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Amorphization of a Heisenberg ferromagnet with anisotropically distributed exchange couplings

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The amorphization of a crystalline ferromagnet with anisotropically distributed exchange parameters is investigated. The amorphous ferromagnet is treated in the framework of a lattice model with fluctuating exchange couplings. With the use of the single-site approximation in the coherent-potential method, equations are found for the parameters of the coherent exchange matrix by means of which the magnon states of the amorphized ferromagnet are described on the average. The case of the amorphization of a quasi-two-dimensional ferromagnet with intraplanar (J_0) and interplanar (K_0) exchange parameters when the exchange interactions become isotropic is investigated. The coherent exchange parameter and the modified density of magnon states are found by using a distribution function corresponding to the mixing of the J_0 and K_0 couplings on amorphization. It is shown that the Curie temperature increases substantially on amorphization of a quasi-two-dimensional crystal.

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1. INTRODUCTION

The problem of magnetic order in amorphous materials was raised by Gubanov[1] and has undergone considerable development since then. Important results have been obtained in the papers of Handrich,[2] Montgomery et al.,[3] Foo and Bose,[4] Gubernatis and Taylor,[5] and others. A characteristic feature of these theoretical papers is that they treat magnetically and structurally stable systems of the cubic-ferromagnet type. In the crystalline state, such substances are characterized by only one exchange-coupling parameter, the magnitude of which is fixed over the whole crystal. The amorphization of such crystals is accompanied by the appearance of fluctuating exchange. Therefore, the results of the aforementioned papers reduce principally to a decrease of the magnetization and Curie temperature T_C of the ferromagnets as they become amorphous. An important aspect is that the ferromagnetism can disappear completely when the exchange fluctuations reach a certain critical size.[4] For this class of substances the existing experiments basically confirm the theoretical ideas.[6-8]

It has been postulated[9] that the strongest effects will arise in the amorphization of magnetic crystals whose magnetic structure is determined in an essential way by the geometry of the distribution of exchange couplings. Such a situation obtains, e.g., in quasi-low-dimensional magnets. The description of such magnets requires the introduction of at least two different exchange parame-

ters. The type of magnetic order and the temperature of the magnetic phase transition in quasi-low-dimensional magnets are determined by the weak exchange that couples the magnetic chains or layers.[10] At the same time, the same characteristics of the amorphized substance are more likely to be determined by a certain averaged exchange. Consequently, it is reasonable to expect that the amorphization of quasi-low-dimensional systems can lead to both a change in the type of magnetic order and a sharp increase in the temperature of the magnetic phase transition. Of course, the traditional effects of amorphization will remain,[2] but in a number of cases their role becomes secondary.

The most important consequence of the amorphization of a substance is the disappearance of the periodic crystalline structure. This is the reason why the theoretical description of the magnets encounters great difficulties of a fundamental character. Because of the absence of translational invariance in the amorphous substance, the traditional methods developed in the theory of solids for perfect crystals do not work.

For an approximate description of the properties of an amorphous substance we can start from the assumption that, after averaging over all possible realizations, translational invariance is re-established on the average. The substance can then be described in terms of a certain ideal crystal with certain effective parameters. The procedure for averaging over the realizations should, in the general case, take into account fluctuations of the

atomic positions and of the parameters describing the interaction in the system. In addition, it should take into account in a self-consistent manner the fluctuations of the thermodynamic quantities. However, because of the absence of sufficiently rigorously substantiated averaging methods, this problem is exceptionally difficult.

An amorphous ferromagnet (FM) can be described by means of a simpler lattice model, in the framework of which the amorphous FM is described by a Heisenberg Hamiltonian on an ideal crystal lattice with fluctuating exchange parameters. This model has been used in a number of papers,[3-5] and Foo and Bose[4] have used the coherent-potential approximation (CPA) to investigate the ground state of an amorphous FM as a function of the magnitude of the exchange-parameter fluctuations.

In the present work an attempt is made to describe some of the properties of an amorphous FM that has arisen from an FM with anisotropically distributed exchange couplings. We consider, as a general case, the amorphization of a cubic crystal with three different exchange-coupling parameters. The results obtained are then applied to the more particular case of the amorphization of a quasi-two-dimensional FM. The latter is described by two positive exchange parameters (for nearest neighbors), J_0 and K_0, where J_0 is the exchange parameter in the xy plane and K_0 is the exchange parameter along the z axis, with $K_0/J_0 \equiv \lambda_0 \ll 1$ for a quasi-two-dimensional crystal. Using the lattice model and the CPA, we show that on amorphization of a quasi-two-dimensional FM the Curie temperature can increase (in contrast to the case of the decrease of T_C on amorphization of an isotropic FM), and that for small values of λ_0 this increase can be very large. Although there are no data on the amorphization of quasi-two-dimensional FMs at the present time, the experiment of Ref. 9 shows that on the amorphization of the planar antiferromagnet $Bi_2Fe_4O_9$ the Néel temperature increases. The presence in an amorphized antiferromagnet of negative and fluctuating exchange parameters leads to certain difficulties even in the description of the ground state, and so the rather simpler problem of the amorphization of an FM is considered here.

2. THE EFFECTIVE HAMILTONIAN. THE COHERENT-POTENTIAL APPROXIMATION

In accordance with the lattice model, we shall describe an amorphous FM by a Hamiltonian with fluctuating exchange parameters A_{fm} that are nonzero for nearest neighbors only:

$$\mathscr{H} = -\sum_{fm} A_{fm} S_f S_m, \quad A_{fm} = A_{mf}, \quad A_{ff} = 0. \quad (1)$$

For simplicity we consider a simple cubic lattice with $z = 6$ and $S = \frac{1}{2}$.

With an ideal crystal, in the general case, we can associate a translationally invariant exchange matrix

$$A^0(f-m) = A^0(\mathbf{h}) = \begin{cases} A_x^0, & \text{if} \quad \mathbf{h} = \mathbf{h}_x, \\ A_y^0, & \text{if} \quad \mathbf{h} = \mathbf{h}_y, \\ A_z^0, & \text{if} \quad \mathbf{h} = \mathbf{h}_z, \end{cases} \quad (2)$$

where the $\mathbf{h}_i$ are the vectors linking nearest neighbors in the directions $i = x, y, z$. In the case of a quasi-two-dimensional crystal, $A_x^0 = A_y^0 \equiv J_0$ and $A_z^0 \equiv K_0$. We shall confine ourselves to treating the case when the fluctuating exchange parameters $A_{fm} > 0$, so that the FM criterion is fulfilled.

Since an exact solution of the problem is absent, even for an ideal isotropic FM with the Hamiltonian (1), we shall make use of the random-phase approximation (RPA) which corresponds to Tyablikov's decoupling[11] in the linearization of the equations of motion for the spin-deviation operators. We represent the model Hamiltonian that reproduces these equations of motion in the form

$$\mathscr{H}_{\text{RPA}} = \sum_{f,m} H_{fm} b_f^+ b_m, \quad H_{fm} = \delta_{fm} \sum_n A_{fn} \sigma_n - \sigma_f A_{fm}, \quad (3)$$

where b_f and b_f^* are Pauli operators, and $\sigma_f = 2\langle S_f^z \rangle = 1 - 2\langle b_f^* b_f \rangle$. For an ideal FM, because of the translational invariance, $\sigma_f = \sigma$ does not depend on the site label. In an amorphous FM this situation occurs strictly at $T = 0$ only, when all $\sigma_f = 1$. With increase of temperature, because of the fluctuations of the exchange parameters, the spin deviations that arise are distributed nonuniformly, and at low temperatures the spin deviations will arise primarily in those regions in which the minimum A_{fm} are realized. However, because of the exchange coupling and the random nature of the fluctuations, the spin deviations are spread over the whole substance. Bearing this spread of the spin deviations in mind, we can assume that $\sigma_f = \sigma$ for an amorphous FM too, i.e., we can assume the relative magnetization $\sigma = \sigma(T)$ to be a self-averaging thermodynamic quantity.

We shall describe the amorphous substance as an effective translationally invariant medium in which the exchange interactions between nearest neighbors are realized by a certain self-consistent (coherent) exchange matrix $A^c(f-m)$. A way of determining A^c is given below. Introducing fluctuations away from the coherent matrix:

$$A_{fm} = A^c(f-m) + [A_{fm} - A^c(f-m)] \equiv A^c(f-m) + \xi_{fm} \quad (4)$$

and using (3), we find

$$H_{fm} = \sigma[H^c(f-m) + V_{fm}],$$

$$H^c(f-m) = \delta_{fm} \sum_n A^c(f-n) - A^c(f-m), \quad V_{fm} = \delta_{fm} \sum_n \xi_{fn} - \xi_{fm}. \quad (5)$$

We shall define the two-time temperature Green function and its Fourier transform by the relations

$$G_{mn}(t) = -i\theta(t)\langle [b_m(t), b_n^+(0)] \rangle_T, \quad G_{mn}(E) = \int_{-\infty}^{+\infty} dt\, e^{iEt} G_{mn}(t), \quad (6)$$

where $\langle \ldots \rangle_T$ denotes statistical averaging with the Hamiltonian (3) in the approximation (5). From (3) and (5) follows the equation

$$\sum_m \left[\frac{E}{\sigma} \delta_{fm} - H^c(f-m) - V_{fm} \right] G_{mn}(E) = \delta_{fn}, \quad (7)$$

which has the matrix form ($\Omega = E/\sigma$, $\hat{I}$ is the unit matrix)

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$$(\Omega I - \hat{H}^c - \hat{V})G = I. \tag{8}$$

The zeroth approximation corresponds to the translationally invariant matrix $\hat{H}^c$ and is described by the Green function

$$G^c = (\Omega I - \hat{H}^c)^{-1}, \tag{9}$$

which, in the site representation, has the form

$$G_{\mathbf{m}\mathbf{n}}^c(\Omega) = \frac{1}{N} \sum_{\mathbf{k}} \frac{e^{i\mathbf{k}(\mathbf{m}-\mathbf{n})}}{\Omega - \omega_c(\mathbf{k}, \Omega)}, \tag{10}$$

where the dispersion law of the spin waves is expressed in terms of the coherent matrix A^c:

$$\omega_c(\mathbf{k}, \Omega) = \sum_{\mathbf{h}} A^c(\mathbf{h}, \Omega)(1 - e^{i\mathbf{k}\mathbf{h}}). \tag{11}$$

The solution of (8) can be represented in the form

$$G = G^c + G^c \hat{T} G^c, \tag{12}$$

where the scattering matrix $\hat{T}$ obeys the equation

$$\hat{T} = \hat{V} + \hat{T} G^c \hat{V}. \tag{13}$$

In the description of the amorphous substance we can use only averaged characteristics. We denote averaging over realizations (configurational averaging) by the symbol $\langle \dots \rangle$. Averaging (12), we obtain

$$\langle G \rangle = G^c + G^c \langle \hat{T} \rangle G^c,$$

where $\langle \hat{G} \rangle$ describes the single-particle properties of the effective translationally invariant crystal which, on the average, reproduces the amorphous substance. The value of the T-matrix (its matrix elements) depends on the choice of A^c, i.e., $\hat{T} = \hat{T}[A^c]$, or $\hat{T} = \hat{T}[G^c]$. In accordance with the ideas of the CPA, the coherent matrix A^c can be found from the condition

$$\langle \hat{T}[G^c(A^c, \Omega)] \rangle = 0. \tag{14}$$

Thus, in this approximation the amorphous substance is modeled by an ideal crystal whose properties are described by the function $\hat{G}^c$. The physical meaning of $A^c(\mathbf{h}, \Omega)$ is that it is the self-consistent exchange matrix that ensures, on the average, zero scattering of spin waves with the dispersion law (11) by fluctuations.

If the coherent matrix A^c has been found, the spin-wave density of states $g_c(\Omega)$ corresponding to the dispersion law $\omega_c(\mathbf{k}, \Omega)$ can be expressed in terms of the imaginary part of the Green function:

$$g_c(\Omega) = -\pi^{-1} \operatorname{Im} G_{00}^c(\Omega + i0),$$

To investigate the self-consistent equation for $\sigma = \sigma(T)$ it is convenient to return to the previous variable $E = \sigma\Omega$. Then,

$$g_c(E) = -\frac{1}{\pi} \operatorname{Im} G_{00}^c(E + i0)$$

$$= \sigma \frac{1}{N} \sum_{\mathbf{k}} \left(-\frac{1}{\pi}\right) \operatorname{Im} \frac{1}{E + i0 - \sigma\omega_c(\mathbf{k}, E + i0)} \equiv \sigma \tilde{g}_c(E; \sigma). \tag{15}$$

The equation for σ has the form[11]

$$1/\sigma = 1 + 2P_c(\sigma), \tag{16}$$

where

$$P_c(\sigma) = \int_{-\infty}^{+\infty} dE\, f(E)\, \tilde{g}_c(E; \sigma) \quad f(E) = (e^{E/T} - 1)^{-1},$$

and makes it possible both to investigate the low-temperature behavior of σ and to estimate the Curie temperature.

3. THE T-MATRIX AND THE APPROXIMATION OF INDEPENDENT PAIR FLUCTUATIONS

Because of the absence of translational invariance, we cannot write the solution of Eq. (13) for a T-matrix with dimensions $N \times N$ in explicit form (because of the enormous dimensions of the matrices, the symbolic solution $\hat{T} = \hat{V}(\hat{I} - \hat{G}^c \hat{V})^{-1}$ is not visualizable and is useless for practical calculations). To elucidate the structure of the T-matrix, using the definition (5) we write the fluctuation matrix $\hat{V}$ in the form

$$\hat{V} = \sum_{\alpha} \hat{v}_{\alpha}, \tag{17}$$

where the summation is taken over all pairs α of nearest neighbors. The partial fluctuation matrix $\hat{v}_{\alpha}$, corresponding, e.g., to the nearest-neighbor pair $\alpha = (\mathbf{f}, \mathbf{m})$, is represented in the form $\hat{v}_{\alpha} = \xi_{\alpha} \hat{l}_{\alpha}$, where $\hat{l}_{\alpha}$ is a matrix of dimensions $N \times N$ in which only the following four matrix elements are nonzero:

$$(\hat{l}_{\alpha})_{\mathbf{m}\mathbf{m}} = (\hat{l}_{\alpha})_{\mathbf{f}\mathbf{f}} = -(\hat{l}_{\alpha})_{\mathbf{f}\mathbf{m}} = -(\hat{l}_{\alpha})_{\mathbf{m}\mathbf{f}} = 1, \quad \alpha = (\mathbf{f}, \mathbf{m}). \tag{18}$$

Using (17), we write (13) in the form[12]

$$\hat{T} = (I + \hat{T} G^c) \sum_{\alpha} \hat{v}_{\alpha} = \sum_{\alpha} \hat{Q}_{\alpha}. \tag{19}$$

For $\hat{Q}_{\alpha}$ it is easy to obtain the representation

$$\hat{Q}_{\alpha} = \left(I + \sum_{\beta \neq \alpha} \hat{Q}_{\beta} G^c\right) \hat{t}_{\alpha}, \tag{20}$$

where $\hat{t}_{\alpha}$ is the partial scattering matrix for scattering by fluctuations of the exchange parameter for the nearest-neighbor pair α:

$$\hat{t}_{\alpha} = \hat{v}_{\alpha}(\hat{I} - G^c \hat{v}_{\alpha})^{-1}, \tag{21}$$

and the sum in (20) takes multiple-scattering effects into account.

Iterating (20), we obtain the following representation for the T-matrix:

$$\hat{T} = \sum_{\alpha} \hat{t}_{\alpha} + \sum_{\alpha \neq \beta} \hat{t}_{\beta} G^c \hat{t}_{\alpha} + \dots \tag{22}$$

To find $\langle \hat{T} \rangle$ we average the expression (20); we then obtain

$$\langle \hat{Q}_{\alpha} \rangle = \left(I + \sum_{\beta \neq \alpha} \langle \hat{Q}_{\beta} \rangle G^c\right) \langle \hat{t}_{\alpha} \rangle$$

$$+ \left\langle \sum_{\beta \neq \alpha} (\hat{Q}_{\beta} - \langle \hat{Q}_{\beta} \rangle) G^c \hat{t}_{\alpha} \right\rangle. \tag{23}$$

A radical simplification of the problem of calculating $\langle \hat{T} \rangle$ is achieved by assuming that each fluctuating coupling between nearest neighbors can be regarded as independent of all the others. Neglecting, in this approximation, the second term in (23), which takes into account the effects of multiple scattering and correlations of fluctuations,[12] we obtain

$$\langle \hat{T} \rangle \approx \hat{\Pi}(I - \hat{G}^c \hat{\Pi})^{-1}, \quad \hat{\Pi} = \sum_\alpha \langle \hat{t}_\alpha \rangle (I + \hat{G}^c \langle \hat{t}_\alpha \rangle)^{-1}. \tag{24}$$

Because of the statistical independence of the individual scatterers the requirement (14) is satisfied when

$$\langle \hat{t}_\alpha \rangle = 0 \tag{25}$$

for all pairs α, and corresponds to the single-site approximation in the coherent-potential method.[12-14]

Using the explicit form of the matrix $\hat{v}_\alpha = \xi_\alpha \hat{l}_\alpha$, from the relation (21) we find $\hat{t}_\alpha$ for any pair of nearest neighbors ($\alpha = (\mathbf{f}, \mathbf{m})$):

$$\hat{t}_{\mathbf{fm}} = \frac{\xi_{\mathbf{fm}}}{1 - 2\xi_{\mathbf{fm}}(G_0^c - G_{\mathbf{f-m}}^c)} \hat{l}_{\mathbf{fm}} \equiv t_{\mathbf{fm}} \hat{l}_{\mathbf{fm}}, \tag{26}$$

where $G_{\mathbf{ff}}^c = G_{\mathbf{mm}}^c \equiv G_0^c$ and $\xi_{\mathbf{fm}} = A_{\mathbf{fm}} - A^c(\mathbf{f} - \mathbf{m})$, in accordance with (4).

The structure of the coherent matrix is not given *a priori* and should be found self-consistently as a function of the degree of amorphization of the substance from the conditions $\langle t_{\mathbf{m+h,m}} \rangle = 0$. Since the configurational averaging restores the translational invariance,

$$\langle t_{\mathbf{m+h,m}} \rangle = \frac{1}{N} \sum_{\mathbf{m}} t_{\mathbf{m+h,m}} \equiv \bar{t}(\mathbf{h}), \tag{27}$$

i.e., in the general case, the average can depend only on the direction of the vector $\mathbf{h}$ linking the pair of nearest neighbors. The summation in (27) means, in effect, summation over all fluctuations of the exchange parameter in the direction specified by the vector $\mathbf{h}$.

To perform the configurational averaging explicitly we introduce the distribution functions $\rho_i(A)$ of the fluctuations of the exchange parameters in the directions $i = x, y, z$. When all three distributions are different, which corresponds to inequivalence of the distributions of exchange couplings along x, y, and z, it is reasonable to assume the existence of three coherent parameters $A_i^c \equiv A^c(\mathbf{h}_i)$. In this case, from (25) and (26) we obtain three equations for the self-consistent determination of A_i^c:

$$\bar{t}_i \equiv \langle t_{\mathbf{m+h}_i,\mathbf{m}} \rangle = \int \rho_i(A) \frac{A - A_i^c}{1 - 2(A - A_i^c)(G_0^c - G_{\perp i}^c)} dA = 0. \tag{28}$$

When a quasi-two-dimensional crystal becomes amorphous the equivalence of the distributions of the exchange couplings along x and y is preserved, i.e., $\rho_x = \rho_y \equiv \rho_\perp$. From symmetry considerations it follows that $G_{h_x}^c = G_{h_y}^c$, and we have two equations:

$$\bar{t}_\perp \equiv \langle t_{\mathbf{m+h}_\perp,\mathbf{m}} \rangle = \int \rho_\perp(A) \frac{A - J_c}{1 - 2(A - J_c)(G_0^c - G_\perp^c)} dA = 0, \tag{29}$$

$$\bar{t}_z \equiv \langle t_{\mathbf{m+h}_z,\mathbf{m}} \rangle = \int \rho_z(A) \frac{A - K_c}{1 - 2(A - K_c)(G_0^c - G_z^c)} dA = 0$$

for the determination of two coherent-matrix parameters: $A_x^c = A_y^c \equiv J_c$ and $A_z^c \equiv K_c$. Here,

$$G_\perp^c = \frac{1}{2}(G_{h_x}^c + G_{h_y}^c) = \frac{1}{4}\sum_{h_\perp} G_{h_\perp}^c, \quad G_z^c = \frac{1}{2}\sum_{h_z} G_{h_z}^c.$$

The characteristics of the original quasi-two-dimensional crystal can be obtained from (29) using the distribution functions corresponding to zero degree of amorphization:

$$\rho_\perp^0(A) = \delta(A - J_0), \quad \rho_z^0(A) = \delta(A - K_0). \tag{30}$$

If the amorphization of the quasi-two-dimensional crystal leads to the result that its exchange properties become isotropic, i.e., $\rho_\perp = \rho_z \equiv \rho$ and $G_{h_x}^c = G_{h_y}^c = G_{h_z}^c$, we have the equation

$$\bar{t} = \int \rho(A) \frac{A - I_c}{1 - 2(A - I_c)(G_0^c - G_1^c)} dA = 0 \tag{31}$$

for the determination of the one coherent parameter $J_c = K_c \equiv I_c$. Here,

$$G_1^c = \frac{1}{3}(G_{h_x}^c + G_{h_y}^c + G_{h_z}^c) = \frac{1}{6}\sum_h G_h^c.$$

The equation (31) coincides with the result of Foo and Bose.[4]

4. THE ISOTROPIC SOLUTION FOR FLUCTUATIONS WITH A δ-FUNCTION DISTRIBUTION

It is obvious that the average characteristics of an amorphous substance modeled by an ideal crystal will depend in an essential way on the distribution functions ρ_i of the exchange-parameter fluctuations.

Foo and Bose[4] considered the isotropic case with the fluctuation distribution function

$$\rho_{FB}(A) = \begin{cases} 1, & \text{if } I_0 - \Delta \leqslant A \leqslant I_0 + \Delta, \\ 0, & \text{otherwise}, \end{cases}$$

where I_0 is the exchange parameter of the ideal isotropic FM and Δ is the range of its fluctuations. As an example, we shall consider another ideal case for the distribution function and assume that the fluctuating exchange parameter can take only the two values J_0 and K_0 corresponding to the original quasi-two-dimensional ideal crystal:

$$\rho_\perp(A) = P_\perp(J_0)\delta(A - J_0) + P_\perp(K_0)\delta(A - K_0),$$

$$\rho_z(A) = P_z(J_0)\delta(A - J_0) + P_z(K_0)\delta(A - K_0). \tag{32}$$

Here, $P_\perp(J_0)$ and $P_z(J_0)$ are the possibilities of the appearance of a J_0 coupling in the plane and along the z-axis, $P_\perp(K_0)$ and $P_z(K_0)$ are the corresponding probabilities of the appearance of a K_0 coupling, and $P_i(J_0) + P_i(K_0) = 1$. Thus, we consider a model in which the amorphization of the quasi-two-dimensional FM leads only to mixing of the exchange couplings K_0 and J_0, without any change in their values. Taking the latter into account

does not lead to fundamental changes in the results, so long as the changes in the values are not too great.

In the framework of the given model, the degree of amorphization of the substance can be characterized, e.g., by the ratio $P_\perp(K_0)/P_\perp(J_0) \equiv \eta$. Since the total number of both the J_0 and the K_0 couplings is conserved, the parameter η will define the degree of amorphization uniquely. Thus, for $\eta = 0$ we obtain the distribution (30).

In the isotropic case we have the distribution function

$$\rho(A) = {}^2\!/_3\delta(A - J_0) + {}^1\!/_3\delta(A - K_0), \qquad (33)$$

since there are twice as many J_0 couplings as K_0 couplings for the crystal under consideration. The following analysis will be devoted to precisely this case. Using (31) and (33), we obtain

$$\frac{K_0 - I_c}{1 - 2a(K_0 - I_c)} + \frac{2(J_0 - I_c)}{1 - 2a(J_0 - I_c)} = 0, \qquad (34)$$

where

$$a = G_0^c(\Omega) - G_1^c(\Omega),$$
$$G_0^c(\Omega) = \frac{1}{N}\sum_k G_k^c(\Omega), \quad G_1^c(\Omega) = \frac{1}{N}\sum_k G_k^c(\Omega)\gamma_k,$$
$$G_k^c(\Omega) = [\Omega - \omega_c(k, \Omega)]^{-1},$$
$$\omega_c(k, \Omega) = 6I_c(\Omega)(1 - \gamma_k), \quad \gamma_k = \frac{1}{6}\sum_h e^{ikh}.$$

We shall characterize the degree of quasi-two-dimensionality of the original ideal crystal by the ratio $K_0/J_0 = \lambda_0$. For $\lambda_0 = 1$ we have an ideal isotropic crystal with the spin-wave dispersion law $\omega_0(k) = 6J_0(1 - \gamma_k)$. The properties of a ferromagnetic crystal describable by the coherent exchange parameter $I_c(\Omega)$ will be considered in relation to those of an FM with exchange parameter J_0. We note that, in the framework of the model used, the amorphized isotropic FM could be characterized by an average exchange parameter $\bar{I} = 2J_0/3 + K_0/3$. However, such a description is crude, although it does take into account the tendency of the Curie temperature to increase on amorphization of the quasi-two-domensional FM.

We introduce some notation: $W_0 = 6J_0$ is the half-width of the spin-wave band of the ideal isotropic FM, $\omega = \Omega/W_0$, $I_c/J_0 \equiv x$ is the dimensionless coherent exchange parameter, and $\varepsilon_k = 1 - \gamma_k$. In these variables, Eq. (34) for x becomes

$$\frac{1}{N}\sum_k \frac{\varepsilon_k}{\omega - x(\omega)\varepsilon_k} = \psi[x(\omega)], \quad \psi(x) = \frac{2 + \lambda_0 - 3x}{(1 - x)(\lambda_0 - x)}. \qquad (35)$$

If $g_0(\varepsilon)$ is the density of states corresponding to the dispersion law ε_k for the ideal FM, the left-hand side of Eq. (35) can be written in the form

$$\frac{1}{N}\sum_k \frac{\varepsilon_k}{\omega - x(\omega)\varepsilon_k} = \int g_0(\varepsilon)\frac{\varepsilon\, d\varepsilon}{\omega - x(\omega)\varepsilon} = \frac{1}{x(\omega)}[zG_0(z) - 1],$$

where $z(\omega) = \omega/x(\omega)$ and $G_0(z)$ is the Green function of the ideal FM:

$$G_0(z) = \frac{1}{N}\sum_k \frac{1}{z - \varepsilon_k} = \int g_0(\varepsilon)\frac{d\varepsilon}{z - \varepsilon}.$$

It is convenient to choose x as the independent variable, i.e., to assume that $\omega = \omega(x)$ and $z = z(x) = \omega(x)/x$. In the new variables, Eq. (35) takes the form

$$zG_0(z) = 1 + x\psi(x). \qquad (36)$$

Having determined the dependence $z = z(x)$ from (36) we can find $\omega(x) = xz(x)$; the coherent potential as a function of ω is the inversion of $\omega(x)$.

In the general case we can assume that the coherent parameter has real and imaginary parts, i.e., $x(\omega + i0) = x_1(\omega) - ix_2(\omega)$. Then the density of states is equal to

$$g_c(\omega) = -\frac{1}{\pi}\operatorname{Im} G_0^c(\omega + i0) = \int g_0(\varepsilon)\frac{1}{\pi}\frac{x_2(\omega)\varepsilon}{[\omega - x_1(\omega)\varepsilon]^2 + [x_2(\omega)\varepsilon]^2}\, d\varepsilon.$$

We shall start by imagining that the imaginary part of $x(\omega)$, responsible for the damping of the spin waves, goes to zero. Indeed, the CPA corresponds to the approximation of an ideal crystal with a self-consistent exchange parameter that ensures zero scattering on the average. In this case

$$g_c(\omega) = \int g_0(\varepsilon)\delta[\omega - x_1(\omega)\varepsilon]\, d\varepsilon = \frac{1}{|x(\omega)|}g_0\left(\frac{\omega}{x(\omega)}\right), \qquad (37)$$

where the modulus sign can be omitted, since $x(\omega) \in [\lambda_0, 1]$.

To calculate $x(\omega)$ we shall use the approximation of an elliptical band for the density of states, corresponding to the dispersion law $\omega_0(k) = 6J_0\varepsilon_k = W_0\varepsilon_k$:

$$g_0(\Omega) = \begin{cases} \dfrac{2}{\pi W_0^2}[\Omega(2W_0 - \Omega)]^{1/2}, & 0 \leqslant \Omega \leqslant 2W_0 \\ 0 & \text{outside this interval} \end{cases} \qquad (38)$$

This approximation correctly reflects the character of the behavior of the true density of states at the bottom and top of the band but does not take into account the van Hove singularity. In dimensionless variables, on the basis of (38) we obtain the following expression for $G_0(z)$ in the interval $0 \leqslant z \leqslant 2$:

$$G_0(z + i0) = P\int \frac{g_0(\varepsilon)}{z - \varepsilon}\, d\varepsilon - i\pi g_0(z) = 2(z - 1) - 2i[z(2 - z)]^{1/2}. \qquad (39)$$

Treating the left-hand side of Eq. (36) in the principal-value sense and using the expressions (35) and (39), we find the solutions of (36):

$$z_\pm(x) = \frac{1}{2}[1 \pm C^{1/2}(x)], \quad C(x) = 3\frac{x^2 - x(1 - \lambda_0)/3 - \lambda_0}{(1 - x)(x - \lambda_0)}$$

Real solutions correspond to the region $x_1 \leqslant x \leqslant 1$ (the physically accessible range of variation of the coherent parameter is $[\lambda_0, 1]$), where x_1—the positive root of the equation $C(x) = 0$—is the minimum value of x:

$$x_1 = (x)_{min} = \frac{1}{6}\{1 - \lambda_0 + [(1 - \lambda_0)^2 + 36\lambda_0]^{1/2}\}, \quad 0 \leqslant \lambda_0 \leqslant 1.$$

The value of the coherent parameter at $\omega = 0$ is easily found from (35) and corresponds to $z = 0$:

$$x(0) = \frac{1}{4}[1 + (1 + 8\lambda_0)^{1/2}].$$
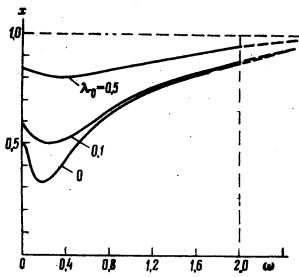
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FIG. 1. Behavior of the dimensionless coherent parameter $x = I_c/J_0$ as a function of $\omega = \Omega/W_0$.

The solutions $x(\omega)$ for different values of the parameter λ_0 are presented in Fig. 1.

The densities of the spin-wave states are depicted in Fig. 2. The boundary point of the spectrum for an FM with a coherent parameter is determined by the relation $z(x_{max}) = 2$ or $2W_c/x(2W_c) = 2$.

The equation (16) for $\sigma(T)$ is conveniently rewritten in terms of the dimensionless variables

$$\nu = E/W_0, \quad \omega = \Omega/W_0 = \nu/\sigma, \quad \tau = T/W_0 .$$

We have

$$\frac{1}{\sigma} = 1 + 2 \int \tilde{g}_c(\nu) f(\nu) d\nu,$$

where

$$\tilde{g}_c(\nu) = \frac{1}{\sigma x(\nu)} g_0\left(\frac{\nu}{\sigma x(\nu)}\right), \quad f(\nu) = (e^{\nu/\tau} - 1)^{-1}.$$

Regarding $z = \nu/\sigma x$ as the independent variable ($x = x(z)$, $\nu = \sigma z x(z)$), we bring the equation to the form

$$\frac{1}{\sigma} = 1 + 2 \int g_0(z) f(\sigma x z)\left(1 + \frac{z}{x} x'\right) dz. \tag{40}$$

In the low-temperature case, when σ is close to unity, Eq. (40) can be solved by iterations, by substituting the value $\sigma = 1$ into the right-hand side of the equation in the first step. Taking into account that only the low-energy part of the spin-wave spectrum is important in this temperature region, we can put $g_0(z) \simeq (2^{3/2}/\pi) z^{1/2}$; then,

$$\int g_0(z) f(\sigma x z)\left(1 + \frac{zx'}{x}\right) dz \approx \frac{2^{3/2}}{\pi} \tau^{3/2} \int_0^\infty \frac{1}{[x(\tau u)]^{3/2}} \frac{u^{1/2} du}{e^u - 1}. \tag{41}$$

From (41) and (40) there follow the Bloch $\tau^{3/2}$ law for the low-temperature magnetization of an ideal FM ($\lambda_0 = 1$, $x = 1$) and the small corrections ($\propto \tau^{5/2}$) to it that arise from the frequency dependence of the coherent parameter for $\lambda_0 < 1$.

To estimate the Curie temperature it is convenient to represent Eq. (40) in the form

$$\frac{1}{\sigma} = \int g_0(z)\left[\text{cth}\frac{\sigma x z}{2\tau} + 2f(\sigma x z)\frac{zx'}{x}\right] dz. \tag{42}$$

Inasmuch as $\sigma \to 0$ as $\tau \to \tau_C = T_C/W_0$, and

$$\text{cth}\frac{\sigma x z}{2\tau} \approx \frac{2\tau}{\sigma x z}, \quad 2f(\sigma x z) \approx \frac{2\tau}{\sigma x z}$$

for $\sigma x z/2\tau \ll 1$, from (42) we find

$$\frac{1}{2\tau_C(\lambda_0)} = \int \frac{g_0(z)}{z}\left[\frac{1}{x(z)} + \frac{zx'(z)}{x^2(z)}\right] dz . \tag{43}$$

Thus, the expression (43) gives an estimate of the Curie temperature $T_C^{am}(\lambda_0) = \tau_C(\lambda_0) W_0$ of the isotropic FM obtained as a result of amorphization of an ideal quasi-two-dimensional ferromagnetic crystal with exchange parameters K_0 and J_0 with ratio λ_0. In the limiting case $\lambda_0 = 1$ (the original crystal is an isotropic FM with exchange parameter J_0), in the framework of our model the crystal does not change its properties ($x = 1$) and (43) goes over into the well-known expression for the Curie temperature of an ideal isotropic FM:

$$T_C^0 = \tau_C^0 W_0,$$

where

$$\frac{1}{2\tau_C^0} = \frac{1}{2\tau_C(\lambda_0 = 1)} = \int \frac{g_0(z)}{z} dz.$$

For the density of states (38) that we have used, the integral is equal to 2 (for the true density of states, the value of the integral is 1.51[11]). The ratio of the Curie temperatures of the amorphous and ideal FMs is equal to

$$\frac{T_K^{am}(\lambda_0)}{T_C^0} = 2 \bigg/ \int \frac{g_0(z)}{z}\left[\frac{x + zx'}{x^2}\right] dz \leqslant 1. \tag{44}$$

It is convenient to compare the expression obtained for $T_C^{am}(\lambda_0)$ with the result of Lines[10] for the Curie temperature of an ideal quasi-two-dimensional ferromagnetic crystal, also obtained in the spin-wave approximation: $T_C^{id}(\lambda_0) = \Phi(\lambda_0) W_0$. The function $\Phi(\lambda_0)$ (we do not give its explicit form) vanishes as $\lambda_0 \to 0$, in agreement with the theorem of Mermin and Wagner[15] on the absence of magnetic order in one- and two-dimensional spin systems, and $\Phi(\lambda_0 = 1) = \tau_C^0$. In Fig. 3 we give Lines' result for $T_C^{id}(\lambda_0)/T_C^0$ (curve 1) and our result (44) (curve 2). The relative increase $T_C^{am}(\lambda_0)/T_C^{id}(\lambda_0)$ of the Curie temperature on amorphization of a quasi-two-dimensional ferromagnetic crystal is depicted as a function of λ_0 by the curve 4. If we use the approximation of an average exchange parameter $\bar{I} = K_0/3 + 2J_0/3$, the corresponding Curie temperature of the amorphized FM will be equal to $\bar{T}_C(\lambda_0)$; the ratio $\bar{T}_C(\lambda_0)/T_C^0 = 2(1 + \frac{1}{2}\lambda_0)/3$ corresponds to the curve 3.

5. CONCLUSIONS

Thus, the CPA method can be used rather effectively in the analysis of the problem of the amorphization of
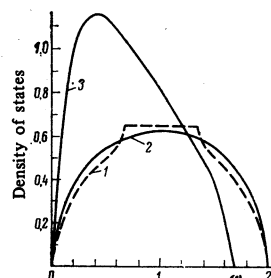

FIG. 2. The density of spin-wave states: 1) the true density of states for the dispersion law ε_k; 2) approximation of the density of states by the function $g_0(\omega)$; 3) the density of states $g_c(\omega) = g_0[\omega/x(\omega)]/x(\omega)$.
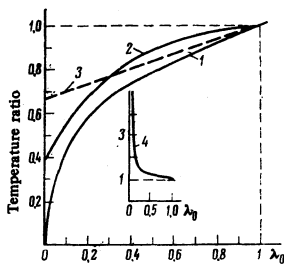
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FIG. 3. Curve 1) the ratio $T_C^{id}(\lambda_0)/T_C^0$ of the Curie temperatures of ideal quasi-two-dimensional and isotropic ferromagnetic crystals according to Lines[10]; 2) the ratio $T_C^{am}(\lambda_0)/T_C^0$ of the Curie temperatures of the amorphous isotropic and ideal isotropic ferromagnets; 3) the ratio $\overline{T}_C(\lambda_0)/T_C^0$ of the Curie temperatures of the amorphous isotropic FM (in the approximation of an average $\overline{I}$) and the ideal isotropic FM; 4) relative increase $T_C^{am}(\lambda_0)/T_C^{id}(\lambda_0)$ of the Curie temperature on amorphization of a quasi-two-dimensional FM.

ferromagnets with anisotropically distributed exchange couplings, such as quasi-two-dimensional magnets. We have shown, e.g., that the amorphization of a quasi-two-dimensional FM leads to a substantial increase of T_C, especially for small values of the interplanar exchange. In real substances we must, of course, also take into account the decrease of the quantities K_0 and J_0 themselves, which leads to a certain lowering of the curve 2 in Fig. 3. For example, for $\lambda_0 = 1$ the ratio T_C^{am}/T_C^0 will be less than unity. However, the results are not qualitatively changed. A similar remark can also be made concerning the magnetization of an amorphous FM. It is clear, of course, that the amorphization of a quasi-one-dimensional FM is not qualitatively different from the case considered. As already stated, the situation is considerably more complicated in the case of amorphization of quasi-two-dimensional anti-

ferromagnets. However, we may expect that for small values of $|\lambda_0|$ the amorphization of systems with $J_0 > 0$ and $K_0 < 0$ can lead to ferromagnetism with a relatively high T_C. In view of the fact that quasi-low-dimensional magnets constitute a broad class of magnets, we may suppose that their amorphization could turn out to be an important way of obtaining new magnetically ordered substances.

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Contribution to the theory of electromechanical forces in metals

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A mechanism is considered for production of electromechanical forces caused by electric current and concentrated near crystallite interfaces. The order of magnitude of the forces is ascertained for simple models of the interface and of the dispersion law.

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An electric field **E** applied to a metal produces an electron current and motion of lattice defects: atoms, dislocations, inclusions, grain boundaries. It does not disturb the mechanical equilibrium of an ideal crystal lattice. This is a dynamic equilibrium: the forces of

the external field, acting on the ions of the "skeleton" of the metal, are exactly compensated by the forces produced on scattering of the electrons in the lattice (the electronic wind[11]). The observable manifestations of the force of the electronic wind depend on the mech-