Theory of rapid (nonadiabatic) rotation of nonspherical nuclei

V. G. Nosov and A. M. Kamchatnov

I. V. Kurchatov Atomic Energy Institute

On the basis of the concept of the growing role of nonadiabatic effects of the nonconservation of the quantum number K a theory has been developed of the phenomenon which has been given the name of backbending. Above the transition point, for $J \geq J_c$, all the values $-J \leq K \leq J$ are equally probable. An investigation is made of the singularities possessed by the ordering parameter (proportional to the spectroscopic quadrupole moment of a nonspherical nucleus), the rotational angular velocity and the moment of inertia of a nucleus at the Curie point. Formulas have been derived for the intensity of quadrupole radiation in the more symmetric n-phase $J > J_c$. By analyzing the experimental values of the moments of inertia belonging to the n-phase the radius of the mass distribution in the nucleus was determined. It agrees with the radius of the proton distribution derived from data on the scattering of electrons by nuclei. On the basis of the simplest form of the singularity of the parametric derivative of the Hamiltonian of the system a general theory of zero-temperature second-order phase transitions is developed in the Appendix.

PACS numbers: 21.10.Hw

1. INTRODUCTION. ESTIMATE OF THE CRITICAL VALUE J_c OF THE ROTATIONAL QUANTUM NUMBER. ROTATIONAL DENSITY MATRIX

In recent decades there perhaps has not been in nuclear physics an event more outstanding than the discovery made at the beginning of the 1950's by A. Bohr and B. Mottelson of rotational levels in nonspherical nuclei. Having confirmed as a subsidiary result the well-known assertion of quantum mechanics concerning the inability of a perfectly spherical body to undergo purely mechanical rotation (cf., for example, Refs. 1, 2), this discovery became the starting point for a detailed study of the so-called rotational bands in the energy spectra of nonspherical nuclei. The initial simplest theoretical treatment is expressed by the widely known formula

$$E(J) \cong \frac{\hbar^2}{2I'} J(J+1) \tag{1}$$

for the energy levels. Here J is the rotational quantum number, i.e., the total angular momentum of the whole system (nuclear spin); I' is the "adiabatic" moment of intertia which refers to the neighborhood of the origin of the band and the value of which is determined from experiment. Avoiding as far as possible complications which do not have particular significance as to principle we shall in future as a rule, have in mind the case of a rotational band based on the ground state of an even-even nucleus. Then in fact the allowable values are $J = 0, 2, 4, \ldots$.

Progress of the experiment upward along the band stimulated an ever more critical attitude to formula (1). One of the methods of generalizing it consists of the following: we adopt the point of view which appears to be natural that the conserved quantity J is the only significant physical (thermodynamic) characteristic of which the energy E is a function within the band under consideration in the case of not too light a nucleus.

Any scalar constructed from the components of the vector **J** will be reduced to the combination $J(J+1)$. Therefore

$$E_m(J) = \sum_{s=1}^{\infty} C_s [J(J+1)]^s \tag{2}$$

(the reason for indexing the rotational energy of the nucleus by m will become clear from subsequent discussion). From a practical point of view doubt immediately arises whether the understandable tendency to break off the series (2) after a finite number of terms would lead to a situation that is sufficiently stable from a purely computational point of view. However with time an even deeper difficulty involving matters of principle became apparent. About six years ago due to the application of new experimental methodology penetration began into the domain of still greater values of J.[3–6] This led to the discovery of a very characteristic phenomenon which was called backbending in foreign literature.[1] When a noticeable effect was observed it consisted of the following: the principal characteristics

$$\frac{dE}{dJ} = \hbar\Omega, \quad \frac{d^2E}{dJ^2} = \frac{d(\hbar\Omega)}{dJ} = \frac{\hbar^2}{I} \tag{3}$$

(Ω is the rotational angular velocity, I is the moment of inertia) of the rotational motion of a nucleus undergo a sharp change over a very narrow region of the rotational band. The angular velocity of rotation Ω here falls off precipitously in spite of an increase in the total angular momentum J. It is also very characteristic that in some cases immediately beyond the transition point the rotational spectrum turns out to be close to an equidistant one. For example, quite typical in this regard is the rotational band based on the ground state of the nuclide W^{170}. The segment $J = 10$–14 in this case contains only two intervals. But over this segment the angular velocity $\hbar\Omega$ (measured in energy units) manages to decrease by ~ 100 keV—or by ap-

proximately 30%. The sector $J = 12–16$ which is minimally displaced in the positive direction also contains two energy intervals. But the second of them exceeds in the first one by only 3%, and the increase in angular velocity of rotation here is just as insignificant (cf., formula (3)).

At first glance it might appear that the discovery of the phenomenon of backbending compromises the very basis for arguing in favor of relations of the type (1) or (2). However it appears to us to be not quite logical and excessively hasty to cast doubt on the determining role of the independent (and, with macroscopic accuracy, essentially unique) thermodynamical variable J. In particular, the arguments leading to formula (2) by no means exclude the possibility that this series would have a finite radius of convergence. In accordance with the spirit of experimental data the point $J = J_c$ of the intersection of the boundary of the circle of convergence with the real axis could then have an important physical meaning. If nuclear states situated on opposite sides of this point differ by some qualitiative property of a symmetry type then for $J > J_c$ the energy $E(J)$ must be a different function which does not coincide with the analytic continuation of expression (2). Due to the presence of a singularity at $J = J_c$ such an analytic continuation could, strictly speaking, turn out to be not single-valued and even complex.

In order to discuss the specific physical nature of the phenomenon we turn to the properties of the component $\mathbf{J \cdot n} = K$ ($\mathbf{n}$ is a unit vector along the axis of the non-spherical nucleus) of the total angular momentum of the nucleus along its symmetry axis. Although this is not a matter of principle and does not in any manner affect exact calculations, in order to make the subsequent discussion more easily visualizable one can, if one so wishes, call upon a crudely classical vector model; one can set $J_z = (J_z)_{max} = J$ and then $K \cong J\cos\theta$. The quantum number K is not conserved since the operator $\mathbf{J \cdot n}$ does not commute with the Hamiltonian of the system. However for $J \ll J_c$ near the origin of the rotational band the commutator is small. It is well known that this corresponds to adiabatic slowness of rotation. As a result we obtain

$$K \cong K_0 = \text{const}, \quad w_K \cong \delta_{KK_0}, \quad J \ll J_c, \tag{4}$$

where w_K is the probability of having the corresponding value of K in the quantum state under consideration of the nucleus as a whole. Here formula (1) is valid. In practice we most frequently have to deal with the simplest case $K_0 = 0$ for even-even nuclei.

But as one moves along the rotational band the admixture of components with $K \neq K_0$ increases more and more rapidly. A rough estimate making use of perturbation theory leads to a relation of the type

$$w_K \propto [2|K|/(J - |K|)]^{J - |K|} e^{J - |K|}$$

to express the law of growth of a given component near the threshold $J = |K|$ where it first appears. High above the threshold we obtain the expression $w_k(J)$

$\propto J^{2|K|}$, which remains sufficiently steep and also favors the appearance of large $K \sim J$. As long as perturbation theory is applicable the initiating component $K = K_0$ appears to serve, in agreement with unitarity, as a source of growth for the others. However, when the latter cease being relatively small, then all the components become qualitatively on the same footing in the process of their, so to say, interaction with each other. The result will be a kind of equilibrium, an equidistribution with respect to K ($w_K = 1/(2J + 1) = \text{const}$), which will be attained, say, at the point $J = J_c$. Of course, here one should not expect any appreciably sharp change in the state of the system as such, for example a discontinuity in its energy. Physically the important feature of such a new situation is contained in a different attribute—in the change in the symmetry of the rotational state of the nucleus. It can be easily understood that an equidistribution with respect to K corresponds to isotropy, to an equal probability of all spatial directions of the vector $\mathbf{n}$ (we recall that we are dealing with an individual specific quantum state of the nucleus as a whole: J and J_z are given quantities!). At the same time in the Hamiltonian of the system as a whole one also should not be able to discern any favored directions for the vector $\mathbf{n}$ in empty space. It would be naive to think that this correspondence will be violated in the course of a further increase in J when physically there are even fewer grounds for some values of the quantum number K to become favored compared to others. Having acquired stability the more symmetric iosotropic phase will not lose it for $J > J_c$.

The attainment of isotropy with respect to the directions of $\mathbf{n}$ can also be interpreted from a somewhat different point of view. In a sufficiently strong rotational "field" the mechanical angular momenta of the individual quasiparticles line up parallel to the vector $\mathbf{J}$ and cease to line up along the vector $\mathbf{n}$. Therefore the latter turns out to be "free," i.e., it is in fact distributed isotropically (for $J \gtrsim J_c$). This constitutes the difference from the old adiabatic region $J \ll J_c$ where the internal state of the nucleus is determined, roughly speaking, by its deformation and, in turn, determines the approximately conserved value $K = \mathbf{J \cdot n} \simeq K_0$. At the transition point itself the number of lined-up quasiparticle is of order unity, i.e.,

$$J_c \sim l \sim k_f R \gg 1 \tag{5}$$

(k_f is the limiting momentum of the Fermi distribution; R is the nuclear radius). This estimate does not contradict experimental data (cf., also Sec. 5).

Thus, $J = J_c$ is the point of a phase transition of the second kind—the zero-temperature Curie point. In analogy with the terminology adopted in Ref. 7 we shall refer to the states of a nonspherical nucleus with a completely isotropic distribution of its axis in space as the n-phase, and we shall call the region $J < J_c$ of "spontaneous symmetry violation"—the m-phase. We then have

$$w_K = 1/(2J + 1), \quad \overline{\cos^2\theta} = 1/3.$$

$$\overline{K^2} = \frac{1}{2J+1}\sum_{K=-J}^{J} K^2 = \frac{J(J+1)}{3}, \quad J \gtrsim J_c \tag{6}$$

V. G. Nosov and A. M. Kamchatnov   412

(θ and φ are the spherical angles defining the direction of the vector $\mathbf{n}$ with respect to fixed axes). We emphasize that these properties of the n-phase presuppose a most essential indeterminancy in the value of the polar angle θ which does not in any way diminish with increasing J. In the m-phase the relations (6) are violated as a result of the "ordering" of the rotational state. Having in mind, primarily, the special case $K_0 = 0$ and taking into account the fact that in the adiabatic region $J \ll J_c$ (cf., formulas (1) and (4)) a change in the quantum number J has little effect on the quantity $\overline{K}^2$, we shall, in order to be specific, set

$$\overline{K^2} = \sum_{K=-J}^{J} w_K K^2 < J(J+1)/3, \quad J < J_c. \tag{7}$$

It can be easily verified that for a state which is a superposition of components with different values of K it is not possible to construct any purely rotational wave function which depends only on $\mathbf{n}$. For $J \gtrsim J_c$ the quantum mechanical description of the rotation of a nonspherical nucleus is achieved with the aid of the density matrix $\rho(\mathbf{n}, \mathbf{n}')$. However, due to the conservation of the z-component of the total angular momentum the "azimuthal" wave function

$$\varphi_{rot} = (2\pi)^{-1/2} e^{iM\varphi}, \quad M = J_z \tag{8}$$

nevertheless exists. In the quasiclassical case $J_z = (J_z)_{max} = J \gg 1$ it corresponds to the so-called regular precession of an ordinary symmetric top (cf., for example, Ref. 8). As regards the polar angle θ, sufficiently far above the Curie point the density matrix which depends upon it must have the following properties: $\rho(\theta, \theta') = 0$ for $\theta \neq \theta'$, while the angular dependence of the diagonal element $\rho(\theta, \theta')$ corresponds to isotropy in three-dimensional space. However the formula appears more attractive if one utilizes a representation which is based not on angles, but on the more customary quantum numbers J, M and K. Then we have

$$\rho_{J_0 J_z}(J, M, K; J', M', K') = \delta_{J J_0} \delta_{J' J_0} \delta_{M J_z} \delta_{M' J_z} \delta_{K K'} / (2J_0 + 1) \tag{9}$$

in a state of angular momentum J_0 and its component J_z. For diagonal elements the region of applicability of this relation is wider and encompasses the whole n-phase including the Curie point. But for $J \to J_c + 0$, apparently, nondiagonal (with respect to K) elements of the density matrix which reflect the correlations between different K also become significant. It is even more difficult to form judgements concerning the specific form of the rotational density matrix in the m-phase $J < J_c$. For $J = J_c$ it has some kind of a singularity which hinders the formal continuation of the corresponding expression into the "foreign" region $J > J_c$. The physically important functional of the rotational density matrix—the so-called ordering parameter—has a radical singularity as $J \to J_c - 0$ (cf., next section).

2. THE AVERAGE ("SPECTROSCOPIC") QUADRUPOLE MOMENT AS AN ORDERING PARAMETER. DISCONTINUITY IN THE ROTATIONAL ANGULAR VELOCITY OF THE NUCLEUS.

If we keep (6) and (7) in mind the following definition suggests itself

$$\eta = 1 - 3\overline{K^2}/J(J+1) \simeq 1 - \overline{3\cos^2\theta} \tag{10}$$

for the ordering parameter which characterizes the m-phase from isotropy (the approximate expression involving the angle refers to the quasiclassical case $J_z = J \gg 1$ and is stated here only for ease of visualization). In the important special case $K_0 = 0$ it in fact varies from unity to zero at the Curie point. Naturally, for a given J and η quite different nuclear states are in principle conceivable—both rotational states, and ones differing microscopically. However, we shall assume that for $J < J_c$ one can utilize the concept of the energy of that one of these states which is "most nearly in equilibrium" and which we shall denote by $E(J, \eta)$.

Intending to analyze the situation for sufficiently small $J_c - J$ in the neighborhood of the Curie point we expand the general expression for the energy in powers of η and limit ourselves to three terms:

$$E(J, \eta) = E_n(J) - A(J)\eta + \tfrac{1}{2} D(J)\eta^2. \tag{11}$$

The function $E_n(J)$ in fact refers to the unordered n-phase. The inequalities

$$A(J) > 0, \quad D(J) \simeq D(J_c) \equiv D > 0 \tag{12}$$

express the advantages of positive η and the stability of the energy minimum with respect to this quantity. Substitution into (11) of the true value of η determined from the equilibrium condition

$$\partial E / \partial \eta = 0 \tag{13}$$

transforms $E(J, \eta)$ into the energy $E_m(J)$ of the rotational levels of the m-phase

$$\eta = A(J)/D, \quad E_n(J) - E_m(J) = \tfrac{1}{2}[A(J)]^2/D. \tag{14}$$

Figure 1, which shows the curve: for the energies of the phases, requires some explanation. At the "endpoint" $J = J_c$ the function $E_m(J)$ has a certain singularity. However, the question is: which of the derivatives of the function will be first affected by this, which of them will become infinite at this point? According to (3) this cannot happen in the case of the first derivative: there are no slightest theoretical or experimental bases for supposing that the rotational velocity of nuclei becomes infinite. Moreover, as may be seen from
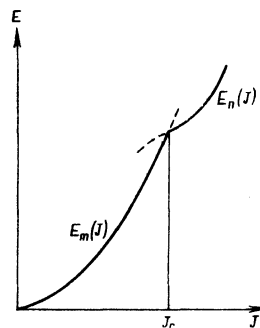

FIG. 1.

V. G. Nosov and A. M. Kamchatnov   413

formula (3), the second derivative also does not become infinite. A zero value of the moment of inertia agrees neither with the fact that the nucleus is nonspherical, nor with the experimentally observed tendency for moments of inertia in the m-phase to vary (cf. Sec. 5). Thus, although the dotted line extrapolation of the $E_m(J)$ curve is somewhat arbitrary, at least the first two derivatives have meaning along it. As a result of this assumption illustrated by Fig. 1 that at $J=J_c$ a simple point of intersection

$$E_n - E_m \propto J_c - J \tag{15}$$

of the curves under investigation occurs is not contradictory and agrees well with the nature of the experimental data.

Comparison of (15) with (14) yields

$$\Lambda(J) = a(J_c - J)^{1/2}, \quad a > 0. \tag{16}$$

Consequently

$$\eta = a(J_c - J)^{1/2}/D \tag{17}$$

and

$$E_n - E_m = \frac{1}{2}\frac{a^2}{D}(J_c - J). \tag{18}$$

Differentiating relation (18) we obtain a discontinuity in the rotational angular velocity of the nucleus at the point of "backbending"

$$\Delta(\hbar\Omega) = a^2/2D. \tag{19}$$

Here $\Delta(\hbar\Omega) = \hbar\Omega_{mc} - \hbar\Omega_{nc}$. As far as one can judge according to experimental data the thermodynamic inequality

$$\Omega_{mc} > \Omega_{nc} \tag{20}$$

is never violated.

It is well known[1,2] that for the value averaged over the state $J_z = J$ of the z-component of the quadrupole moment (the so-called spectroscopic quadrupole moment) of a nonspherical nucleus the following formula is valid

$$Q = \frac{3\overline{K^2} - J(J+1)}{(J+1)(2J+3)}Q_0. \tag{21}$$

Here, Q_0 is the component of the "collective," macroscopic quadrupole moment along the nuclear axis wholly determined by its axially-symmetric deformation. Taking (10) into account we have

$$Q = -\frac{J}{2J+3}Q_0\eta. \tag{22}$$

Now comparing this with (17) we verify that both the quadrupole moment Q and also the ordering parameter vanish at the Curie point according to $(J_c - J)^{1/2}$. In the n-phase $J \geq J_c$ there can no longer be any collective (spectroscopic) quadrupole moment because of its isotropy.

3. POSITION OF THE ROTATIONAL LEVELS IN THE n-PHASE $J > J_c$. MOMENT OF INERTIA.

First of all we touch in a couple of words upon the problem of the stability of either of the phases. The corresponding condition

$$d^2E/dJ^2 > 0 \tag{23}$$

has a fairly natural appearance.[2] The symmetric n-phase becomes unrealizable for $J < J_c$ because of the absolute instability of the state in which the mechanical angular momenta of the quasiparticles lining up along the vector $\mathbf{J}$ vector ignore the direction of the vector $\mathbf{n}$ determined by the deformation (cf., the in a certain sense opposite nature of the states which are in fact realized in the adiabatic region $J \ll J_c$; cf., also the Introduction). Therefore we have

$$\left.\frac{d^2E_n}{dJ^2}\right|_{J=J_c} = 0 \tag{24}$$

and

$$d^2E_n/dJ^2 \propto J - J_c \tag{25}$$

near the Curie point. Substitution into the second of formulas (3) yields

$$I \cong j/(J - J_c), \quad J - J_c \ll j/I_0. \tag{26}$$

Here j is a certain constant coefficient;

$$I_0 = \tfrac{2}{5}MR^2 \tag{27}$$

is the solid body value of the moment of inertia; $M = m_n A$ is the nuclear mass; m_n is the nucleon mass;

$$R = r_0 A^{1/3} \tag{28}$$

is the nuclear radius. We arrive at an important conclusion that the moment of inertia of a nonspherical nucleus becomes infinite in accordance with the law (26) if one approaches the Curie point from above. This is what explains the observed approximate uniform spacing of levels in corresponding segments of the rotational spectra (cf., the Introduction). In the opposite limiting case of large $J - J_c$, for a sufficiently clearly pronounced lining-up of mechanical angular momenta of the quasiparticles along the direction of the vector $\mathbf{J}$ the moment of inertia of the Fermi system becomes the same as that for a solid body:

$$I \cong I_0, \quad J - J_c \gg j/I_0. \tag{29}$$

If the right-hand sides of (26) and (29) are added, then an interpolation formula will be obtained for the moment of inertia of the n-phase, which satisfies both limiting cases. By itself it, of course, is devoid of any to some extent deep physical foundations. Moreover, in the case of a literal acceptance of this interpolation a situation would arise of a monotonic approach to the solid body limit I_0 from above, while experimentally, apparently, a minimum was observed in the

variation of the moment of inertia (cf., Sec. 5). However, in a purely technical respect the interpolation formula is a convenient route for obtaining correct limiting expressions for $\Omega_n(J)$ and $E_n(J)$ by means of integrating it twice. Retaining everywhere in addition to the terms containing constants of integration, a single term of the corresponding expansion we obtain the following formulas

$$\hbar\Omega = \hbar\Omega_{nc} + \frac{\hbar^2}{2j}(J-J_c)^2,$$

$$E = E_c + \hbar\Omega_{nc}(J-J_c) + \frac{\hbar^2}{6j}(J-J_c)^3, \tag{30}$$

for $J - J_c \ll j/I_0$ and

$$\hbar\Omega = \hbar\Omega_{nc} + \frac{\hbar^2}{I_0}(J-J_c),$$

$$E = E_c + \hbar\Omega_{nc}(J-J_c) + \frac{\hbar^2}{2I_0}(J-J_c)^2 \tag{31}$$

for $J - J_c \gg j/I_0$.

It is desirable to keep in mind that, generally speaking, the last formula predicts the position of the rotational levels with good relative but not absolute accuracy. We note that for nuclides for which $j/I_0 \sim 1$ the domain of applicability of formulas (31) is broadened encompassing pratically the whole n-phase.

4. ELECTRIC QUADRUPOLE RADIATION IN THE n-PHASE

The basic property of the n-phase can be formulated as the absence of an observable (average) quadrupole moment;

$$\langle Q_{ik} \rangle = 0. \tag{32}$$

This does not mean that its matrix elements nondiagonal with respect to the levels of the band also vanish. They are responsible for the quadrupole radiation the probability of which we can write in the form

$$w = (e^2\omega^5/90\hbar c^5)\langle\langle Q^2 \rangle\rangle. \tag{33}$$

We have for the sake of brevity symbolically denoted by $\langle\langle Q^2 \rangle\rangle$ (reduced intensity of the transition) the square of the absolute value of the matrix element between the final state $J - 2$ and the initial state J summed over the final orientations of the nuclear spin. The transition frequency of interest to us amounts to $\omega = 2\Omega$.

For the subsequent discussion it is convenient to express this collective quadrupole moment in terms of the components of the vector **n**:

$$Q_{ik} = \frac{1}{2}Q_0(3n_in_k - \delta_{ik}). \tag{34}$$

From this it can be seen that it is immaterial over which state the square of the tensor is averaged:

$$Q_{ik}^2 = \frac{3}{2}Q_0^2. \tag{35}$$

With the aid of the usual rules for matrix multiplication we verify the validity of the relation

$$\sum_b |(Q_{ik})_a^b|^2 = \frac{3}{2}Q_0^2. \tag{36}$$

Here the indices a and b enumerate the individual states of a nonspherical nucleus irrespectively of to which rotational band they happen to belong.

Above the Curie point the quadrupole radiation can be calculated quasiclassically. Setting $\varphi = \Omega t$ we calculate in accordance with (34) the classically varying quadrupole moment. Then in accordance with the isotropy of the n-phase it is averaged uniformly over $\cos\theta$ ($K = J\cos\theta$ in the quasiclassical case). Finally the square of the absolute value of the spectral component $\omega = 2\Omega$ is substituted in formula (33) in place of $\langle\langle Q^2 \rangle\rangle$. As a result of elementary calculations we obtain[3]

$$\langle\langle Q^2 \rangle\rangle = \frac{1}{4}Q_0^2, \quad w = (e^2\omega^5/360\hbar c^5)Q_0^2, \quad J > J_c. \tag{37}$$

Thus, within the bounds of the n-phase all the radiative transitions have the same reduced intensity. In addition to the transition responsible for the real radiation there also exists an analogous transition in the opposite direction upwards along the band. The sum of the reduced intensities of both transitions is obtained by doubling the right hand side of the first of formulas (37). Comparing this with the sum rule (36) we find that it is saturated to one-third within the band under consideration, while the remaining two-thirds refer to transitions to other rotational bands.

5. COMPARISON WITH EXPERIMENT

Data on the states of nonspherical even-even nuclei with high spins were taken primarily from Ref. 11. The variation of the rotational angular velocity $\hbar\Omega_m$ in the m-phase was extrapolated graphically into a relatively small region near the Curie point (cf., also comments on Fig. 1 in Sec. 2), while the latter was identified with the position of the energy interval which already belongs, as far as the available data enable us to judge, to the n-phase. The results of this treatment of data are shown in Table I. The two cases marked by an asterisk refer to the rotational band originating from the 0^+ level of the excitation of β oscillations (the fully symmetric oscillation of a nonspherical nucleus). The quite unsystematic variations in the magnitude of the increment $\Delta(\hbar\Omega)$ of the rotational velocity from one nucleus to the next are noteworthy. The critical angular velocity $\hbar\Omega_{mc}$ behaves in a much more stable manner, but it also is subject to random fluctuations. It is

TABLE I.

Nucleus	J_c	$\hbar\Omega_{mc}$, keV	$\Delta(\hbar\Omega)$, keV	Nucleus	J_c	$\hbar\Omega_{mc}$, keV	$\Delta(\hbar\Omega)$, keV	Nucleus	J_c	$\hbar\Omega_{mc}$, keV	$\Delta(\hbar\Omega)$, keV
Ba¹²⁶	13	530	200	Dy¹⁵⁸	15	335	45	Yb¹⁶⁸	15÷17	330	~30
Ce¹³⁰	11	410	175	Dy¹⁶⁰	15	310	~25	Yb¹⁷⁰	17	365	60
Ce¹³²	13	490	235	Er¹⁵⁶	13	380	120	Hf¹⁶⁸	15	340	110
Ce¹³⁴	11	600	365	Er¹⁵⁸	15	360	125	Hf¹⁷⁰	15÷17	300	~5
Gd¹⁵⁴	≥17	340	≥25	Er¹⁶⁰	15	345	80	Hf¹⁷²	≥17	360	≥40
Gd¹⁵⁴*	13	275	75	Er¹⁶²	15	345	70	W¹⁷⁰	13	320	105
Gd¹⁵⁶	≥15	310	≥15	Er¹⁶⁴	17	370	120	W¹⁷⁶	≥15	350	≥40
Dy¹⁵⁴	≥15	320	≥45	Er¹⁶⁶	≥15	310	≥20	Os¹⁸²	15	340	100
Dy¹⁵⁶	≥17	370	≥40	Yb¹⁶⁴	15	350	110	Os¹⁸⁴	≥13	385	≥30
Dy¹⁵⁶*	13	275	95	Yb¹⁶⁶	15	340	90	Os¹⁸⁶	≥15	460	≥210

V. G. Nosov and A. M. Kamchatnov

somewhat more difficult to judge how real on the average is the tendency to a certain amount of decrease in $\hbar\Omega_{mc}$ with increasing atomic weight.

In addition to those given in Table I there are six more nuclides in which rotational levels have been discovered up to $J_{max} = 14-18$: W^{172}, W^{174}, Os^{176}, Os^{176}, Os^{180}, and Th^{232}. No significant effect was observed. However, let us turn to the rotational velocities attained. For the three osmium isotopes mentioned above we have $\hbar\Omega_{max} = 280-315$ keV. This is lower than the values characteristic of the end of Table I and is close to the lower limit of critical velocities in the whole table. For both tungsten isotopes $\hbar\Omega_{max} = 300$ keV. The thorium nucleus in the experiment attained the value $\hbar\Omega_{max} = 200$ keV. A conclusion suggests itself that in the nuclides "suspected" of not showing the effect there was simply a lack of sufficient rotational velocity to equalise the intensities of the components of the wave function with different values of K. Therefore there is every reason to suppose that with a further increase in J the situation will become clarified and, as usual, a phase transition will be found. We note, in order to avoid misunderstanding, that even in the case of a vanishingly small increment $\Delta(\hbar\Omega)$ (a situation close to this already occurred in the case of Hf^{170}, cf., Table I) it still will be possible to recognize the n-phase by its moment of inertia.

The study of the variation of the moment of inertia of a nonspherical nucleus in both phases is of considerable interest. The most characteristic and complete data in this respect are those concerning the principal rotational band for W^{170}; cf., Fig. 2. The increase in the moment of inertia in the m-phase can be qualitatively understood in the spirit of the Le Chatelier–Brown principle as a "resistance to an external action." Indeed, the rotational perturbation responsible for the growth in the components with $K \neq K_0$ is determined not directly by the mechanical angular momentum J, but by the angular velocity Ω_m. Therefore the system tries to
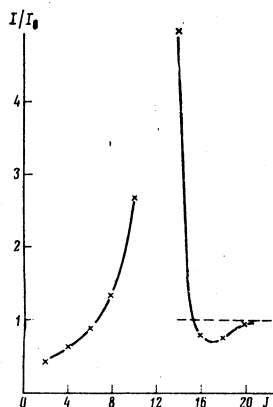

FIG. 2. Dependence of the moment of inertia of the W^{170} nucleus on the angular momentum. The point $J = 12$ which falls just on the discontinuity of the curve has descended into the negative region and is not shown in the diagram. This exception to the general picture should not be ascribed a physical meaning of a negative moment of inertia. The origin of the paradox becomes clear from relationship (25).

TABLE II.

	Nucleus							
	Ce¹³⁰	Gd¹⁵⁴	Dy¹⁵⁸	Er¹⁶²	Yb¹⁶⁴	Yb¹⁶⁶	W¹⁷⁰	Os¹⁸²
J_c	11.0	12.4	14.8	15.7	13.0	15.5	13.6	11.9
j/I_0	2.11	2.12	6.76	2.63	3.98	2.11	1.88	4.20

diminish the derivative $d(\hbar\Omega)/dJ$.[4] On the other side of the Curie point in the n-phase, the moment of inertia drops somewhat below I_0, passes through a minimum and then rapidly tends to approach the solid body asymptote from below. Analogous nonmonotonic behavior in the variation of the quantity d^2E_n/dJ^2 was also observed for the nuclide Yb^{164}. An interesting case was encountered in the β-vibrational band of dysprosium (Dy^{156*} according to the notation of Table I). From all indications here the very first two experimental points belonging to the n-phase turned out to be situated on opposite sides of the minimum in the moment of inertia. A final verification of the hypothesis will have to be postponed until the position of the rotational levels of the band with $J > 18$ is established.

On the whole the data concerning the n-phase are so far relatively scarce. When both experimental values closest to the Curie point belong to the left wing of the moment of inertia curve formula (26) yields

$$\frac{\hbar^2}{j} = \frac{(\hbar^2/I)_2 - (\hbar^2/I)_1}{J_2 - J_1}, \quad J_c = \frac{J_1(\hbar^2/I)_2 - J_2(\hbar^2/I)_1}{(\hbar^2/I)_2 - (\hbar^2/I)_1}. \tag{38}$$

These relations enable us to obtain the "extrapolated" (from the domain of the n-phase) position of the point of phase transition, and also to evaluate j/I_0. For the eight nuclides the results are shown in Table II. Basically good agreement is observed with values of J_c shown in Table I determined by a different method. The greatest discrepancy occurs in the case of osmium. However, it must be stated that the value of $J_c = 12$ given in Table II falls just in the middle of that segment of the rotational band where a decrease in the rotational angular velocity occurs. The region occupied by the phase transition is here anomalously wide and encompasses three energy intervals. Possibly this is in some way associated with the closeness of osmium to the point of phase transition of nonspherical nuclei into spherical ones[7] (cf., also the striking anomalies at the beginning and at the end of Table I). We also note that for specific nuclides the values of the ratio j/I_0 vary no less randomly than the increments $\Delta(\hbar\Omega)$.

The universal character of the asymptotic properties of the n-phase far from the Curie point in principle opens a path to the determination of the radius of the mass distribution in a nucleus. Unfortunately at the present time such high spins of rotational states have not yet been attained for which the relation (29) could be confidently regarded as a sufficiently strict equality. However one can try to circumvent this difficulty in the following manner. On the right wing of the moment of inertia curve for the n-phase where it is increasing the corresponding values are lower than the rigid body values. This very fact provides us with a lower bound on

V. G. Nosov and A. M. Kamchatnov

the possible values of the nuclear radius. A less rigorous but still quite plausible assumption consists of the following: in cases of the type selected for Table II the experimental values of the moment of inertia of the n-phase closest to the Curie point are higher than the rigid body values. This provides an upper bound on the radius. Making use of the fact that data are available concerning eight different nuclides we narrow down as much as possible the bounds of the inequality:

$$1.08 \, F < r_0 < 1.11 \, F \qquad (39)$$

($1 F = 10^{-13}$ cm). Thus, with reasonable accuracy we have

$$r_0 = 1.1 \, F. \qquad (40)$$

This agrees with the measurements of the radius for the distribution of electrical charge in the nucleus from electron scattering.[12] In the preceding discussion the value (40) was utilized everywhere in the calculations of the rigid body moment of inertia using formulas (27) and (28).

For a comparison of formulas (30) and (31) with experiment one must have in the n-phase a sufficiently large number of experimental points referring to the same nuclide. The most favorable case is the one of W^{170}. Since the value of j/I_0 given in Table II is not large we may use formula (31). It is not difficult to understand the reason for the good agreement over the whole n-phase demonstrated in Table III. Being quite well founded under the condition $J - J_c \gg j/I_0$ formula (31) at the same time has a correct interpolation behavior: as $J \to J_c$ it gives an experimental value Ω_{nc} for the rotational angular velocity of the nucleus which is known to be correct (cf., also the remark at the end of Sec. 3). In order to illustrate the application of formula (30) we in contrast select a nuclide with the greatest value of j/I_0; in accordance with Table II this will be Dy^{158}. As Table III shows, the theory agrees well with experiment.

We now pass to the question of the electric quadrupole radiation from a nonadiabatically rapidly rotating nucleus. In a recently published experimental paper[13] a study was made of the radiation within the limits of the principal rotational band of Ce^{134}. Two observed transitions belong to the n-phase. For comparison with previous formulas we reproduce the value of $\langle\langle Q^2 \rangle\rangle$ for $K = K_0 = 0$:

$$\langle\langle Q^2 \rangle\rangle_{K=0} = {}^9/_{16} Q_0^2 (1 - 1/J) \qquad (41)$$

TABLE III.

	W^{170} Nucleus			Dy^{158} Nucleus	
	$\hbar\Omega$, keV			$\hbar\Omega$, keV	
J	experiment	theory	J	experiment	theory
13	216.4	–	15	289.0	–
15	223.2	240	17	295.5	295.7
17	265.0	273	19	312.9	313.3
19	308.3	306	21	339.0	342.0
21	342.6	339			

TABLE IV.

Nucleus	J_c	$\hbar\Omega_{mc}$, keV	$\Delta(\hbar\Omega)$, keV	Nucleus	J_c	$\hbar\Omega_{mc}$, keV	$\Delta(\hbar\Omega)$, keV
Ho^{157}	$\geqslant 37/2$	355	$\geqslant 100$	Er^{159}	$39/2$	380	~ 25
Ho^{159}	$\geqslant 37/2$	340	$\geqslant 70$	Yb^{163}	$35/2$	305	70
Ho^{161}	$\geqslant 37/2$	325	$\geqslant 55$	Lu^{167}	$\geqslant 27/2$	300	$\geqslant 15$

(cf., for example, Ref. 14; we neglect the corrections $\sim 1/J^2$). Dividing the right hand side of the first of formulas (37) by (41) we obtain the reduced intensity F measured relative to its purely adiabatic value:

$$F = \frac{\langle\langle Q^2 \rangle\rangle}{\langle\langle Q^2 \rangle\rangle_{K=0}} = \frac{4}{9}\left(1 + \frac{1}{J}\right). \qquad (42)$$

Setting $J = 12$ we have $F_{theor} = 0.48$. According to Ref. 13, $F_{exp} = 0.65 \pm .13$ for the transition $12^+ \to 10^+$, while the next transition $14^+ \to 12^+$ has the intensity $F_{exp} = 0.64 \pm 0.28$.

In conclusion we touch upon the case of odd nuclei. Because it is difficult to sort out the numerous rotational bands until now there are relatively few data concerning them. We know that the principal source of experimental information concerning the problem of interest to us are the quadrupole γ quanta corresponding to transitions with $\Delta J = 2$. Therefore the usual adiabatic "rotational band" characterized by certain K_0 and parity should be separated into two appropriate bands and treated as distinct bands.[5] Results referring to six nuclides and based on the experimental papers[15-17] are given in Table IV. They give no cause for doubting the universal character of the phenomenon.

We express our gratitude to A. I. Baz', I. I. Gurevich, I. M. Pavlichenkov and K. A. Ter-Martirosyan for discussions of questions touched upon in this paper and its results. The authors are particularly grateful to I. M. Pavlichenkov for many stimulating discussions of the nature of experimental data concerning the phenomenon of backbending.

APPENDIX

It appears to be of some interest to analyze certain specific features of zero-temperature phase transitions of the second kind from a more general point of view.

Let the energy E of the ground state of a macroscopic body and the state $\Psi(\xi)$ itself depend on a certain thermodynamic quantity x, and the corresponding condition for stability be represented for the sake of brevity and simplicity in the form

$$d^2E/dx^2 > 0, \qquad (A.1)$$

analogous to (23). We shall assume that for $x < x_c$ a spontaneous breaking of symmetry occurs which is characterized by the ordering parameter η. An analytic continuation of the symmetric n-phase into the region $x < x_c$ is not physically realizable because of

$$d^2E_n/dx^2 \big|_{x=x_c} = 0. \qquad (A.2)$$

Formally this can be only the result of some kind of singularity of the parametric derivatives of the Hamiltonian $\hat{H}(x)$ acting on the variables ξ of the system. In accordance with

$$\hat{H}(x)\Psi_i(\xi;x)=E_i(x)\Psi_i(\xi;x) \qquad (A.3)$$

it determines the eigenfunctions and the energy eigenvalues (the Latin subscript enumerates the stationary states of the system which parametrically depend on x). The simplest imaginable possibility is the following: the singularity in the derivative $\partial\hat{H}/\partial x$ leads to the "break" in the curve of $E(x)$ of the type shown in Fig. 1. Then proceeding in the same manner as in Sec. 2, i.e., actually replacing J formulas (11) and (14)–(18) by x we obtain the following results:

$$\eta=a(x_c-x)/D,$$
$$E_n-E_m=\frac{1}{2}\frac{a^2}{D}(x_c-x),$$
$$\left.\frac{dE_m}{dx}\right|_{x=x_c}-\left.\frac{dE_n}{dx}\right|_{x=x_c}=\frac{1}{2}\frac{a^2}{D}>0. \qquad (A.4)$$

However, we emphasize that so simple a possibility exists only when the following important condition is satisfied: the thermodynamic quantity $dE/dx|_{x=x_0}$ must not reduce to a functional of the wave function $\Psi(\xi;x_0)$. In the opposite case a discontinuity in the functional can only be the result of a discontinuous variation of the function itself. But this would already be a first-order transition and not a second-order phase transition in which we are interested for which we always have

$$\Psi_m(\xi;x_c)=\Psi_n(\xi;x_c). \qquad (A.5)$$

We consider the case when the quantity x has the properties of a dynamic variable (this is possible—cf., for example, Ref. 7—although not necessarily so). In the presence of a velocity $\dot{x}$ an additional kinetic energy proportional to its square appears, and the total energy can be expressed in the form

$$\mathscr{E}=\tfrac{1}{2}B(x)\dot{x}^2+E(x). \qquad (A.6)$$

Here $B(x)$ is a mass coefficient. Suppose that the system is moving in the negative direction along the x axis. At first sight the hypothesis seems plausible that it, generally speaking, will approach the point x_c with a certain finite velocity $\dot{x}_n$. But then in the relation

$$\dot{\eta}=\gamma\dot{x} \qquad (A.7)$$

the condition

$$\gamma=0, \quad x\geqslant x_c \qquad (A.8)$$

must hold also for $x=x_c$: the domain of the more symmetric n-phase extends up to the point of phase transition inclusively. [9] Having all this in mind we apply the law of conservation of energy directly at the Curie point:

$$\tfrac{1}{2}B_m\dot{x}_m^2+E_m=\tfrac{1}{2}B_n\dot{x}_n^2+E_n, \quad x=x_c. \qquad (A.9)$$

In accordance with the second of formulas (A.4) (cf., also (A.5)), we ignore the "potential energy" $E(x)$. The system cannot pass to the left with a finite velocity $\dot{x}_m$: differentiation of the first of formulas (A.4) shows that in violation of condition (A.8) at $x=x_c$ the coefficient $\gamma(x)$ would change discontinuously from zero to infinity. Thus, $\dot{x}_m=0$ within the framework of the foregoing assumption and, consequently,

$$B_n(x_c)=0. \qquad (A.10)$$

Switching on adiabatically smoothly at $t\to-\infty$ the variation $x(t)$ of the coordinate and solving the time-dependent Schrödinger equation with the Hamiltonian $\hat{H}(x(t))$ we obtain the well-known expression

$$B_n(x)=2\hbar^2\sum_{i\neq n}\frac{1}{E_i-E_n}\left|\left\langle\Psi_i\left|\frac{\partial\Psi_n}{\partial x}\right\rangle\right|^2 \qquad (A.11)$$

for the mass coefficient (in order to ascribe to the notation a more concrete character we denote by the index n the ground states $i=0$ within the domain of the n-phase $x>x_c$). Since all the denominators are positive substitution into (A.10) yields

$$\left\langle\Psi_i\left|\frac{\partial\Psi_n}{\partial x}\right\rangle=0, \quad x=x_c. \qquad (A.12)$$

Regarding the amplitude $\langle\Psi_n|\partial\Psi_n/\partial x\rangle$, it as a result of normalization turns out to be purely imaginary and can be everywhere made equal to zero with an appropriate choice of the indefinite phase multiplier $e^{i\alpha(x)}$ in the wave function of the ground state. Then all the components of the derivative $\partial\Psi_n/\partial x$ along the basis vectors of the Hilbert space turn out to have zero values at the Curie point

$$\partial\Psi_n/\partial x|_{x=x_c}=0. \qquad (A.13)$$

Expanding the parametric derivative $\partial\Psi_n/\partial x$ in series and now measuring the coordinate x from the position of the point of phase transition we represent the mass coefficient of the n-phase in the form

$$B_n(x)=8bx^2, \qquad (A.14)$$

where

$$b=\frac{\hbar^2}{4}\sum_{i\neq n}\frac{1}{E_i-E_n}\left|\left\langle\Psi_i\left|\frac{\partial^2\Psi_n}{\partial x^2}\right\rangle\right|^2. \qquad (A.15)$$

For the solution of equations of classical mechanics it is simplest of all to make use of their first integral (A.6)

$$x(t)=\begin{cases}(\mathscr{E}/b)^{1/4}(\pm t)^{1/2} & \text{if} \quad x>0\\ \pm(2\mathscr{E}/B_m)^{1/2}t & \text{if} \quad x<0\end{cases}. \qquad (A.16)$$

Here the time is so reckoned that at the instant $t=0$ the system would be passing through the point $x=0$ of the phase transition and the energy $\mathscr{E}$ is measured from the "potential energy" $E(0)$. The upper sign corresponds to motion in the positive direction of the x axis and the lower sign to motion in the negative direction.

V. G. Nosov and A. M. Kamchatnov

By B_m we have denoted the limiting value of the mass coefficient of the m-phase as $x \to -0$. The singularity which the solution of (A.16) in fact has as $x \to +0$ deprives the proportionality relationship (A.7) of a strict mathematical interpretation directly at the origin. By this the paradox is removed concerning the simultaneous retention both of the law of conservation of energy and the nature of the phase transition as the system passes through the Curie point.

Strictly speaking the states that have been considered are not entirely equilibrium states, since the macroscopic motion is accompanied by inevitable friction. In the general case in order to achieve complete equilibrium it is necessary to add to the Hamiltonian a term of the form $-\lambda x$ which does not affect the nature of the phase transition. Equilibrium at $x = x_0$ corresponds to $\lambda = dE/dx|_{x=x_0}$. If the zero-temperature second-order phase transitions have to a large extent a common nature, then the result (A.13) must not depend on whether the quantity x does in fact have the properties of an autonomous dynamical variable. Indeed, it is sufficient to require complete thermodynamic equilibrium—"rest," in order that the abstract capability in principle of the system to be "in motion" would in no way manifest itself physically. From this point of view the condition (A.13) imposed on the wave function of the n-phase at the Curie point appears to be of a sufficiently general nature.

We now return to the dynamic case. The solution (A.16) was classical. The question arises whether the limiting expansion (A.14) for the variation of the mass coefficient of the n-phase might not lose its concrete physical content when quantum effects are taken into account. In order to check this, we turn to the indeterminacy relation

$$\Delta \mathscr{E} \Delta x \sim \hbar \dot{x} \tag{A.17}$$

(cf., for example, Ref. 1). According to (A.16) we have

$$\dot{x} = \pm (\mathscr{E}/b)^{1/2}/2x,$$

i. e.,

$$\Delta x \sim \left(\frac{\mathscr{E}}{b}\right)^{1/2} \frac{\hbar}{x} \frac{1}{\Delta \mathscr{E}} \gg \frac{\hbar}{(b\mathscr{E})^{1/2}} \frac{1}{x}. \tag{A.18}$$

From this it can be seen that the requirement $\Delta x \ll x$ cannot be satisfied only for

$$x \lesssim (\hbar^2/b\mathscr{E})^{1/4} \to 0 \quad \text{as} \quad \mathscr{E} \to \infty. \tag{A.19}$$

Thus, there exists a possibility in principle to check by a purely classical method the validity of the expression (A.14) down to the smallest values of x.

In the foregoing friction accompanying motion was not taken into account explicitly. Generally speaking, the question concerning dissipative processes is quite complicated and requires a more concrete investigation in each individual case. However, in the region wher the n-phase is sufficiently close to the point of

phase transition the situation is simplified. Indeed, projecting the time-dependent Schrödinger equation onto the base vector system defined by (A.3), we establish that the connection between the amplitudes of states with different i is realized exclusively by means of coefficients which in addition to the velocity $\dot{x}$ contain matrix elements of the form $\langle \Psi_k | \partial \Psi_i / \partial x \rangle$. They must also be responsible for the irreversible quantum jumps from level to level which are the cause of dissipation. But, as is clear from (A.13), such matrix elements vanish at the Curie point and near it friction is suppressed. Therefore results which concern the limiting behavior of the n-phase in the neighborhood of the phase transition point remain valid.

Finally, we briefly touch upon the relation of the foregoing to the special case $x = J$ of nuclear rotation considered in the present paper. The complete wave function of a nonspherical nucleus can be written as a product

$$\Psi_{J_0 J_z}(J, M, \xi) = \delta_{JJ_0} \delta_{MJ_z} \psi_{J_0}(\xi). \tag{A.20}$$

By ξ we have denoted variables commuting with J and M and with one another (among their number is contained also the macroscopic quantity $K = \mathbf{J} \cdot \mathbf{n}$). The wave functions $\Psi_J(\xi)$ depend on J as on a parameter. They obey an equation of the form (A.3) in which the role of the "Hamiltonian" is played by the block diagonal with respect to J and M of the complete Hamiltonian for the nucleus (parametrically dependent on J), while rotational energy levels serve as eigenvalues. Since the quantity $\hbar\Omega_0 = dE/dJ|_{J=J_0}$ does not reduce to a functional of $\Psi_{J_0}(\xi)$, the rotational velocity Ω can undergo a discontinuity at the transition point as the energy itself $E(J)$ undergoes a continuous variation along the band, and this is in fact observed. The discontinuous diminution of Ω signifies in essence a change in the "coupling scheme" of angular momenta in the system (cf., Introduction).

---

[1] This is due to the external appearance of graphs that illustrate the position of rotational levels of the band and constructed in terms of somewhat artificially chosen coordinates. In the present work we shall not make use of such coordinates.

[2] It is not difficult to verify that negative moments of inertia correspond to absolute instability both thermodynamic and purely mechanical. The latter in the present case has the following meaning: free rotation of a body corresponds, as is well known, to thermodynamic equilibrium and is not accompanied by friction (cf., for example, Refs. 9 and 10). If the regular precession with respect to the angle φ is treated in accordance with the laws of classical mechanics (cf. the Introduction, formula (8) and the text referring to it), then for negative moments of inertia the principle of least action is violated.

[3] Since the operator for collective quadrupole moment is diagonal with respect to K the correlations mentioned at end of the Introduction do not affect the result.

[4] An analogous interpretation can also be given to the increase in the moment of inertia of the n-phase on the right wing of its curve (cf., Fig. 2)—only here the system resists the lining up which begins to acquire a macroscopic scale of the mechanical angular momenta of the individual quasiparticles along the direction of the vector J (cf., the Introduction).

V. G. Nosov and A. M. Kamchatnov   419

This is what explains the existence of the minimum in the variation of the moment of inertia of the n-phase which will be discussed subsequently.

5) The division into two "sub-bands" is equally well-founded also from a purely theoretical point of view. Indeed, one does not expect any considerable differences in principle from even-even nuclei in the present case, while it is desirable to exclude the not entirely clear and nonmacroscopic effect of the "lining up" of the odd nucleon. It is well known that for $K_0 = \frac{1}{2}$ a similar effect occurs even in the adiabatic region where it is more easily susceptible of being investigated (cf., for example, Ref. 1).

[1] L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika (Quantum Mechanics) Nauka, 1974.

[2] L. D. Landau and Ya. Smorodinskiĭ, Lektsii po teorii atomnogo yadra (Lectures on the Theory of Atomic Nuclei), Gostekhizdat, 1955.

[3] A. Johnson, H. Ryde, and J. Starkier, Phys. Lett. 34B, 605 (1971).

[4] J. Krumlinde, Nukleonika 19, 251 (1974).

[5] A. Johnson and S. Szymanski, Phys. Rep. 7C, 181 (1973).

[6] F. S. Stephens, Rev. Mod. Phys. 47, 43 (1975).

[7] V. G. Nosov, Zh. Eksp. Teor. Fiz. 53, 579 (1967) [Sov. Phys. JETP 26, 375 (1968)].

[8] L. D. Landau and E. M. Lifshitz, Mekhanika (Mechanics), Nauka, 1973.

[9] L. D. Landau and E. M. Lifshitz, Statisticheskaya fizika (Statistical Physics), 1976.

[10] L. D. Landau and E. M. Lifshitz, Mekhanika sploshnykh sred (Mechanics of Continuous Media), Gostekhizdat, 1954.

[11] R. O. Sayer, J. S. Smith, and W. J. Milner, Atomic Data and Nuclear Data Tables 15, 85 (1975).

[12] A. Bohr and B. Mottelson, The Structure of Atomic Nuclei (Russ. Transl., "Mir," 1971).

[13] D. Husar, J. S. Mills, H. Gräf, U. Neumann, D. Pelte, and J. C. Merdinger, Phys. Rev. Lett. 36, 1291 (1976).

[14] V. B. Berestetskiĭ, E. M. Lifshitz, and L. P. Pitaevskiĭ, Relyativistskaya kvantovaya teoriya (Relativistic Quantum Theory), part 1, Nauka, 1968 [Pergamon, 1971].

[15] E. Grosse, F. S. Stephens, and R. M. Diamond, Phys. Rev. Lett. 31, 840 (1973); 32, 74 (1974).

[16] L. L. Riedinger, G. J. Smith, P. H. Stelson, E. Eicher, G. B. Hagemann, D. C. Hensley, N. R. Johnson, R. L. Robinson, and R. O. Sayer, Phys. Rev. Lett. 33, 1346 (1974).

[17] C. Foin and D. Barnéoud, Phys. Rev. Lett. 33, 1049 (1974).

Translated by G. Volkoff

Possibility of studying the structure of weak neutral currents in optical transitions in heavy atoms

V. N. Novikov, O. P. Sushkov, V. V. Flambaum, and I. B. Khriplovich

Institute of Nuclear Physics, Siberian Division, USSR Academy of Sciences

Effects of parity nonconservation, caused by the weak interaction of neutral electronic vector and nucleonic axial currents, are examined in optical transitions in thallium, lead, bismuth, and cesium. Degrees of circular polarization of the light are calculated for ordinary and strongly forbidden $M1$ transitions.

PACS numbers: 31.90.+s, 12.30.Cx

1. Experiments on parity nonconservation effects in heavy atoms (first discussed in[11]) are now being conducted by several research groups.[2-6] From our point of view, the most promising of these experiments are those on the rotation of the plane of polarization of light from heavy-metal vapors, as proposed in[7-9]. They have already given a significant limit on the constant for the weak interaction of the neutral vector current (VC) of nucleons with the axial-vector current (AC) of electrons. Parity nonconservation effects in heavy atoms can also be produced by another type of weak interaction, that between nucleonic AC and electronic VC. The investigation of this interaction would not be unimportant even if there were no interaction of electronic AC with nucleonic VC (which is at present the most popular interpretation of the preliminary experimental results). In fact, the absence of an axial part in the neutral electronic current by no means indicates that it is also absent for nucleons. Moreover, neutrino experiments evidently show that the neutral hadronic current contains both a vector and an axial part.

We have previously pointed out[10,11] the possibility of studying the interaction of the nucleonic AC and the electronic VC via the measurement of the angle of rotation of the plane of polarization of the light at frequencies corresponding to optical transitions between individual hyperfine components of levels in heavy atoms. A complicating feature of the observation of this interaction is that its size is smaller than that of the interaction between the nuclear VC and the electronic AC by roughly a factor Z, the nuclear charge, since only the valence nucleon contributes to the nucleonic AC. However, the accuracies attainable in experiments on the optical activity of heavy-metal vapors are already close to that needed to detect the effect in question. In this paper we make a detailed calculation of the size of the effect. In addition to this, we find the contribution of this interaction to the circular polarization of the radiation emitted in strongly forbidden $M1$ transitions.