Elastic anomalies near a phase transition point

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The problem of a phase transition in an anisotropic compressible lattice is considered. Solutions to the "fast" parquet equations are found. Anomalies in the elastic constants at the transition point are computed.

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1. INTRODUCTION

Near a second-order phase transition point the fluctuations exert a decisive influence on the behavior of the system. In this case it may turn out that the stable—in the self-consistent field approximation—Hamiltonian loses its stability when allowance is made for the fluctuations. Such an instability obtains in the case when there is a strictive coupling between the order parameter of the transition and the elastic degrees of freedom. This question has been considered by various authors.[1–3] In the case of an elastically isotropic crystal, the problem is rigorously soluble, and, as has been shown by Larkin and Pikin,[1] the transition is of first order when a nonzero shear modulus is present and the specific heat C_v diverges. In the case of anisotropic elastic properties Khmel'nitskiĭ and Shneerson[2] have shown that the renormalization-group equations describing the transition do not possess stable solutions. The physical causes of the two instabilities are different. In the present paper we obtain the general form of the solutions to the "fast" parquet equations that have been investigated for stability by Khmel'nitskiĭ and Shneerson,[2] when the equations depend not only on a slow logarithmic variable, but also on fast angular variables. This allowed the computation of those anomalies in the elastic properties that arise as a result of the fluctuations in the order parameter. Since in the case under consideration the transition is of first order (though close to a second-order transition), as the transition point is approached from above, the elastic constants (the stiffness) first decrease according to the specific-heat law and then undergo a finite jump downwards.

In the paper we compute the values of the elastic constants at the transition point both before and after the jump. The obtained values are determined by the various angle-averaged bare values of the elastic and striction constants, as well as by the fourth-order constants in the expansion of the thermodynamic potential. Relations between the anomalies of the various constants are also obtained. Approximate computations are carried out for the specific example of the uniaxial ferroelectric: triglycine sulfate (TGS).

2. THE RENORMALIZATION-GROUP EQUATIONS

Let us consider the simplest example of a phase transition with a scalar order parameter in a compressible anisotropic lattice. The phase transitions in the uniaxial ferroelectrics can serve as an example. The Hamiltonian for the transition has the form

$$H=\sum_{\mathbf{q}} {}^{1}/_{2}(\tau+s_{lm}q_lq_m)P(\mathbf{q})P(-\mathbf{q})$$
$$+\lambda \sum_{\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3+\mathbf{q}_4=0} P(\mathbf{q}_1)P(\mathbf{q}_2)P(\mathbf{q}_3)P(\mathbf{q}_4)$$
$$+ \sum_{\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3=0} \alpha_{lm}u_{lm}(\mathbf{q}_1)P(\mathbf{q}_2)P(\mathbf{q}_3)$$
$$+\sum_{\mathbf{q}} {}^{1}/_{2}c_{iklm}u_{ik}(\mathbf{q})u_{lm}(-\mathbf{q}). \qquad (1)$$

Here the s_{lm} characterize the anisotropy of the gradient term; the α_{lm}, the striction constants. In them only two indices have been written out since the other two are determined by the orientation of the order parameter in the lattice; the

$$u_{ik}=u_{ik}^{(0)}+{}^{1}/_{2}(\partial u_i/\partial x_k+\partial u_k/\partial x_i)$$

are the elastic deformations and P is the order parameter.

In the approximation under consideration the elastic-deformation Hamiltonian is quadratic; therefore, the corresponding functional integral is Gaussian, and can be integrated over all the elastic variables. As a result, there arise two different interactions of the fluctuations via the phonons: via the homogeneous deformations, i.e., with momentum $\mathbf{q}$ equal to zero, this interaction being isotropic; and via the inhomogeneous deformations. The latter depends on the angles in the case of an anisotropic lattice.

Let us now analyze the behavior of the system in the presence of such interactions within the framework of the parquet approximation in the four-dimensional model.[4] The results can be extended to the three-dimensional situation by Wilson's ε-expansion method.[5] In addition, below we shall analyze the case of uniaxial ferroelectric crystals, which are effectively four-dimensional.[4]

It is convenient to write out the parquet equations for each of the interactions separately. Let λ be the vertex corresponding to the point interaction, ν the vertex corresponding to the interaction via the homogeneous deformations, and μ the vertex corresponding to the interaction via the inhomogeneous deformations. The equations are represented graphically in Fig. 1. The heavy

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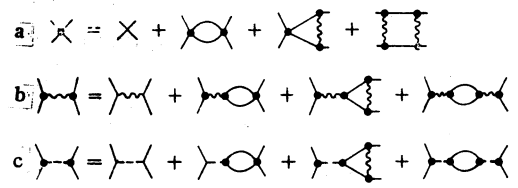

FIG. 1. The renormalization-group equations.

lines correspond to the P field; the dashed lines, to the homogeneous deformations; and the wavy lines, to the inhomogeneous deformations. Thus, the analytic expression for the equations has the form

$$-d\lambda/d\xi=36\lambda^2\langle g^2\rangle-48\lambda\langle\mu g^2\rangle+16\langle\mu^2 g^2\rangle, \tag{2}$$

$$-d\mu/d\xi=24\lambda\mu\langle g^2\rangle-16\mu\langle\mu g^2\rangle-4\mu^2\langle g^2\rangle, \tag{3}$$

$$-d\nu/d\xi=24\lambda\nu\langle g^2\rangle-16\nu\langle\mu g^2\rangle-4\nu^2\langle g^2\rangle, \tag{4}$$

$$\frac{\partial G^{-1}}{\partial \mathbf{q}}=\frac{\partial G_0^{-1}}{\partial \mathbf{q}}-8\int\frac{d^dp}{(2\pi)^d}\mu(\mathbf{q}-\mathbf{p})G^2(\mathbf{p})\mathbf{p}. \tag{5}$$

Here $G(q)=g/q^2$, $\xi=\ln(\max\{\tau,q^2\})$, $\mathbf{q}$ is the momentum, $\lambda(\xi=0)=\lambda_0$, $\mu(\xi=0)=\mu_0$, $\nu(\xi=0)=\nu_0$, and the symbol $\langle\ \rangle$ denotes averaging over the angles.

As is well known, usually in the theory of phase transitions the Green function is renormalized in the second approximation. The presence of the long-range interaction connected with the acoustic phonons leads to a situation in which the renormalization appears even in the first approximation.[1)] However, Eqs. (2)–(5) have been constructed such that this circumstance will not play any role below, although Eq. (5) must be taken into account when carrying out the averaging in specific calculations. Let us first consider the Eqs. (2) and (3). Let us divide both sides of Eq. (3) by μ^2. We obtain

$$\frac{d}{d\xi}\frac{1}{\mu}=24\lambda\frac{1}{\mu}\langle g^2\rangle-16\frac{1}{\mu}\langle\mu g^2\rangle-4\langle g^2\rangle. \tag{6}$$

Let us now average (6) over the angles and subtract the averaged equation from (6). We have

$$\frac{d}{d\xi}\left(\frac{1}{\mu}-\left\langle\frac{1}{\mu}\right\rangle\right)=(24\lambda\langle g^2\rangle-16\langle\mu g^2\rangle)\left(\frac{1}{\mu}-\left\langle\frac{1}{\mu}\right\rangle\right). \tag{7}$$

It is clear from (7) that $1/\mu-\langle 1/\mu\rangle=\varphi\eta$, where φ depends only on the angles and does not depend on ξ, while η does not depend on the angles.

Thus, the angular and logarithmic variables separate. This allows us to rewrite the Eqs. (2)–(5) in the following form:

$$-d\lambda/d\xi=36\lambda^2\langle g^2\rangle-48\lambda\langle\mu g^2\rangle+16\langle\mu^2 g^2\rangle, \tag{8}$$

$$-dv/d\xi=24\lambda v\langle g^2\rangle-16v\langle\mu g^2\rangle-4v^2\langle g^2\rangle, \tag{9}$$

$$dz/d\xi=4(1-z)v\langle g^2\rangle, \tag{10}$$

$$-d\nu/d\xi=24\lambda\nu\langle g^2\rangle-16\nu\langle\mu g^2\rangle-4\nu^2\langle g^2\rangle, \tag{11}$$

$$\frac{\partial G^{-1}}{\partial \mathbf{q}}=\frac{\partial G_0^{-1}}{\partial \mathbf{q}}-8\int d\xi\, v\int\frac{d\Omega}{(2\pi)^d}\frac{(1-z)\gamma}{1-z\gamma}g^2\mathbf{p}. \tag{12}$$

Here $\mu=v(1-z)\gamma/(1-z\gamma)$; z and v do not depend on the angles; $z(\xi=0)=0$; and γ characterizes the dependence on the angles: $\gamma=\mu(\xi=0)/v(\xi=0)$, where $v(\xi=0)=\mu_{\max}(\xi=0)$. The angular factor γ is chosen such that $\gamma_{\max}=1$; then the factor $(1-z)$ in the numerator will eliminate the singularity in the denominator at $z\gamma_{\max}=1$. Since it follows from Eq. (10) that $z<1$, pole singularities will not arise in the angular integrals.

It is convenient to go over to the variables $x=\lambda/v$ and z. As a result, for the Eqs. (8)–(10) and (12), we have

$$(z-1)\frac{dx}{dz}=3x^2-x\left\{8\frac{\langle\mu g\rangle^2}{v\langle g^2\rangle}-1\right\}+4\frac{\langle\mu^2 g^2\rangle}{v^2\langle g^2\rangle}, \tag{13}$$

$$\frac{\partial G^{-1}}{\partial \mathbf{q}}=\frac{\partial G_0^{-1}}{\partial \mathbf{q}}-2\int\frac{dz}{\langle g^2\rangle}\int\frac{d\Omega}{(2\pi)^d}\frac{\gamma g^2}{1-z\gamma}\mathbf{p}. \tag{14}$$

Notice that the angular integrals can be assumed to be known functions of the parameters z and g, since the angular dependence of γ is determined by the bare striction and elastic constants, while the angular dependence of g is determined by the point symmetry of the crystal. Therefore, the system (13)–(14) can easily be integrated numerically. The behavior of the vertex ν is completely determined by the variable z. Let us introduce $y=\nu/v$. Then for y we have the equation

$$dy/d\xi=4y(y-1)v\langle g^2\rangle. \tag{15}$$

Now, using (15) and (10), we obtain

$$y=y_0(1-z)/(1-y_0z). \tag{16}$$

In Khmel'nitskiĭ and Shneerson's paper[2] it is shown that the system (8)–(12) does not have stable solutions if γ depends on the angles. The question of the behavior of the solutions is closely tied with the question of the stability of the thermodynamic potential.

It is customarily assumed that stability is lost when the effective interaction between the fluctuations changes sign and becomes negative. If we are discussing a transition to a homogeneous state and we leave out the question of the influence of the boundary conditions, then the quantity $\lambda-\nu$ can be assumed to be the effective interaction, since the vertex μ corresponds to a nonzero momentum transfer. The Larkin-Pikin effect[1] corresponds, in the formulation under consideration, to the growth of the vertex $\nu=v(1-z)/(1-zy_0)$, since $y_0>1$ ($y_0=(K_0+\frac{4}{3}L_0)/K_0$ in the isotropic case, where K_0 and L_0 are the bulk and shear moduli). There is therefore a pole at $z=y_0^{-1}$, which leads to the loss of stability.

In the general case the system (13)–(16) is solved numerically. It makes sense to analyze such a solution for a specific experimental situation. However, if $\langle\gamma\rangle$ and $\langle\gamma^2\rangle$ vary slowly right up to the transition point, then we can obtain an analytic solution approximating the solution to the system (13)–(16). If the coupling between the fluctuations and the acoustic phonons is strong, then stability is lost even in the case of weak renormalization of the interaction, and therefore $\langle\gamma\rangle$ and $\langle\gamma^2\rangle$ almost do not change. Apparently, it is precisely such a situation that is realized in the uniaxial ferroelectrics: triglycine sulfate and triglycine selenate (TGSe). Assuming $\langle\gamma\rangle$ and $\langle\gamma^2\rangle$ to be constants, we obtain from Eq. (13)

$$z=1-\exp\left\{-\frac{2}{A}\operatorname{arctg}\frac{6A(x_0-x)}{A^2+(6x-8\langle\gamma\rangle+1)(6x_0-8\langle\gamma\rangle+1)}\right\},$$
$$A=(48\langle\gamma^2\rangle-(8\langle\gamma\rangle-1)^2)^{1/2}. \tag{17}$$

The solution (17) has meaning only when $48\langle\gamma^2\rangle$

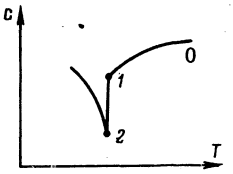

FIG. 2. Temperature dependence of the stiffness.

$>(8\langle\gamma\rangle-1)^2$, i.e., under conditions of sufficiently strong anisotropy.

3. COMPUTATION OF THE ELASTIC-CONSTANT ANOMALIES

Let us now consider the question of the elastic-constant anomalies. Figure 2 shows a typical behavior of the stiffness. The behavior of the stiffness along the segment 0–1 corresponds to the temperature dependence of the specific heat, since, owing to the strictive coupling, the equations for the fluctuation anomalies of the stiffness and for the specific heat coincide up to a constant factor. At the point 1 there occurs a first-order transition, and the stiffness decreases discontinuously to the point 2. We can compute in terms of the bare values of the elastic and striction constants and the fourth-order constant in the expansion of the thermodynamic potential the values c_1 and c_2 of the stiffness at the points 1 and 2.

Let us consider as an example the elastic modulus c_{xxxx}. Because of the striction, the fluctuations in P will make a contribution to c_{xxxx}. A calculation shows that the fluctuation contribution is determined only by the striction constant α_{xx}. The equations for the quantities $\nu_x=\frac{1}{2}\alpha_{xx}^2/c_{xxxx}$ and $c_x=c_{xxxx}$ are represented graphically in Fig. 1c and Fig. 3. Their analytic expression corresponds to the equations

$$-dv_x/d\xi=24\lambda v_x\langle g^2\rangle-16v_x\langle\mu g^2\rangle-4v_x^2\langle g^2\rangle, \tag{18}$$

$$-d\ln c_x/d\xi=4v_x\langle g^2\rangle. \tag{19}$$

Using Eqs. (9), (10), (18), and (19), we easily obtain

$$c_x=c_{0x}(1-zv_{0x}/v_0). \tag{20}$$

Here c_{0x} is the bare value of c_x and ν_{0x} is the bare value of ν_x.

It can be seen from (20) that, first, to determine the elastic-modulus anomalies at the point 1, we need to know only the parameter z. Secondly, we have the following invariant for the anomalies of the various elastic moduli:

$$(c_{0x}-c_x)/v_{0x}=\text{const}. \tag{21}$$

Since the dimensionality of the space in no way enters explicitly into (21), we should expect that such a simple relation will obtain also in three-dimensional space.

The computation of the magnitudes of the elastic constants after the jump is closely tied with the problem of the computation of the condensate that separates out during the fluctuation instability of a second-order transition. The instability is connected with the fact that the coupling constant attached to the fourth power of the parameter in the expansion of the thermodynamic potential becomes negative. In the case of the self-consistent field theory a sixth-order positive constant Γ was usually added, which restored the stability at large values of the condensate. However, near a second-order phase transition point the behavior of this constant is determined by the behavior of the fourth-order constant and, as it turns out, Γ decreases rapidly in the case of a second-order transition.[5]

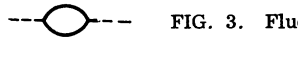

FIG. 3. Fluctuation correction to the stiffness.

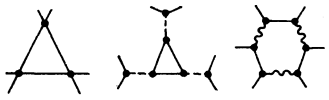

FIG. 4. Examples of ring diagrams.

In the case of an unstable behavior of the fourth-order constant, this constant can also change sign. Therefore, the question arises of the stabilization of the thermodynamic potential at large values of the condensate. It has been shown[6] that of greatest importance are the anharmonicities represented by the ring diagrams constructed from the fourth-order vertices. Some of the diagrams making contributions to the sixth-order vertex are shown in Fig. 4. Summing the diagrams of all orders higher than the fifth, we easily obtain the result for the additional contribution to the potential from the ring diagrams:

$$\Delta\Phi=\frac{1}{16\pi^2}\left\langle\Lambda^2\ln\frac{12\Lambda}{\tau}P^2\right\rangle P^4,\quad \Lambda=\lambda-\frac{1}{3}\nu-\frac{2}{3}\mu.$$

Here μ depends on the angles and the symbol $\langle\ \rangle$ denotes averaging over the angles. Let us show that the quantity $\lambda-\frac{1}{3}\nu-\frac{2}{3}\mu$ arising here is positive. As was noted above, the transition occurs at $\lambda\sim\nu$, so that we obtain

$$\Lambda=\frac{1}{3}\nu\frac{y_0-\gamma}{(1-z\gamma)(1-zy_0)}.$$

We have set here $\lambda=\nu$ and $x=y_0(1-z)/(1-zy_0)$. Since $\gamma_{max}=1$ and $y_0>1$, the quantity $\lambda-\frac{1}{3}\nu-\frac{2}{3}\mu$ is always positive.

Equating the thermodynamic potential and its derivative at the transition point to zero, we can obtain two equations from which we can determine the value of x at the transition point, as well as the magnitude of the condensate:

$$P^2=8\pi^2\tau/\langle\Lambda^2\rangle$$

$$x-\frac{y_0(1-z)}{1-zy_0}=-\frac{1}{16\pi^2}\frac{1}{v}\left[\langle\Lambda^2\rangle+\left\langle\Lambda^2\ln\frac{96\Lambda\pi^2}{\langle\Lambda^2\rangle}\right\rangle\right]. \tag{22}$$

It is clear that, to find the transition point, it is necessary to simultaneously use the relations (22) and the solutions to the system (13)–(16). Using the obtained results, we can compute the jumps in the elastic constants due to the condensate. Thus, for the stiffness $c_{xxxx}=c_x$ we have

$$\frac{1}{c_{2x}}=\frac{1}{c_{1x}}\left[1+\frac{8\pi^2\nu_{x1}}{\langle\Lambda^2\rangle}\right]. \tag{23}$$

Here the indices 1 and 2 respectively correspond to the points 1 and 2 in Fig. 2. Thus, we can derive simple expressions for the jumps in the elastic constants. The most attractive for comparison with experiment is the relation (21), since for its verification it is necessary to know only the elastic moduli and the striction constants. This relation is not trivial and arises from the renormalization-group equations. In contrast to this, the analogous expression obtained from the formula (23) will express a trivial fact that follows from the presence of the strictive coupling. Besides the indicated verification of the relation (21), we can carry out calculations directly for the magnitudes of the jumps, but this requires either a numerical, or an approximate, integration of the system (13)–(16).

4. ELASTIC ANOMALIES IN TGS

As is well known, an effectively four-dimensional situation obtains in uniaxial ferroelectrics owing to the dipole-dipole interaction.[4] Therefore, the considered theoretical ideas can be verified on these materials. The most suitable objects for comparison are TGS and its analog TGSe. According to the experimental results obtained in Ref. 7, the fluctuation correction to the elastic constants in TGS behaves like $\ln\tau$, which should testify to the effective four-dimensional nature of the fluctuations (strictly speaking, the dependence should have the form $\ln^{1/3}\tau$,[4] but this is difficult to detect experimentally). The elastic and striction constants have also been measured[8,9] below the transition point. An estimate for the magnitude of the effective constant $\lambda-\nu$ in the ferroelectric phase is given in Jona and Shirane's book.[10]

This allows us to estimate the fluctuation corrections at the transition point. Naturally, using, instead of data above the transition point, data measured in the ferroelectric phase, besides far from the transition point, we can obtain only order-of-magnitude estimates. The inaccuracies in the determination of the elastic moduli are an important source of errors here. Thus, for the moduli c_{15}, c_{25}, c_{35}, and c_{64} the error can be of the order of the constants themselves. Such is the error for the striction constants, for they are computed, using the elastic moduli.

Therefore, it is reasonable to perform only approximate computations. As is well known, TGS belongs to the monoclinic system, and the spontaneous moment in it is directed along the Y axis. Therefore, a logarithmic divergence will occur in the case of momenta lying in the XZ plane. As noted above, the averaging over the angles should be done with some weight connected with the anisotropy of the Green function. Since in the present case the calculations are approximate calculations, it is reasonable to consider the Green function to be isotropic. Let us now compute the bare values μ_0 and ν_0. The computations are carried out by an elementary Gaussian integration over the inhomogeneous and homogeneous deformations, respectively. Unfortunately the literal expression, especially for μ_0, is too unwieldy to give here, especially as the integration is trivial. Numerically, these quantities have the following form:

$$\mu_0=\frac{46q_z^6+48q_z^6+90q_z^4q_z^2+76q_z^2q_z^4+28q_z^3q_z^3+67q_z^5q_z-144q_z^5q_z}{3{,}0q_z^6+2.5q_z^6+7{,}3q_z^4q_z^2+5{,}4q_z^3q_z^4+1{,}6q_z^5q_z^3-2{,}3q_z^5q_z+0{,}2q_z^5q_z}.$$

The quantity μ_0 has been expressed in units of 10^{-11} cm^2/dyn, $\mu_{0\max}\sim 15\times10^{-11}$ cm^2/dyn, $\nu_0\sim 25\times10^{-11}$ cm^2/dyn. The computation of the mean values of μ_0 is also performed numerically and yields $\langle\mu_0\rangle\sim 8\times10^{-11}$ cm^2/dyn, $\langle\gamma\rangle\sim 0.53$, $\langle\gamma^2\rangle\sim 0.35$. The value of the effective constant $\lambda_0-\nu_0\sim 5\times10^{-11}$ cm^2/dyn. Hence we obtain $x_0\sim 2$, $y_0\sim\frac{5}{3}$.

Thus, the situation in TGS corresponds to the case of strong coupling. Therefore, we can assume that x_n at the transition point differs little from x_0 and that z_n at the transition point will be small. In that case $\langle\gamma\rangle$ and $\langle\gamma^2\rangle$ will be almost constants, and we can use the approximate solution (17). Substituting the values of x_0, $\langle\gamma\rangle$, and $\langle\gamma^2\rangle$, we obtain from (17) the stability conditions:

$$x=(4+{}^7/_2z)/(2+9z),\quad x={}^5/_3(1-z)/(1-{}^5/_3z).$$

From here we find $z_n\sim 0.05$; $x_n\sim 1.7$. The correction to $\langle\gamma\rangle$, $\Delta\langle\gamma\rangle=z_n(\langle\gamma\rangle-\langle\gamma^2\rangle)=0.01$, so that the initial assumptions are correct.

Knowing z_n, we can estimate the fluctuation corrections to the elastic constants. We have $\Delta c_{11}/c_{11}\sim 0.005$, $\Delta c_{22}/c_{22}\sim 0.01$, and $\Delta c_{33}/c_{33}\sim 0.003$. The experimental value for $\Delta c_{33}/c_{33}$ (~ 0.05), determined from the c_{11} and c_{22} anomalies, is considerably higher. In comparing with experiment, we should take into account the fact that, for the reasons indicated above, the above-used relations between the striction constants bear no relation to reality, although the order of their magnitude should be substantially closer to the experimentally measurable order. Therefore, the greatest experimental value should be compared with the greatest computed value, i.e., the experimental value for $\Delta c_{33}/c_{33}$ should be compared with the computed value of $\Delta c_{22}/c_{22}$. On the whole, on account of the above-indicated reasons, the question of comparison with experiment remains open.

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[1]This circumstance was pointed out to the author by D. E. Khmel'nitskiĭ.

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Amorphization of a Heisenberg ferromagnet with anisotropically distributed exchange couplings

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The amorphization of a crystalline ferromagnet with anisotropically distributed exchange parameters is investigated. The amorphous ferromagnet is treated in the framework of a lattice model with fluctuating exchange couplings. With the use of the single-site approximation in the coherent-potential method, equations are found for the parameters of the coherent exchange matrix by means of which the magnon states of the amorphized ferromagnet are described on the average. The case of the amorphization of a quasi-two-dimensional ferromagnet with intraplanar (J_0) and interplanar (K_0) exchange parameters when the exchange interactions become isotropic is investigated. The coherent exchange parameter and the modified density of magnon states are found by using a distribution function corresponding to the mixing of the J_0 and K_0 couplings on amorphization. It is shown that the Curie temperature increases substantially on amorphization of a quasi-two-dimensional crystal.

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1. INTRODUCTION

The problem of magnetic order in amorphous materials was raised by Gubanov[1] and has undergone considerable development since then. Important results have been obtained in the papers of Handrich,[2] Montgomery *et al.*,[3] Foo and Bose,[4] Gubernatis and Taylor,[5] and others. A characteristic feature of these theoretical papers is that they treat magnetically and structurally stable systems of the cubic-ferromagnet type. In the crystalline state, such substances are characterized by only one exchange-coupling parameter, the magnitude of which is fixed over the whole crystal. The amorphization of such crystals is accompanied by the appearance of fluctuating exchange. Therefore, the results of the aforementioned papers reduce principally to a decrease of the magnetization and Curie temperature T_C of the ferromagnets as they become amorphous. An important aspect is that the ferromagnetism can disappear completely when the exchange fluctuations reach a certain critical size.[4] For this class of substances the existing experiments basically confirm the theoretical ideas.[6–8]

It has been postulated[9] that the strongest effects will arise in the amorphization of magnetic crystals whose magnetic structure is determined in an essential way by the geometry of the distribution of exchange couplings. Such a situation obtains, e.g., in quasi-low-dimensional magnets. The description of such magnets requires the introduction of at least two different exchange parameters. The type of magnetic order and the temperature of the magnetic phase transition in quasi-low-dimensional magnets are determined by the weak exchange that couples the magnetic chains or layers.[10] At the same time, the same characteristics of the amorphized substance are more likely to be determined by a certain averaged exchange. Consequently, it is reasonable to expect that the amorphization of quasi-low-dimensional systems can lead to both a change in the type of magnetic order and a sharp increase in the temperature of the magnetic phase transition. Of course, the traditional effects of amorphization will remain,[2] but in a number of cases their role becomes secondary.

The most important consequence of the amorphization of a substance is the disappearance of the periodic crystalline structure. This is the reason why the theoretical description of the magnets encounters great difficulties of a fundamental character. Because of the absence of translational invariance in the amorphous substance, the traditional methods developed in the theory of solids for perfect crystals do not work.

For an approximate description of the properties of an amorphous substance we can start from the assumption that, after averaging over all possible realizations, translational invariance is re-established on the average. The substance can then be described in terms of a certain ideal crystal with certain effective parameters. The procedure for averaging over the realizations should, in the general case, take into account fluctuations of the

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