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Alfven and magnetosonic vortices in a plasma

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It is shown that Alfven and magnetosonic waves can exist in a plasma in the form of two- and three-dimensional vortices. The dispersion spreading of such vortices is impeded by nonlinear effects. Magnetosonic waves form a toroidal vortex that travels along the magnetic field with Alfven velocity. Nonlinear Alfven waves form along the magnetic field an axially symmetrical waveguide. Inside of which the plasma executes vortical oscillations in the azimuthal direction. MHD vortices of this kind are observed in the earth's magnetosphere.

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1. INTRODUCTION

It is known that various types of vortices can propagate in an incompressible liquid. A quantitative description of this phenomenon, however, encounters considerable difficulties because there is no small expansion paramter.[1] An examination of such vortices in a low-pressure plasma ($\beta\equiv 8\pi p/H_0^2 \ll 1$, where p is the plasma pressure and H_0 is the unperturbed magnetic field), undertaken in the present paper, is facilitated by the presence of small parameters. This makes it possible to reduce the magnetohydrodynamics equations to simpler equations of non-one-dimensional solitons with the aid of perturbation theory, by assuming the dispersion and nonlinear terms to be small quantities of the same order (an expansion of the Korteweg–de Vries type). The importance of solutions in the form of non-one-dimensional solitons lies primarily in the fact that in contrast to simple wave packets they do not spread out when they propagate in the plasma, as a result of which they accumulate much energy and can therefore be relatively easily observed. The amplitude and dimensions of such formations are related by simple equations, which make it possible to distinguish them in experiment from perturbations of other types.

There are known solutions of the plasma equations in the form of one-dimensional magnetosonic[2] and Alfven[3] solitons. The energy of the one-dimensional solitons, however, is very large, and they can be produced only by large plasma perturbations, such as solar flares. In addition, it can be shown that in the cases considered in the present paper the one-dimensional solitons are stable. Taking this circumstance into account, we are interested in solitons with maximum possible dimensionality. There are a number of known solutions of the plasma equations in the form of three-dimensional solitons.[4,5] All the cited studies, however, were confined to potential oscillations. Yet many observations in the magnetosphere, which is still the best object for the study of waves in a plasma, point to the existence of non-one-dimensional solitons of the Alfven and of the magnetosonic type. They constitute vortices that travel along the magnetic field with a velocity close to the Alfven velocity.

Recently the interest in Alfven and magnetosonic perturbations of plasma has increased because they can be easily made to build up in tokomaks of the future, where the condition $\beta > m/M$ must be satisfied if controlled nuclear fusion (CNF) is to be realized. At low amplitudes these perturbations have a rather large localization region and can therefore be easily stabilized by shear of the force lines. Allowance for the nonlinearity leads to self-focusing—to a decrease of the characteristic dimensions of the perturbations. The influence of the shear of the force line is therefore decreased and the perturbations can increase to amplitudes that are dangerous for plasma containment.

MHD vortices might also be observed in a solid-state plasma, where MHD wave propagation is possible. We note that the existence of one-dimensional MHD solitons traveling along the magnetic field is impossible because of the absence of the nonlinearity which is needed to compensate for the dispersion spreading of the wave

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packet. It is shown in this paper that the nonlinearity is produced by the non-one-dimensionality of these wave packets. The presence of nonlinearity impedes their dispersion spreading and leads to formation of MHD vortices of both the magnetosonic and the Alfven type. These vortices have axial symmetry about the force line of the constant magnetic field and their velocity is somewhat lower than the Alfven velocity. The difference between them is that a magnetosonic vortex is bounded on all sides and is oblate in the propagation direction. The velocity circulation takes place in it just as in vortex ring. The perturbed magnetic field has a predominantly radial direction. An Alfven vortex is a waveguide elongated in the magnetic field direction. The wavelength in the waveguide is much less than the waveguide radius, and the perturbed magnetic field and the velocity have mainly an azimuthal direction. The current lines are closed in the zero-order approximation.

2. FUNDAMENTAL EQUATIONS

The dispersion equation for fast magnetosonic (FMS) and Alfven waves in a plasma with $\beta \ll 1$ is of the form[6]

$$\omega^2=k_z^2c_A^2+{}^1/_2k_\perp^2c_A^2\pm(k_z^6c_A^6\Omega^{-2}+k_\perp^4c_A^4/4)^{1/2},\qquad k_\perp^2/k_z^2\lessdot|k_z|c_A/\Omega\ll1. \tag{2.1}$$

The z axis is directed here along the magnetic field, Ω is the cyclotron frequency of the ions, and the upper sign in (2.1) corresponds to magnetic sound. It is seen from (2.1) that a FMW has positive dispersion.[5] This indicates that if nonlinearity is present, then FMW can produce three-dimensional vortices. The dispersion of Alfven waves along z is negative and they can therefore be localized only in a plane transverse to the magnetic field.

We use the MHD equations written in a cylindrical coordinate system assuming axial symmetry $(\partial/\partial\varphi=0)$ and neglecting the plasma pressure and the dispersion:

$$\frac{\partial v_r}{\partial t}+\mathbf{v}\nabla v_r=\frac{v_\varphi^2}{r}+\frac{H_z}{D}\left(\frac{\partial H_r}{\partial z}-\frac{\partial H_z}{\partial r}\right)-\frac{H_\varphi}{Dr}\frac{\partial}{\partial r}rH_\varphi, \tag{2.2}$$

$$\frac{\partial v_\varphi}{\partial t}+\mathbf{v}\nabla v_\varphi+\frac{v_rv_\varphi}{r}=\frac{1}{D}\left(H_z\frac{\partial H_\varphi}{\partial z}+\frac{H_r}{r}\frac{\partial}{\partial r}rH_\varphi\right), \tag{2.3}$$

$$\frac{\partial v_z}{\partial t}+\mathbf{v}\nabla v_z=\frac{H_r}{D}\left(\frac{\partial H_z}{\partial r}-\frac{\partial H_r}{\partial z}\right)-\frac{H_\varphi}{D}\frac{\partial H_\varphi}{\partial z}, \tag{2.4}$$

$$\frac{\partial H_r}{\partial t}=\frac{\partial}{\partial z}(v_rH_z-v_zH_r), \tag{2.5}$$

$$\frac{\partial H_\varphi}{\partial t}=\frac{\partial}{\partial z}(v_\varphi H_z-v_zH_\varphi)-\frac{\partial}{\partial r}(v_rH_\varphi-H_rv_\varphi), \tag{2.6}$$

$$\frac{\partial H_z}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}r(v_zH_r-v_rH_z), \tag{2.7}$$

$$\frac{\partial\ln D}{\partial t}+\mathbf{v}\nabla\ln D=-\operatorname{div}\mathbf{v}. \tag{2.8}$$

Here $D\equiv 4\pi nM$ is proportional to the plasma density. This system can be simplified by assuming the v is small: $v\ll c_A\equiv H_0/D^{1/2}$, and the values of H_φ and H_z-H_0 are much less than the constant magnetic field H_0.

3. MAGNETOSONIC VORTEX

We consider first a FMS vortex. Let $v_\varphi=0$, $H_\varphi=0$, $|\partial/\partial r|\ll|\partial/\partial z|$, and let H_r and v_r be much larger than the remaining perturbed quantities. The nonlinear terms in (2.2) and (2.5) are small, so that we can substitute in them the expressions obtained from the other equations by successive approximation. We assume that the nonlinearity results in small corrections on the order of the dispersion terms in (2.1). We therefore neglect in the calculation of the nonlinear corrections the dispersion and use the ordinary system of the MHD equations (2.2)—(2.8). The vortex travels along z with a velocity close to c_A, so that we can put in small terms $\partial/\partial t\approx -c_A\partial/\partial z$. We then have in the first nonvanishing approximation

$$v_r\approx -c_A\frac{\partial\xi}{\partial z},\quad H_r\approx H_0\frac{\partial\xi}{\partial z},\quad H_z\approx-\frac{H_0}{r}\frac{\partial}{\partial r}r\xi,\qquad v_z\approx\frac{c_A}{2}\left(\frac{\partial\xi}{\partial z}\right)^2,\quad \frac{D-D_0}{D_0}\approx\frac{v_z}{c_A}-\frac{1}{r}\frac{\partial}{\partial r}r\xi. \tag{3.1}$$

The integration constants are equal to zero because the perturbations vanish at infinity; we introduce the plasma displacement in accord with the formula

$$\xi=\xi(z-c_At,r,t),\quad v_r=\frac{d\xi}{dt}=\frac{\partial\xi}{\partial t}-c_A\frac{\partial\xi}{\partial z}. \tag{3.2}$$

The dependence of ξ on the first argument is assumed small.

Substituting (3.1) and (3.2) in Eqs. (2.2) and (2.5), we obtain after simple transformations

$$\frac{2}{c_A}\frac{\partial^2\xi}{\partial z\,\partial t}+\Delta_\perp\xi-\frac{1}{2}\frac{\partial}{\partial r}\left(\frac{\partial\xi}{\partial z}\right)^2-\frac{\partial}{\partial z}\left(\frac{\partial\xi}{\partial z}\frac{1}{r}\frac{\partial}{\partial r}r\xi\right)+\frac{1}{6}\frac{\partial}{\partial z}\left(\frac{\partial\xi}{\partial z}\right)^3=0. \tag{3.3}$$

This equation describes the evolution of a magnetosonic packet in a reference frame that moves along z with a velocity c_A, neglecting dispersion effects. Assuming the dispersion corrections to be small, on the order of the nonlinear corrections, we refine (3.3) by comparing it with (1.1). We get

$$\frac{2}{c_A}\frac{\partial^2\xi}{\partial z\,\partial t}+\frac{1}{2}\Delta_\perp\xi+\frac{c_A}{\Omega}\hat{K}\xi-\frac{1}{2}\frac{\partial}{\partial r}\left(\frac{\partial\xi}{\partial z}\right)^2-\frac{\partial}{\partial z}\left(\frac{\partial\xi}{\partial z}\frac{1}{r}\frac{\partial}{\partial r}r\xi\right)+\frac{1}{6}\frac{\partial}{\partial z}\left(\frac{\partial\xi}{\partial z}\right)^3=0. \tag{3.4}$$

Here $\hat{K}$ is an operator whose Fourier spectrum is equal to $-(k_z^6+\Omega^2k_\perp^4/4c_A^2)^{1/2}$. Equation (3.4) has a stationary solution in the form

$$\xi=-\frac{1}{A}\frac{c_A}{\Omega}f(\zeta,\rho),\quad \zeta=\frac{\Omega A^2}{c_A}\left(z+\frac{A^2c_At}{2}\right),\quad \rho=\frac{\Omega A^3}{c_A}r; \tag{3.5}$$

A is the dimensionless amplitude of the vortex, $A\ll1$, and f satisfies the equation

$$\frac{\partial^2 f}{\partial\zeta^2}+\frac{1}{2}\frac{\partial}{\partial\rho}\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho f+\hat{K}_0f+\frac{1}{2}\frac{\partial}{\partial\rho}\left(\frac{\partial f}{\partial\zeta}\right)^2+\frac{\partial}{\partial\zeta}\left(\frac{\partial f}{\partial\zeta}\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho f\right)+\frac{1}{6}\frac{\partial}{\partial\zeta}\left(\frac{\partial f}{\partial\zeta}\right)^3=0, \tag{3.6}$$

where the operator $\hat{K}_0$ has a Fourier spectrum in the form $-(k_\zeta^6+k_\rho^4/4)^{1/2}$.

Equation (3.6) is the Euler equation for the following functional:

$$H(f)=\int\rho\,d\rho\,d\zeta\left[\left(\frac{\partial f}{\partial\zeta}\right)^2+\frac{1}{2}\left(\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho f\right)^2 -f\hat{K}_0 f+\frac{1}{12}\left(\frac{\partial f}{\partial\zeta}\right)^4+\frac{1}{2}\left(\frac{\partial f}{\partial\zeta}\right)^2\frac{\partial}{\rho\,\partial\rho}\rho f\right]. \tag{3.7}$$

It is seen from (3.7) that H is bounded from below. We can therefore conclude that the solution of (3.6) realizes the absolute minimum of this functional. According to the Lagrange theorem, this suffices for the solution of (3.4) in the form of a magnetosonic vortex described by (3.5) and (3.6) to exist and to be stable. To prove the stability we note that Eq. (3.4) conserves two integrals: the Hamiltonian (3.7) and the particle number $\int(\partial\xi/\partial z)^2 d\mathbf{r}$. The proof of the stability of the vortex can therefore be obtained by a method used by Zakharov and Kuznetsov,[7] who investigated the stability of a three-dimensional ion-sound soliton in a magnetic field.

The numerical solution of (3.6) is made difficult by the fact that it has a cubic nonlinearity on top of the quadratic one. In the laboratory coordinate frame, such a vortex travels with a velocity $c_A(1-A^2)$ and has the following parameters:

$$L_z\sim c_A/\Omega A^2,\quad L_r\sim c_A/\Omega A^3,\quad H_r\sim H_0 A,\quad v_r\sim c_A A, \tag{3.8}$$

where L_z is the characteristic dimension of the vortex along z, and L_r is the dimension across z. The dependence of the characterisitc parameters of the vortex on the coordinates is obtained from formulas (3.8). The total energy of the FMS vortex is given by

$$W\approx\frac{1}{4\pi}\int H_r^2\,d\mathbf{r}=\frac{1}{2}\left(\frac{c_A}{\Omega}\right)^3\frac{H_0^2}{A^6}\int\left(\frac{\partial f}{\partial\zeta}\right)^2\rho\,d\rho\,d\zeta. \tag{3.9}$$

It follows from a comparison of (3.8) with (3.9) that when the vortex energy increases its dimensions increase and the amplitude decreases. The amplitude depends little on the vortex energy.

4. ALFVEN VORTEX

In the case of Alfven waves the difference of the signs of the longitudinal and transverse dispersions in the dispersion equation (1.1) indicates that three-dimensional localization of an Alfven wave packet is impossible.[4] What is possible, however, is the formation of a stable cylindrical waveguide elongated along a force line of the constant magnetic field, under the condition that a corresponding nonlinearity can be found.

The transverse dispersion is appreciable in the case when the ratio of the waveguide radius to the wavelength is large enough. Then the dispersion equation (1.1) for the Alfven wave can be rewritten in the form

$$\omega\approx k_z c_A[1+k_\perp^2/2k_z^2-c_A|k_z|/2\Omega],\quad k_\perp^2 k_z^{-2}\ll c_A|k_z|/\Omega\ll 1. \tag{4.1}$$

We seek the nonlinear corrections. We assume in first order that only v_φ and H_φ differ from zero and that $\partial/\partial t\approx -c_A\partial/\partial z$. In the second approximation we have from (2.2)–(2.8) that

$$v_\varphi\approx -c_A h,\quad v_z\approx c_A[h^2/2-\psi(r,t)],\quad h=H_\varphi/H_0,\quad H_z-H_0\approx -H_0h^2/2,\quad H_r\approx -H_0 v_r/c_A,\quad c_A\frac{\partial H_z}{\partial z}\approx\frac{H_0}{r}\frac{\partial}{\partial r}rv_r,\quad \operatorname{div}\mathbf{v}=0,\quad D=\text{const}. \tag{4.2}$$

Here $\psi(r,t)$ is an arbitrary function which will be determined later on. We substitute these expressions in the nonlinear terms of (2.3) and (2.6). We get

$$\frac{\partial v_\varphi}{\partial t}\approx c_A(1-\psi)\frac{\partial h}{\partial z},\quad \frac{\partial h}{\partial t}\approx c_A(1-\psi)\frac{\partial v_\varphi}{\partial z}. \tag{4.3}$$

We seek the solution of the system (4.3) in the form of a harmonic wave traveling along z:

$$h=\frac{1}{2}h_0(r,t)\exp[ik_0(z-c_At)],\quad k_0>0, \tag{4.4}$$

where the dependence of h_0 on t and r is assumed to be weak. We can now determine ψ by starting from (4.4) and the approximate equation obtained from (2.4):

$$\frac{\partial v_z}{\partial t}\approx-\frac{c_A}{2}\frac{\partial}{\partial z}h^2\approx ik_0c_Ah_0^2\exp[2ik_0(z-c_At)]. \tag{4.5}$$

This equation shows that v_z has no dc component, so that we must put in (4.2)

$$\psi=|h_0|^2/4. \tag{4.6}$$

Then, taking (4.1) and (4.3)–(4.6) into account, we obtain an equation that describes the evolution of an Alfven packet in a reference frame that moves at the Alfven velocity:

$$-\frac{2i}{c_Ak_0}\frac{\partial h_0}{\partial t}=|h_0|^2h_0+\frac{ck_0}{\Omega}h_0+\frac{1}{k_0^2}\Delta_\perp h_0. \tag{4.7}$$

It is assumed that the characteristic dimension across the magnetic field is much larger than $1/k_0$.

It is seen from (4.7) that in the absence of nonlinearity the packet would spread transversely to the magnetic field because of the dispersion effect, within a time on the order of $L_r^2k_0/c_A$, where L_r is the transverse dimension of the packet. The nonlinear impedes the spreading, so that (4.7) has a stationary solution in the form

$$h_0=Af(\rho)\exp\left[ic_Ak_0\left(A^2+\frac{c_Ak_0}{\Omega}\right)\frac{t}{2}\right],\quad \rho=Ak_0r, \tag{4.8}$$

where f satisfies the equation

$$\frac{\partial}{\partial\rho}\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho f=f-f^3. \tag{4.9}$$

A plot of the solution (4.9) is given in the paper of Litvak and Fraĭman.[8]

Thus, under the natural condition that the wave does not drag the plasma with it, self focusing of a harmonic Alfven wave takes place along the force line of the magnetic field. The magnetic field oscillates in azimuth with an amplitude H_φ, and the effective radius is equal to L_r:

$$L_r \sim 1/Ak_0, \quad H_\varphi \sim AH_0, \tag{4.10}$$

where the wave number k_0 along z and the dimensionless amplitude A should satisfy the inequality given in (4.1), which we write for the case of a waveguide, taking (4.10) into account, in the form

$$A^2 \ll k_0 c_A/\Omega \ll 1. \tag{4.11}$$

The waveguide is stable to small perturbations because of the difference between the signs of the dispersion in (4.1). The relation between the signs of the nonlinearity and the dispersion along z is such that localization of an Alfven wave along z is impossible. Since the wave is focused by the nonlinearity of the H_z oscillation, the waveguide is tied to the magnetic-field force line, whereas the plasma can drift because of the curvature of the force lines of the magnetic field. Since the frequencies excited are not quite equal, beats appear in the waveguide and are observed in experiment in the form of "pearls." We note that the resonant character of the interaction of particles with vortices can also lead them to modulation along z.[10]

The phase velocity of the wave in the waveguide depends both on the amplitude and on the wave number

$$u = c_A(1 - A^2 - c_A k_0/\Omega)^{1/2}. \tag{4.12}$$

The inequality (4.11) shows that the dependence on k_0 is stronger than the dependence on the amplitude.

5. DYNAMICS OF THE VORTICES

In the presence of a weak instability localized in space, waves and solitons will build up in the plasma. However, the wave energy remains small because of the dispersion spreading. Solitons, on the other hand, do not spread and can therefore accumulate a large energy. Let us estimate the growth of the soliton energy density at a given growth rate $\gamma_{\mathbf{k}}$. In a plasma, such solitons can build up in the presence of a group of fast particles with an anisotropic velocity distribution.[11] The growth rate of this instability is

$$\gamma_{kz} = -\varepsilon\sqrt{\pi}\frac{\Omega^3 \exp(-\Omega^2/k_z^2 v_{T\parallel}^2)}{2k_z^2 v_{T\parallel} c_A}\frac{T_\perp}{T_\parallel}\left(\frac{|k_z|c_A}{\Omega} \mp \frac{T_\perp - T_\parallel}{T_\perp}\right), \tag{5.1}$$

where the upper sign corresponds to the case of Alfven waves and the lower to magnetosonic waves, ε is the ratio of the density of the fast particles to the density of the cold plasma, $T_{\perp,\parallel}$ are the transverse and longitudinal temperatures of the fast particles, and $v_{T\parallel}$ is their longitudinal thermal velocity.

It follows from (5.1) that to excite Alfven wave it is necessary that the transverse temperature of the fast particles exceed the longitudinal one, $T_\perp > T_\parallel$. The magnetosonic waves build up if the anisotropy is reversed, $T_\parallel > T_\perp$. This instability can play an important role in a two-component tokamak with injection of neutral atoms. The conditions for its appearance are frequenctly realized in the magnetosphere plasma. An example of the excitation of Alfven waves in the magnetosphere plasma is the generation of geomagnetic pulsations of the type $P_c - 1$ (pearls). The magnetosonic waves can be produced in the nighttime sector of the magnetosphere in the quasicapture region, where satellite measurements[12] and theoretical estimates[13] demonstrate the possibility of a distribution of fast protons with $T_\parallel > T_\perp$. In particular, development of cyclotron instability of magnetosonic waves can correspond to generation of geomagnetic pulsations of the type $P_i - 1B$.

The interaction of the plasma particles with solitons can be divided into two types. In one case the particle interacts with the soliton just as with a monochromatic sinusoidal wave (in resonant fashion[10]), and in the other it does not feel the phase correlation of the harmonics in the soliton. The first type of interaction occurs in the case of an Alfven vortex. It calls for a special analysis, and we confine ourselves here to the following remark: in an unstable plasma the energy density of the vortices should increase, and this is possible when their number per unit area transverse to the magnetic field increases; if the vortices are closely packed, this can take place with decreasing vortex dimensions, accompanied by an increase of their amplitude.

In the second case, which is much simpler, as shown by Rudakov,[14] the change of the soliton amplitude in a weakly unstable medium can be described by the equation

$$\partial W/\partial t = 2\int \gamma_{\mathbf{k}} W_{\mathbf{k}}\, d\mathbf{k} \approx 2\bar{\gamma} W, \tag{5.2}$$

where $W = \int W_{\mathbf{k}} d\mathbf{k}$ is the soliton energy, $W_{\mathbf{k}}$ is the energy of the harmonic of the Fourier expansion of the soliton solution, $\gamma_{\mathbf{k}}$ is the linear growth rate, and $\bar{\gamma}$ is its averaged value with weight $W_{\mathbf{k}}$. Starting with this formula, let us examine the behavior of a magnetosonic vortex in a medium with $\gamma_{\mathbf{k}} \neq 0$. Using the expression calculated for W in Sec. 3 (formula (3.9)), and substituting it in (5.2), we obtain in order of magnitude

$$\frac{\partial}{\partial t}\frac{1}{A^6} \approx 2\bar{\gamma}\frac{1}{A^6}. \tag{5.3}$$

It is seen from (5.3) and (3.8) at $\bar{\gamma} > 0$ the dimensions of the FMS vortex increase in coordinate space, and its amplitude decreases (this corresponds to contraction of the soliton in **k**-space). This expansion can be halted by interaction with neighboring vortices or because the effective wave number of the vortex decreases upon expansion, and the vortex can go out of the instability region in **k**-space.

It follows from (5.1) that the growth rate of the cyclotron instability decreases exponentially at $k_z \ll \Omega v_{T\parallel}$. From this we get that the expansion of the vortex slows down if $A^2 \ll c_A/v_{T\parallel}$. We can then obtain the characteristic parameters of the vortex with the aid of (3.8).

6. CONCLUSION

The foregoing analysis shows that low-frequency potential waves can exist in a plasma in the form of three-dimensional and two-dimensional vortices. A connection exists between their amplitude and the characteristic dimensions. It turns out that magnetosonic vortices are three-dimensional, and in an unstable medium they expand until their Fourier spectrum goes out of the instability region in the wave-number space. Alfven vortices are two-dimensional, and in a homogenous unstable plasma their amplitude increases and the dimensions decrease.

In a recent paper, Hasegawa and Mima[15] also investigated one-dimensional Alfven solitons. The nonlinearity obtained by them is proportional to the longitudinal electric field. It has been shown by others,[16,17] however, that the longitudinal electric field in Alfven waves is $\sim k_\perp^2 \rho_s^2$ ($\rho_s = c_s/\Omega$, where c_s is the speed of the ion sound), a quantity regarded in[15] as the small expansion parameter. Consequently, the equilibrium between the dispersion and nonlinearity, which must take place in the soliton, is reached at $k_\perp^2 \rho_s^2 \approx 1$, in contradiction to the initial assumptions of Hasegawa and Mima.[15]

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Experimental investigation of the mechanism of turbulent heating of a plasma carrying a transverse current

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A stationary model of a plasma with a transverse current was realized in a system consisting of an electron-ionized ion beam moving perpendicular to a magnetic field. It is established that the produced turbulence is due to excitation of ion-sound oscillations that propagate across the magnetic field. The oscillations are unstable in a wide wavelength range bounded from below by the average Larmor radius of the electrons. The characteristics of the instability are investigated in the saturation regime. It is established that effective linear transformation of the noise spectrum takes place in the direction towards decreasing frequencies (wave numbers). The structure of the wave process in k-space is experimentally investigated, and it is established that the excited oscillations are three dimensional and that their phase correlation is disturbed. At large wave numbers, the steady-state nonlinear spectrum is characterized by an exponential decrease of the noise amplitude with increasing k. In the case of advanced instability, the ions and electrons are found to be heated. The final state of the plasma is characterized by a relation $T_i \lesssim T_e$ between the ion and electron thermal energies, owing to the rapid heating of the ions as a result of capture by the ion-sound waves.

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The turbulence produced by development of instability in a current-carrying plasma (anomalous resistance) can lead to heating of the electrons and ions. In systems in which the current flows across the magnetic field, these effects are attributed to excitation of low-frequency oscillations.[1–10] Plasma heating by a transverse current was observed in experiments with shock, magnetosonic, and ion-cyclotron waves of large am-

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