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Paramagnetic relaxation of conduction electrons, due to indirect spin-spin interaction via a phonon field

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The longitudinal paramagnetic conduction-electron relaxation times due to electron-electron interaction via the phonon field are calculated. In the case considered, the contribution to the relaxation process is made by that part of the indirect phonon interaction which leads to a change in the spin states of the interacting electrons. It is found that, in the temperature range $T \geq 10$ K, the rate of relaxation via the mechanism under consideration is independent of the temperature and is proportional to $T_1^{-1} \propto N^{4/3}$, where N is the electron concentration. At temperatures below 10 K, $T_1^{-1} \propto T^2 N^{2/3}$. The effect of multiple phonon exchange and of collective excitations of the electron system by indirect phonon interaction on the magnitude of T_1 is analyzed.

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Paramagnetic relaxation of conduction electrons in metals and semiconductors is due to the scattering of electrons with spin flip. The basic scattering mechanisms are: interaction with phonons[1] or with impurities[2] and scattering of electrons by one another.[3] The paramagnetic relaxation of the conduction electrons in metals and semiconductors, due to electron-electron interaction, is described by the function $f(\mathbf{p}, \sigma; \mathbf{p}', \sigma')$, which characterizes the amplitude of scattering of particles by one another with spin flip. For a system that is invariant relative to rotations, the general form of this function is[4]

$$f(\mathbf{p}_i, \sigma_i; \mathbf{p}_j', \sigma_j') = \sum_{\alpha,\beta} [f(\mathbf{p}_i, \mathbf{p}_j') + \sigma_i g(p_i, p_j') + \sigma_j' g'(p_i, p_j') + \sigma_{i\alpha}\sigma_{j\beta}' \xi^{\alpha\beta}(\mathbf{p}_i, \mathbf{p}_j')], \quad (1)$$

where $\alpha, \beta = x, y, z$. The terms in (1) that are linear in the spin describe the spin-orbit interaction of the spin of the i-th (or j-th) electron with the magnetic field produced by the orbital motion of the j-th (or i-th) electron. Terms that are quadratic in the spin operator describe the magnetic dipole-dipole and exchange interactions between electrons.

It is known that the indirect exchange interaction of electrons via the phonon field is the principal real mechanism, leading to the appearance of the superconducting state. In works on the theory of superconductivity, the process of exchange of phonons without change in the ground states of the interacting electrons is considered. However, in materials with strong spin-orbit interaction $\mathcal{H}_{c0}$ of the conduction electrons with the ion core of the lattice,

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$$\mathcal{H}_{c0}=\frac{\hbar}{4m^2c^2}[\nabla\times V\mathbf{p}]\sigma\,, \tag{2}$$

(V is the potential of the Coulomb interaction of the ion core of the lattice with the electrons, σ is the Pauli spin operator, $\mathbf{p}$ is the momentum operator of the electron), the operator that describes the direct interaction of the conduction electrons via the phonon field contains, together with the term $\mathcal{H}_{eff}^{(1)}$ that does not depend on the spins of the interacting particles, also a term $\mathcal{H}_{eff}^{(2)}$ that depends on the spin:

$$\mathcal{H}_{eff}(\alpha,i;\gamma,j)=\sum_{\mathbf{q},\omega}\Gamma_0^0(\mathbf{q},\omega)a_\alpha^+(i)a_\gamma^+(j)a_\gamma(j)a_\alpha(i)+\sum_{\mathbf{q},\omega}\Gamma_{\alpha\beta}^{\gamma\delta}(\mathbf{q},\omega)a_\alpha^+(i)a_\gamma^+(j)a_\delta(j)a_\beta(i), \tag{3}$$

where $\alpha\neq\beta$ in the second term, $\Gamma_{\alpha\beta}^{\gamma\delta}(\mathbf{q},\omega)$ and $\Gamma_0^0(\mathbf{q},\omega)$ are the effective amplitude of the scattering of the electrons by one another with and without spin flip.

Since the paramagnetic relaxation times $T_1\gg\tau_p$, where τ_p is the momentum relaxation time of the conduction electrons, are connected with the processes of scattering of the electrons with spin flip, it follows that the investigation by EPR on conduction electrons can give information on the magnitude and character of the indirect exchange interaction of electrons via the phonon field with spin flip.

In the present work, we consider the contribution of the indirect spin-spin interaction of conduction electrons via the phonon field to the processes of paramagnetic relaxation. In principle, the indirect spin-spin interaction can lead to the appearance of a correlation between the spins of the interacting electrons. However, since $\Gamma_0^0(\mathbf{q},\omega)\gg\Gamma_{\alpha\beta}^{\gamma\delta}(\mathbf{q},\omega)$, the phase transition of the conductor from the normal to the superconducting state takes place at temperatures T_c that are much greater than the temperature at which possible effects of spin-spin correlation can appear.

The times T_1, which are governed by the dipole-dipole and spin-orbit interactions, were calculated theoretically by Overhauser[3] and Yafet.[1] The obtained values of T_1 turned out to be several orders of magnitude larger (10^3 to 10^4 times) than the experimentally observed $T_{1\,exp}\approx10^{-8}$ sec because the constants of the interactions that correspond to the relaxation processes are in themselves small.

The time of longitudinal relaxation T_1 is determined by the relation[3]

$$\frac{1}{T_1}=\frac{2(W_{+-}-W_{-+})}{D_0-D} \tag{4}$$

where $D=N_--N_+$, $D_0=N_-^0-N_+^0$, $N_\pm^0$ is the number of particles (electrons) with spin directed along the opposite to the external magnetic field, respectively:

$$N_\pm=\sum_{\mathbf{p}}n_\pm(\mathbf{p}),\quad W_{+-}=\partial N_+/\partial t$$

is the total transition rate of particles from the state with spin up to the state with spin down. The quantity $\partial N_\alpha/\partial t$ is connected with the imaginary part of the self-energy operator of the Green's function $\Sigma(\mathbf{p}_\alpha,\omega)$, which is due to the scattering of the carrier with spin flip, by the relation

$$\frac{\partial N_\alpha}{\partial t}=\frac{1}{(2\pi)^3}\int n(\mathbf{p}_\alpha)\,\mathrm{Im}\,\Sigma(\mathbf{p}_\alpha,\omega_p)\,d\mathbf{p}. \tag{5}$$

Under the condition $\tau_p\ll T_1$, $\Sigma(\mathbf{p}_+)=\Sigma(\mathbf{p}_-)$ and $kT\ll\varepsilon_F$, where τ_p is the momentum relaxation time, we get the following for the time T_1 from the relations (4) and (5):

$$\frac{1}{T_1}=2\int\frac{\partial n(\mathbf{p})}{\partial\varepsilon}\,\mathrm{Im}\,\Sigma(\mathbf{p}_\alpha,\omega_p)\,d\mathbf{p}\Big/\int\frac{\partial n(\mathbf{p})}{\partial\varepsilon}\,d\mathbf{p}. \tag{6}$$

In the case in which we can neglect the quantization of the orbital motion of the conduction electrons in the external magnetic field, the Hamiltonian of the system $\mathcal{H}_0$ is invariant relative to the translations of the lattice. Therefore the eigenfunctions of the conduction electrons will be of the Bloch type. The spin-orbit interaction leads to the result that the functions will not correspond to pure spin states $\chi_\pm$ with σ_z equal to 1 or -1. In the general case, the wave function of the conduction electrons is written down in the following form:

$$\Psi_{\mathbf{p}\uparrow}=\psi_{\mathbf{p}\uparrow}\chi_++\varphi_{\mathbf{p}\downarrow}\chi_-. \tag{7}$$

In this formula, the arrow indicates the spin state; for example, the value $\langle\Psi_{\mathbf{p}\uparrow}|\sigma_z|\Psi_{\mathbf{p}\uparrow}\rangle$ is positive. The quantity $\varphi_{\mathbf{p}\downarrow}$ is proportional to the shift Δg of the g factor of the electron.

The electron-phonon coupling comes about by modulation, by means of the thermal lattice vibrations, of the Coulomb and spin orbit interaction $W(\mathbf{q},\omega,\mathbf{r})$, where $\mathbf{q}$ and ω_q are the wave vector and the frequency of the phonon, and $\mathbf{r}$ is the radius vector of the conduction electron. As a consequence of the fact that the spin-orbit interaction leads to a mixing of the spin functions of the conduction electrons in (7), the term in the matrix element

$$\langle\Psi_{\mathbf{p}\uparrow}|W(\mathbf{q},\omega,\mathbf{r})|\Psi_{\mathbf{p}'\downarrow}\rangle=M_{\mathbf{p}\uparrow;\mathbf{p}'\downarrow}$$

which represents the modulation by thermal oscillations of the Coulomb interaction, has a nonzero value proportional to Δg. For metals and semiconductors with a structure possessing a center of inversion,[1]

$$M_{\mathbf{p}\uparrow;\mathbf{p}'\downarrow}=A|\mathbf{p}+\mathbf{p}'||\mathbf{p}-\mathbf{p}'|^2u_{\mathbf{q}}, \tag{8}$$

where the interaction constant $A\approx C\Delta g\hbar^2(mE_g)^{-1}$, C is the deformation potential, E_g is the width of the forbidden band, and $u_{\mathbf{q}}$ is the amplitude of the phonon with wave vector $\mathbf{q}=\mathbf{p}-\mathbf{p}'$. For structures that do not have a center of inversion,

$$M_{\mathbf{p}\uparrow;\mathbf{p}'\downarrow}=B|\mathbf{p}+\mathbf{p}'||\mathbf{p}-\mathbf{p}'|u_{\mathbf{q}}, \tag{9}$$

where $B\approx Cf\Delta g\hbar(amE_g)^{-1}$—for the deformation mechanism of interaction, and $B=4\pi eu\Delta g\hbar^2(\varepsilon_0mE_g)^{-1}$—for the polarization mechanism, a is a constant of the order of the period of the lattice, f is the relative intensity of the crystal potential, e is the electron charge, u is the piezoelectric constant, and ε_0 is the dielectric constant of the sample.

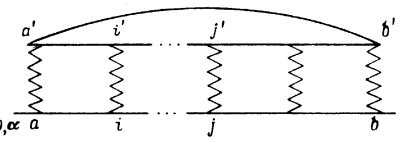

FIG. 1.

Finally,

$$M_{\mathbf{p}'\downarrow,\mathbf{p}\uparrow}=g_0(\hbar s|\mathbf{p}-\mathbf{p}'|)^{1/2}, \tag{10}$$

where $g_0=ea^2C/s\rho^{1/2}$, s is the speed of sound, and ρ is the density.

With account of the relations (8)–(10) for $M_{\mathbf{pp}'}$ written down above, we can represent the amplitudes of scattering of electrons by one another Γ_0^0 and $\Gamma_{\alpha\beta}^{\gamma\delta}$, due to indirect interaction of electrons via the phonon field in (3), in the following form:

$$\Gamma_0^0(\mathbf{p},\mathbf{p}')=D(\mathbf{q},\omega)g_0^2,\quad \Gamma_{\alpha\beta}^0(\mathbf{p},\mathbf{p}')=\frac{g_0BD(\omega,\mathbf{q})}{s(2\rho)^{1/2}}\left|\mathbf{p}-\frac{\mathbf{q}}{2}\right|,$$

$$\Gamma_{\alpha\beta}(\mathbf{p},\mathbf{p}')=\frac{B^2D(\omega,\mathbf{q})}{s^2 2\rho}\left|\mathbf{p}-\frac{\mathbf{q}}{2}\right|\left|\mathbf{p}'+\frac{\mathbf{q}}{2}\right| \tag{11}$$

$$D(\omega,\mathbf{q})=-\frac{s^2q^2}{\omega^2+s^2q^2},$$

where s is the speed of sound in the sample, $D(\omega,\mathbf{q})$ is the Green's function of the phonon, and $\mathbf{q}=\mathbf{p}-\mathbf{p}'$ is the wave function of the phonon.

The simplest diagram for the self-energy operator $\Sigma^{(2)}(\mathbf{p}_\alpha,\omega)$, which describes the scattering of electrons with spin flip, due to indirect spin-spin interaction via the phonon field, is represented in Fig. 1 by a diagram of second order in the indirect interaction. The determination of $\Sigma^{(2)}(\mathbf{p}_\alpha,\omega)$ reduces to the calculation of the sum over ω_i and integration over $\mathbf{p}_2$ and $\mathbf{q}$ in the right side of the relation

$$\Sigma^{(2)}(\mathbf{p}_\alpha,\omega)=\frac{T^2}{(2\pi)^6}\sum_{\omega_1,\omega_2}\int d\mathbf{q}\,d\mathbf{p}_2\,\Gamma_{\alpha\beta}^0(\omega_1,\mathbf{q},\mathbf{p})G_\beta(\mathbf{p}-\mathbf{q},\omega-\omega_1)$$
$$\times G_\gamma(\mathbf{p}_2,\omega_2)G_\gamma(\mathbf{p}_2+\mathbf{q},\omega_2+\omega_1)\Gamma_{\beta\alpha}^0(\omega_1,\mathbf{q},\mathbf{p}), \tag{12}$$

where $G_\beta(\mathbf{p},\omega)=[i\omega-\varepsilon_\beta(\mathbf{p})+\mu_0]^{-1}$ is the Green's function of the free electron with spin β, $\omega_1=2\pi nT$, and $\omega_2=(2k+1)\pi T$.

The quantity T_1 is determined, according to the relation (6), by the value $\Sigma(\mathbf{p}_0,\omega)$, where $\mathbf{p}_0$ is the wave vector of the conduction electron with Fermi energy μ_0. Summing over ω_1 and ω_2 in (12), and separating the imaginary part of $\Sigma^{(2)}(\mathbf{p},\omega)$, which describes the process of paramagnetic relaxation, we get with the help of the procedure $\mathrm{Im}(i\omega+b+c)^{-1}=-\pi\delta(\omega+b+c)$, where $b=\mu_0-\varepsilon(\mathbf{p}-\mathbf{q},\beta)$, $c=-\varepsilon(\mathbf{p}_2+\mathbf{q},\gamma)+\varepsilon(\mathbf{p}_2,\gamma)$, after integration over $\mathbf{p}_2$,

$$\mathrm{Im}\,\Sigma^{(2)}(p_0,\omega)=\frac{1}{(2\pi)^3}\frac{g_0^2B^2}{2\rho\hbar^2}\left(\frac{m^*}{\hbar}\right)^3J(T,p_0); \tag{13}$$

$$J(T,p_0)=-\frac{\alpha^2}{p_0}\int_0^{q_0}q^4\,dq\int_{a_-}^{a_+}\left[p_0^2-\frac{q^2}{4}+\frac{zmkT}{\hbar^2}\right]\frac{z\,dz}{\mathrm{sh}\,(z)\,(z^2-\alpha^2q^2)^2},$$

$$\alpha=\frac{\hbar s}{kT},\quad a_\pm=\frac{\hbar^2}{2mkT}(q^2\pm 2p_0q).$$

The main contribution to this integral is made by the region of integration $z\sim\alpha q$. Consequently, if the inequality $a_-<\alpha q<a_+$ is not satisfied for αq, the value of the integral $J(T,p_0)$ is negligibly small. It follows from this condition that, in the calculation of $J(T,p_0)$, it is sufficient to limit ourselves to the region of integration $0<q<2p_0+2ms/\hbar$.

Depending on the value of αq, it is expedient to consider two cases: $\alpha q\leq 1$—the high temperature approximation, and $\alpha q>1$—the low-temperature approximation. In the case of high temperatures, after integration over $\mathbf{q}$ in (13), we obtain for the relaxation time, in accord with (6),

$$\frac{1}{T_1}=\frac{1}{5\pi^3}\frac{B^2g_0^2}{\rho}\left(\frac{m^*}{\hbar}\right)^3\frac{p_0^4}{\hbar^2}. \tag{14}$$

The contribution of the region of integration $2p+2ms/\hbar<q\leq q_0$ to the value of T_1^{-1} is of the order of s/T_1v_0, where $s/v_0\sim 10^{-2}$ at concentrations of the conduction electrons $n\geq 10^{16}$ cm^{-3}.

At temperature $T<2\hbar sp_0k^{-1}$ we obtain with the help of the relations (13) and (6)

$$\frac{1}{T_1}=\frac{1}{3\pi^3}\frac{B^2g_0^2}{\hbar^2\rho}\left(\frac{m^*}{\hbar}\right)^3\left(\frac{kT}{\hbar s}\right)^2p_0^2\left[5+8\frac{m^*s^2}{kT}\right]. \tag{15}$$

Contributions to the value of $\Sigma(\mathbf{p}_\alpha,\omega)$ are considered below for diagrams of higher order. Of all the forms of possible diagrams, the greatest contribution to $\Sigma(\mathbf{p}_\alpha,\omega)$ is made by diagrams of the two types represented in Figs. 1 and 2. Diagrams of the ladder type (Fig. 1) describe the process of multiple exchange of phonons. This process, without account of the change in the spin state, is considered in the theory of superconductivity. The considered interaction makes a contribution to the processes of paramagnetic relaxation only if at least in two points, i and j in Fig. 1, the absorption or emission of a phonon by the considered electron takes place with change in its spin state, i.e., in $\Gamma_{\alpha\beta}^{\gamma\delta}(i,i')$ and $\Gamma_{\alpha'\beta'}^{\gamma'\delta'}(j,j')$, $\alpha\neq\beta$ and $\alpha'=\beta'$, while γ, δ and γ', δ' can be arbitrary. For the subsequent calculations it is convenient to represent $\Gamma_{\alpha\beta}^{\gamma\delta}$ in the following form:

$$\Gamma_{\alpha\beta}^{\gamma\delta}(\mathbf{p},\mathbf{p}')=g_{\alpha\beta}(\mathbf{p},\mathbf{q})D(\mathbf{q},\omega)g_{\gamma\delta}(\mathbf{p}',\mathbf{q}), \tag{16}$$

where $g_{\alpha\beta}(p,q)=g_0$ at $\alpha=\beta$ and $g_{\alpha\beta}=|\mathbf{p}-\mathbf{q}/2|B((2\rho)^{1/2}s)^{-1}$ at $\alpha\neq\beta$.

Since $|\Gamma_{\alpha\alpha}^{\gamma\gamma}|\gg|\Gamma_{\alpha\beta}^{\gamma\gamma}|\gg|\Gamma_{\alpha\beta}^{\gamma\delta}|$, where $\alpha\neq\beta$ and $\gamma\neq\delta$, then the contribution to $\Sigma(\mathbf{p}_\alpha,\omega)$ from ladder diagrams will be a maximum when the diagram of the n-th order $(n\to\infty)$ contains the two operators $\Gamma_{\alpha\beta}^{\gamma\gamma}$ and $\Gamma_{\beta\alpha}^{\gamma\gamma}$; the remaining $n-2$ operators can be represented in the form of a set of m operators $\Gamma_{\alpha\alpha}^{\gamma\gamma}$ and u operators $\Gamma_{\beta\beta}^{\gamma\gamma}$, with $m+u=n-2$.

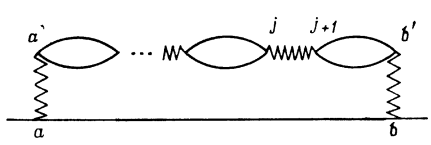

FIG. 2.

From the rules of calculation of the diagram elements,[5] it follows that, since $G^0_{\alpha\beta}=0$ at $\alpha\neq\beta$, two operators $\Gamma^{\gamma\delta}_{\alpha\beta}$ and $\Gamma^{\gamma'\delta'}_{\alpha'\beta'}$, located successively in the diagram give a nonzero contribution under the conditions $\alpha'=\beta$ and $\gamma'=\delta$. Therefore, in accord with the permutation relation for the operator $\Gamma^{\gamma\gamma}_{\alpha\beta}$ in the diagram of n-th order for $\Sigma(\mathbf{p}_\alpha,\omega)$ the diagram consisting of the $n-2$ operators $\Gamma^{\gamma\gamma}_{\beta\beta}$ has the maximum value:

$$\Sigma(\mathbf{p}_\alpha,\omega)=\sum_{\omega_1\omega_2}\sum_{n=2}^{\infty}\frac{T^2}{(2\pi)^6}\int d\mathbf{p}_2\,d\mathbf{q}\,\Gamma_{\alpha\beta}{}^0(\mathbf{p},\mathbf{q},\omega_1)G_\gamma(\mathbf{p}_2,\omega_2)G_\beta(\mathbf{p}-\mathbf{q},\omega-\omega_1)\times G_\gamma(\mathbf{p}_2+\mathbf{q},\omega_2+\omega_1)\frac{2(-1)^nX^n}{n(n-1)}\Gamma_{\beta\alpha}{}^0(\omega_1,\mathbf{p},\mathbf{q}),\qquad(17)$$

where

$$X=\frac{T}{(2\pi)^3}\sum_{\omega_3}\int d\mathbf{p}_3\,\Gamma_0{}^0(\mathbf{q},\omega_1)G_\beta(\mathbf{p}_2+\mathbf{p}-\mathbf{p}_3,\omega_2+\omega-\omega_3)G_\gamma(\mathbf{p}_3,\omega_3).$$

Summation over n in (17) gives

$$I(X)=2\left[\frac{1+X}{X^2}\ln(1+X)-\frac{1}{X}\right].$$

The dependence of the quantity X on the value of $\mathbf{p}+\mathbf{p}_2$ at $\Gamma^0_0=-2\pi^2\zeta\hbar^2(p_0m)^{-1}$, where the nondimensional quantity $\zeta\approx1$, is well known.[5,6] In particular, at arbitrary values of $|\mathbf{p}+\mathbf{p}_2|\leq2p_0$ we have $|X|\leq1$. At such values of X, the function I depends weakly on X: $\frac{2}{3}<I<\frac{4}{3}$. Then the relation (17) can be represented in the form

$$\Sigma(\mathbf{p}_\alpha,\omega)=I(\bar X)\Sigma^{(2)}(\mathbf{p}_\alpha,\omega).\qquad(18)$$

In the case of an indirect phonon interaction between the electrons $-1\leq\bar X\leq0$,[6] we have $I(\bar X)\approx1-\bar X/3$. Consequently, account of the multiple exchange of phonons between the conduction electrons leads to antiscreening of the effective interaction, as a result of which the quantity T_1 is contracted by a factor of $(1-X/3)\approx\frac{3}{4}$. Physically, this circumstance is explained by the fact that the indirect phonon interaction without spin flip leads to correlation of the motion of the electrons relative to one another, which in turn causes an increase in the probability of scattering of the electrons by one another with spin flip.

In the calculation of the contribution to the quantity $\Sigma(\mathbf{p}_\alpha,\omega)$ from the diagrams represented in Fig. 2, it is convenient to consider three types of chain diagrams: the diagram in which the indirect phonon interaction between the electrons at all points except the points a and b takes place without spin flip; the diagram in which $\Gamma^{\gamma\delta}_{\alpha\beta}(j,j+1)$, $j=1,2,\ldots,n$ at all points, including the end points aa' and bb', $\gamma\neq\delta$, $\alpha\neq\beta$; and, finally, the diagram with any set of values γ, δ and α, β except the points a, b, at which, by virtue of the conditions of the considered problem, $\alpha\neq\beta$.

Summation of the chain diagram of the first type for $\Sigma(\mathbf{p}_\alpha,\omega)$ leads to the following expression:

$$\Sigma(\mathbf{p}_\alpha,\omega)=\frac{T}{(2\pi)^3}\sum_{\omega_1}\int d\mathbf{q}G_\beta(\mathbf{p}-\mathbf{q},\omega-\omega_1)g_{\alpha\beta}{}^2(\mathbf{p},\mathbf{q})\times D(\omega,\mathbf{q})\tilde v(\mathbf{q},\omega_1)\tilde{\mathcal{T}}_{11}(\mathbf{q},\omega_1),\qquad(19)$$

where

$$\tilde{\mathcal{T}}_{11}(\mathbf{q},\omega_1)=\frac{\mathcal{T}_{11}{}^0(\mathbf{q},\omega_1)}{1+\tilde v(\mathbf{q},\omega_1)\mathcal{T}_{11}{}^0(\mathbf{q},\omega_1)},$$

$\tilde{\mathcal{T}}^0_{\gamma\gamma}(\mathbf{q},\omega_1)$ is the ordinary polarization loop, $\tilde v(\mathbf{q},\omega_1)=g_0^2D(\mathbf{q},\omega_1)$.

If we include the Coulomb electron-electron interaction $v(q)=4\pi e^2/q^2$ in $\Gamma^{\gamma\gamma}_{\delta\delta}(j,j+1)$, then the expression for $\tilde v(q,\omega_1)$ becomes equal to $v(q)+\tilde v(\mathbf{q};\omega_1)$ and the factor $1+V(\mathbf{q},\omega_1)\tilde{\mathcal{T}}^0_{\gamma\gamma}(\mathbf{q},\omega)$ is none other than the dielectric constant of the medium, calculated with account of the Coulomb and indirect phonon interactions.

In the scattering, the change of energy of the electrons is of the order $\omega\leq\omega_F\leq\omega_p$, where $\omega_p=(4\pi e^2N/m\varepsilon_0)^{1/2}$ is the plasma frequency, $\omega_F=\mu_0/\hbar$. In this range of frequencies $\varepsilon(q,\omega)\approx1$,[7] i.e., the summation of such a chain of diagrams with accuracy to terms of order ω/ω_p leads to the value of $\Sigma^{(2)}(\mathbf{p}_\alpha,\omega)$. Account of the correction to the value of $\varepsilon(q,\omega)$ and the dispersion law of the plasma frequency $\omega_p(q)$ of the indirect phonon interaction $\tilde v(\mathbf{q},\omega)$ is not the aim of the present work and will be given elsewhere.

Summation of the diagrams of the second type should obviously lead to account of the contribution of the interaction of the conduction electron via the phonon field with spin waves to the quantity T_1. In the case considered, the spin waves are formed not by the exchange interaction between the electrons in the expression (1) for $f(\mathbf{p}_i,\sigma_i,\mathbf{p}'_j,\sigma'_j)$, but by the indirect phonon interaction with spin flip. The value of $\Sigma(\mathbf{p}_\alpha,\omega)$ for such a process is expressed by a relation similar to the relation (19), in which $\tilde v(\mathbf{q},\omega)$ is replaced by $B^2D(\mathbf{q},\omega_1)/2\rho s^2$ and $\tilde{\mathcal{T}}(\mathbf{q},\omega_1)-\Pi_{\gamma\delta}(\mathbf{q},\omega)$, where

$$\Pi_{\gamma\delta}(\mathbf{q},\omega_1)=-\frac{T}{(2\pi)^3}\sum_{\omega_2}\int\left|\mathbf{p}_2-\frac{\mathbf{q}}{2}\right|^2G_\gamma(\mathbf{p}_2,\omega_2)G_\delta(\mathbf{p}_2+\mathbf{q},\omega_2+\omega_1)\,d\mathbf{p}_2.\qquad(20)$$

Calculation of the relation (20) in the spin-wave frequency range $qv_0<\omega<\omega_F$ leads to the following expression:

$$V(q,\omega_1)\Pi_{\gamma\delta}(q,\omega_1)=-u^iq^i/[(i\omega_1)^i-u^iq^i].\qquad(21)$$

The poles of this expression describe the dispersion law of the spin waves due to the indirect phonon interaction of the electrons with spin flip,

$$\omega_{sv}=(B^2p_0{}^5/10\pi^2\rho m^*)^{1/4}q.\qquad(22)$$

As a consequence of the fact that the dispersion curve of single particle excitations of the conduction electrons $\varepsilon_{\mathbf{p}+\mathbf{q}}-\varepsilon_{\mathbf{p}}\approx v_0q$ in the region $q<p_0$ does not intersect the dispersion curve for spin waves (22), substitution of (21) in (19) leads to the result that the contribution of the spin waves to the process of paramagnetic relaxation is absent.

We can majorize the sum of diagrams of the mixed type under the condition

$$|\Gamma_0{}^0|>|\Gamma_{\alpha\beta}{}^0|>|\Gamma_{\alpha\beta}{}^{\gamma\delta}|\quad\text{and}\quad|\Gamma_0{}^0+\Gamma_{\alpha\beta}{}^0+\Gamma_{\alpha\beta}{}^{\gamma\delta}|\approx{}^3/_2|\Gamma_0{}^0|$$

by means of the expression $\tilde{\pi}_{rr}(q, \omega_1)$ in (19), where $V(q, \omega_1) = v(q) + \frac{3}{2}\Gamma_0^0$. Thus the quantity T_1 and its dependence on the temperature and concentration of conduction electrons are determined by the expressions (14) and (15).

Metals in which the paramagnetic resonance of conduction electrons have been investigated (Li, Na, Rb, Mg, Cu, Al)[18] possess a structure with a center of inversion. The largest values of the shift of the g factors in these metals, $\Delta g \approx 10^{-3}$, attest to the smallness of the spin-orbit interaction constant. A compound with a crystal structure without a center of inversion with strong spin-orbit interaction, is the III–V semiconductor n-InSb with $\Delta g \approx -50$, in which the electron paramagnetic resonance has been relatively well studied.[9] With account of the weak non-parabolic character of the dispersion law of the energy of the conduction electrons at a carrier concentration $N \geq 10^{16}$ cm^{-3} the relation (14) for T_1 in the high-temperature region $T \geq 2\hbar s p_0 k^{-1}$ is written in the following fashion:

$$\frac{1}{T_1} = \frac{B^2 g_0^2}{5\pi^3 \rho} \frac{(3\pi^2 N)^{4/3}}{\hbar^5} m^{*3}(0) \left[1 + \frac{2\pi^2\hbar^2}{m^*(0) E_g} N^{2/3}\right]^{3/2}, \quad (23)$$

where $m^*(0) = 0.0132\, m$ is the effective mass of the electron at the bottom of the conduction band, m is the mass of the free electron, $E_g = 3.76 \cdot 10^{-13}$ erg is the energy gap between the bottom of the conduction band and the valence band. In this region of temperatures, the value of T_1 does not depend on T. For numerical estimates, we used the following values of the constants entering in (23): $B = 10^{-16}$ erg-cm,[1,10] $N = 10^{16}$ cm^{-3}, $g_0^2 = 2.5 \times 10^{-30}$ erg-cm^3, $\rho = 5.78$ g-cm^{-3}, $s = 3 \times 10^5$ cm-sec^{-1} and T_1 becomes equal to 6×10^{-7} sec. This quantity is of the same order as the value of T_1 due to the inelastic scattering of the electron with spin flip by thermal phonons.[1] The experimentally measured value of $T_{1e} \approx 5 \times 10^{-7}$ sec.[9]

The concentration dependence $T_{1e}(N)$ obtained by Isaacson[9] in the region $N \sim 1.8 \times 10^{15} - 1.5 \times 10^{16}$ cm^{-3} agrees qualitatively with the dependence $T_1^{-1} \sim N^{4/3}$ that follows from (23). Upon increase in the concentration of carriers or decrease in the temperature to the value $T < 2\hbar s p_0 k^{-1}$, Eq. (15) yields $T_1^{-1}(T) \sim T^2$.

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