TABLE II. Temperature dependence of the degree of polarization $\rho_L$ for an exciting beam directed along [100].

| $\varphi$, deg | $T$, K | $\rho_{\text{Л}}$, % (calculation) I | II | III | $\rho_L$, % (experiment) |
|---|---|---|---|---|---|
| 0 | 100 | 4.7 | 3.6 | 3.2 | 1.5±1.0 |
| | 300 | 7.4 | 6.3 | 5.6 | 2.0±1.0 |
| | 400 | 7.8 | 6.8 | 6.1 | 4.5±1.0 |
| | $10^5$ | 10.5 | – | – | – |
| 45 | 100 | 21.4 | 22.3 | 22.6 | 24.0 |
| | 300 | 19.5 | 20.3 | 20.8 | 22.0 |
| | 400 | 19.1 | 19.9 | 20.4 | 20.0 |
| | $10^5$ | 17.1 | – | – | – |
| Values of parameters used in the calculation. | | | | | |
| | $m_c/m_0$ | 0.0655 | 0.11 | 0.0665 | – |
| | $\gamma$ | 0.85 | 0.85 | 1.23 | – |

*Note.* In the calculations we used the values $\gamma = 0.85$, $\gamma_1 = 7.65$, $|\gamma_2| = 2.41$, $\gamma_3 = 3.28$[6].

trons due to the non-parabolicity of the conduction band (see, e.g., [7]). To illustrate this, column II to Table II shows the results of the calculations $m_c = 0.1m_0$. A noticeable increase of $\gamma$ (Table II, column III) does not improve significantly the agreement between the calculated and experimental data at $\varphi = 0$. The reason for some discrepancy between them still remains unclear.

In the high-temperature limit, $T \gg \Delta\mathscr{E}_{vh}$, the populations for the different directions become equalized. However, as shown by the corresponding calculations (see the data for $T = 10^5$ °K in Table II), the degrees of polarization for different excitation directions still differ noticeably. This "residual" anisotropy is obviously due to the anisotropy of the matrix elements for the interband transitions and leads to values of $\rho_L$ equal to 0.105 and 0.17 for 0 and 45°, respectively.

We note in conclusion that the experimental results obtained in this paper serve as a direct confirmation of the correctness of the interpretation proposed in[3] for the polarization effects in the hot-photoluminescent spectrum.[1]

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# Modulational instability of magnetohydrodynamic waves in a plasma

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We study the interaction between magnetohydrodynamic waves and a plasma with $\beta < 1$. We show that this leads to a modulational instability for the fast magnetosonic waves at a well defined level of oscillation energy. As a result of the development of this instability the particle density and wave energy density start to increase in a certain region. The modulational instability of a beam of almost parallel waves can be stabilized in the weakly non-linear stage and a wave channel is then formed with an increased plasma density to which the waves are confined due to refraction. In the case of an isotropic wave distribution the compression of the plasma may proceed until the energy of the oscillations and of the particles becomes comparable with the energy of the stationary magnetic field. We discuss the possibility of observing the modulational instability of fast magnetosonic waves under natural conditions.

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Modulational instabilities play an important role in the formation of plasma turbulence. They affect the spectrum of the turbulent oscillations, the acceleration and heating of particles, the emission of electromagnetic waves, and so on. It is of interest in that connection to consider the possibility of the development of the modulational instability of magneto-hydrodynamic waves, since MHD turbulence is widespread in cosmic and laboratory plasmas. The modulational instability of MHD waves must qualitatively differ strongly from the modulational instability of Langmuir waves which has been studied in much more detail.[1–4]

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In the first place this difference is connected with the fact that the spatial dispersion of the frequencies is linear.

One of the peculiarities of the modulational instability of MHD waves has a rather trivial nature. It refers to Alfvén waves and beams of parallel MHD waves of other types. In a system of coordinates moving with the wave velocity, the dispersion disappears almost completely and even weak non-linear effects can cause their modulation. Longitudinal modulations of this nature move with the waves and are responsible for the formation of shock waves, discontinuities, and solitary waves. Transverse modulations lead to the self-focusing of waves.

Somewhat unexpected is the fact that, as we shall show below, in one of the MHD branches—that of the fast magnetosonic oscillations—the modulational instability can develop also for an isotropic spectrum. This instability is accompanied by an increase in the plasma density and of the wave energy. If the waves are sufficiently strongly mixed in directions, for instance, due to scattering, the process may lead to a very high concentration of oscillation energy.

We study in the present paper the modulational instability of fast magnetosonic waves in a low density plasma. We determine in Sec. 1 the ponderomotive forces averaged over the first oscillations which act on the plasma particles as a result of the MHD waves. We calculate in Sec. 2 the growth rates of the modulational instability in the geometric optics approximation. Section 3 is devoted to a study of the transverse modulation of a monochromatic fast magnetosonic wave. We give in Sec. 4 qualitative discussions of the development of the instability for the case of an isotropic oscillation spectrum, taking into account the induced scattering of the waves. We discuss in the Conclusion possible applications of the theory.

## 1. INTERACTION BETWEEN MAGNETO-HYDRODYNAMIC WAVES AND PLASMA PARTICLES

In what follows in this paper we deal with a two-component isothermal plasma with a strong stationary uniform magnetic field $\mathbf{H}_0$ and an oscillating field of the MHD waves—the electric field $\mathbf{E} = -\dot{\mathbf{A}}/c$ and the magnetic field $\mathbf{H} = \operatorname{curl}\mathbf{A}$. We assume everywhere that the energy of the stationary magnetic field is much larger than the kinetic energy of the particles and than the wave energy:

$$\beta = 8\pi\rho T/H_0^2 \ll 1, \quad H/H_0 \ll 1; \tag{1}$$

here $\rho$ is the ion density and $T$ the plasma temperature. We also assume that the longitudinal conductivity is sufficiently large so that $\mathbf{E}$ and $\mathbf{A}$ are at right angles to $\mathbf{H}_0$.

Three kinds of MHD waves can propagate in a magnetized isothermal plasma. These are sound waves, Alfvén waves and fast magnetosonic (FMS) waves. We consider only waves of the last two types. Their velocity is equal to the Alfvén speed $c_A = (H_0^2/8\pi\rho m_i)^{1/2}$.

If condition (1) is satisfied, the Alfvén speed is much higher than the velocity of the plasma motion. We can split the motion of the plasma into two parts: high-frequency drift pulsations at right angles to the magnetic field $\mathbf{H}_0$ and low-frequency motions along $\mathbf{H}_0$ under the action of the pressure of the waves.

The transverse motion of the particles is caused by the electric field of the waves $\mathbf{E}$. To a first approximation the electric drift does not create an electric current as electrons and ions move together. A current appears only in the next approximation in the ratio of the wave frequency to the ion cyclotron frequency $\omega/\omega_{hi}$ due to the difference in the electron and ion inertia. Up to small terms of the order of $\omega/\omega_{hi}$, $v_{Ti}/c_A$, $H/H_0$ the current density is equal to[1)]

$$\mathbf{j} = \frac{\partial}{\partial t}(\dot{\mathbf{A}}/4\pi c_A^2). \tag{2}$$

Substituting (2) in the Maxwell equations we get

$$\operatorname{rot}_\perp \operatorname{rot}\mathbf{A} = \frac{\partial}{\partial t}\frac{1}{c_A^2}\frac{\partial}{\partial t}\mathbf{A}. \tag{3}$$

The symbol $\operatorname{rot}_\perp$ indicates here the part of the curl which is at right angles to $\mathbf{H}_0$.

Equation (3) corresponds to a Lagrangian density

$$\mathscr{L} = -\frac{1}{8\pi}\left[(\operatorname{rot}\mathbf{A})^2 - \frac{1}{c_A^2}\dot{\mathbf{A}}^2\right] \tag{4}$$

and an energy density

$$w = \frac{1}{8\pi}\left[(\operatorname{rot}\mathbf{A})^2 + \frac{1}{c_A^2}\dot{\mathbf{A}}^2\right]. \tag{5}$$

The second term in (4) is proportional to the particle density. It is clear that it can be interpreted as the Lagrangian of the interaction of the ions with the wave field. Starting from this we find that the ponderomotive force acting upon the ions is given by a effective potential

$$U = -\frac{\dot{\mathbf{A}}^2}{8\pi\rho c_A^2} = -\frac{m_i\dot{\mathbf{A}}^2}{2H_0^2} = -\frac{w}{\rho}. \tag{6}$$

The density of the force component in the direction of the stationary magnetic field

$$f = -\rho\nabla_{H_0}U = \frac{\rho}{2}\nabla_{H_0}\frac{w}{\rho} \tag{7}$$

determines the slow plasma displacement.

## 2. GEOMETRIC OPTICS APPROXIMATION

In this section we consider the evolution of inhomogeneities in the plasma density with sizes $l$ which are much larger than the characteristic wavelength $\lambda$ of the oscillations. We can describe the propagation of the waves in the geometric optics approximation.

If there occurs an inhomogeneity in the particle density in the plasma, there arises a redistribution of the waves due to the change in their velocity; the wave energy density $w$ becomes inhomogeneous and the force (7) begins to act upon the plasma. It follows from (7) that when $w$ increases faster than $\rho$ this force is directed in the direction of the density increase so that the initial state turns out to be unstable.

Alfvên waves propagate only along $\mathbf{H}_0$. Therefore, we have for them

$$w \propto 1/c_A \propto \rho^{1/2}, \quad w/\rho \propto \rho^{-1/2}. \tag{8}$$

Hence, a uniform distribution of Alfvên waves is stable and their interaction with the plasma leads to the possible appearance of second-sound type oscillations.

The situation changes for FMS waves, which can propagate in all directions. The wave propagation proceeds almost in stationary conditions as their velocity is appreciably larger than the plasma velocity. It is well known that in that case the density of waves in momentum (wavevector) and coordinate phase space must be constant along a trajectory. When the wave velocity changes the momentum volume occupied by them changes $d^3k \propto 1/c_A^3$. If the waves are at all points distributed more or less isotropically their density changes as $1/c_A^3 \propto \rho^{3/2}$. From this it follows that a uniform, isotropic distribution of FMS waves must be unstable with respect to modulations.

We find the frequency dispersion law for modulations of Alfvên and FMS waves. To do this we need a set of equations which determine simultaneously the motion of the waves and of the particles. We shall describe the waves by the kinetic equation

$$\frac{\partial n_{\mathbf{k}}}{\partial t}+\frac{\partial \omega_{\mathbf{k}}}{\partial \mathbf{k}}\frac{\partial n_{\mathbf{k}}}{\partial \mathbf{r}}-\frac{\partial \omega_{\mathbf{k}}}{\partial \mathbf{r}}\frac{\partial n_{\mathbf{k}}}{\partial \mathbf{k}}=0, \tag{9}$$

where $n_{\mathbf{k}}$ is the spectral density of the waves, while $\omega_{\mathbf{k}} = c_A k_{\parallel}$, respectively, for Alfvên and FMS waves. To describe the motion of the plasma along $\mathbf{H}_0$ we can use the one-dimensional hydrodynamic equations

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}=-\frac{1}{m_i\rho}\frac{\partial P}{\partial x}+\frac{1}{2m_i}\frac{\partial}{\partial x}\frac{w}{\rho}, \quad \frac{\partial}{\partial x}v\rho=-\frac{\partial \rho}{\partial t}. \tag{10}$$

To close the set (9), (10) we still must give the connection between kinetic pressure $P$ and the density. For the sake of simplicity we shall assume that

$$P \sim \rho^{\alpha}. \tag{11}$$

We consider small perturbations of the density of the waves and of the particles, which are proportional to $\exp[i(\mathbf{q}\cdot\mathbf{r}-\Omega t)]$ on a background of a distribution $n_{\mathbf{k}}$, $\rho$ which is uniform is space. Linearizing (9) to (11) we find

$$\left(\frac{\Omega}{q_x}\right)^2=\frac{1}{m_i}\left(\alpha T+\frac{w}{2\rho}-\frac{1}{4\rho}\sum_{\mathbf{k}}\frac{\omega_{\mathbf{k}}}{\Omega-\mathbf{q}\mathbf{u}_{\mathbf{k}}}\mathbf{q}\frac{\partial w_{\mathbf{k}}}{\partial \mathbf{k}}\right), \tag{12}$$

where $\mathbf{u}_{\mathbf{k}} = \partial\omega_{\mathbf{k}}/\partial\mathbf{k}$ is the group velocity and $w_{\mathbf{k}} = \omega_{\mathbf{k}} n_{\mathbf{k}}$ the spectral density of the wave energy.

Assuming that $\Omega \ll \mathbf{q}\cdot\mathbf{u}_{\mathbf{k}}$ we can obtain from (12) the dispersion equation for modulations for a well defined spectral composition of the waves. In the case of Alfvên waves it takes the form

$$\left(\frac{\Omega}{q_x}\right)^2=\frac{1}{m_i}\left(\alpha T+\frac{w}{4\rho}\right). \tag{13}$$

For FMS waves the dispersion equation depends strongly on their angular distribution. For an isotropic spectrum we have

$$\left(\frac{\Omega}{q_x}\right)^2=\frac{1}{m_i}\left(\alpha T-\frac{w}{4\rho}\right), \tag{14}$$

whence it follows that a uniform isotropic field of FMS waves is unstable, if

$$w>4\alpha\rho T. \tag{15}$$

For a beam of FMS waves with an angular spread $\delta \ll 1$ we get

$$\left(\frac{\Omega}{q_x}\right)^2=\frac{1}{m_i}\left(\alpha T+\frac{w}{4\rho\cos^2\beta}\right), \tag{16}$$

if $|1/2-\beta|>\delta$, and

$$\left(\frac{\Omega}{q_x}\right)^2=\frac{1}{m_i}\left(\alpha T-\frac{w}{4\delta^2\rho}\right), \tag{17}$$

if $|1/2\pi-\beta|<\delta$. In Eqs. (16), (17) $\beta$ is the angle between $\mathbf{H}_0$ and the beam axis. A beam of FMS waves is unstable with respect to transverse modulation when

$$w>4\delta^2\alpha\rho T. \tag{18}$$

## 3. TRANSVERSE MODULATION OF A MONOCHROMATIC FMS WAVE

It is clear from (18) that the modulation of a beam of almost parallel waves occurs even when the energy density of the waves is considerably lower than the thermal one. In that case the process can proceed for the case of a relatively small non-linearity. We consider the propagation of a monochromatic weakly non-linear FMS wave in the plasma. Let the wave propagate along the $z$-axis and let the stationary magnetic field lie in the $yz$-plane at an angle $\vartheta$ to the $z$ axis. We write the field of the wave in the form

$$\mathbf{A}(\mathbf{r},t)=\frac{1}{\sqrt{2}}\left[\mathbf{A}_k(\mathbf{r})e^{i(kz-\omega t)}+\text{c. c.}\right], \tag{19}$$

where $\mathbf{A}_k(\mathbf{r})$ is a slowly varying function of the coordinates such that $\mathbf{A}_k(\mathbf{r})\to 0$ as $r_\perp\to\infty$.

The plasma density is given by the Boltzmann distribution

$$\rho(\mathbf{r})=\rho_0\exp\left(m_i\langle\dot{\mathbf{A}}^2\rangle/2TH_0^2\right). \tag{20}$$

The pointed brackets indicate averages over the fast oscillations. Together with (3) Eq. (20) leads to the equation

$$\text{rot}_\perp\,\text{rot}\,\mathbf{A}=\frac{1}{c_{A0}^2}\frac{\partial}{\partial t}\exp\left(\frac{m_i\langle\dot{\mathbf{A}}^2\rangle}{2TH_0^2}\right)\frac{\partial}{\partial t}\mathbf{A}. \tag{21}$$

This equation corresponds to the variational principle $\delta\int\mathscr{L}d^3r=0$ with

$$\mathscr{L}=-\frac{1}{8\pi}\left[(\text{rot}\,\mathbf{A})^2-\frac{2TH_0^2}{m_ic_{A0}^2}\exp\left(\frac{m_i\langle\dot{\mathbf{A}}^2\rangle}{2TH_0^2}\right)\right]. \tag{22}$$

In a uniform plasma the vector potential of the FMA waves satisfies the condition

$$\text{div}\,\mathbf{A}=0. \tag{23}$$

Within the accuracy sufficient for our needs this condition is also satisfied in the case considered. We use it to find the connection between the components of the vector potential. As the vector potential is perpendicular to $\mathbf{H}_0$ and satisfies (23), it is mainly directed along the $x$ axis, provided the angle $\vartheta$ is not too small. Apart from small corrections of the order of $\lambda/l$, where $\lambda$ is the wavelength and $l$ the size of the modulation, we get from (23)

$$A_{kz}\approx\frac{i}{k}\frac{\partial A_{kx}}{\partial x},\quad A_{ky}\approx\frac{i}{k}\frac{\partial A_{kx}}{\partial x}\,\text{tg}\,\vartheta. \tag{24}$$

We introduce a new variable

$$\psi=\left(\frac{m_i\omega^2}{2TH_0^2}\right)^{1/2}\left(1+\frac{1}{k\cos\vartheta}\frac{\partial}{\partial x}\right)A_{kx}. \tag{25}$$

We choose as the unit of length along $z$ the quanity $kc_{A0}^2/\omega^2$, and in the transverse direction $c_{A0}/\omega$. We assume further that we are considering a wave with a modulation along the $z$ axis which is much weaker than the transverse modulation, so that we can neglect terms containing second derivatives of $\psi$ with respect to $z$.

Substituting (24), (25) into (22) and dropping divergent and small terms we get with an accuracy $O(|\psi|^6)$

$$\begin{aligned}\mathscr{L}&=\mathscr{L}_z+\mathscr{L}_1+\mathscr{L}_2,\\ \mathscr{L}_z&=2i\mu\psi^*\frac{\partial}{\partial z}\psi,\\ \mathscr{L}_1&=-\mu\psi^*(\xi^2-\nabla_\perp^2-{}^1/{}_2|\psi|^2)\psi,\\ \mathscr{L}_2&=\mu\psi^*\left(\frac{1}{2\cos^2\vartheta}\frac{\partial^2|\psi|^2}{\partial x^2}+\frac{1}{6}|\psi|^4\right)\psi,\end{aligned} \tag{26}$$

where $\mu=\omega^2/8\pi c_{A0}^2$ while $\xi^2=c_{A0}^2k^2/\omega^2-1\ll1$. When the assumption of weak non-linearity of the wave is satisfied the part $_2$ of the Lagrangian is small compared to $\mathscr{L}_1$. We retain it nonetheless as it will become clear from what follows that the corresponding small corrections appreciably affect the behavior of the wave.

From (26) we get an equation for the field

$$i\frac{\partial\psi}{\partial z}=\frac{1}{2}\left(\xi^2-\nabla_\perp^2-|\psi|^2-\frac{1}{2\cos^2\vartheta}\frac{\partial^2|\psi|^2}{\partial x^2}-\frac{1}{3}|\psi|^4\right)\psi. \tag{27}$$

Equation such as (27) rather often occur in problems about the propagation of waves in a non-linear medium so that it will be pertinent here to analyze its solution in more detail.

Beforehand we consider the solution of the truncated equation

$$i\partial\psi/\partial z={}^1/{}_2(\xi^2-\nabla_\perp^2-|\psi|^2)\psi. \tag{28}$$

It is obtained from (27) by dropping the small non-linear terms corresponding to $\mathscr{L}_2$.

Equation (28) has two integrals which are conserved along the $z$-axis:

$$I=\int|\psi|^2d^2r, \tag{29}$$

$$I_1^0=-\int\psi^*(\nabla_\perp^2+\frac{1}{2}|\psi|^2)\psi d^2r. \tag{30}$$

Here and henceforth in this section we understand by $\mathbf{r}$ that part of the radius vector which is at right angles to the $z$ axis.

We define the square of the width of the wave by the relation

$$D=\frac{1}{I}\int|\psi|^2r^2d^2r. \tag{31}$$

Using (28) we find

$$Id^2D/dz^2=4I_1^0, \tag{32}$$

whence

$$D=D_0+D_0'z+4\frac{I_1^0}{I}z^2. \tag{33}$$

It follows from (33) that the behavior of the solutions of Eq. (28) depends strongly on $I_1^0$. If $I_1^0>0$ the wave widens without bounds with increasing $z$. If, however $I_1^0<0$, it contracts and for finite $z$ the quantity $D$ would become negative which, clearly, means the appearance of a non-integrable singularity in $|\psi|^2$.

Solutions of Eq. (27) which are independent of $z$ (stationary) are possible only when $I_1^0=0$. Stationary solutions $\psi_\xi$ corresponding to different values of $\xi$ are connected through a similarity transformation so that we can express them in terms of $\psi_1$, the solution for $\xi=1$:

$$\psi_\xi(z)=\xi\psi_1(\xi r). \tag{34}$$

Hence it follows that in the stationary case the integral $I$, like $I_1^0$, is independent of $\xi$. It is just because of this degeneracy that the small non-linear corrections in Eq. (27) may appreciably change the nature of the solutions.[2)]

For Eq. (27) Eqs. (30), (32) are replaced by

$$\begin{aligned}I_1&=I_1^0+I_1^1,\\ I_1^1&=-\frac{1}{2}\int\psi^*\left(\frac{1}{\cos^2\vartheta}\frac{\partial^2|\psi|^2}{\partial x^2}+\frac{1}{3}|\psi|^4\right)\psi d^2r\end{aligned} \tag{35}$$

and

$$Id^2D/dz^2=4(I_1+I_1^1). \tag{36}$$

We consider the change of the solution of Eq. (27) along the $z$ axis. Let $I_1<0$, and the width of the wave be sufficiently large: $D\gg|I/I_1|$. The right-hand side of (36) is then negative and the wave contracts as $z$ increases. The field amplitude then increases: $|\psi|^2\sim I/D$ and at the same time the absolute magnitude of the second term in

(36) increases while the first term stays constant. The contribution of $I_1^1$ to the integral $I_1$ consists of two parts. The first of those is positive-definite and the second negative-definite. If the first one dominates, ultimately the whole right-hand part of (36) can become positive and the contraction of the wave stops. This occurs when $D \sim I/I_1$ and if $|I/I_1| \ll 1$ the wave will still be weakly non-linear.

The small non-linear corrections to Eq. (27) can thus prevent the collapse of the wave. A consideration similar to the one just given shows that stable stationary weakly non-linear solutions of Eq. (27) can exist.

We shall look for stationary solutions of Eq. (27) in the form $\psi_\xi = \psi_\xi^0 + \delta\psi_\xi$, where $\psi_\xi^0$ is a stationary solution of Eq. (28) and $\delta\psi_\xi$ a small correction. It follows from (36) that

$$I_1 = -I_1^1. \tag{37}$$

Hence we find

$$I_1 \approx -\xi^4 (a/\cos^2\vartheta - b), \tag{38}$$

where

$$a = \frac{1}{2}\int \left(\frac{\partial |\psi_1^0|^2}{\partial x}\right)^2 d^2r, \quad b = \frac{1}{6}\int |\psi_1^0|^6 d^2r. \tag{39}$$

Comparing (35) and (37) we get

$$I_1^0(\psi_\xi^0 + \delta\psi_\xi) \approx -2I_1^1(\psi_\xi^0). \tag{40}$$

The functional $I_1^0$ vanishes when $\psi \equiv \psi_\xi^0$. By virtue of (28) its variation also vanishes when $I = \text{const}$. Therefore only remains on the left-hand side of (40) the variation

$$\delta I_1^0 = \frac{\delta I_1^0}{\delta I}\delta I = -\frac{\delta I}{I}\int \psi^*(\nabla_\perp^2 + |\psi|^2)\psi d^2r = -\xi^2 \delta I. \tag{41}$$

One can easily check the last equation using (28) and (30). From (38), (40), (41) we get

$$I = I^0 + \delta I = I^0 + 2\xi^2 (a/\cos^2\vartheta - b), \quad I^0 = I(\psi_\xi^0). \tag{42}$$

It is clear from (38), (42) that the small corrections in Eq. (28) lift the degeneracy and the integrals $I$, $I_1$ begin to depend on $\xi$. When $\xi^2 \ll 1$ this dependence is determined by the three constants $I^0$, $a$, $b$ which can be found by numerically integrating Eq. (28). Some of their smallest values obtained for solutions of the form $\psi(r) = \psi_m(r)e^{im\varphi}$ are given in Table I.

In order that the stationary solutions of Eq. (27) are stable, the integral $I_1$ must be negative for them, and the first term in $I_1^1$ must dominate over the second one. It follows from (38) that these conditions are satisfied *simultaneously*, if $a > b\cos^2\vartheta$. For those states which are given in the table axially symmetric waves ($m = 0$) are stable for all angles of propagation for which Eq. (27) is applicable. For other $m$ there occur regions of instability. The interaction of the plasma with the field of a monochromatic FMS wave can thus lead to

TABLE I.

| $m$ | $I^0$ | $a$ | $b$ |
|---|---|---|---|
| 0 | 2.18 | 2.39 | 2.06 |
| 0 | 14.30 | 16.7 | 14.1 |
| 1 | 7.61 | 5.19 | 6.25 |
| 1 | 23.6 | 20.7 | 21.5 |
| 2 | 14.3 | 8.9 | 11.4 |
| 2 | 36.3 | 26.7 | 30.9 |

the formation of a stationary wave channel with a somewhat increased plasma density. Thanks to refraction the wave is contained in the channel, while the plasma is under the influence of the force (7). However, the wave is not always stabilized. If the condition for stabilization is not satisfied, the wave contracts, at least until it becomes strongly non-linear.

## 4. PLASMA COLLAPSE UNDER THE ACTION OF FAST MAGNETOSONIC WAVES

If condition (15) is satisfied a uniform isotropic wave distribution is unstable. When fluctuations in the density arise the plasma starts to contract into a region of enhanced density. The compression occurs along the magnetic field so that the shape of the blob turns out to be anisotropic. We noted already earlier that the increase in the energy density of the FMS waves with increasing plasma density is caused by the change in the momentum volume occupied by the waves. If the waves are distributed relatively isotropically at all points of space, their density changes as $\rho^{3/2}$. For small plasma inhomogeneities a mixing of the waves with respect to directions is guaranteed by refraction. This also supports the modulational instability in its initial stage. However, when the density modulation turns out to be large, the role of the refraction reduces to confining the waves trapped in the region of compression and one needs include other mechanisms for the trapping of the waves. Scattering may serve as one of such mechanisms.

We consider how wave-wave scattering affects the development of the process. The rate of induced scattering of two FMS waves with wavevectors $\mathbf{k}_1$ and $\mathbf{k}_2$ into waves with $\mathbf{k}_3$ and $\mathbf{k}_4$ is important when $|\omega_1 - \omega_3| \lesssim |\mathbf{k}_1 - \mathbf{k}_3| v_{Ti}$. One can show that in that case

$$\dot{w}_1 \sim \frac{\omega}{(\rho T)^2}(w_3 - w_1) w_2 w_4. \tag{43}$$

Let the compression region have a characteristic size $l$ and let the velocity of the compression be $u \sim v_{Ti}$. For sustaining the compression the energy density of the waves must increase faster than the particle density:

$$\frac{\dot{\rho}}{\rho} \sim \frac{u}{l} < \frac{\dot{w}}{w} \sim \omega\left(\frac{w}{\rho T}\right)^2 \frac{v_{Ti}}{c_A}. \tag{44}$$

Hence

$$kl\left(\frac{w}{\rho T}\right)^2 \gg 1. \tag{45}$$

This condition is always satisfied when the condition

(15) for the occurrence of the modulational instability is satisfied. Hence, the scattering of the waves guarantees that they are trapped sufficiently fast in the compression region, and the compression of the plasma must continue until the condition for the applicability of Eq. (7) is violated, i.e., until the density of the kinetic energy of the plasma or of the energy becomes comparable to $H_0^2/8\pi$.

## CONCLUSION

Two distinctive characteristics of MHD waves give particular importance to modulational processes in them.

1. MHD waves are oscillations with very long wavelengths which propagate in a plasma with relatively weak dissipation. The modulational instability of MHD waves can therefore proceed as a large-scale process.

2. Magneto-hydrodynamic turbulence often contains in it an appreciable amount of energy with a transfer velocity $c_A$ which for small $\beta$ is much larger than the thermal velocity. This guarantees a fast supply of energy to the compression region so that a large amount of energy can be released when the modulational instability develops.

These characteristics give us a basis for expecting that the modulational instability of MHD waves can be observed directly in natural conditions. One of the most appropriate objects where the effects considered might take place is the solar plasma, especially in the chromospheric region. If we take the following values for the parameters for the chromospheric plasma: magnetic field $H_0 \sim 10$ to 100 gauss, density $\rho \sim 10^{12}$ cm$^{-3}$, temperature $T \sim 10^4$ K, turbulent velocity of the order of 10 to 30 km/s, which are, probably, rather typical[5] we get $\beta \sim 10^{-3}$, $w/\rho T \gtrsim 1$. In that case all conditions for the occurrence of the modulational instability are satisfied in the chromosphere. In this connection it is fully realistic that one can identify some of the chromospheric phenomena with the modulational instability of MHD waves.

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1) In (2) we have dropped non-linear terms of the order $(H/H_0)^2$. This does, however, not contradict the fact that we shall in what follows when we consider the low-frequency longitudinal motion of the plasma take into account non-linearities caused by the pressure of the waves which are, as will be clear, determined by the parameter $H^2/\rho T$.

2) We note that the above-mentioned degeneracy of Eq. (28) is connected with the fact that the space orthogonal to the $z$-axis is two-dimensional.


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# Kinetic cooling of a $CO_2$–$N_2$ gas mixture by $CO_2$-laser radiation

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We report a theoretical analysis and an experimental investigation of the cooling of a molecular gas by resonant intermode absorption of laser radiation. The actual results pertain to cooling of a $CO_2$–$N_2$ gas mixture by $CO_2$-laser radiation. The rate and depth of the cooling are investigated as functions of the partial composition of the mixture (including pure $CO_2$ gas) at different intensities and waveforms of the laser pulse. Good agreement is obtained between the theoretical and experimental results.

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It was shown theoretically by a number of workers[1–4] that when $CO_2$-laser radiation is absorbed in air, the gaseous medium may be cooled rather than heated during the initial instants of time. The appearance of the kinetic-cooling effect is due to intermode resonant absorption of the $CO_2$-laser radiation by the molecules of the carbon dioxide. The initial interest in this effect was due to the possibility of resonant

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