An investigation of optical, oscillation, and galvanomagnetic effects in doped semiconducting $Bi_{1-x}Sb_x$ alloys

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A comprehensive study has been made of galvanomagnetic, oscillation, and plasma effects in the far infrared at liquid-helium temperatures in single-crystal samples of semiconducting $Bi_{0.924}Sb_{0.076}$ doped with small amounts of donor and acceptor impurities. A computational procedure is proposed and all the main Abrikosov energy-spectrum parameters are calculated for carriers at the L_a and L_s extrema. It is shown that the carrier spectra at the L_a and L_s extrema are specular for momentum directions along the short axes of the L ellipsoids and are described well by the Lax model. The carrier spectra along the prolate direction of the L ellipsoids (the bisector axis) are not specular. Evidence is obtained that the Fermi surface for the L_a electrons in $Bi_{0.924}Sb_{0.076}$ deviates from ellipsoidal and in the general case has a dumb-bell shape ($M_1 > M_2$ in the Abrikosov dispersion law). The degree of deviation of the equal-energy surface from ellipsoidal is discussed for the doped $Bi_{0.924}Sb_{0.076}$ samples investigated. The optical characteristics (high-frequency dielectric constant ϵ_∞, plasma frequency ω_p) of the materials studied are given and the characteristic optical relaxation times estimated.

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The Abrikosov–Fal'kovskiĭ[1,2] theory of the energy spectrum of bismuth and $Bi_{1-x}Sb_x$ alloys contains parameters whose values cannot be fully determined individually from optical, oscillation, or galvanomagnetic measurements. In the present work it is shown that reflection measurements in the far infrared on semiconducting $Bi_{1-x}Sb_x$ alloys doped with donor and acceptor impurities, in conjunction with the results of an investigation of the Shubnikov–de Haas quantum oscillations and of galvanomagnetic effects, permit all the basic parameters of the dispersion law in the above theory[1,2] to be determined. A computational procedure is presented and the spectrum parameters are calculated for the semiconducting alloy $Bi_{0.924}Sb_{0.076}$. In addition, the high-frequency dielectric constants and plasma frequencies at liquid-helium temperatures are determined from reflection in the plasma effect region for $Bi_{0.924}Sb_{0.076}$ doped with small amounts of impurities, and the characteristic optical relaxation times are estimated.

MEASUREMENT TECHNIQUE AND SAMPLES

Reflection measurements in the 20–250 μm wavelength region were carried out on the freshly cleaved (basal plane) surfaces of large single-crystal samples of $Bi_{0.924}Sb_{0.076}$ in nonpolarized light with a single-beam infrared spectrophotometer type FIS-21 by the technique described in[3]. It is well known that when light is normally incident on the basal plane, the reflecting power $R(\lambda)$ does not depend on its polarization.[3,4] The area of the reflecting surface of the samples was 1–1.5 cm^2. The temperature of the samples in the liquid-helium optical cryostat was measured with copper–constantan thermocouple. The construction of the cryostat ensured that the temperature was kept at 12 K in the liquid-helium experiments.

The measurements of the galvanomagnetic coefficients in weak magnetic fields were carried out by a null method. The accuracy of the electrical measurements was $\pm 5\times10^{-9}$ V.

The investigations of the Shubnikov–de Haas effect were carried out on equipment which permitted automatic recording of the $\rho(H)$ curves and of the derivatives $\partial\rho/\partial H$ and $\partial^2\rho/\partial H^2$ in magnetic fields in the range 0–55 kOe at liquid-helium temperatures. The samples for the galvanomagnetic measurements, in the form of right-angled parallelepipeds 2.5–3 mm long and 0.7×0.8 mm in cross section, were cut from oriented single-crystal billets by electric-spark machining. Before mounting the samples were etched in a mixture of 50% C_2H_5OH and 50% HNO_3. The current contacts were soldered over the whole area of the end faces, while the potential contacts (40 μm in diameter) were attached by electric-spark welding.[5]

Perfect single crystals of $Bi_{0.924}Sb_{0.076}$ alloys doped with small amounts of tellurium and tin were grown at the Baukov Institute of Metallurgy of the U.S.S.R. Academy of Sciences. The technique of preparing and inspecting them has been reported earlier.[6,7] We investigated 8 samples of $Bi_{0.924}Sb_{0.076}$ alloys; their parameters are shown in Table I.

MEASUREMENT RESULTS

The infrared reflection spectra of $Bi_{0.924}Sb_{0.076}$ alloys doped with tellurium and tin have the form characteristics of plasma reflections (Fig. 1). The dependences of the position of the plasma minimum in the $R(\lambda)$ curves and of the galvanomagnetic tensor components (see the table) on the concentration of impurities can be explained on the basis of the $Bi_{1-x}Sb_x$ energy diagram[8,9] if one bears in mind that the undoped alloy with $x = 0.076$ is a semiconductor.

The introduction of tellurium as an impurity causes an increase in the electron density in the conduction band, while tin increases that of holes in the valence band. This leads to a monotonic shift of the plasma minimum towards the shorter wavelengths and to a decrease in the absolute magnitude of the Hall component R_{231}. As is evident from Fig. 1, doping with tin leads

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TABLE I. Values of the various parameters of the $Bi_{0.924}Sb_{0.076}$ samples at liquid-helium temperatures.*

Sample no.	Impurity at.%		$\rho_{11}\cdot10^4$, Ω·cm	R_{231}, cm³/K	ε_∞	$\omega_p\cdot10^{-12}$, sec⁻¹	$S^L_{2min}\cdot10^{48}$, g·cm²·sec⁻²	$\frac{m_{2c}}{m_0}\cdot10^3$	$S^T_{min}\cdot10^{48}$, g·cm²·sec⁻²	$N_L\cdot10^{-17}$, cm⁻³
	Te	Sn								
1	–	–	8.7	−2500 **	–	–	–	–	–	0.025
2	0.0001	–	0.98	−165	120	1.40	0.26	0.50	–	0.38
3	0.001	–	0.37	−14.0	120	3.08	1.77	1.26	–	4.5
4	0.005	–	0.33	−4.8	120	4.18	3.85	1.86	–	13.0
5	–	0.004	0.88	+15.5	120	2.98	1.33	1.14	13.2	–
6	–	0.01	0.80	+7.4	110	3.36	2.60	1.44	17.7	–
7	–	0.05	1.75	+2.0	90	5.53	3.8	–	26.5	–
8	–	0.1	1.5	+1.4	90	6.72	6.6	–	35.4	–

*The errors in S^L_{2min} do not exceed 5%, S^T_{min} −10%, m_{2c} −15%, and N_L −10%.
**The value of the Hall coefficient is given in a strong magnetic field, when $R_{231}=R_{123}$.

to some smoothing out of the plasma minimum in the reflection spectra, which can be attributed to a decrease in the effective relaxation times as a consequence of the increase in the number of scattering centers and, evidently, of an enhancement of the role of heavy holes in the transport phenomena when the Fermi level is displaced downwards.

The temperature dependences of the Hall components R_{123} and R_{231} tend to saturation at low temperatures,[10] indicating that in the doped alloys the carrier gas is fairly strongly degenerate even at liquid-nitrogen temperatures. As a result, the spectral position and shape of the plasma minimum for the doped alloys undergo little change when the temperature is decreased from 82 to 12 K.[7,10]

The reflection spectra of the doped alloys are satisfactorily described by the phenomenological expression for the dielectric constant in the zero-pole representation[11]:

$$\varepsilon(\omega)=\varepsilon_\infty(\omega^2-\omega_p^2+i\gamma_p\omega)/\omega(\omega+i\gamma), \quad (1)$$

where $\gamma_p=\tau_p^{-1}$ characterizes the plasma mode damping near $\omega\approx\omega_p$, and $\gamma=\tau^{-1}$ is the damping corresponding to its pole at $\omega=0$, τ of necessity being $\geq\tau_p$, ε_∞ is the high-frequency dielectric constant, and ω_p is the resulting frequency of the collective (plasma) oscillations.

The relaxation time τ was determined from the value of the static electrical conductivity: $\tau=4\pi/\rho_{11}\varepsilon_\infty\omega_p^2$, and the value of the parameter τ_p from the expression for the imaginary part of the dielectric constant $\varepsilon_2(\omega)$ at $\omega=\omega_p$: $\tau_p=\varepsilon_\infty/\varepsilon_2(\omega_p)\omega_p$. At the same time the relationship

$$(\omega\tau_p)^2\gg1 \quad (2)$$

was fulfilled with a considerable margin for all the alloys in the entire range of frequencies we investigated. With condition (2) fulfilled, the squares of the plasma frequencies ω_p^2 were determined by using for the real part of the dielectric constant the expression which follows from (1):

$$\varepsilon_1(\omega)=\varepsilon_\infty(1-\omega_p^2/\omega^2) \quad (3)$$

with an error of less than 5% for all the alloys regardless of the scattering mechanism.

Expression (3) for $\varepsilon_1(\omega)$ is the same as the analogous expression for a classical plasma and is in good qualitative agreement with the theory of the dielectric constants of materials of the bismuth type.[4] The Kramers-Kronig dispersion analysis we made of the reflection spectra confirms the dependence (3) for all the doped alloys. The values of ε_∞ and ω_p are shown in the table. The error in determining ε_∞ does not exceed 10%.

The plasma frequencies depend appreciably on the amount of impurity, owing to the change in the filling of the L_a, L_s, and T_{45} extrema in the energy spectra of the alloys. To investigate the characteristics of the Fermi surfaces at the L and T points in the doped alloys, measurements were made of the Shubnikov–de Haas quantum oscillations in fields of up to 55 kOe at liquid-helium temperatures.

The periods of the reciprocal-field oscillations $\Delta(1/H)$, which are related to the cross section of the Fermi surface by the formula[12]

$$S=eh/c\Delta(1/H), \quad (4)$$

where e is the electron charge, h is Planck's constant and c is the velocity of light in vacuum, were determined from the dependences of the quantum number on the reciprocal field using the oscillation curves. No special allowance was made for the change in the oscillation period $\Delta(1/H)$ near the quantum limit, since the very last peaks in the oscillation curves were excluded from the analysis.

With the magnetic field H directed along the binary axis C_2 oscillations were observed in the longitudinal magnetoresistance $\rho(H)$ from near-minimum cross sections $S^L_{2\,min}$ of the light-carrier Fermi surface at L, and the effective cyclotron masses m_{2c} corresponds to them were determined from the temperature dependence of the oscillation amplitudes using the formulae in[13,14].

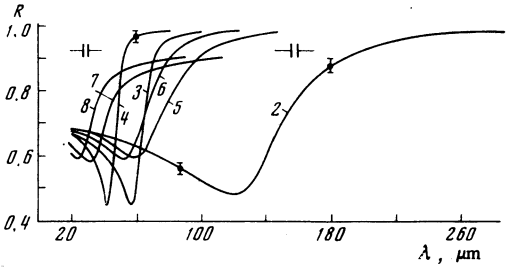

FIG. 1. Reflection spectra for doped $Bi_{0.924}Sb_{0.076}$ at $T=12$ K. The figures beside the curves correspond to the numbers of the samples in the table.

Oscillations in the transverse magnetoresistance from small cross sections $S^T_{\min}$ of the hole ellipsoid at T_{15} were also observed in alloys doped with tin when the field was directed along the trigonal axis C_3. The table shows the values of $S^L_{2\min}$, m_{2c}, and $S^T_{\min}$ for the alloys studied at various doping levels, i.e., in reality, for different Fermi energies.

It will be shown below that there are sufficient data in the table to determine all the parameters of the Abrikosov[2] dispersion law for the alloys studied.

THE ABRIKOSOV DISPERSION LAW AND THE PROCEDURE FOR CALCULATING ITS PARAMETERS

The energy spectrum of the carriers at the point L of the Brillouin zone in $Bi_{1-x}Sb_x$ alloys is characterized by a narrow gap E_{gL},[8] and several theoretical models[1,2,15,16] have been proposed to describe it. All the models give a good description of the dependences of the electron energies on the momentum projections on the short axes of the electron ellipsoids. A detailed theoretical analysis of the electron and hole spectra at L in materials of the bismuth type based on a four-band model was carried out by Abrikosov and Fal'kovskiĭ[1] and using it as a starting point, Abrikosov[2] showed that with a small distance between two bands at the point L, allowing for the possibility of inversion, the spectrum has the form:

$$\left(E-\frac{E_{gL}}{2}-\frac{p_z^2}{2M_1}\right)\left(E+\frac{E_{gL}}{2}+\frac{p_z^2}{2M_2}\right)=V_x^2p_x^2+V_y^2p_y^2, \tag{5}$$

where the energy E is referred to the middle of the gap (of the pseudogap in the case of an inversion spectrum at L), the x and y axes are directed along the short axes of the ellipsoids (the x axis is parallel to the binary axis), while the z axis coincides with the elongation direction, and M_1, M_2, V_x, and V_y are positive constants of the spectrum.

If the energy is referred to the bottom of the conduction band, then (5) may be re-written in the form:

$$\left(E-\frac{p_z^2}{2M_1}\right)\left(1+\frac{E}{E_{gL}}+\frac{p_z^2}{2M_2E_{gL}}\right)=\frac{p_x^2}{2m_{xL}^0}+\frac{p_y^2}{2m_{yL}^0} \tag{6}$$

where $m^0_{kL}=E_{gL}/2V_k^2$ are the components of low effective masses at the bottom of the band L.

In accordance with the theoretical model,[2] Eq. (6) determines the dependences of the minimum cross section $S^L_{1\min}(E_F)$ of the electron ellipsoid and the low cyclotron mass $m_{1c}(E_F)$ corresponding to it on the Fermi energy E_F:

$$S^L_{1\min}(E_F)=\frac{\pi}{V_xV_y}\left[\left(E_F+\frac{E_{gL}}{2}\right)^2-\frac{E_{gL}^2}{4}\right], \tag{7}$$

$$m_{1c}(E_F)=\frac{1}{2V_xV_y}(2E_F+E_{gL}). \tag{8}$$

From (7) and (8) we obtain the relationship

$$[m_{1c}(E_F)]^2=\frac{2\beta}{\pi}S^L_{1\min}(E_F)+\beta^2E_{gL}^2, \tag{9}$$

where $\beta=1/2V_xV_y$. The equation for the experimental dependences of $[m_{1c}]^2$ on $S^L_{1\min}$ for the L_a electrons and the L_s holes permits one to determine the degree of specularity of the energy spectrum at L for momentum directions along the short x and y axes of the equal-energy surfaces, and using formula (9), to determine the product of the matrix elements V_xV_y.

The Fermi energies in the L bands can be determined from formulae which follow from (7) and (8):

$$E_0=S^L_{1\min}/2\pi m_{1c}, \tag{10}$$

$$E_F=E_0-E_{gL}/2+(E_0^2-E_{gL}^2/4)^{1/2}. \tag{11}$$

The value of the energy gap E_{gL} in the alloy $Bi_{0.924}Sb_{0.076}$ is determined in[8,9]. The parameters V_x and V_y can be determined separately if the plasma frequencies and carrier densities are known. With the optical experiment geometry we used, the plasma frequency is expressed by the formula[3]

$$\omega_p^2=(2\pi e^2/\varepsilon_\infty m_0)\sum_k N_k[\alpha_{xk}(E_F)+\alpha_{zk}(E_F)]. \tag{12}$$

The summation is carried out over all the band extrema, N_k is the carrier density at the extremum k, m_0 is the free-electron mass, and $\alpha_{jk}(E_F)=m_0/m_{jk}$ are the reciprocal effective masses of carriers at the Fermi level (in units of $1/m_0$) along the binary ($j=x$) and bisector ($j=z$) axes.

The quantity $\alpha_{xL}(E_F)$ in (12) can be neglected because its value is in practice small for any value of E_F in comparison to $\alpha_{xL}(E_F)$, since the small cross sections of the L ellipsoids vary little in Bi and $Bi_{1-x}Sb_x$ alloys, and the angle of inclination of the ellipsoids to the basal plane is small.[17-20] It is convenient to plot the quantity

$$N\alpha\equiv\omega_p^2\Big/\left(\frac{2\pi e^2}{\varepsilon_\infty m_0}\right)$$

on the graphs rather than ω_p^2.

The electron density at the extremum L_a described by the dispersion law (6) is given by the formula

$$N_L=[(g\cdot8\pi\sqrt{2M_1})/(2\pi\hbar)^3V_xV_y]E_F^{3/2}(E_F+E_{gL})\varphi(l), \tag{13}$$

where g is the number of extrema at the point L of the Brillouin zone, $l=(1+E_{gL}/E_F)M_1^{-1}M_2$, and $\varphi(l)=(1+1/5l)$ is the correction for the deviation of the equal-energy surface from ellipsoidal.

The formula for the hole density P_L is analogous to formula (13). The low effective mass components at the Fermi level are defined by the expression

$$m_{iL}(E_F)=\frac{1}{V_i^2}E_F+m_{iL}^0, \quad i=x,y. \tag{14}$$

In the semiconducting $Bi_{1-x}Sb_x$ alloys doped with donor-type impurities only L_a electrons take part in the transport phenomena at low temperatures. In that case the values of $N_L\alpha_{xL}(E_F)$ are determined directly from the plasma frequency values using formula (12). Making use of the data for the electron densities N_L obtained from Hall measurements, one can determine α_{xL} and consequently the effective mass components $m_{xL}(E_F)$

$= N_L m_0/[N_L \alpha_{xL}(E_F)]$ at the Fermi level. The matrix elements V_x can then be determined from formula (14). The values of the parameter V_y is easy to calculate once V_x and $V_x V_y$ are known.

Numerous data in the literature indicated that in Bi and $Bi_{1-x}Sb_x$ alloys the carrier spectra at the L_a and L_s extrema are specular at least along the short x and y axes. In the procedure given above for determining the parameters V_x and V_y the specularity of the spectra at L_a and L_s (for directions along x and y) is established by analysis of the dependences of $[m_{1c}(E_F)]^2$ on $S^L_{1\min}$. According to the data in[21-25] the values of the matrix elements X_x and V_y vary little with the composition of the $Bi_{1-x}Sb_x$ alloys in the range $x \lesssim 15\%$ and with temperature up to the liquid-nitrogen temperature.

The calculation of the parameters V_x and V_y for light holes at the L_s extrema in $Bi_{1-x}Sb_x$ alloys doped with acceptor-type impurities is made difficult by the presence of heavy holes at the T_{45} extremum. If one bears in mind that in the first approximation the hole extremum T_{45} is described by the ellipsoidal parabolic model, with $\alpha_{xT} = \alpha_{yT}$,[18] then in accordance with formula (12) the light-hole contribution to the total value of $N\alpha$ can easily be separated:

$$P_L \alpha_{xL}(E_F) = N\alpha(E_F) - 2P_T \alpha_{xT}, \qquad (15)$$

where P_T is the hole density at the T_{45} extremum and α_{xT} is the component of their reciprocal effective mass along the binary axis. The density P_T can be calculated from the experimentally determined small cross sections $S^T_{\min}$, using the formula

$$P_T = 8a(S^T_{\min})^{3/2}/[3\sqrt{\pi}(2\pi\hbar)^3], \qquad (16)$$

where $a = S^T_{\max}/S^T_{\min}$ is the anisotropy of the hole Fermi surface at T_{45}. The values of a and α_{xT} in formulae (15) and (16) for $Bi_{1-x}Sb_x$ alloys are close to the corresponding values for Bi. The small difference possible in the values of a and α_{xT} for Bi and $Bi_{1-x}Sb_x$ alloys cannot lead to an appreciable error for comparable densities P_L and P_T since α_{xT} is small compared to α_{xL}. The densities P_L and P_T can be calculated from the galvanomagnetic coefficients using the formulae in[26], or using the data for the large cross sections if the latter can be measured.

Comparison of the experimental dependences of $N_L\alpha_{xL}$ and $P_L\alpha_{xL}$ on E_F with those calculated from formulae (13)–(16) (making use of the parameters V_x and V_y determined above) allows the parameters M_1 and M_2 in the dispersion law (6) to be determined. Let us note that in calculating, for example, the product $N_L\alpha_{xL}(E_F)$, the matching parameter is in fact M_1, since M_2 comes into the correction $\varphi(l)$ and has little influence on the result of the calculation.

CALCULATION OF THE ENERGY SPECTRUM PARAMETERS FOR $Bi_{0.924}Sb_{0.076}$

In accordance with the scheme for the energy spectrum rearrangement in $Bi_{1-x}Sb_x$ alloys as a function of the antimony concentration,[8,9] the alloy with $x = 0.076$ has a noninverted band spectrum: the term L_a is the bottom of the conduction band, the term L_s—the top of the valence band and $L_a - L_s = E_{gL} > 0$. The parameters M_1 and M_2 (6) have the sense of electron and hole masses respectively in the direction of elongation of the L ellipsoids. At full specularity of the spectra at L we have $M_1 = M_2$ (let us note that for $L_a - L_s > 0$ the spectrum (6) is formally the same as Cohen's dispersion law[16]).

According to the data in[8,9] the value of the energy gap at L in alloys with $x = 0.076$ is $E_{gL} = 10$ meV. This value is the same as that of the thermal gap determined from an analysis of the temperature dependences of the galvanomagnetic tensor components for the undoped alloy No. 1.[10]

The experimental dependence of the square of the cyclotron mass m^2_{2c} on the corresponding cross sections $S^L_{2\min}$ for alloys Nos. 2–6 is shown in Fig. 2, a. The experimental points fit the linear dependence (9) well. The best agreement with theory is achieved for $\beta = 1.13\times10^{-16}$ sec^2/cm^2. The value of β obtained belongs to the cross sections of the equal-energy surfaces at L close to $S^L_{1\min}$. By introducing the appropriate correction one can calculate the value of β for $S^L_{1\min}$ and determine the product of the matrix elements $V_x V_y$ in the Abrikosov dispersion law as $V_x V_y = (0.51 \pm 0.08)\times10^{16}$ cm^2/sec^2.

The fact that the experimental values of $m_{2c}(E_F)^2$ for the electron (alloys Nos. 2–4) and hole (alloys Nos. 5 and 6) ellipsoids at L fit the single theoretical curve (9) well indicates that the spectra along the short x and y axes are specular within the limits of measurement accuracy, and the dispersion law (6) describes the small cross sections in all the alloys well. Let us note that the dispersion law (6) is the same as the Lax[15] dispersion law for momentum directions along the short x and

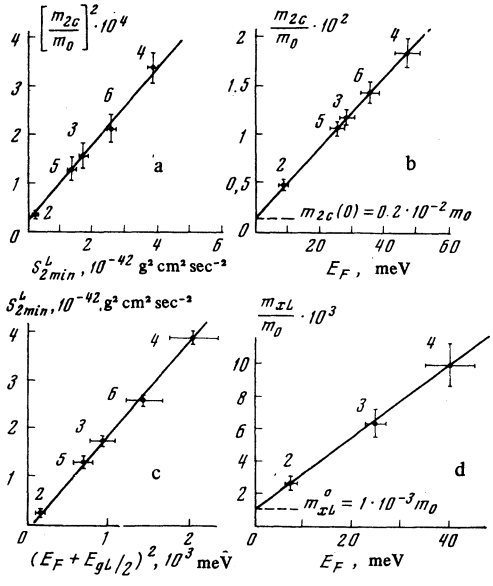

FIG. 2. The dependence (a) of the cyclotron mass m_{2c} on $S^L_{2\min}$ and (b) on E_F, (c) of $S^L_{2\min}$ on $(E_F + E_{gL}/2)^2$, and (d) of m_{xL} on E_F for carriers at the L_a and L_s extrema in $Bi_{0.924}Sb_{0.076}$ at liquid-helium temperatures. The figures indicate the sample numbers.

y axes. The results are in good agreement with all the currently available experimental data for the dispersion law in the direction of the short axes of the equal-energy surfaces at L.

In the alloys investigated the values of the Fermi energies in the L bands, calculated from formulae (10) and (11), do not depend on the band model chosen to describe the spectrum along the prolate direction of the L ellipsoids, and are determined only by the inaccuracies inherent in the experiment.

In this connection the dependences of the cyclotron masses m_{2c} on the Fermi energy E_F and the dependences of the cross sections $S^L_{2\min}$ on $(E_F+E_{gL}/2)^2$, shown in Figs. 2b and 2c, are of interest. The straight lines are plots of (7) and (8) using the values of E_{gL} and V_xV_y determined above. It is seen that the experimental points are in good agreement with the corresponding theoretical curves. This was to be expected since relations (7) and (8) follow from (9)–(11).

The Fermi energies in the L bands for the two alloys most heavily doped with tin (Nos. 7 and 8) can be calculated from the cross sections $S^L_{2\min}$ using formula (7), or can be determined from the graph in Fig. 2c. To determine V_x and V_y it is only necessary to determine one of them, e.g., for the L_a electrons, since the energy spectrum at L along the x and y directions is specular. To this end let us calculate the values of the low-effective-mass components $m_{xL}(E_F)$ along the binary axis for the n-type alloys Nos. 2–4 from the plasma frequencies ω_p and the carrier densities N_L. The values of m $m_{xL}(E_F)$ at the Fermi level shown in Fig. 2d fit the straight line calculated from formula (14) well. The best agreement is achieved for $V_x^2=0.83\times10^{16}$ cm^2/sec^2 and a low-mass component $m^0_{xL}=1\times10^{-3}m_0$ at the bottom of the L_a band. Knowing $V_x=(0.91\pm0.06)\times10^8$ cm/sec and V_xV_y, we find $V_y=(0.56\pm0.13)\times10^8$ cm/sec.

The values of V_x and V_y found in this work agree with the analogous values found in[21–24] in an investigation of Bi and $Bi_{1-x}Sb_x$ alloys. This obviously indicates that the parameters V_x and V_y do not vary markedly in $Bi_{1-x}Sb_x$ alloys with increasing antimony content up to 15%.

To calculate the parameters M_1 and M_2 in the energy spectrum (6) it is necessary to compare the experimental and calculated plots of $N_L\alpha_{xL}(E_F)$ and $P_L\alpha_{xL}(E_F)$. In alloys 1–4 the L_a electrons take part in the transport phenomena at liquid-helium temperatures. Consequently, as was shown above, the values of $N_L\alpha_{xL}(E_F)=N\alpha(E_F)$ can be calculated directly from the plasma frequencies. In

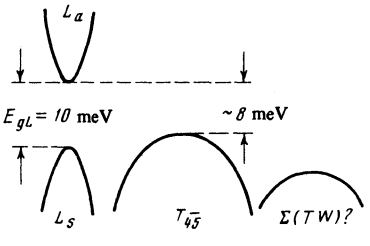

FIG. 3. Diagram of the arrangement of the main extrema in $Bi_{0.924}Sb_{0.076}$.

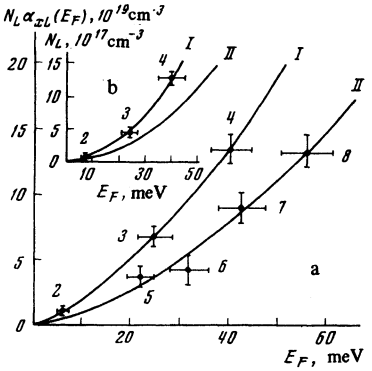

FIG. 4. (a) The dependence of $N_L\alpha_{xL}(E_F)$ (of $P_L\alpha_{xL}(E_F)$ for p-type alloys) on the Fermi energy E_F referred to the bottom of the appropriate band at L at $T=4.2$ K, and (b) the dependence of the electron density N_L in $Bi_{0.924}Sb_{0.076}$ alloys doped with tellurium on the electron Fermi energy E_F. The continuous lines are drawn from the Abrikosov model using $M_1=1.2m_0$ (curve I) and $M_2=0.6m_0$ (curve II). The figures beside the points indicate the sample numbers.

determining $P_L\alpha_{xL}(E_F)$ for p-type alloys from formulae (15) and (16) we shall assume that, in a first approximation, the hole extremum T_{45} is described by the ellipsoidal parabolic model with parameters a and α_{xT} that are the same as those of pure bismuth, i.e., $a=3.32$,[27] and $\alpha_{xT}=\alpha_{yT}=16.5$.[18] In doing this we are neglecting the possible nonparabolic nature of the T_{45} band and also the contribution of the other bands to $N\alpha(E_F)$[10] (e.g., from holes in the extrema $\Sigma(TW)$).[28]

The values of the Fermi energies in the T_{45} band calculated from $S^T_{\min}$ for alloys 5 and 6 were respectively 23 ± 3 meV and 32 ± 4 meV, which enables us to construct for the arrangement of the main extrema in $Bi_{0.924}Sb_{0.076}$ (Fig. 3) a diagram that is in good agreement with data[29] the magnetic susceptibility in alloys of this composition, and with the scheme for the rearrangement of the energy spectrum in $Bi_{1-x}Sb_x$.[8,9]

The experimental values of $N_L\alpha_{xL}(E_F)$ and $P_L\alpha_{xL}(E_F)$ for n- and p-type alloys respectively are shown in Fig. 4a. The theoretical curves (continuous lines) are calculated from formulae (13) and (14) using the parameters determined in this work. The experimental values of $N_L\alpha_{xL}(E_F)$ for alloys doped with tellurium fit the curves calculated with $M_1=(1.2\pm0.3)m_0$ (curves I) well. The curves of electron density N_L vs E_F calculated from formula (13) with $M_1=1.2m_0$ are shown in Fig. 4b (continuous line I). The points in this figure show the values of the electron densities N_L determined from Hall measurements (the Hall factor was taken as 1). Good agreement is found between the measured values of N_L and those calculated from formula (13).

A similar analysis for alloys doped with tin (curves II) showed that the best agreement with experiment for samples Nos. 5–8 is obtained with $M_2=(0.60\pm0.20)m_0$. This value is in good agreement with data from an investigation of p-type semiconducting $Bi_{1-x}Sb_x$ alloys in[23].

The values of the parameters M_1 and M_2 obtained in

the present work are worthy of note. Inequality of these parameters:

$$M_1 > M_2 \qquad (17)$$

means that the energy spectra of carriers at the L_a and L_s extrema in $Bi_{0.924}Sb_{0.076}$ are not specular for momentum directions along the elongation of the equal-energy surfaces.

According to the Abrikosov theory, when the inequality (17) is fulfilled and the parameter $f = E_F/E_{gL}$ is finite, the equal-energy surface for the L_a electrons must, in the general case, be dumb-bell shaped. The extent to which the equal-energy surface differs from an ellipsoid depends appreciably on the values of f and $\mu = M_1/M_2$, and it may be characterized by the ratio $\eta = S^a_{max}/S^a_{min}$ of the areas of the intersections of the "dumb-bell" with planes perpendicular to the z axis at the widest point (S^a_{max}) and at the neck (S^a_{min}). A calculation (the appropriate formula for η is given in[30]) with allowance for the experimental errors shows that the deviation of the equal-energy surface from ellipsoidal in alloys 2–4 is slight, and that η cannot in any event exceed 1.01, 1.03, and 1.06 respectively.

The maximum deviations of the electron equal-energy surfaces from ellipsoids with the same values of the long axes do not exceed (in terms of the ratios of the extremal cross sections) 1.04, 1.15, and 1.20 respectively for the alloys 2–4 (with $M_1 = 2M_2$).

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