Parametric resonance in films on antiferromagnetic backings

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A nonuniform distribution of magnetization, which varies between wide limits, is induced in a ferromagnetic layer on an antiferromagnetic backing by a steady external magnetic field. The threshold for excitation of parametric resonance in such a structure by an alternating magnetic field parallel to the stationary field is found. The dependence of the threshold on the strength of the stationary field is compared with the corresponding dependences of the line width and susceptibility for the ordinary magnetic resonance. The threshold rises to infinity and the resonance susceptibility vanishes at a certain value of the stationary external field strength.

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Studies of the parametric excitation of spin waves in ferromagnets have mainly been concerned with the case in which the magnetization of the specimen is uniform or has random nonuniformities.[1] Instabilities of the spin waves in ferromagnets have also been investigated in connection with certain spatial nonuniformities of the magnetic moment, e.g., domain structure[3] or helicoidal structure.[4] A peculiar nonuniformity arises in a ferromagnetic layer on an antiferromagnetic backing. In such a layer, as a result of the exchange interaction at the ferromagnet-antiferromagnet interface the magnetic moment vector $\mathbf{M}$, in external magnetic fields weaker than the effective antiferromagnetic anisotropy field, remains fixed in a direction parallel to the stationary vector $\mathbf{M}_1$ ($\mathbf{M}_1$ is the magnetization nearest to the surface of contact with the antiferromagnetic sublattice). Under the influence of a constant external magnetic field there arises in the ferromagnetic layer a nonuniform distribution of magnetization $\mathbf{M}$ that varies between wide limits. In this paper we investigate the parametric excitation of spin waves in such a ferromagnetic layer.

Let us consider a ferromagnetic layer of thickness d whose plane is normal to the z axis (Fig. 1). The external constant and variable magnetic fields are directed along the x axis. The behavior of the magnetic system of this layer will be described in a phenomenological approximation by the Landau-Lifshits equation for the magnetization vector $\mathbf{M}$ of the ferromagnet:

$$\dot{\mathbf{M}} = g[\mathbf{M}\times\mathbf{H}^{(e)}] - \frac{\xi}{M}[\mathbf{M}\times\dot{\mathbf{M}}] \tag{1}$$

with the boundary conditions

$$\mathbf{M}\parallel\mathbf{M}_1 \quad \text{if} \quad z=0, \quad \partial\mathbf{M}/\partial n=0 \quad \text{if} \quad z=d. \tag{2}$$

If the magnetic anisotropy of the ferromagnetic film be neglected, the effective magnetic field will be given by $\mathbf{H}^{(e)} = \alpha\nabla^2\mathbf{M}+\mathbf{H}$, where α is the exchange constant of the ferromagnet. Introducing the normalized fields $\mathbf{h} = \mathbf{H}/M$ and magnetizations $\mathbf{m}' = \mathbf{M}/M$ and expressing $\mathbf{m}'(z,t)$ in the form $\mathbf{m}'(z,t) = \mathbf{m}(z) + \mu(z,t)$, we obtain a static set of equations for $\mathbf{m}(z)$ whose solution is known[5,6]:

$$\begin{gathered} m_x=-1, \quad m_y=0 \text{ when } h\leqslant h_u, \\ m_x=-1+2k^2s^2, \quad m_y=2ks(1-k^2s^2)^{1/2} \text{ when } h\geqslant h_u, \end{gathered} \tag{3}$$

where $s \equiv \mathrm{sn}(Kz/d, k)$, the modulus k of the elliptic integral K is given in terms of the magnetic field strength h by the formula

$$K^2(k) = (\pi/2)^2 h/h_u \tag{4}$$

and $h_u = (\pi/2)^2\alpha/d^2$ is the effective unidirectional anisotropy field.

The set of dynamical equations for $\mu(z,t)$ can be linearized, but the term $\mu_y h_x(t)$ responsible for the parametric excitation remains. Then under the condition $|h| \ll |4\pi m_x|$ the equations can be reduced (much as was done in Ref. 5) to a single equation for μ_y, which, after the substitution $\mu_y = m_x v$, takes the form

$$\frac{\partial^2 v}{\partial\eta^2} - m_x\left(1+\frac{h_{x0}}{h}\cos\omega t\right)v - \frac{1}{\omega_M\omega_H}\frac{\partial^2 v}{\partial t^2} - \frac{\xi}{\omega_H}\frac{\partial v}{\partial t} = m_y\frac{h_{x0}}{h}\cos\omega t, \tag{5}$$

with the boundary conditions

$$v|_{\eta=0}=0, \quad \left.\frac{\partial v}{\partial\eta}\right|_{\eta=K}=0, \tag{6}$$

where $\eta = Kz/d$, $\omega_M = 4\pi gM$, $\omega_H = gMh$, and h_{x0} is the amplitude of the high-frequency field h_x of frequency ω.

It is evident that when $h < h_u$ (so that $m_y = 0$) only the parametric resonance, whose minimum threshold corresponds to the frequency $\omega/2$, can be excited, but that when $h > h_u$, not only the parametric resonance, but also the ordinary magnetic resonance at the exciting frequency (or at multiples of it) can be excited.

Considering only the fundamental modes, we shall seek

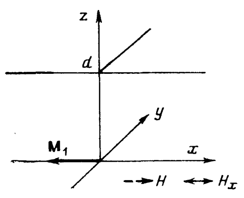

FIG. 1.

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a solution in the form

$$v(\eta,t)=a_1(\eta)e^{i\omega t/2}+a_1^*(\eta)e^{-i\omega t/2}+a_2(\eta)e^{i\omega t}+a_2^*(\eta)e^{-i\omega t}+c(\eta), \quad (7)$$

where $a_1 = a + ib$ and $a_2 = a' + ib'$. We take the function a_1 independent of time since we seek only the limits of the instability region.

On substituting (7) into (5) we obtain two independent sets of equations which, by ordinary linear processes, can be put in the form

$$\frac{\partial^2 a}{\partial\eta^2}+\left[\frac{(\omega/2)^2}{\omega_M\omega_H}-m_x(1+\nu)\right]a+\frac{\xi\omega}{2\omega_H}b=0,$$
$$\frac{\partial^2 b}{\partial\eta^2}+\left[\frac{(\omega/2)^2}{\omega_M\omega_H}-m_x(1-\nu)\right]b-\frac{\xi\omega}{2\omega_H}a=0; \quad (8)$$

$$\frac{\partial^2 a'}{\partial\eta^2}+\left(\frac{\omega^2}{\omega_M\omega_H}-m_x\right)a'+\frac{\xi\omega}{\omega_H}b'-m_x\nu c=m_y\nu.$$
$$\frac{\partial^2 b'}{\partial\eta^2}+\left(\frac{\omega^2}{\omega_M\omega_H}-m_x\right)b'-\frac{\xi\omega}{\omega_H}a'=0, \quad (9)$$
$$\frac{\partial^2 c}{\partial\eta^2}-m_x c-2m_x\nu a'=0.$$

Here $\nu = h_{x0}/2h$ and all the functions of η satisfy the boundary conditions (6).

Let us consider Eqs. (8), which describe the parametric resonance. When $h \leq h_u$, these equations have constant coefficients. On substituting a solution of the form

$$a=A_L\sin\varkappa_m\eta, \quad b=B_L\sin\varkappa_m\eta,$$

into Eqs. (8), where $\varkappa_m^2 = (2m-1)^2 h_u/h$, we find that the region in which the parametric resonance exists is defined by the formulas

$$\omega/2=[(h_u/h-1)\omega_M\omega_H]^{1/2}\pm\varepsilon/2,$$
$$\varepsilon=[\nu^2\omega_M\omega_H/(\varkappa_m^2-1)-\omega_M^2\xi^2]^{1/2}, \quad (10)$$

and that the parametric-resonance threshold is given by

$$h^{\text{thr}}=4\xi(\pi h_u)^{1/2}[(2m-1)^2-h/h_u]^{1/2}. \quad (11)$$

When $h \geq h_u$, Eqs. (8) constitute a set of two coupled Lamé equations. When $\xi = 0$ and $\nu = 0$, these equations reduce to a single equation, which was investigated in Ref. 5. Its solutions are the Lamé polynomials[7] $\mathrm{Ec}_1^{2m-1}(\eta)$; for $m = 1$ we have $\mathrm{Ec}_1^1(\eta) = s$, and the dispersion equation for the first resonance mode becomes

$$(\omega/2)^2=\omega_M\omega_H k^2. \quad (12)$$

When $\xi = 0$ but $\nu \neq 0$ we have two uncoupled equations, which differ in the sign of ν and have the form

$$\frac{\partial^2 a}{\partial\eta^2}+\left[\frac{(\omega/2)^2}{\omega_M\omega_H}+1+\nu-n(n+1)k^2s^2\right]a=0, \quad (13)$$

the degree n of the Lamé function $\mathrm{Ec}_n^{2m-1}(\eta)$ being determined by the formula

$$n(n+1)=2(1+\nu). \quad (14)$$

It is evident that n is not always an integer when $\nu \neq 0$, and for the case $\nu \ll 1$ of interest to us, we have $n \approx 1 + \frac{2}{3}\nu$. The Lamé function Ec_n^{2m-1} for noninteger n can be represented[8] by an infinite series of odd powers of s, which converges when $|ks| < 1$. We shall seek the solution to Eqs. (8) with $\nu \neq 0$ and $\xi \neq 0$ in the form of such series:

$$a(\eta)=\sum_{r=0}^{\infty}A_{2r+1}s^{2r+1}, \quad b(\eta)=\sum_{r=0}^{\infty}B_{2r+1}s^{2r+1}. \quad (15)$$

On substituting (15) into (8) and neglecting terms of order ν^2 and higher we obtain an infinite set of algebraic equations for the A_i and B_i, which consists of the equations

$$\left[\frac{(\omega/2)^2}{\omega_M\omega_H}-k^2+\nu\right]A_1+6A_3+\frac{\xi\omega}{2\omega_H}B_1=0,$$
$$2\nu k^2A_1+8(1+k^2)A_3-20A_5=0, \quad (16)$$
$$(2r-2)(2r+1)k^2A_{2r-1}-[(2r+1)^2-1](1+k^2)A_{2r+1}$$
$$+(2r+2)(2r+3)A_{2r+3}=0 \text{ when } r\geq 2$$

and analogous equations obtained by making the substitutions $A_i \rightleftarrows B_i$, $\nu \to -\nu$, and $\xi \to -\xi$.

The first of Eqs. (16) yields

$$\frac{A_3}{A_1}=-\frac{\nu k^2}{4(1+k^2)-10A_5/A_3},$$

and we express the quantity A_5/A_3 in the denominator in terms of the quantity A_7/A_5 obtained in a similar way from the third equation. Continuing this process, we finally obtain for A_3/A_1 a nonterminating continued fraction. We substitute the expression obtained for A_3/A_1 in the first equation (and proceed analogously for B_3/B_1 in the second group), and obtain for the system

$$\left[\frac{(\omega/2)^2}{\omega_M\omega_H}-k^2+\nu(1-R)\right]A_1+\frac{\xi\omega}{2\omega_H}B_1=0,$$
$$-\frac{\xi\omega}{2\omega_H}A_1+\left[\frac{(\omega/2)^2}{\omega_M\omega_H}-k^2-\nu(1-R)\right]B_1=0. \quad (17)$$

The continued fraction

$$R(k^2)=\cfrac{3k^2/2(1+k^2)}{1-\cfrac{c_2}{1-\cfrac{c_3}{1-\ldots}}}, \quad (18)$$
$$c_n=(2n+1)^2k^2/[(2n+1)^2-1](1+k^2)^2, \quad n=2,3,\ldots.$$

in (17) can be easily tabulated ($0 \leq R(k^2) \leq 1.64$ when $0 \leq k^2 \leq 1$).

The region in which the parametric resonance exists is now defined by the formulas

$$\omega/2=k(\omega_M\omega_H)^{1/2}\pm\varepsilon/2,$$
$$\varepsilon=[\nu^2\omega_M\omega_H k^{-2}(1-R)^2-\xi^2\omega_M^2]^{1/2}. \quad (19)$$

The threshold is given by

$$h^{\text{thr}}/4\xi(\pi h_u)^{1/2}=2kK(k)(1-R)^{-1}/\pi, \quad (20)$$

where the dependence of k and K on the field h/h_u is determined by (4), from which it follows that $k^2 \approx 2(h/h_u - 1)$ when $h \gtrsim h_u$. The dependence of the threshold $h^{\text{thr}}/4\xi(\pi h_u)^{1/2}$ on the constant magnetic field h/h_u is shown graphically in Fig. 2 (curve 1). The dependence of the square of the normalized frequency on the constant mag-

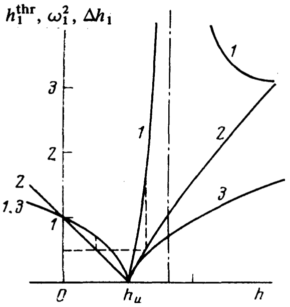

FIG. 2. The parametric resonance threshold $h_1^{th} = h^{thr}/4\xi(\pi h_u)^{1/2}$ (1), the first resonant mode $\omega_1^2 = \omega^2/\omega_M gMh_u$ (2), and the magnetic resonance line width $\Delta h_1 = \Delta h/4\xi(\pi h_u)^{1/2}$ (3) vs the stationary field strength.

netic field is also shown in the figure (curve 2) to make it possible to determine the threshold fields for any frequency. The dashed lines on the figure show how the threshold fields may be found for the frequency $\omega^2/\omega_M gMh_u = 0.5$. It will be seen that when $h > h_u$, i.e., when the film is nonuniformly magnetized, the parametric-resonance excitation threshold at a given frequency is much higher than the corresponding threshold for a uniformly magnetized film ($h < h_u$) and becomes infinite at $h/h_u = 1.61$ ($k^2 = 0.64$). As h/h_u increases further, $|h^{thr}|$ reaches a minimum at $h/h_u \approx 3$, the minimum value being ~3 in the units used in the figure.

It is desirable to compare the behavior of the parametric excitation threshold as a function of the field strength h with the corresponding behavior of the line width and susceptibility for the linear magnetic resonance in the same situation. It is known that in uniformly magnetized specimens the threshold is proportional to the line width; this is a simple expression of the energy balance (see, e.g., Ref. 1). To find the shape and width of the ordinary magnetic resonance line in this situation we turn to Eqs. (9).

It is evident from the third of Eqs. (9) that $c \sim \nu a'$, and hence that the term $m_x \nu c$ should be dropped from the first equation; and since a' and b' are of the order of ν, $c \sim \nu^2$ and the third equation itself may be dropped. This leaves a set of two equations, which is inhomogeneous when $h > h_u$. We may seek the solution of the inhomogeneous set as a series of the solutions $\mathrm{Ec}_1^{2m-1}(\eta)$ of the corresponding homogeneous set. Retaining only the first term of the expansion for the first resonant mode (this is justified if the magnetic resonance line is narrow enough), we obtain

$$a' = A_1' s, \quad b' = B_1' s. \tag{21}$$

We evaluate the coefficeints A_i' and B_i' by multiplying Eqs. (9) (without c) by $\mathrm{Ec}_1^1(\eta) = s$ and integrating over η from zero to K; this gives

$$\begin{aligned} A_1' &= \frac{\pi k \nu}{2D} \frac{(\omega^2 - k^2\omega_M\omega_H)\,\omega_M\omega_H}{(\omega^2 - k^2\omega_M\omega_H)^2 + (\omega\omega_M\xi)^2}, \\ B_1' &= \frac{\pi k \nu}{2D} \frac{\omega\omega_M^2\omega_H\xi}{(\omega^2 - k^2\omega_M\omega_H)^2 + (\omega\omega_M\xi)^2}, \end{aligned} \tag{22}$$

where $D = (K - E)/k^2$ and E is the complete elliptic integral of the second kind. Then, taking into account the fact that $\mu_y = m_x \nu$ and $\mu_x = -m_y \nu$ (this follows from the conservation of $\mathbf{m}'$) and averaging μ_x and μ_y over the thickness of the film, we obtain the resonance susceptibilities $\chi''_{xx\,\mathrm{res}}$ and $\chi_{xy\,\mathrm{res}}$ for $h > h_u$:

$$\xi(4\pi h_u)^{1/2}\chi''_{xx\,\mathrm{res}} = \frac{\pi^3 k}{8K^2 D}, \tag{23}$$

$$\xi(4\pi h_u)^{1/2}\chi_{xy\,\mathrm{res}} = \frac{\pi^2}{4K^2 D}\left(1 - k\,\mathrm{Arsh}\frac{k}{k'}\right). \tag{24}$$

These susceptibilities vanish for $h < h_u$.

Salansky and Eruchimov[9] discussed the linear magnetic resonance induced by a high frequency field h_y in such an inhomogeneous system and obtained the following expression for $\chi''_{yy\,\mathrm{res}}$ for the first resonance mode:

$$\xi(4\pi h_u)^{1/2}\chi''_{yy\,\mathrm{res}} = \begin{cases} \dfrac{8}{\pi^2}\left(1 - \dfrac{h}{h_u}\right)^{-1/2}, & h < h_u, \\ \dfrac{\pi}{2kK^2 D}\left(1 - k\,\mathrm{Arsh}\dfrac{k}{k'}\right)^2, & h > h_u. \end{cases} \tag{25}$$

Equations (23), (24), and (25) are graphed in Fig. 3. (curves a, b, and c, respectively). We note that $\chi''_{yy\,\mathrm{res}}$ and $\chi_{xy\,\mathrm{res}}$ vanish at $h/h_u = 1.72$ ($k^2 = 0.69$).

The peculiar behavior of the resonance susceptibilities as functions of the field strength is due to the field-strength dependence of the static-magnetization distribution $\mathbf{m}(z)$. When $h < h_u$ the static magnetization has no component perpendicular to the high-frequency field h_x, so the susceptibilities χ_{xx} and χ_{xy} vanish. When $h > h_u$ the average of m_y over the thickness of the film first rises and then falls; and accordingly, we find that the susceptibility χ_{xx} first rises, and then falls.

The width of the linear magnetic resonance line is determined by the formula $\Delta\omega = \xi\omega_M$ for all orientations of the high frequency field, both when $h < h_u$ and when $h > h_u$. To find the line width Δh we use the approximation $\Delta h \approx (d\omega/dh)^{-1}\Delta\omega$, and obtain

$$\Delta h \approx (d\omega/dh)^{-1}\Delta\omega:$$
$$\Delta h = \begin{cases} 4\xi(\pi h_u)^{1/2}(1 - h/h_u)^{1/2}, & h < h_u, \\ 4\xi(\pi h_u)^{1/2}(2/\pi)\,kK(K - D)/E, & h > h_u. \end{cases} \tag{26}$$

The h/h_u dependence of $\Delta h/4\xi(\pi h_u)^{1/2}$ is shown in Fig. 2 (curve 3). When $h < h_u$ the width of the linear magnetic resonance line coincides with the parametric-resonance excitation threshold. When $h > h_u$ the threshold exceeds the line width and tends to infinity at the values of h at which $\chi''_{xy\,\mathrm{res}}$ vanishes. This is reasonable since both $\chi''_{yy\,\mathrm{res}}$ and the interaction of the magnetization with the high frequency field, which is responsible for the parametric excitation, are connected with μ_y.

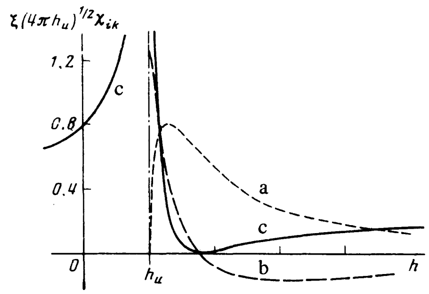

FIG. 3. The resonance susceptibilities χ''_{xx} (a), χ_{xy} (b), and χ''_{yy} (c) vs the stationary field strength.

Thus, in the system under consideration the nonuniformity of the magnetization distribution at certain values of the stationary field strength sharply limits the possibility of exciting parametric oscillations.

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Bragg diffraction of 14.1-keV resonant γ quanta by a mosaic α-$^{57}Fe_2O_3$ crystal in an oblique magnetic field

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The energy dependence of the intensity of the scattering of 14.4-keV resonant γ radiation is measured for all even (from the second through the tenth) orders of the Bragg reflection ($2n\ 2n\ 2n$) from a single crystal of hematite placed in an oblique magnetic field. The experimental results are compared with calculations performed for models of a mosaic and an ideal crystal.

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We have previously[1,2] observed and investigated systematically the polarization-induced dependences of the interference between nuclear-resonance and electron Rayleigh scattering on the scattering angle, on Δm of the nuclear transition, and on the direction of the magnetic field at the iron nuclei. We measured in these studies the energy dependences of the scattering of 14.4-keV resonant γ radiation of ^{57}Fe for all even (from the second through the tenth) orders of the Bragg reflection from a single crystal of hematite (α-Fe_2O_3) at two mutually perpendicular directions of the external magnetic field applied to the crystal. The experimental curves were compared with the theoretical calculations based on the model of an ideal crystal. The agreement with experiment was good.

The obtained curves turned out to be very sensitive to the orientation of the external magnetic field. It was therefore of interest to perform the measurements at an intermediate orientation of the magnetic field. This case is of particular interest because, in contrast to the heretofore considered simpler cases,[1,2] the π and σ polarizations of the incident beam are no longer the natural ones. Namely, when and if the incident radiation is, say, π-polarized, the scattered radiation contains γ quanta with the other polarization (σ). The polarization picture is here therefore greatly complicated.

The measurments were performed with a Mössbauer diffractometer[3] at room temperatures. The γ rays from a single-line Mössbauer source (^{57}Co in Cr, 200 μCi) were incident on a single crystal of α-Fe_2O_3 (85% ^{57}Fe, mosaic angle ~30") placed in one of the positions of the symmetrical Bragg scattering ($2n\ 2n\ 2n$). The crystal was in a magnetic field ~1 kOe situated in the plane of the crystal and making an angle 45° with the scattering plane (Fig. 1).

Figure 2 shows the measured dependences of the integrated intensity of the reflection of 14.4-keV quanta on the source velocity for all the existing orders of the reflection. As already mentioned, the polarization picture under these conditions becomes much more complicated. It turns out that the general system of dynamic equations that describe the propagation of the γ quanta in the crystal[4] no longer breaks up into two independent subsystems for the π and σ polarizations of the incident radia-

