Stabilizing action of high frequency fields in orientational transitions

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Just as the upper equilibrium position of a pendulum can be made stable by vibrating the point of its suspension, high-frequency fields can be used to stabilize one symmetry or another of the spontaneous polarization of a medium. In phase transitions of the "order-order" type, the effect should be revealed by the appearance of a hysteresis shift of the transition point. Attention is called in this paper to the possibility of appreciably enhancing the stabilizing action of the field if the polarization-oscillation mode is of the activating type. The effect is traced with spin-orientational transition in a magnet as an example.

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1. It is known that a force that oscillates rapidly in time can transform an unstable equilibrium state of a system into a stable one. Thus, by vibrating the suspension point of a pendulum it is possible to stabilize its upper equilibrium position. Something of the sort is typical also of the spin, i. e., for the system $\dot{\mathbf{m}}=g[\mathbf{m}\times\mathbf{H}]$ in an alternating field $\mathbf{H}(t)$. For example, in a circularly polarized field $H(t)$ of frequency $\Omega \gg gH$ the quantity $\mathbf{m}(t)$ behaves literally as if a constant field of magnitude $gH^2/2\Omega$ is applied along the circulation axis.

An interesting object in which these known dynamic effects can occur is a solid near an "order–order" phase transition, i. e., when the spontaneous polarization of the medium (electric, magnetic, antiferromagnetic, elastic, etc.) has a finite value in both phases, and the transition is connected with a change of its symmetry. An alternating field can be used to stabilize one polarization symmetry or another, and the critical point should experience a shift with hysteresis (just as in a constant field).

It is important, however, that in these transitions, as the polarization-oscillation frequencies decrease, their interaction with the acoustic mode of the spectrum usually manifests itself. As a result the velocity of the long-wave sound decreases to zero, and the polarization-oscillation mode has activation at the critical point (see Fig. 1). Under conditions of such an interweaving of the polarization oscillations with the others, the stabilizing action of the field is not so evident.

We shall verify below, using as an example a spin-orientational transition in a magnetically ordered substance, that this effect does take place and its magnitude even increases when the alternating-field frequency is close to the frequency Ω_0 of the orientational gap at the critical point.

In magnets, the gap in the spin-oscillation spectrum is due primarily to magnetoelastic interactions.[1,2] The abrupt decrease of the speed of sound in magnets near the magnetic-reorientation points, being of interest both as a research tool and for applications, has been investigated experimentally (see, e.g.,[2–5]) and theoretically (see, e.g.,[6–12]) under conditions that are constant in time. When estimating the influence of an alternating field, we shall disregard effects due to heating and redistribution of the heat in the system. In this approximation, the analysis can be carried out directly on the basis of macroscopic models. For the example under considerations, these are well known and will serve as the basis here.

2. Consider a ferromagnet described by two interacting subsystems, magnetic with uniaxial anisotropy and elastic isotropy, with energy

$$\begin{gathered}\mathscr{H}=\mathscr{H}_m+\mathscr{H}_u+\mathscr{H}_{mu},\\ \mathscr{H}_m=\int\left(\frac{1}{2}\alpha_{ik}\frac{\partial\mathbf{m}}{\partial x_i}\frac{\partial\mathbf{m}}{\partial x_k}-\frac{1}{2}\beta(\mathbf{mn})^2-\mathbf{mh}\right)dv,\\ \mathscr{H}_u=\int\left(\frac{1}{2}\rho\dot{u}^2+\frac{1}{2}\lambda(u_{ii})^2+\mu u_{ik}^2\right)dv,\\ \mathscr{H}_{mu}=\gamma\int m_i m_k u_{ik}\,dv.\end{gathered}\tag{1}$$

Here $\mathbf{m}(\mathbf{r}, t)$ is the magnetization density at the point $\mathbf{r}=(x_1, x_2, x_3)$ at the instant t, α_{ik} are exchange integrals (the matrix α_{ik} is positive-definite), β is the uniaxial-anisotropy constant, $\mathbf{n}$ is a unit vector along the anisotropy axis, $\mathbf{h}$ is the external magnetic field, ρ is the density of the medium, λ and μ are Lamé coefficients, $u_{ik}(\mathbf{r}, t)$ is the strain tensor, and γ is the magnetoelastic constant.

In model (1) we have $|\mathbf{m}|=m=\text{const}$, i. e., it is assumed that $T \ll T_c$, where T_c is the Curie point. Nor is account taken of the magnetic-dipole interaction. These approximations are typical of models usually resorted to for the description of spin-orientational transitions under conditions of constant external fields (and pressures) in antiferromagnets (see, e.g.,[11,12]). In contrast to (1), the magnetic system is simulated here by two magnetic sublattices. In order not to clutter up the exposition, we consider the case of sublattice, since this case preserves the main features of the action of the alternating field in the transition; we shall discuss antiferromagnets briefly in the conclusion.

Let $h=0$. The ground state of the system, determined from the condition that the energy (1) be a minimum, is known[3,4] corresponds to a homogeneous magnetization m_0 and to homogeneous static deformations $(u_{ik}^0=u_{ii}^0\delta_{ik})$

$$u_{11}^0=-\frac{\lambda+\mu}{3\lambda+2\mu}\frac{\gamma m^2}{\mu},\quad u_{22}^0=u_{33}^0=\frac{\lambda}{3\lambda+2\mu}\frac{\gamma m^2}{2\mu}.$$

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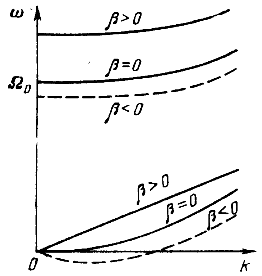

FIG. 1. Variation of polarization-oscillation spectrum (upper curve) and of the acoustic-branch oscillations (lower curve), which interact with the polarization oscillations, near the transition. At the critical point $\beta=0$ the acoustic branch has a horizontal segment, and then at $\beta<0$ its long-wave excitations become unstable and the system relaxes to another equilibrium phase.

Here (and hereafter throughout), the direction $\mathbf{m}_0$ is chosen to be the axis 1. At $\gamma>0$, the ferromagnet is compressed in a direction parallel to $\mathbf{m}_0$ and is stretched in the transverse plane. At $\beta>0$ the equilibrium magnetization is aligned along the $\mathbf{n}$ axis, and at $\beta<0$ the vector $\mathbf{m}_0$ lies in the basal plane $\mathbf{m}_0\perp\mathbf{n}$.

At the critical point $\beta=0$, $\mathbf{m}_0$ is in an indifferent equilibrium position. In this case the frequency Ω_0 of the homogeneous $\mathbf{m}$ oscillations that are not accompanied by deformation does not vanish (if $\gamma\neq 0$). These oscillations initiate a spin-wave mode with a dispersion law (at $\mathbf{m}_0\parallel\mathbf{n}$)

$$\Omega(\mathbf{k})\approx gm(\varkappa+\beta+\alpha_{ij}k_ik_j), \tag{2}$$

where

$$\varkappa=2\gamma(u_{22}{}^0-u_{11}{}^0)=\gamma^2m^2/\mu,$$

$\mathbf{k}=(k_1, k_2, k_3)$ is the wave vector of the wave. We note that $\alpha_{ij}k_ik_j\geq 0$.

At $\beta<0$ ($\beta>0$) the orientation $\mathbf{m}_0\parallel\mathbf{n}$ ($\mathbf{m}_0\perp\mathbf{n}$) becomes an unstable equilibrium position, and the first to increase are magnetoelastic waves with transverse polarization of the deformations ($\mathbf{u}\perp\mathbf{k}$) with $\mathbf{k}\to 0$, propagating along $\mathbf{m}_0$ and in the directions $\mathbf{k}\perp\mathbf{m}_0$. The dispersion law for these and the other waves at small k in the phase $\mathbf{m}_0\parallel\mathbf{n}$ is given by

$$\omega^2(\mathbf{k})\approx s_t^2k^2\frac{\beta+\alpha_{ij}k_ik_j}{\varkappa+\beta+\alpha_{ij}k_ik_j}, \tag{3}$$

where $s_t^2=\mu/\rho$ is the velocity of the transverse elastic waves at $\gamma=0$. It is seen from (3) that at $\beta=0$ the speed of sound $\partial\omega/\partial k$ vanishes, at $\beta<0$ the quantity $\omega^2(\mathbf{k})$ on a certain interval of k becomes negative (see Fig. 1), and instability sets in and leads to a change of the ground state. For oscillations about the ground state $\mathbf{m}_0\perp\mathbf{n}$ it is necessary to replace β in expressions (2) and (3) by $-\beta$.

3. We discuss now the action exerted on the system by a homogeneous alternating field $\mathbf{h}(t)$ with a frequency Ω much higher than the frequency of the normal oscillations $\omega(\mathbf{k})$ that lose stability in the transition. The amplitude h is assumed to be small, so that after the field is turned on the system relaxes to a homogeneous state, against the background of which small forced oscillations are produced with a period $2\pi/\Omega$. When $\mathbf{m}$ and $\mathbf{u}$ deviate from the equilibrium regime, additional forces of pulsation origin arise. They cause a renormalization of the speed of sound and a shift, in either direction, of the point where the speed of sound vanishes and the equilibrium phase changes.

The problem consists of investigating the averaged action of the alternating field and of the oscillations of $\mathbf{m}$ and of $\mathbf{u}$, with frequencies $\sim\Omega$, on the dynamics of the smooth forms of motion, whose stability is greatly weakened near the transition. Such motions are the already mentioned transverse-sound waves with small k. To estimate the effect in first-order approximation in the small quantity h^2, to which we confine ourselves, we can propose the following rather simple and physically meaningful analysis procedure.

The direct action of the field is determined by the energy $-\mathbf{m}\cdot\mathbf{h}$ in (1). Since the field $\mathbf{h}(t)$ is homogeneous, it follows that the magnetoelastic waves are connected with the field only by the quadratic (in u) part of this energy, which is contained in $-m_1h_1\approx(1/2m)(m_2^2+m_3^2)h_1-mh_1$, where $h_1(t)$ is the field component along the axis 1, i.e., in the direction of $\mathbf{m}_0$. It is easily found from (1) that for magnetoelastic waves with $k\to 0$ ($s_t^2k^2\ll\Omega_0^2$) the polarization of $\mathbf{m}$ and $\mathbf{u}$ is such that in the phase $\mathbf{m}_0\parallel\mathbf{n}$ we have

$$m_2\approx-\frac{2\gamma m}{\varkappa+\beta}u_{12},\quad m_3\approx-\frac{2\gamma m}{\varkappa+\beta}u_{13}. \tag{4}$$

The considered binding energy with the field is therefore

$$\mathscr{H}_d\approx h_1\frac{2\gamma^2m^2}{(\varkappa+\beta)^2}\int(u_{12}{}^2+u_{13}{}^2)\,dv. \tag{5}$$

For the oscillations in the phase $\mathbf{m}_0\perp\mathbf{n}$ it is necessary to replace β in expressions (4) and (5) by $-\beta$. We note that proximity to the transition point will be understood in the sense of $|\beta|\ll\varkappa$.

The energy (5) introduces into the equation for the complex amplitudes of the waves $b_k(t)$ a term $\sim h_1$:

$$\dot{b}_\mathbf{k}=-i\omega_\mathbf{k}b_\mathbf{k}+A(\mathbf{k})h_1b_\mathbf{k}+\ldots, \tag{6}$$

where $A(\mathbf{k})$ is a coefficient. The symbol ... stands for nonlinear proportional to h^2 and due to the contribution from the energy $\mathscr{H}_{mu}$ in (1), principal among which is $i\delta\mathscr{H}_{\rm int}/\delta b_\mathbf{k}^*$, where $\mathscr{H}_{\rm int}$ is defined in (9). Let $h_1(t)=h_0\cos\Omega t$, where $\Omega\gg|\omega_\mathbf{k}|$. In the first-order approximation in h we have $b_\mathbf{k}^{(1)}\approx\Omega^{-1}A(\mathbf{k})h_0\bar{b}_\mathbf{k}\sin\Omega t$, where $\bar{b}_\mathbf{k}(t)$ is the smooth part of $b_\mathbf{k}(t)$, with frequencies much lower than Ω. Therefore in the second order approximation in h the value averaged over the time $2\pi/\Omega$ is

$$\overline{h_1b_\mathbf{k}}\approx\frac{1}{\Omega}A(\mathbf{k})h_0{}^2\bar{b}_\mathbf{k}\overline{\sin\Omega t\cos\Omega t}=0,$$

This means that in first order in h^2 the field $\mathbf{h}(t)$ does not influence directly the dynamics of $\bar{b}_\mathbf{k}(t)$ of the soft modes. This can be deduced also more rigorously. Consider the indirect influence—via nonlinear interaction of the waves.

4. The binding energy of the normal waves is contained in the term $\mathcal{H}_{mu}$ in (1). It contains terms of third and fourth degree in the wave amplitudes; these terms are equal to

$$\mathcal{H}_3=\gamma\int(m_2^2u_{22}+m_3^2u_{33}-(m_2^2+m_3^2)u_{11})\,dv, \tag{7}$$

$$\mathcal{H}_4=-\frac{\gamma}{m}\int(m_2^2+m_3^2)(m_2u_{12}+m_3u_{13})\,dv. \tag{8}$$

In the first order in h, the homogeneous field $\mathbf{h}(t)$ can excite only the mode of homogeneous precession of $\mathbf{m}$. The oscillations of other forms come into play in the second order in h, and accordingly their averaged action on the transverse-sound waves can manifest itself only in the higher approximations in h.

Consequently, out of the entire reservoir of interacting waves, it suffices to take into account only the interaction of soft modes with homogeneous precession, and this simplifies the analysis greatly. Taking (4) into account, we find that the main contribution is determined by the energy $\mathcal{H}_4$ and amounts to

$$\mathcal{H}_{int}=\frac{2\gamma^2}{\varkappa+\beta}\int[(3M_2^2+M_3^2)u_{12}^2+(3M_3^2+M_2^2)u_{13}^2+4M_2M_3u_{12}u_{13}]dv. \tag{9}$$

Here $M_{2,3}(t)$ are the amplitudes of the oscillations of the homogeneous precession. We have written out terms up to the second power in the deformation u_{ik}; this is sufficient for an analysis of the stability in the small.

In terms of the canonical variables, the most significant in $\mathcal{H}_{int}$ is the term with structure $\sim c_0^*c_0b_k^*b_k$, where $c_0(t)$ is a variable that represents the homogeneous-precession mode. We note that in the expansion of the nonlinear part of the magnet energy in terms of the canonical variables of the waves, the typical terms are usually those linear in the small amplitudes b_k of the magnetoelastic waves (see[14]). In our case, however, the energy $\mathcal{H}_{mu}$ does not contain such terms for the soft modes (for the "hard" waves, such as e.g., longitudinal-sound waves, such a part is contained in $\mathcal{H}_3$). The fact that the structure of the coupling of the soft modes with other, "hard" ones is bilinear in b_k is no accident, and reflects a characteristic feature of phase transitions of second order and of first order close to second order. The reason is that the soft modes have a symmetry different from the other modes, and this symmetry is connected with the transition to the other phase.

The averaging defined in (9) over the time $2\pi/\Omega$ in the case of a nonlinear coupling of the motions, is carried out in elementary fashion: in the first-order approximation in h^2 it is necessary simply to replace the quantities M_2^2, M_3^2, and M_2M_3 in (9) by their mean values. The averaged behavior of the small-amplitude transverse sound waves with $k\to 0$ is thus described by the energy

$$\overline{\mathcal{H}}\approx\mathcal{H}_u-2\mu\frac{\varkappa}{\varkappa+\beta}\int(u_{12}^2+u_{13}^2)\,dv+2\mu\frac{\varkappa}{\varkappa+\beta}\int(\varepsilon_1u_{12}^2+\varepsilon_2u_{13}^2+\varepsilon_3u_{12}u_{13})\,dv, \tag{10}$$

where

$$\varepsilon_1=\frac{1}{m^2}(3\overline{M_2^2}+\overline{M_3^2}),\quad \varepsilon_2=\frac{1}{m^2}(3\overline{M_3^2}+\overline{M_2^2}),\quad \varepsilon_3=\frac{4}{m^2}\overline{M_2M_3}.$$

The second term from the right in (10) is due to the linear magnetelastic interaction. Expression (10) corresponds to the phase $\mathbf{m}_0\parallel\mathbf{n}$. In the phase $\mathbf{m}_0\perp\mathbf{n}$, in addition to the reversal of the sign of β, there appears also a small correction $\sim\beta$ in the second term of (10).

5. The quadratic form $\varepsilon_1u_{12}^2+\varepsilon_2u_{13}^2+\varepsilon_3u_{12}u_{13}$ is, as can be readily verified, positive-definite. This means that the hardness of the soft modes due to the action of the alternating field can only increase, regardless of the frequency and polarization of the field $\mathbf{h}(t)$. The change of the hardness is proportional to the intensity of the excitation of the $\mathbf{m}$ homogeneous-precession mode by the field and increases resonantly at $\Omega\sim\Omega_0$.

According to (10) the soft waves are those having the same direction as at $h=0$. The renormalization of their velocity Δs under the action of a linearly polarized field $\mathbf{h}\perp\mathbf{m}_0$ at $|\Omega-\Omega_0|\ll\Omega+\Omega_0$ is equal to

$$\frac{\Delta s}{s_t}\approx\frac{|\Delta\beta|}{\varkappa}\approx\frac{g^2\overline{h^2}}{(\Omega_0-\Omega)^2+\Gamma^2}, \tag{11}$$

where $s_t=(\mu/\rho)^{1/2}$, and $\Delta\beta$ is the shift of the transition point with respect to the constant β. We have written out the result with account taken of linear damping in the model, and Γ is the relaxation frequency of the homogeneous ferromagnetic resonance. We note that allowance for the damping of the soft modes, if their characteristic relaxation frequencies are much less than the frequencies Ω and Ω_0, does not affect the shift of the transition point.

The region where the obtained estimates are valid corresponds to sufficiently small amplitudes h, such that[1)]

$$|\Delta\beta|\ll\Omega/gm \text{ and } |\Delta\beta|\ll\varkappa.$$

We note that the case $\varkappa=0$ (i.e., $\gamma=0$) differs strikingly from that considered (the shift of the reorientation point under the influence of the alternating field for models of type (1) at $\gamma=0$ was considered in[15]); there is no resonance effect, there is no stabilization at all in a linearly polarized field, and even destabilization is possible if the circulation axis of the field $\mathbf{h}(t)$ is antiparallel to $\mathbf{m}_0$.

Figure 2 shows the variation of the equilibrium phase following the application of a linearly polarized field $\mathbf{h}\parallel\mathbf{n}$ (a) and $\mathbf{h}\perp\mathbf{n}$ (b). If the transition is approached from the "easy axis" phase ($\beta>0$), then this phase re-

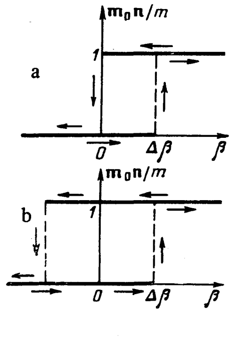

FIG. 2. Variation of equilibrium orientation $\mathbf{m}_0$: a) $\mathbf{h}\parallel\mathbf{n}$, b) $\mathbf{h}\perp\mathbf{n}$.

mains stable (metastable) even past the point $\beta=0$ up to a value $\beta=-|\Delta\beta|$. On the other hand if the transition is approached from the "easy plane" side ($\beta<0$), then the phase $\mathbf{m}_0\perp\mathbf{n}$ remains metastable up to the point $\beta=|\Delta\beta|$. The difference between the behaviors in cases a and b is due to the fact that at $\mathbf{h}\parallel\mathbf{n}$ the field $\mathbf{h}$ in the phase $\mathbf{m}_0\parallel\mathbf{n}$ is longitudinal and therefore $\Delta\beta=0$, while at $\mathbf{h}\perp\mathbf{n}$ we have $\mathbf{h}\perp\mathbf{m}_0$ both in the phase $\mathbf{m}_0\parallel\mathbf{n}$ and in the phase $\mathbf{m}_0\perp\mathbf{n}$ (in the latter case the orientation $\mathbf{h}\perp\mathbf{m}_0$ is regarded as preferred, since it is stabilized by the field).

As the stability-loss points are approached, the speed of sound decreases rapidly, and the susceptibility and fluctuations increase as a result, thereby contributing to the jump to the ground state. Reverse jumps should be less frequent near these points, since the decrease in the speed of sound is smaller in the ground state (the difference Δs is of the order $\varkappa^{-1}|\Delta\beta|s_t$), and accordingly the fluctuations are smaller and the stability margin larger.

Thus, superposition of an alternating field deforms the character of the transition: a region of metastable phases appear, the transition is subject to hysteresis, and the fluctuation level is limited. A typical picture of a first-order transition is obtained.

6. The effect is similarly analyzed for orientational transitions in a system with two magnetic sublattices. The direct action of a homogeneous alternating field on the dynamics of magnetoelastic waves whose propagation velocity changes abruptly (these can now be also waves with a longitudinal deformation component) is negligible in first order in h^2. To estimate the effect it is necessary to separate in the system energy the nonlinear part of the interaction between the sound and the modes of the homogeneous ferro- and antiferromagnetic resonance.

Consider, for example, a magnetically uniaxial antiferromagnet with mirror-symmetry magnetic sublattices (without allowance for the weak ferromagnetism) and with energy (1), where now $\mathcal{H}_m$ and $\mathcal{H}_{mu}$ are given by

$$\mathcal{H}_m=\int\Big[\frac{1}{2}\alpha_{ik}\Big(\frac{\partial\mathbf{m}_1}{\partial x_i}\frac{\partial\mathbf{m}_1}{\partial x_k}+\frac{\partial\mathbf{m}_2}{\partial x_i}\frac{\partial\mathbf{m}_2}{\partial x_k}\Big)+\alpha_{ik}'\frac{\partial\mathbf{m}_1}{\partial x_i}\frac{\partial\mathbf{m}_2}{\partial x_k}+\delta\mathbf{m}_1\mathbf{m}_2$$

$$-\frac{1}{2}\beta((\mathbf{m}_1\mathbf{n})^2+(\mathbf{m}_2\mathbf{n})^2)-\beta'(\mathbf{m}_1\mathbf{n})(\mathbf{m}_2\mathbf{n})-(\mathbf{m}_1+\mathbf{m}_2)(\mathbf{H}+\mathbf{h})\Big]dv, \qquad (12)$$

$$\mathcal{H}_{mu}=\frac{1}{2}\int(\gamma_1L_iL_ku_{ik}+\gamma_2M_iM_ku_{ik}+\gamma_3(M^2-L^2)u_{ii})\,dv. \qquad (13)$$

Here $\mathbf{L}=\mathbf{m}_1-\mathbf{m}_2$, $\mathbf{M}=\mathbf{m}_1+\mathbf{m}_2$, $\mathbf{m}_{1,2}$ are the sublattice magnetizations, $|\mathbf{m}_1|=|\mathbf{m}_2|=m$; α_{ik}, α_{ik}' and δ are exchange integrals, β and β' are the anisotropy constants, $\gamma_{1,2,3}$ are the magnetostriction constants, $\mathbf{H}$ is the constant field and $\mathbf{h}(t)$ is the alternating field.

Orientational transitions in such a model were considered in[12] at $h=0$ (and in the presence of a constant external pressure). If, for example, the field $\mathbf{H}$ is directed along the $\mathbf{n}$ axis and $\beta-\beta'>0$, then the collinear phase $\mathbf{L}\parallel\mathbf{n}$ is stable in the field interval $0<H<H_1$, and the noncollinear phase ($\mathbf{M}\neq0$) is stable in the interval $H_2<H<H_E$, with $\mathbf{L}\perp\mathbf{n}$ and $\mathbf{M}\parallel\mathbf{H}$ in this case. A state $\mathbf{L}=0$ is produced at $H>H_E$. The field H_E is usually strong ($\sim10^6$ Oe) and greatly exceeds the anisotropy and the magnetostriction fields, and

$$H_1\approx H_2\approx(H_EH_A)^{1/2},\quad H_E\approx2\delta m,$$

where $H_A=(\beta-\beta')m$.

We estimate now the effect of the alternating field at $H\approx(H_EH_A)^{1/2}$. The transition at this point is of first order and is close to second order at $H_A\ll H_E$. In this transition, the speed of the transverse sound with $\mathbf{k}\parallel\mathbf{L}$ and $\mathbf{k}\perp\mathbf{L}$ vanishes. As $k\to0$ in such waves, it can be easily verified from (12) and (13) that the oscillations of $\mathbf{M}$, $\mathbf{L}$, and $\mathbf{u}$ about the ground state are connected by the relations

$$L_2\approx-\frac{4\gamma_1m}{\varkappa_1+\beta_1}u_{12},\quad L_3\approx-\frac{4\gamma_1m}{\varkappa_1+\beta_1}u_{13},\quad M_{2,3}\approx\frac{H}{H_E}L_{2,3}, \qquad (14)$$

where

$$\varkappa_1=2\gamma_1(u_{22}^0-u_{11}^0)=\frac{2\gamma_1^2m^2}{\mu},\quad \beta_1=\frac{H_1^2-H^2}{mH_E}.$$ [2]

The main contribution to the nonlinear part of the coupling energy of the soft magnetoelastic modes with the inhomogeneous precession $\mathbf{m}_{1,2}$ is made by the term $\sim\gamma_1$ in (13). Taking (14) into account, we obtain for $\mathcal{H}_{\rm int}$ an expression of the form (9)

$$\mathcal{H}_{int}\approx\frac{\gamma_1^2}{\varkappa_1+\beta_1}\int[(3l_2^2+l_3^2)u_{12}^2+(3l_3^2+l_2^2)u_{13}^2+4l_2l_3u_{12}u_{13}]\,dv,$$

where $l_{2,3}$ are the components of the homogeneous oscillations of $\mathbf{L}$. As a result, we arrive after the averaging to expression (10), where $\varkappa$, β, and $\varepsilon_{1,2,3}$ must be replaced respectively by $\varkappa_1$, β_1, and

$$\varepsilon_1=\frac{1}{4m^2}(3\overline{l_2^2}+\overline{l_3^2}),\quad \varepsilon_2=\frac{1}{4m^2}(3\overline{l_3^2}+\overline{l_2^2}),\quad \varepsilon_3=\frac{1}{m^2}\overline{l_2l_3}.$$

Thus the picture of the deformation of the transition by the field $\mathbf{h}(t)$ is the same as in the previously analyzed case of a ferromagnet. Now, however, it must be recognized in the estimates of the intensities $\overline{l_{2,3}^2}$ and $\overline{l_2l_3}$ that two spin modes, rather than one, have now nonzero gaps at the transition point. Their activation energies are

$$\Omega_{1,2}\approx g[(H_E(H_A+H_\varkappa))^{1/2}\pm(H_EH_A)^{1/2}]$$

where $H_\varkappa=\varkappa_1m$. When the alternating-field frequency is tuned to either of these frequencies, the stabilizing action of the field will increase in resonant fashion. For the particular case when $\beta-\beta'=0$ (i.e., $H_{1,2}=0$) we have $\Omega_1=\Omega_2$ and we obtain exactly the estimate (11), where Ω_0 and Γ now characterize the homogeneous antiferromagnetic resonance.

We note in conclusion that not only in magnets but also in ferroelectrics and in other systems, in the vicinity of the "order–order" transition, an alternating field that excites oscillations of a spontaneous polarization whose symmetry changes during the transition, should lead to stabilization of the ground state and to the onset of the associated hysteresis. If the natural frequency of the polarization oscillations is different from zero at the transition point, then a resonant amplification of the effect is possible, and the alternating field stabilizes each time the initial symmetry of the system.

One can expect to observe this phenomenon in materials that are perfect in the sense that the resonances of the polarization oscillations is of high Q, and the transition is not masked by the domain structure (i.e., the strong decrease of the speed of sound in the vicinity of the transition is well pronounced).

1) In addition, in the analysis it was implicitly assumed that $h < h_p$, where h_p is the field amplitude at which parametric oscillations are produced in the system. At $\mathbf{h} \perp \mathbf{m}_0$ and $\Omega = \Omega_0$ the threshold of the parametric excitation of the spin waves is according to[13] $h_p \sim (\Delta H/4\pi m)^{1/2}\Delta H$, $\Delta H = \Gamma/g$. At $h = h_p$ we obtain from (11) $\Delta s/s_t \sim \Delta H/4\pi m$.

2) It is assumed for the sake of argument that the system is in a collinear phase, $\mathbf{L} \parallel \mathbf{n} \parallel 1$. For a noncollinear phase, choosing as before the axis 1 in the direction of the equilibrium value of $\mathbf{L}$, we have (14), where β_1 must be replaced by $\beta_2 = (H^2 - H_2^2)/mH_E$. All the calculations that follow must be correspondingly modified.

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Diffraction of electromagnetic waves by a domain wall in a ferroelectric

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An analytic solution is obtained of the problem of diffraction of an electromagnetic wave by a domain wall in a ferroelectric. It is shown that the picture of the interference fringes is observed only in diffraction of light of sufficiently small wavelength. The distance between the interference fringes is determined not only by the geometrical dimensions of the wall, but also by the difference between the values of the refractive index inside the wall and far from it.

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1. INTRODUCTION

We solve here the problem of diffraction of an electromagnetic wave by a ferroelectric domain wall. This problem arises in connection with the possibility of using the diffraction of light for a direct measurement of the domain-wall thickness. In addition, the solution of this problem is also of independent interest, since analytic solutions of diffraction problems encounter as a rule great mathematical difficulties.

The use of optical methods to measure domain-wall thicknesses is particularly pressing because of the substantial discrepancies that exist between the *a priori* theoretical estimates and the data obtained from x-ray scattering. Theoretical estimates lead as a rule to a domain-wall thickness on the order of 10^{-6}–10^{-7} cm. Yet measurements made on sodium nitrate[4] and triglycine sulfate[2] yield values larger than 10^{-5} cm.

The general solution of the diffraction problem poses no fundamental difficulties. The formulas for the diffracted-wave amplitudes in quadratures are derived in Sec. 2. An investigation of these formulas, however, for the purpose of deriving expressions useful to experimenters, entails great technical difficulties. This investigation is the subject of an appreciable part of the paper. In the last section we discuss the form of the diffraction pattern in various cases and the possibility of extracting from it information on the structure of the domain wall.

We consider a plane 180° domain wall in a cubic uniaxial ferroelectric, or one belonging to a rhombic system, and exhibiting no piezoelectric effect in the paraphase. The wall thickness is assumed to be much larger than the lattice structure, so that its structure is described by a phenomenological theory. The length of the electromagnetic wave is also assumed to be much larger

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