Linear interaction of electromagnetic waves in a neutral current layer of a plasma

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Linear interaction of ordinary and extraordinary waves in a neutral current layer of a plasma is considered. The interaction is the result of violation of the geometric-optics approximation in a region adjacent to a plane with a zero magnetic field. The effectiveness of the interaction is determined by a parameter whose value is independent of the frequency. Analytic expressions for the degree of transformation are derived for the limiting cases of small and large values of the interaction parameter in comparison with unity; for arbitrary values of this parameter, the degree of transformation is obtained by numerical methods. The possibility is noted of using the effect in question for diagnostics of laboratory and cosmic plasma.

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Interest has noticeably increased in recent years in a theoretical and experimental investigation of neutral current layers containing a surface with a zero magnetic field. This circumstance is due to a considerable degree to the role played by neutral layers in cosmic plasma, and namely in the origin of magnetospheric substorms[1] and solar flares.[2]

At the same time, current layers in a plasma can alter significantly the polarization of the electromagnetic radiation that passes through them. This phenomenon is closely connected with the new form of linear interaction of ordinary and extraordinary waves in the inhomogeneous magnetic field that is characteristic of the current layer. This interaction effect, which is considered in the present paper, can turn out to be useful for the diagnostics of cosmic or laboratory plasma. We note also that the singularities of the passage of electromagnetic waves through current layers in the solar corona help explain certain polarization characteristics of the radio emission from the corona (see[3,4] on this subject).

1. PASSAGE OF WAVES THROUGH A CURRENT LAYER (QUALITATIVE PICTURE)

In a cold magnetoactive plasma, the refractive index of the wave and its polarization are determined by the values of the parameters $v=\omega_L^2/\omega^2$ and $u=\omega_B^2/\omega^2$ and of the angle α between the propagation direction in a stationary magnetic field $\mathbf{B}$, where ω is the wave frequency, ω_L is the electron plasma frequency, and ω_B is the electron gyrofrequency. The dependence of the square of the refractive index n_j on the quantity $u^{1/2}$ taken with the proper sign[1)] is shown in Fig. 1 (for constant α and v). The numbers I and II on the figure stand for the numbers of the dispersion curves, and the subscripts $j=1, 2$ of the refractive index n_j label the type of the wave (1—extraordinary, 2—ordinary). For the plasma neutral-current layer, in which

$$u^{1/2}=bl, \quad b=\text{const} \tag{1}$$

(l is the coordinate along the propagation direction), the curves of Fig. 1 characterize also the function $n_j^2(l)$. The curves of Fig. 1 were plotted for $\alpha\neq\pi/2$; they are characterized by intersection of the dispersion curves at the point $u^{1/2}=0$. If $\alpha=\pi/2$, the plots are qualitatively different, and the intersection gives way to tangency of the dispersion curves at $u^{1/2}=0$.

The polarization of the waves in a tenuous plasma with $v\ll 1$ depends in the geometric-optics approximation on the parameter

$$q^2=\frac{u\sin^4\alpha}{4\cos^2\alpha}. \tag{2}$$

If $q^2\ll 1$, then the propagation is quasilongitudinal, with circularly polarized ordinary and extraordinary waves; at $q^2\gg 1$ we have the quasitransverse case, which corresponds to linear polarization of the waves of both types. The character of the polarization of these waves on opposite sides of the layer $q^2\sim 1$ is shown in Fig. 1.

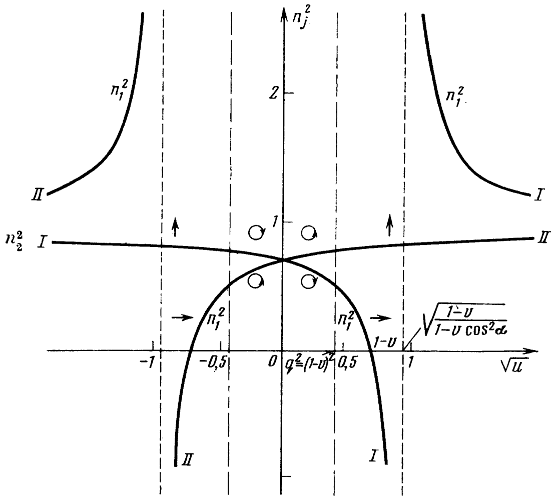

FIG. 1. Distribution of the squares of the refractive indices of the ordinary and extraordinary waves in a neutral current layer ($\alpha\neq\pi/2$). The arrows and the circles near the dispersion curves indicate the character of the polarization of the waves on opposite sides of the layer $q^2=(1-v)^2\approx 1$.

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We assume from now on that in addition to the condition $v \ll 1$, the inequality $u \ll 1$ is also valid within the limits of the current layer. The conditions for quasi-transverse approach $q^2 \gg 1$ are then satisfied only for angles that are close enough to $\pi/2$. Therefore in expression (2) for q^2 and later, in the derivation of Eq. (8), it is assumed that $\sin^2\alpha \approx 1$. We note also that in the case $v \ll 1$, $u \ll 1$, there are no reflection points and singularities of the extraordinary wave n_1^2 in the current layer, and these will not be taken into account below.

An analysis of the geometrical-optics approximation in the plasma current layer shows that it is violated for both waves simultaneously in the vicinity of the points where $n_1 = n_2$. There are three such points on the complex l plane: $l = 0$ and $l = \pm (2i/b)\cos\alpha$. The qualitative character of wave propagation under similar conditions was made clear earlier ([3], §7). It turned out that at sufficiently high frequencies (when $u \ll 1$ and $v \ll 1$) it is determined by the value of the "interaction parameter"

$$G_0 = 8\sqrt{2}\frac{\omega}{c}\frac{v|\cos^3\alpha|}{|du^{1/2}/dl|}. \tag{3}$$

Accurate to a numerical coefficient of the order of unity, this parameter is equal to the integral

$$\int \frac{\omega}{c}(n_2 - n_1)\, dl$$

in the complex l plane between the points $l = 0$ and $l = (2i/b)\cos\alpha$.

If $G_0 \gg 1$, then the geometric-optics approximation holds along the entire propagation route in the current layer. In this case, when an extraordinary wave is incident on the layer, the refractive index of the wave corresponds to the values given by curve II.[2)] The polarization of the wave far from the plane $\mathbf{B} = 0$ (in the region of quasitransverse propagation $q^2 \ll 1$) will be linear, with the vector $\mathbf{E}$ perpendicular to the plane of $\mathbf{B}$ and $\mathbf{l}$ ($\mathbf{l}$ is a unit vector in the wave propagation direction). Closer to the plane $\mathbf{B} = 0$, in layers where $q^2 \ll 1$, the polarization becomes circular; on going through the plane $\mathbf{B} = 0$, the designation of the wave corresponding to the curve II changes to "extraordinary," but the direction of rotation of the vector $\mathbf{E}$ in the wave remains unchanged. The wave then again reaches layers where $q^2 \gg 1$. The polarization again becomes linear, but the vector $\mathbf{E}$ now lies in the plane of $\mathbf{B}$ and $\mathbf{l}$, a feature characteristic of the ordinary wave.

Thus, at $G_0 \gg 1$ the propagation through a current layer takes place along one and the same dispersion curve, and is accompanied by rotation of the polarization plane by $\pi/2$ in comparison with the polarization in the incident wave. The deviation of the character of the solution from the geometric-optic approximation in a narrow layer near $l = 0$, where $q^2 \ll 1$, leads to a suppression of the weak extraordinary wave that propagates "along the curve I" in the region $l > 0$.

In the case of small values of the interaction parameter, $G_0 \ll 1$, the wave propagation through the thin layer is substantially different. A wide region where geometric optics is not valid is located on both sides of the plane $\mathbf{B} = 0$, and $q^2 \gg 1$ on the edges of this region. Consequently, that part of the current layer near the plane $\mathbf{B} = 0$ where $q^2 \gg 1$ lies entirely in the indicated region. A linearly polarized extraordinary wave entering from the left on Fig. 1 into this region does not change its polarization to fit the requirements of geometric optics, as was the case at $G_0 \gg 1$, namely, its polarization remains linear and the orientation of the vector $\mathbf{E}$ remains unchanged all the way to the emergence to the layer with $q^2 \gg 1$, which is located on the right of Fig. 1. Here, however, in the region where geometric optics holds again, this orientation of the vector $\mathbf{E}$ (in the direction of the vector $\mathbf{B} \times \mathbf{l}$) corresponds to an extraordinary wave with dispersion curve I. It follows from the foregoing that as it propagates through a current layer under conditions when $G_0 \ll 1$, the wave goes over from one dispersion curve to the other, but preserves is polarization.

Finally, in the intermediate variant $G_0 \sim 1$, the dimensions of the region where geometric optics is violated are of the order of the distance between the layers $q^2 = 1$ on the two sides of the plane $\mathbf{B} = 0$. In this case, waves of equal intensity, corresponding to curves I and II, emerge to the outside of the current layer. These linearly polarized components are mutally coherent, so that they are subject to the Cotton-Mouton effect. The resultant polarization of the radiation will depend on the phase shift produced between the waves in the current layer of the plasma.

2. CALCULATION OF LINEAR INTERACTION IN TWO LIMITING CASES

A quantitative investigation of the effect of linear interaction in a neutral current layer will be carried out here on the basis of the so-called "quasi-isotropic approximation" developed by Kravtsov[5] for the description of wave propagation in a smoothly inhomogeneous weakly anisotropic tenuous plasma ($u \ll 1$; $v \ll 1$).

In this approximation, the high-frequency electric field in the inhomogeneous plasma is sought in the form

$$\mathbf{E} = \frac{1}{\sqrt{n}}(F_1\mathbf{e}_1 + F_2\mathbf{e}_2)\exp\left(i\frac{\omega}{c}\int n\, dl\right), \tag{4}$$

where $n = (1 - v)^{1/2}$ is the refractive index in the isotropic plasma, dl is a length element of the ray, and $\mathbf{e}_1$ and $\mathbf{e}_2$ are unit vectors of the polarization in a plane perpendicular to the tangent $\mathbf{l}$ to the ray. (The vector $\mathbf{e}_1$ lies in the plane of $\mathbf{B}$ and $\mathbf{l}$, while the vector $\mathbf{e}_2$ is perpendicular to this plane.) The functions $F_1(l)$ and $F_2(l)$ are then described by a linear system of equations[6]

$$\begin{aligned} \frac{dF_1}{dl} &= -i\frac{\omega}{2c}vu\cos^2\alpha F_1 + \frac{\omega}{2c}vu^{1/2}\cos\alpha F_2, \\ \frac{dF_2}{dl} &= -\frac{\omega}{2c}vu^{1/2}\cos\alpha F_1 - i\frac{\omega}{2c}vuF_2. \end{aligned} \tag{5}$$

In (5), α is the angle between $\mathbf{B}$ and $\mathbf{l}$; the torsion of the ray, due to the rotation of the force lines of the field $\mathbf{B}$ relative to the ray, is disregarded.

We approximate the magnetic field in the neutral current layer by the linear relation (1); we assume also that the plasma is homogeneous (v = const) and tenuous enough ($v \ll 1$). The latter assumption allows us to neglect refraction and put α = const. Taking the foregoing into account and changing to new variables

$$K=iF_1/F_2, \quad \xi=Al, \tag{6}$$

$$A=(\omega v b^2/2c)^{1/3}, \tag{7}$$

we obtain from the system (5) a nonlinear equation for the polarization coefficient K:

$$\frac{dK}{d\xi}=ip\xi(1-K^2)+i\xi^2K. \tag{8}$$

Equation (8) contains only one parameter

$$p=\left(\frac{\omega}{2c}\frac{v\cos^2\alpha}{b}\right)^{1/3}=\left(\frac{\omega_L^2\cos^2\alpha}{2c\,d\omega_B/dl}\right)^{1/3}. \tag{9}$$

From a comparison of (9) and (3) it is clear that this quantity is connected with the characteristic interaction parameter G_0 by the relation

$$2^{2/3}p=G_0^{1/3}. \tag{10}$$

Equation (8) can be easily solved at values of p that ensure satisfaction of the inequality $|K^2|\ll 1$ along the entire route, including the interaction region. Then $ip\xi(1-K^2)\approx ip\xi$ and the solution of (8) is obtained in this case in elementary fashion. If the boundary condition is $K(\xi=-\infty)=0$,[3] which correspond to incidence of a linearly polarized extraordinary wave with vector $\mathbf{E}\parallel\mathbf{e}_2$ on the current layer, the solution is

$$K=ip\exp\left(\frac{i\xi^3}{3}\right)\int\limits_{-\infty}^{\xi}\xi\exp\left(-\frac{e\xi^3}{3}\right)d\xi. \tag{11}$$

To find the transformation coefficient that determines the effectiveness of the wave interaction we must know the value of KK^* at $\xi\gg 1$ (see Sec. 3). It follows from (11) that as $\xi\to+\infty$ we have

$$KK^*=4\pi p^2[v'(0)]^2, \tag{12}$$

where $v'(0)\approx -0.459$ is the derivative of the Airy function for zero argument. In the change from (11) and (12) we took into account the fact that

$$\int\limits_{-\infty}^{+\infty}\xi\exp\left(-\frac{i\xi^3}{3}\right)d\xi=2i\pi^{1/2}v'(0). \tag{13}$$

According to (12), in order for the obtained solution to be valid it is necessary to satisfy the condition $p^2\ll 1$.

It is possible to obtain for (8) an asymptotic solution in the other limiting case $p\to\infty$. This solution is close to that obtained by geometric optics practically in the entire current layer, including the interaction region (see Sec. 1).* The foregoing allows us to represent K in the form

$$K=K_{II}+\varkappa, \quad |\varkappa|\ll|K_{II}|, \tag{14}$$

assuming that the wave incident on the current layer from the side of negative ξ corresponds to the dispersion curve II. The value of K_{II} in the geometric-optics approximation should coincide with the corresponding value in a homogeneous medium (with the same parameters u and v as at the given point of the inhomogeneous medium). In a homogeneous medium, however, $dK/d\xi=0$ and consequently the function $K_{II}(\xi)$ in the current layer is subject to the relation

$$ip\xi(1-K_{II}^2)+i\xi^2K_{II}=0. \tag{15}$$

Substituting (14) in (8) we obtain, taking (15) into account, the following equation for $\varkappa$:

$$\frac{d\varkappa}{d\xi}+i(2p\xi K_{II}-\xi^2)\varkappa=-\frac{dK_{II}}{d\xi}. \tag{16}$$

Its solution subject to the boundary conditions $\varkappa=0$ (i.e., $K=K_{II}$) at the point $\xi=-\xi_0$ ($\xi_0>0$) is of the form

$$\varkappa=-\exp\left[-i\int\limits_{-\xi_0}^{\xi}(2p\xi K_{II}-\xi^2)d\xi\right]\int\limits_{-\xi_0}^{\xi}\frac{dK_{II}}{d\xi}\exp\left[i\int\limits_{-\xi_0}^{\xi}(2p\xi K_{II}-\xi^2)d\xi\right]d\xi. \tag{17}$$

To find subsequently the transformation coefficient in the case of large p we shall need the value of $\varkappa\varkappa^*$ at the point $\xi=+\xi_0$; it is equal to

$$\varkappa\varkappa^*=\int\limits_{-\xi_0}^{\xi_0}\frac{dK_{II}}{d\xi}\exp\left[i\int\limits_{-\xi_0}^{\xi}(2p\xi K_{II}-\xi^2)d\xi\right]d\xi \times\int\limits_{-\xi_0}^{\xi_0}\frac{dK_{II}}{d\xi}\exp\left[-i\int\limits_{-\xi_0}^{\xi}(2p\xi K_{II}-\xi^2)d\xi\right]d\xi. \tag{18}$$

The integrals in (18) with stationary point ($\xi=0$) can be evaluated by the stationary-phase method. As a result, at fixed value of ξ_0 and as $p\to\infty$, we get

$$\varkappa\varkappa^*=\frac{\pi}{p|K_{II}(0)|}\left(\frac{dK_{II}}{d\xi}\right)_0^2=\frac{\pi}{4p^3}. \tag{19}$$

The last equation takes into account, in agreement with (15), the fact that at the point $\xi=0$ the values of $|K_{II}|$ and $|dK_{II}/d\xi|$ are respectively l and $l/2p$.

3. COEFFICIENTS OF LINEAR TRANSFORMATION

For a quantitative estimate of the effectiveness of the interaction, we introduce the linear-transformation coefficient Q defined by the relation

$$Q=\frac{|F_1^{(1)}|^2+|F_2^{(1)}|^2}{|F_1|^2+|F_2|^2}. \tag{20}$$

Here $F_1^{(1)}$ and $F_2^{(1)}$ are the amplitudes of the components of the field of the extraordinary wave with polarizations $\mathbf{e}_1$ and $\mathbf{e}_2$, respectively, passing through the interaction region in the current layer. The coefficient Q characterizes the intensity of this wave relative to the total intensity of the ordinary and extraordinary waves $|F_1|^2+|F_2|^2$.[4] The quantity Q can also be expressed in the form

$$Q=\frac{|K-K^{(2)}|^2}{|K^{(2)}-K^{(1)}|^2}\frac{1+|K^{(1)}|^2}{1+|K|^2}, \tag{21}$$

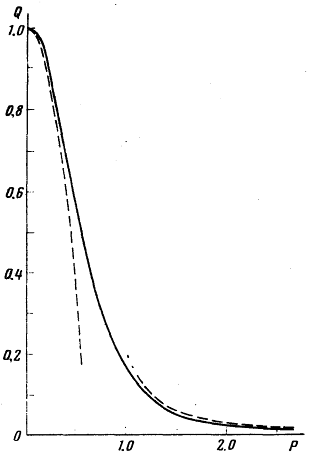

FIG. 2. The transformation coefficient Q vs. the interaction parameter p in the linear layer (1). The dashed and solid curves represent the results of analytic and numerical calculations, respectively.

where

$$K=i\frac{F_2}{F_2},\quad K^{(1)}=i\frac{F_1^{(1)}}{F_2^{(1)}},\quad K^{(2)}=i\frac{F_1^{(2)}}{F_2^{(2)}}$$

(the superscripts 1 and 2 in the parentheses indicate that the amplitudes F_1 and F_2 pertain to the extraordinary and ordinary waves, respectively). In the change from (20) to (21) we took into account the fact that

$$F_2=F_2^{(1)}+F_2^{(2)},\quad F_1=-iKF_2=-iK^{(1)}F_2^{(1)}-iK^{(2)}F_2^{(2)}. \tag{22}$$

In the case $p^2\ll 1$ we have $K^{(1)}=0$ and $K^{(2)}=\infty$ for the waves in the region of the quasitransverse propagation, and $|K|^2$ is defined by formula (12). Substituting these values in the expression (21) for Q, we obtain

$$Q\approx\frac{1}{1+|K|^2}\approx 1-4\pi p^2[v'(0)]^2. \tag{23}$$

In the other limiting case of large p (as $p\to\infty$) we have calculated the value of $\varkappa\varkappa^*$ (19) at the point $\xi=\xi_0$. Since the layer $q^2=1$ corresponds to the value $\xi=2p$, it is clear that as $p\to\infty$ we have $q^2\ll 1$ at a fixed point ξ_0 and the propagation is quasilongitudinal. This means that $K^{(1)}=1$, $K^{(2)}=-1$, and K is determined by relation (14) in which $K_{II}=K^{(2)}=-1$. Taking these equations into account, we obtain from (21) and (19)

$$Q\approx\frac{\varkappa\varkappa^*}{4}=\frac{\pi}{16p^3}=\frac{\pi\sqrt{2}}{G_0}. \tag{24}$$

In the transition to the last expression we took relation (10) into account.

Plots of Eqs. (23) and (24) for $Q(p)$ in the limiting cases of small and large values of the interaction parameter p are shown dashed in Fig. 2. Equation (8) with arbitrary p was solved by numerical means with a computer. The resultant $Q(p)$ dependence is also shown in Fig. 2 (solid curve). It is clear from the figure that the transformation coefficient is close to unity in the region $p\ll 1$ and decreases rapidly with increasing p at $p\gg 1$, in full agreement with the qualitative conclusions deduced in Sec. 1 concerning the character of passage of waves through a neutral current layer.

4. TRANSFORMATION IN A SELF-CONSISTENT CURRENT LAYER

The foregoing results are correct provided that the linear approximation (1) remains valid in the interaction region. In the self-consistent current layer of a plasma, however, the distribution of the magnetic field **B** and of the electron density N is more complicated. Thus, for example, in a collisionless plasma with a Maxwellian distribution of the electron and ion velocities in the plane $\mathbf{B}=0$ the neutral current layer (the Harris layer[7]) is described by the formulas

$$\mathbf{B}=\mathbf{B}_0\,\mathrm{th}\,(z/L),\quad N=N_0\,\mathrm{ch}^{-2}(z/L). \tag{25}$$

Here L is the characteristic thickness of the current layer, $\mathbf{B}_0$ is the value of the magnetic field far from the plane ($\mathbf{B}=0$) (at a distance $|z|\gg L$), and N_0 is the electron density in this plane.

In accord with (25), the variations of u and v along a ray passing through the current layer are given by

$$u=u_0\,\mathrm{th}^2(l/L),\quad v=v_0\,\mathrm{ch}^{-2}(l/L), \tag{26}$$

where $l=z/\sin\alpha\approx z$. The equation for the polarization coefficient K can then be written in the form

$$\frac{dK}{d\xi}=ipg(1-K^2)\,\mathrm{th}\frac{\xi}{g}\,\mathrm{ch}^{-2}\frac{\xi}{g}+ig^2K\,\mathrm{th}^2\frac{\xi}{g}\,\mathrm{ch}^{-2}\frac{\xi}{g}, \tag{27}$$

where

$$g=\left(\frac{\omega v_0u_0L}{2c}\right)^{1/3}=\left(\frac{\omega_L{}^2\omega_B{}^2L}{2\omega^3c}\right)^{1/3}. \tag{28}$$

The quantity ω_L in (28) and (9) now stands for the plasma frequency in the plane $\mathbf{B}=0$, while ω_B stands for the electron gyrofrequency far from this plane. The derivative $d\omega_B/dl$ in (9) is taken at the point $l=0$.

Under the condition $g\gg 1$, Eq. (27) goes over in (8), i.e., in this case the effect of the linear interaction in the self-consistent current layer will be characterized as before by the transformation coefficient Q on Fig. 2.

The results of the numerical solution of (27) are given for several values of the parameter g in Fig. 3. It is seen from the figure that the $Q(p)$ curves become nonmonotonic when $g\sim 1$, i.e., if the width of the effective interaction region $|\xi|\lesssim 1$ is comparable with the characteristic thickness of the current layer g. We note also that the $Q(p)$ curve for $g=10$ practically coincides with the results of the calculation of Q for the layer (1), shown in Fig. 2. From a comparison of the plots of $Q(p)$ for different g it is clear that in tentative estimates of the transformation coefficient Q one can use Fig. 2 not only at $g\gg 1$, but also at $g\sim 1$.

5. CONCLUDING REMARKS

The investigation of the polarization properties of a wave passing through a current layer can serve as a convenient device for plasma diagnostics. In fact, at sufficiently low frequencies, at which the parameter g (28) becomes much larger than unity, the transforma-

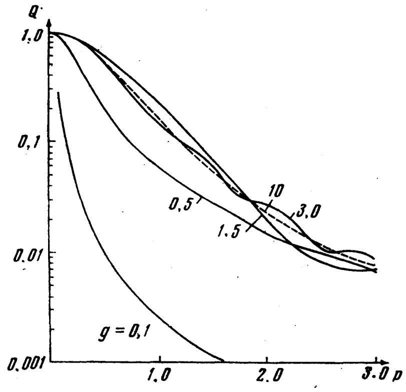

FIG. 3. Plot of $Q(p)$ in a self-consistent neutral current layer at fixed values of the dimensionless layer thickness g. The dashed curve is a plot of $Q(p)$ at $g = 10$.

tion coefficient $Q(p)$ is described by the solid curve of Fig. 2. By measuring the relative intensity of the extraordinary wave emerging from the layer in the case when the wave incident on the layer is also extraordinary, we determine the value of Q by the procedure described in Sec. 3. Finding next the corresponding value of p from the plot of Fig. 2, we determine from the known values of the angle α and of the magnetic field gradient dB/dl, with the aid of (9), the electron density N_0. If, on the contrary, we know N_0, then the indicated procedure allows us to assess the value of dB/dl in the current layer.

It should be noted that the singularities of the propagation of electromagnetic waves considered above are typical of a stable (stationary) neutral current layer. The thickness L of such a layer must be large enough (see[8]). On the other hand if L is comparable with $r_{Bi}(T_e/T_i)^{1/2}$ (at $T_e \gg T_i$) or if $L \sim r_B$ (when $T_e \simeq T_i$), then the layer becomes turbulized as a result of the development of Buneman and modified-Buneman instability on the longitudinal waves produced by the electric current in the layer (In the foregoing relations, r_B and r_{Bi} are the gyrofrequencies of electrons and ions moving with velocities $(T_e/m_e)^{1/2}$ and $(T_i/m_i)^{1/2}$), respectively.) At the same time a current layer of thickness $L \sim r_{Bi}$ is destroyed as a result of the development of an instability at which the layer breaks up into individual clusters with a characteristic perturbation dimension $\lambda > 2\pi L$, and the magnetic-field force lines are reclosed through the layer.

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1)In a current layer with a magnetic field parallel to the plane $B = 0$, the projection of the magnetic field on the propagation direction (i.e., the quantity $u^{1/2}\cos\alpha$) reverses sign on going through this plane. Instead of reversing the sign of $\cos\alpha$, we shall henceforth reverse the sign of $u^{1/2}$, assuming that $\cos\alpha$ is constant.

2)We have in mind here the region $v - 1 < u^{1/2} < 0$ on the left in Fig. 1. It is assumed that in this region there is specified an extraordinary wave propagating towards positive $u^{1/2}$.

3)More accurately speaking, at $\xi < 0$ and $|\xi| \gg 1$. To realize the latter inequality in the region $u \ll 1$, where Eq. (8) holds, it is necessary to satisfy the condition $(\omega v/2bc)^{1/3} \gg 1$.

4)According to the system of equations (5), the quantity $|F_1|^2 + |F_2|^2$ remains constant along the ray, and is consequently equal to the intensity of the extraordinary wave incident on the current layer.

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