive to variation of the laser parameters and increasing with increasing excitation and pressure. The dependence of this region on the output power P_{out} of the radiation for different values of p at a constant distance between the resonator modes is illustrated in Fig. 2. The results obtained with allowance for the fact that $P_{\text{out}} \sim \eta$ agreed qualitatively with expression (9) and showed that the locking region can become quite large. By choosing Ω_{12} and the excitation we can easily lock two modes in the entire detuning range where the gain exceeds the losses.

The measurements have shown that in the locking region the laser radiation has elliptic polarization whose semiaxes are rotated relative to the polarization planes relative to the non-locked mode by an angle $\varphi \approx 15°$, and the ratio $(a/b)^2$ of the ellipse semiaxes was approximately equal to two for the $2s_2$–$2p_1$ transition and to three for the $3s_2 - 3p_2$ transition. In the measurements we did not observe, accurate to 10%, any change in the degree of ellipticity within the limits of the capture region. The fact that the radiation has elliptic polarization in the case of mode locking confirms the results of the theoretical analysis. The less pronounced degree of ellipticity of the polarizations for the transition $2s_2$–$2p_1$ is apparently due to the smaller difference between the values of Q_x and Q_y of the modes with different polarizations in this transition in comparison with the $3s_2$–$3p_2$ transition.

Direct registration of the beat signal with an oscilloscope makes it also possible to obtain some information on the relative phase ψ in the locking region. Since the registered signal is in this case $\sim E_x E_y \cos\psi$ and, as noted above, the degree of ellipticity of the polarization does not change with detuning, it follows that the change of the registered signal is connected only with ψ. Depending on the detuning, the phase remains practically constant within the limits of the locking region, with a value 0 or π determined principally by the adjustments of the resonator and the $\lambda/4$ plates. By varying the adjustment we were also able to obtain a rarer locking regime, in which the phase varied with the tuning and was equal to $\pi/2$ when the modes were symmetrical about the line center. Typical oscillograms of the beat signals for $\psi = 0$ and $\psi = \pi/2$ are shown in Fig. 3. Since the adjustment influences primarily the values of Q_x and Q_y for the different polarizations, it appears that it is precisely the Q values which determine the relative phase shift in the locking region.

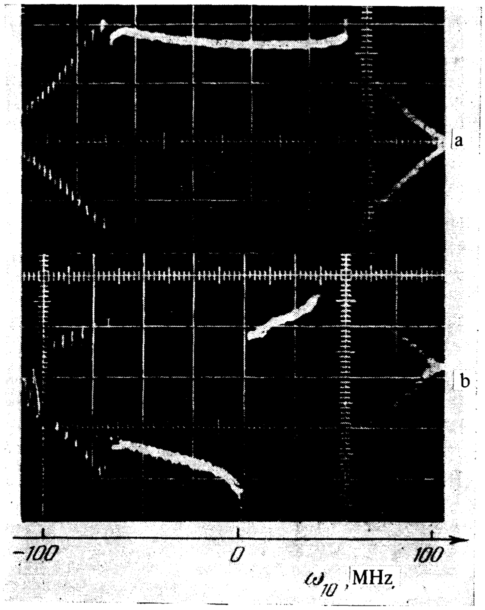

FIG. 3. Oscillograms of the beat signal as a function of the detuning ω_{10} for the neon transition $2s_2 - 2p_1$ at $\Omega_{12} = \text{const}$, $\omega_{10} = \omega_{12}/2$ for a) $\psi = 0$ and b) $\psi = \pi/2$.

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