of the critical parameters (ζ_{cr}, $\bar{r}$, r_{max} etc.) on the proton density n_p.

[1]I. Pomeranchuk and Ya. Smorodinsky, J. Phys. USSR 9, 97 (1945).
[2]Ya. B. Zel'dovich and V. S. Popov, Usp. Fiz. Nauk 105, 403 (1971) [Sov. Phys. Usp. 14, 673 (1972)].
[3]S. Brodsky, Commun. Atom. Molec. Phys. 4, 109 (1973).
[4]L. B. Okun', Commun. Nucl. Part. Phys. 6, 25 (1974).
[5]V. S. Popov, Kvantovaya élektrodinamika v sil'nykh vneshnikh polyakh ($Z>137$) (Quantum electrodynamics in strong external fields ($Z>137$)). Proc. 3rd physics school of IETF, issue 1, Atomizdat, 1975, pp. 5–21.
[6]W. Pieper and W. Greiner, Z. Phys. 218, 327 (1969).
[7]V. S. Popov, Pis'ma Zh. Eksp. Teor. Fiz. 11, 254 (1970) [JETP Lett. 11, 162 (1970)]; Yad. Fiz. 12, 429 (1970) [Sov. J. Nucl. Phys. 12, 235 (1971)].
[8]G. Soff, B. Müller, and J. Rafelski, Z. Naturforsch, 29A, 1267 (1974).
[9]V. L. Eletskiĭ and V. S. Popov, Yad. Fiz. 25, 1107 (1977) [Sov. J. Nucl. Phys. 25, 587 (1977)].
[10]A. B. Migdal, Zh. Eksp. Teor. Fiz. 61, 2209 (1971); 63, 1993 (1972) [Sov. Phys. JETP 34, 1184 (1972); 36, 1052 (1973)].
[11]A. B. Migdal, O. A. Markin, and I. P. Mishustin, Zh. Eksp. Teor. Fiz. 66, 443 (1974); 70, 1592 (1976) [Sov. Phys. JETP 39, 212 (1974); 43, 830 (1976)].
[12]A. B. Migdal, Phys. Lett. 52B, 182 (1974).
[13]A. B. Migdal, O. A. Markin, I. P. Mishustin, and G. A. Sorokin, Zh. Eksp. Teor. Fiz. 72, 1247 (1977) [Sov. Phys. JETP 45, 654 (1977)].
[14]T. D. Lee and G. C. Wick, Phys. Rev. D9, 2291 (1974).
[15]T. D. Lee, Rev. Mod. Phys. 47, 267 (1975).
[16]A. B. Migdal, D. P. Voskresenskiĭ, and V. S. Popov, Pis'ma Zh. Eksp. Teor. Fiz. 24, 186 (1976) [JETP Lett. 24, 163 (1976)]; Zh. Eksp. Teor. Fiz. 72, 834 (1977) [Sov. Phys. JETP 45, 436 (1977)].
[17]V. L. Eletskiĭ and V. S. Popov, Pis'ma Zh. Eksp. Teor. Fiz. 24, 253 (1976) [JETP Lett. 24, 226 (1976)].
[18]V. S. Popov, V. L. Eletskiĭ, and V. D. Mur, Zh. Eksp. Teor. Fiz. 71, 856 (1976) [Sov. Phys. JETP 44, 451 (1976)].
[19]A. I. Akhiezer and V. B. Berestetskiĭ, Kvantovaya élektrodinamika (Quantum Electrodynamics), "Nauka", M., 1969.
[20]V. S. Popov, Zh. Eksp. Teor. Fiz. 59, 965 (1970); 65, 35 (1973) [Sov. Phys. JETP 32, 526 (1971); 38, 18 (1974)].
[21]A. Bohr and B. Mottelson, Nuclear Structure v. 1, W. A. Benjamin, N. Y., 1969.
[22]S. S. Gershtein and V. S. Popov, Lett. Nuovo Cimento 6, 593 (1973).
[23]M. S. Marinov and V. S. Popov, Pis'ma Zh. Eksp. Teor. Fiz. 17, 511 (1973) [JETP Lett. 17, 368 (1973)]; Zh. Eksp. Teor. Fiz. 65, 2141 (1973) [Sov. Phys. JETP 38, 1069 (1974)].
[24]M. Gyulassy, Phys. Rev. Lett. 33, 921 (1974); Nucl. Phys. A244, 497 (1975).
[25]R. P. Feynman, N. Metropolis, and E. Teller, Phys. Rev. 75, 1561 (1949).
[26]S. Kobayashi, T. Matsukuma, S. Nagai, and K. Umeda, J. Phys. Soc. Jpn. 10, 759 (1955).
[27]E. Fermi, Mem. Accad. Italia 1, 149 (1930) (Russ. Transl. E. Fermi, Scientific Works 1, "Nauka", M., 1971, p. 351).
[28]A. C. Coulson and N. H. March, Proc. Phys. Soc. London 63A, 367 (1950).
[29]E. Fermi and E. Amaldi, Mem. Accad. Italia 6, 119 (1934) (Russ. Transl. E. Fermi, Scientific Works 1, "Nauka", M., 1971, p. 542).
[30]G. I. Plindov and I. K. Dmitrieva, Dokl. Akad. Nauk B SSR 19, 788 (1975).

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Single-nucleon absorption of slow pions by atomic nuclei and π condensation

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The problem of the single-nucleon absorption of slow pions by atomic nuclei is solved. The presence of a pion condensate significantly increases the single-nucleon absorption probability. The measurement of the single-nucleon absorption probability may be a critical experiment for the elucidation of the question of the existence of a condensate in nuclear systems.

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1. INTRODUCTION

In 1971 Migdal pointed out the possibility of a reconstruction of the pion field in a sufficiently dense nucleon system, i.e., the formation of a "pion condensate." The main physical consequence of such a phase transition is the possibility in principle of the existence of abnormally dense nuclei.[1] In these nuclei the energy loss due to the change in the nucleon density is compensated by the energy gain from the phase transition. The quantitative theory developed by Migdal[2] led to a critical-density value (the density at which the phase transition occurs) of $n_c \gtrsim 0.6n_0$ for nuclear matter with $N=Z$ and $n_c \gtrsim 0.8n_0$ for a neutron material ($N \gg Z$), where n_0 is the normal nuclear density. This allowed the existence of a condensate in real atomic nuclei to be postulated. In subsequent investigations[3,4] the estimate for the critical density did not change in comparison with the estimate obtained in Migdal's first papers. A more exact computation of the critical density is not possible, since the quantity n_c critically depends on the phenomenological parameters of the theory. These parameters cannot be computed theoretically at present because of the lack of a theory of strong interactions. The values of these parameters found from the analysis of the experimental data on the properties of atomic nuclei are not accurate enough. Therefore, only a direct experiment can give an answer to the question of the existence of a condensate in atomic nuclei. It is important to note that, for any experimental result, the theory allows a more rigorous determination of the critical density and, thereby, a more definite indication of the region of existence of nuclei with an abnormal density.

The existence of a π condensate in nuclei is manifested in the formation in the spin-isospin density of the nucleons in the nuclei of a periodic structure with wave vector $k_0 \gtrsim p_F$ (p_F is the Fermi momentum). An analysis of the experimental data on electron and proton scattering by nuclei,[5] l-forbidden $M1$ transitions, magnetic moments, nuclear spectra,[6] and π-atomic spectra[7,8] does not contradict the possibility of the existence of a condensate in nuclei. However, we cannot unequivocally infer the existence of a condensate in atomic nuclei from this analysis. In view of this, the search for a critical experiment is very important. In our opinion, an experiment on the measurement of the probability for single-nucleon absorption of a slow pion by atomic nuclei can be such an experiment. Theoretical estimates made in Ref. 9 showed that the presence of a periodic structure in the spin-isospin density leads to a significant increase in the probability for single-nucleon absorption of slow pions. The cause of the increase consists in the following. In homogeneous nuclear matter single-nucleon absorption is strictly forbidden by the laws of conservation of energy and momentum. In the case when a π condensate exists in the nuclear matter, the nucleon momentum is not conserved because of rescattering on the periodic structure of the spin-isospin density, which removes the prohibition on the absorption of slow pions.

In the present paper we solve the problem of the single-nucleon absorption of slow pions by real nuclei. In Sec. 2 we consider the absorption of pions by atomic nuclei in the absence of a pion condensate. In this case the prohibition on single-nucleon absorption is removed because of the finiteness of the nucleus, since the finiteness of the nucleus leads to the nonconservation of the nucleon momentum. As a result, the single-nucleon absorption probability is nonzero, and is equal to $\sim 10^{-3}$. In Sec. 3 we compute the probability, due to the existence of a π condensate, for single-nucleon absorption in nuclear matter and in atomic nuclei. It is shown that the role of the condensate remains important in a finite system. In heavy nuclei the presence of a condensate leads to a $\sim$100-fold increase in the probability. Thus, experiments on single-nucleon absorption of slow pions can be critical for the elucidation of the question of the existence of a condensate in nuclear systems. In conclusion, we discuss the available experimental data on pion absorption and point out the need for the performance of further experiments.

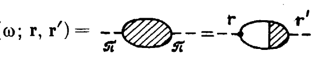
2. SINGLE-NUCLEON ABSORPTION OF SLOW PIONS BY NUCLEI WITHOUT A π CONDENSATE

The Klein–Gordon–Fock (KGF) equation for a pion in the field of a nucleus has the form

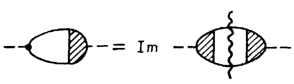
$$[\nabla^2+(\omega-V_Q)^2-m_\pi^2-\hat{\mathscr{P}}]\Psi=0, \tag{1}$$

where V_Q is the potential connected with the electromagnetic interaction of the pion with the nucleus; $\hat{\mathscr{P}}$ is the pion polarization operator arising from the strong π-N interaction in the nuclear material. In view of the possibility of absorption of the pion by the nucleons, $\hat{\mathscr{P}}$ contains not only a real, but also an imaginary, part. For a slow pion we can restrict ourselves to s and p pion scattering by a nucleon. In this case the polarization operator is expressible in terms of a small number of phenomenological parameters whose values are found from an analysis of the experimental data on, for example, the locations and widths of the π-atomic levels.[10] In the majority of papers an optical potential, which is connected in the nonrelativistic limit with the polarization operator by the relation $2m_\pi\hat{V}=\hat{\mathscr{P}}$, is used.

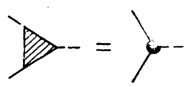
In a finite system $\hat{\mathscr{P}}$ depends on $\mathbf{r}$ and $\mathbf{r}'$, and can be represented graphically:

$\hat{\mathscr{P}}(\omega;\mathbf{r},\mathbf{r}')=$  $+\hat{\mathscr{P}}'(\omega;\mathbf{r},\mathbf{r}')$. (2)

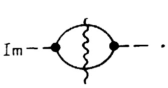
To compute the probability for single-nucleon absorption of a pion, we should separate out in (2) the part, $\hat{\mathscr{P}}_N$, of the polarization operator that contains in the vertical cross section a particle with a hole ($N\bar{N}$) at least once. The continuous line in (2) is the nucleon Green function. The point corresponds to the irreducible amplitude of the $\pi N\bar{N}$ interaction in nuclear matter, i.e., the diagram does not contain a particle with a hole in the vertical cross section. The hatched triangle represents the amplitude of the $\pi N\bar{N}$ interaction in the medium. In computing the imaginary part of the polarization operator $\hat{\mathscr{P}}_N$, responsible for the single-nucleon absorption, we take into account the fact that

$\mathrm{Im}\,\hat{\mathscr{P}}_N(\omega;\mathbf{r},\mathbf{r}')=$  (3)

The vertical wavy line in (3) indicates that we should take the imaginary part only from the product of the single-particle Green functions. It can be verified that, for slow pions ($\omega\sim m_\pi$) in systems with $(N-Z)/A\ll 1$,

 $\left[1+O\left(\frac{g}{A^{2/3}}\left(\frac{\varepsilon_F}{\omega}\right)^2\right)\right]$, (4)

where $g(\sim 1)$ is the amplitude of the interaction of the nucleons. Then

$\mathrm{Im}\,\hat{\mathscr{P}}_N(\omega;\mathbf{r},\mathbf{r}')=$  (5)

In free space the vertex of the pion-nucleon interaction in the nonrelativistic limit is equal to

$$\hat{\tau}_0 = \frac{f}{m_\pi}\left[(\sigma\nabla_\pi) - \frac{m_\pi}{M}(\sigma\nabla_N)\right]\tau, \tag{6}$$

where $f^2/4\pi = 0.08$, M is the nucleon mass, $\nabla_{\pi(N)}$ is the gradient acting on the meson (nucleon) wave function, and τ is the isospin Pauli matrix. The second term in (6) makes, in spite of the smallness of the factor m_π/M, a comparable contribution (< 50%) in the computation of the probability for single-nucleon absorption of a pion, since the momentum of the outgoing nucleon ($\sim (2M\omega)^{1/2}$) is high compared to the pion momentum. The slow pion-nucleon interaction amplitude, i.e., the π-N-$\bar{N}$ vertex, in a medium differs from the corresponding quantity in free space mainly because of the processes of virtual creation of the nucleon hole and the $\Delta_{33}(1232)$ isobar. Since Galilean invariance is not required in a medium, the renormalizations by the medium of the first and second terms in (6) are, generally speaking, different. These renormalizations are local renormalizations, are determined by the $N\Delta \to NN$ and $\Delta\Delta \to NN$ interaction amplitudes, and can be taken into account through the introduction of phenomenological constants. Notice that, since the renormalization is determined by the virtual creations of the Δ-$\bar{N}$ pairs, these constants do not depend on the retained frequency in a wide frequency range (when the frequency is varied in the range $0 \leqslant \omega \leqslant 1$[1)] the screening changes by 20%[8])

$$\hat{\tau} = -\!\!\blacktriangleleft = \frac{f}{m_\pi}\left[e_q^\pi(\sigma\nabla_\pi) - e_q^N(\sigma\nabla_N)\frac{m_\pi}{M}\right]\tau. \tag{7}$$

Here and below an isotopically invariant medium, i.e., the relation $(N-Z)/A \ll 1$, is implied, and the Coulomb interaction is neglected. The parameters e_q^π, e_q^N apparently differ little from the free-space values, which are equal to unity. For example, the parameter $e_q^\pi = 1 - 2\zeta_s$ is known from the analysis of the p-wave terms of the real part of the optical potential, which determines the π-atom spectrum,[8] as well as from the analysis of the magnetic moments and the probabilities for β decay of nuclei,[11] where it is found that $\zeta_s < 0.15$. The parameter e_q^N is unknown. For simplicity, we shall assume it is equal to its free-space value.

For infinite homogeneous nuclear matter the formula (5) has the form

$$\operatorname{Im}\hat{\mathscr{P}}_N(\omega, \mathbf{k}) = -\frac{p_F M}{\pi^2}\{\hat{\tau}\operatorname{Im}\Phi(\omega, \mathbf{k})\hat{\tau}\}. \tag{8}$$

The imaginary part, $\operatorname{Im}\Phi(\omega, \mathbf{k})$, of the product of the Green functions is, in the regions where it is nonzero, equal to

$$\operatorname{Im}\Phi(\omega, \mathbf{k}) = \frac{\pi}{2}\frac{\omega}{kv_F}, \quad 0 \leqslant \omega \leqslant kv_F - \frac{k^2}{2M};$$

$$\operatorname{Im}\Phi(\omega, \mathbf{k}) = \frac{\pi}{4}\frac{p_F}{k}\left\{1 - \frac{(\omega - k^2/2M)^2}{(kv_F)^2}\right\}, \tag{9}$$

$$\left|\frac{k^2}{2M} - kv_F\right| \leqslant \omega \leqslant \frac{k^2}{2M} + kv_F.$$

It can be seen from (8) and (9) that $\operatorname{Im}\hat{\mathscr{P}}_N(\omega, \mathbf{k}) = 0$ for $\omega = 1$, $k \ll 2.2$, and $k > 6.2$ ($p_F = 2$). The equality to zero of $\operatorname{Im}\hat{\mathscr{P}}_N(\omega \to 1, k \to 0)$ corresponds to the strict prohibition by the conservation laws of the single-nucleon absorption of a slow pion in an infinite system. In a finite system the nucleon momentum is not conserved, and single-nucleon absorption becomes weakly allowed. For the computation of the probability for single-nucleon absorption by atomic nuclei, it is convenient to use the coordinate representation; then

TABLE I. Probability for one-nucleon emission from various π-atom levels upon the absorption of stopped π^- mesons by atomic nuclei with allowance for the existence of a pion condensate.

Element	π-atom level	π-atom level width, keV[10]	One-nucleon emission probability: without a condensate ($\times 10^4$)	One-nucleon emission probability: with a condensate ($\times 10^2$)
U^{238}	$5g$	$1.4 \cdot 10^{-2}$	13	4
	$4f$	3.5	8	4
Th^{232}	$4f$	3	9	4
Pb^{208}	$4f$	4.2	12	5
Tl^{204}	$4f$	0.9	13	7
Ta^{181}	$4f$	0.4	20	7
Sn^{119}	$3d$	1.9	30	5
Mo^{96}	$3d$	0.6	34	7
Ni^{60}	$3d$	$1.5 \cdot 10^{-2}$	50	8
	$2p$	12	28	4.5
Ca^{40}	$3d$	$0.7 \cdot 10^{-3}$	36	7
	$2p$	2	16	4

Note. We assume that the condensate exists in that region where the nucleon density $n \geqslant 0.7n_0$, n_0 being the nucleon density at the center of the nucleus ($a^2 = 0.05$, $k_0 = p_F = 2$, where a and k_0 are the amplitude and wave vector of the condensate field; $\hbar = m_\pi = c = 1$). For condensate parameters $k_0 = 2.5$, $a^2 = 0.05$ the single-nucleon absorption probability increases by a factor of four. When allowance is made for the creation of the Δ isobar in the intermediate state in the diagrams (12), the values in the last column increase by a factor of 1.5.

$$\operatorname{Im}\hat{\mathscr{P}}_N(\omega; \mathbf{r}, \mathbf{r}') = -p_F M\pi^{-2}\{\hat{\tau}(\mathbf{r})\operatorname{Im}\Phi(\omega; \mathbf{r}, \mathbf{r}')\hat{\tau}(\mathbf{r}')\}. \tag{10}$$

The method of computing $\operatorname{Im}\hat{\mathscr{P}}_N(\omega; \mathbf{r}, \mathbf{r}')$ in a finite system is similar to the method, expounded in Ref. 12 (see the Appendix), of computing $\operatorname{Re}\Phi(\omega; \mathbf{r}, \mathbf{r}')$. Using the expression (10), we can compute the total cross section for slow-pion absorption accompanied by the emission of one nucleon, as well as the partial π-atom level widths, Γ_1, due to single-nucleon absorption:

$$\Gamma_1 = \int \Psi_{NLM}^{\pi*}(\mathbf{r})\operatorname{Im}\hat{\mathscr{P}}_N(\omega; \mathbf{r}, \mathbf{r}')\Psi_{NLM}^{\pi}(\mathbf{r}')\,d^3r\,d^3r'. \tag{11}$$

In (11) $\Psi_{NLM}^\pi(\mathbf{r})$ is the pion wave function satisfying the KGF equation (1). The formulas necessary for the computation of the Γ_1, (11), are given in the Appendix. The computation of the quantities Γ_1 for a number of nuclei and for the various π-atom levels was performed numerically. Here we used the π-meson optical potential parameters given in Ref. 10. The nucleon wave functions satisfied the Schrödinger equation with the Woods–Saxon potential. For the emitted nucleon we used a complex potential whose parameters can be adjusted to describe the scattering by nuclei of nucleons of energy ~ 140 MeV. The parameters for the nucleon optical potential were taken from Ref. 13, and are given in the Appendix. In Table I we give the values of the probabilities for one-nucleon emission upon the absorption of stopped pions, these probabilities being defined as the ratios of the partial widths Γ_1 to the observed π-atom level width. It can be seen from the table that the probability for one-nucleon emission in finite nuclei without a π condensate is small: $\sim 10^{-3}$.

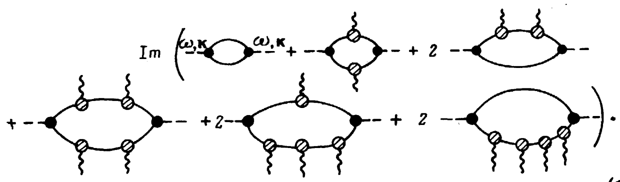
3. INFLUENCE OF THE PION CONDENSATE ON THE PROBABILITY FOR SINGLE-NUCLEON ABSORPTION IN NUCLEAR MATTER AND IN ATOMIC NUCLEI

1. Nuclear matter

Let us consider the case when a pion condensate exists in nuclear matter at a density equal to the density, $n_0 = 0.5$, of real nuclei. In this case the probability for the absorption of a slow pion by one nucleon in infinite nuclear matter is nonzero. This is due to the fact that, because of the processes of rescattering of the nucleons on the spin-isospin structure of characteristic dimension $k_0^{-1} \lesssim p_F^{-1}$, the nucleon momentum is not conserved.

Let us calculate the imaginary part, $\mathrm{Im}\hat{\mathcal{P}}_N(\omega, \mathbf{k})$, of the polarization operator of the slow pion ($\omega \to 1$, $\mathbf{k} \to 0$), which part determines the probability for single-nucleon absorption in infinite nuclear matter with allowance for the existence of the condensate. Let us expand $\mathrm{Im}\hat{\mathcal{P}}_N \times (\omega, \mathbf{k})$ in a power series in the amplitude, a, of the condensate field to fourth-order terms (because of the symmetry of the condensate field, $(\boldsymbol{\sigma} \cdot \mathbf{k}_0)\boldsymbol{\tau}$, the expansion contains only terms with even powers of the amplitude), since terms $\sim a^2$ vanish in the case of a condensate with wave vector $k_0 \approx p_F = 2 < 2.2$. This is due to the nature of the function $\mathrm{Im}\Phi(\omega, \mathbf{k})$, (9). For $6.2 > k_0 > 2.2$ the dominant terms will be the terms $\sim a^2$. The skeleton diagrams for the polarization operator have the following form:

$\mathrm{Im}\,\hat{\mathcal{P}}_N(\omega, \mathbf{k}) =$



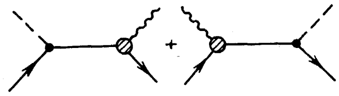
(12)

Here a wavy line designates the static condensate field with wave vector $\mathbf{k}_0$, a hatched nucleon-condensate field interaction vertex takes into account the screening by the Fermi-liquid spin-spin interaction. Analytically, (12) has, in the isotopically invariant system, the form

$$\mathrm{Im}\,\hat{\mathcal{P}}_N(\omega\to 1, k\to 0) = -8\frac{Mp_F}{\pi^2}\frac{f^4 e_q^4 k_0^2}{[1+g^-\Phi(0,k_0)]^2\omega^2}$$
$$\times\Big\{\mathrm{Im}\,\Phi(\omega,k_0)\cdot a^2(\varphi^2 - {}^1/_3\varphi_0^2) + 8\frac{f^2 e_q^2 k_0^2\omega^2}{[1+g^-\Phi(0,k_0)]^2[\omega^2 - 4(k_0^2/2M)^2]^2}$$
$$\times\ \mathrm{Im}\,\Phi(\omega, 2k_0)a^4\varphi^2(2\varphi^2 - {}^1/_3\varphi_0^2)\Big\}k^2. \tag{13}$$

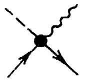
This formula has been written for the case when the "external" pion is charged. Here we have retained only those terms from (12) $\sim a^4$ which do not vanish simultaneously with the terms $\sim a^2$, i.e., we have dropped the terms $\sim a^4 \mathrm{Im}\Phi(\omega, \mathbf{k}_0)$. We have also taken into account the fact that $\mathrm{Im}\Phi(1, 0) = 0$, (9), and have averaged the expression over the angle between the momentum, $\mathbf{k}$, of the "external" pion and the momentum, $\mathbf{k}_0$, of the condensate field. Here $\varphi\{\varphi_+, \varphi_-, \varphi_0\}$ is the isotope vector of the condensate field ($\varphi^2 = 1$). The Fermi-liquid parameters for the nuclear matter are equal to $g^{-1} = 1.6$, $1 \gtrsim e_q \gtrsim 0.8$.[14]

Allowance for diagrams more complicated than the skeleton diagrams leads to additional screening of the terms in the expression (13). This screening can be estimated. Let us consider the terms of the order of a^2. Since in the diagrams determining the screening the departure of the nucleons from the mass shell is $\sim k_0 v_F$, the pion-nucleon scattering amplitude



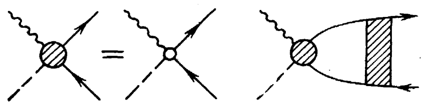
(14)

entering into them can be assumed to be local (i.e., to be nucleon-momentum independent) to within $(k_0 v_F/\omega)^2$. Therefore, it is convenient to rewrite the diagram (14) in the form



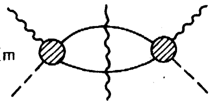
$$= i\frac{2f^2e_q^2}{[1+g^-\Phi(0,k_0)]\,\omega}\{[\mathbf{k}\mathbf{k}_0]\,\boldsymbol{\sigma}(\boldsymbol{\chi}\boldsymbol{\varphi}) + (\mathbf{k}\mathbf{k}_0)[\boldsymbol{\chi}\boldsymbol{\varphi}]\,\boldsymbol{\tau}\}. \tag{15}$$

Here $\boldsymbol{\chi}$ is the isotopic vector of the "external" pion ($\boldsymbol{\chi}^2 = 1$). The screening of the diagram (15) is largely connected with the creation in the intermediate state of a particle and a hole possessing nonzero total energy and momentum ($\omega = 1$, $\mathbf{k} = \mathbf{k}_0$), and is described by the following equation:



(16)

where the hatched rectangle represents the effective interaction of the nucleons for a carried-away energy $\omega \sim 1$ and momentum $\mathbf{k} = \mathbf{k}_0$. The first term on the right-hand side of (16) is the irreducible πN scattering amplitude in the sense that it does not contain a particle and a hole in its vertical section. The main difference between this amplitude and the amplitude (15) is connected with the creation of the Δ-$\bar{N}$ pair. As for the π-N-$\bar{N}$ vertex, (7), this difference is small ($\lesssim 20\%$). With allowance for screening

$\mathrm{Im}\hat{\mathcal{P}}_N(\omega, \mathbf{k}) =$ Im  . (17)

From (15)–(17) it is easy to obtain the expression

$$\mathrm{Im}\,\hat{\mathcal{P}}_N(\omega,k) = -\frac{16}{3}\frac{Mp_F}{\pi^2}\frac{f^4e_q^4k_0^2}{[1+g^-\Phi(0,k_0)]^2\omega^2}a^2\Big\{2\frac{(\boldsymbol{\chi}\boldsymbol{\varphi})^2}{[1+g_{\mathrm{eff}}\Phi(\omega,k_0)]^2}$$
$$+\frac{[\boldsymbol{\chi}\boldsymbol{\varphi}]^2}{[1+f_{\mathrm{eff}}\Phi(\omega,k_0)]^2}\Big\}\mathrm{Im}\,\Phi(\omega,k_0)k^2. \tag{18}$$

The screening in (18) leads to the decrease of $\mathrm{Im}\hat{\mathcal{P}}_N$ by a factor of 2–3 for $g_{\mathrm{eff}} \sim f_{\mathrm{eff}} \sim 1$. The screening of the terms $\sim a^4$ can be roughly estimated if we take into account diagrams of the type (16), which lead to the appearance of screening factors of the type $(1 + g\Phi(\omega, k_0))^{-2} \times (1 + f\Phi(\omega, 2k_0))^{-2}$, which decreases the terms $\sim a^4$ by a factor of 3–4.

Using the foregoing, let us give the lower limit of the effect of the condensate on the probability for single-nucleon absorption of slow pions in infinite nuclear matter. For $k_0=2$

$$\mathrm{Im}\hat{\mathscr{P}}_N \approx -5a^4k^2. \tag{19}$$

Let us compare this expression, using the relation between the polarization operator and the optical potential ($2\hat{V}=\hat{\mathscr{P}}$), with the imaginary part of the potential for the slow pion. The parameters of the optical potential have been found from the experimental data on the π atoms[10]:

$$\mathrm{Im}\,\hat{V} = -2\pi n_0{}^2\left\{0{,}04 + \frac{0{,}08}{1+0.21 n_0{}^{-1/3}\pi}(\mathbf{kk}')\right\}. \tag{20}$$

The second term here is connected with the p-wave absorption of the pion, and is the decisive term for the level widths of the π-mesic atom that are observed in experiment (the widths of the levels $2p$, $3d$, $4f$, ...). Comparing (19) with the second term in (20), we see that, for $a^2=0.05$, the contribution of the single-nucleon absorption due to the π condensate is $\sim 10^{-1}$. Let us also give an estimate for the $k_0>2.2$ case. In this case the terms $\sim a^2$ are nonzero. For example, for $k_0=2.5$

$$\mathrm{Im}\hat{\mathscr{P}}_N \approx -a^2k^2. \tag{21}$$

In this case the contribution of the single-nucleon absorption becomes ~ 1 ($a^2=0.05$). Consequently, the estimates show that there occurs a substantial increase in the single-nucleon absorption probability in the case when a pion condensate exists (~100-fold increase when $a^2=0.05$).[9] As will be shown below, the indicated single-nucleon absorption mechanism connected with the presence of a condensate remains important also in finite nuclei.

2. Atomic nuclei

In a finite system with dimension $R=r_0A^{1/3}$ the single-nucleon absorption probability determined by the existence of a condensate can be computed, using the formulas for the infinite medium, since the condensate-field momentum $k_0 \gtrsim p_F \gg R^{-1} \sim p_F/A^{1/3}$. In doing this, it is necessary to take into account the dependence of the nucleon density and the condensate-field amplitude on the distance to the center of the nucleus. Then[2)]

$$\begin{aligned} \mathrm{Im}\,\hat{\mathscr{P}}_N &\approx -5\left(\frac{n(r)}{n_0}\right)^{2/3} a^4(r)\,(\mathbf{kk}'), \quad k_0=p_F=2, \\ \mathrm{Im}\,\hat{\mathscr{P}}_N &\approx -\left(\frac{n(r)}{n_0}\right)^{2/3} a^2(r)\,(\mathbf{kk}'), \quad k_0=2.5. \end{aligned} \tag{22}$$

In experiment the emission of one nucleon with energy in the 140-MeV region from a nucleus upon the absorption of a stopped pion is observed. The appearance of this nucleon is connected with single-nucleon absorption.

Let us estimate the probability for emission of one nucleon from an atomic nucleus, assuming that a condensate exists in the real nucleus. Having absorbed a stopped pion, a nucleon acquires an energy ~140 MeV, and is emitted from the nucleus with probability less than unity. This is connected with the fact that the mean free path of a nucleon with energy ~140 MeV in nuclear matter is less than the nuclear dimension, and is ~2 F.

Bearing in mind that the probability for the appearance of a nucleon with energy ~140 MeV at the point $\mathbf{r}$ is given by the formulas (22), the partial width, Γ_1, connected with the one-nucleon emission can be estimated from the formula

$$\Gamma_1 = \int \Psi^{\pi *}_{NLM}(\mathbf{r})\,\mathrm{Im}\,\hat{\mathscr{P}}_N(\mathbf{r})\,\Psi^{\pi}_{NLM}(\mathbf{r})\,P(\mathbf{r})\,d^3r, \tag{23}$$

where $P(\mathbf{r})$ gives the probability for the emission of a nucleon from the nucleus:

$$\begin{aligned} P(\mathbf{r}) &= \int_0^1 p(r,x)\,dx, \\ p(r,x) &= \exp\left\{-\int_0^\infty d\rho\,\delta^{-1}\left(\sqrt{r^2+\rho^2+2r\rho x}\right)\right\}, \quad \delta^{-1}(r)=\delta_0{}^{-1}n(r)/n_0. \end{aligned} \tag{24}$$

Here δ_0 is the mean free path of the nucleon in the central region of the nucleus, and is equal to 2 F. We have computed with the aid of (22)–(24) the probabilities for one-nucleon emission (the ratio of the partial width to the experimental π-atom level width) for a number of nuclei with allowance for the π condensate. The pertinent results are given in Table I. It can be seen from the table that the presence of the condensate increases substantially the probability for one-nucleon emission.

Let us discuss a few questions that arise in the analysis of the mechanism of single-nucleon absorption of pions by finite nuclei.

1. It is known that the wave functions of pions for the levels given in Table I lie largely outside the nucleus, and are damped inside the nucleus. The phenomenon of pion condensation is a volume effect,[3,4] and the amplitude of the condensate field is nonzero inside the nucleus. However, in spite of this fact, as the numerical calculations show, the one-nucleon emission probability is sensitive to the existence of a pion condensate in atomic nuclei, which fact is demonstrated by Fig. 1. In Fig. 1 we show the dependence on the condensate radius of the probability for the emission of a nucleon from the $4f$ level in lead upon the absorption of a π meson. If we assume that the existence domain of the condensate extends to the nuclear-matter density, which is equal to $0.7n_0$, then the presence of the condensate leads to a ~100-fold increase in the probability. Similar situa-

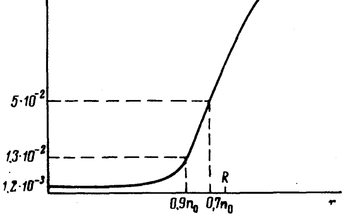


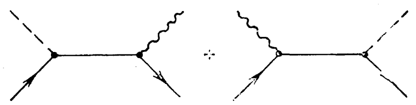
FIG. 1. Dependence of the probability for the emission of one nucleon upon the absorption of a stopped pion (from the $4f$ level) for ^{208}Pb on the condensate radius. The quantities $0.9n_0$ and $0.7n_0$ are the nucleon densities in the nucleus at different distances to the center, n_0 being the nucleon density at the center of the nucleus.

tions are observed in the other cases given in Table I.

2. In a finite system there arises a problem connected with the fact that the condensate field has a quantum character.[4] The meaning of this is as follows. In an infinite system the condensate field can be considered to be classical, i. e., to have a definite value at each point in space. In a finite system the ground state is characterized by a wave function with zero mean field value. As has been shown in Refs. 1 and 4, the condensation manifests itself in the fact that the positive and negative values of the field at a given point are equally probable, and only the mean square of the field has a nonzero value. To the ground state of the finite system corresponds a symmetric wave function, $\chi^s(q)$, while to the first excited state corresponds an antisymmetric wave function, $\chi^a(q)$. The function $\chi(q)$ describes motion in two identical wells separated by a potential barrier.[3)] The level spacing decreases exponentially with increasing volume, and in a sufficiently large system there arises degeneracy, which allows us to speak of a definite field value at each point. In real nuclei the distance to the first excited level can be estimated[4] as $\delta \sim 28 \times \exp\{-0.96A/100\}$ (for $A = 50$, $\delta \sim 20$ MeV; $A = 200$, $\delta \sim 4$ MeV).

In the case of interest to us, when the frequency of the external influence (in the present case the frequency of the "external" pion) is high: $\omega \sim 140$ MeV $\gg \delta$, we can assume that the system is degenerate, and use the representation of the condensate as a classical field, i. e., the use of the above-presented formulas is legitimate.

3. In a finite system there exist fluctuations with characteristic dimension $\sim (2p_F)^{-1}$ in the scalar density—Kohn density oscillations. The effect of these oscillations on single-nucleon absorption has been taken into account in that in (10) and (11) we use nucleon Green's functions found for a finite system. The probabilities computed in this case are many times smaller than the probabilities obtained when the condensate is taken into account (see Table I). The question arises why these oscillations do not give the same contribution as the π condensate, in spite of the fact that their amplitudes are of the same order of magnitude. This is connected with the symmetry of the oscillations. The vanishing of the bare amplitude of the scattering on the Kohn oscillations:


(25)

(the wavy lines correspond to the scalar field) leads to a situation in which the imaginary part of the skeleton diagrams (12) $\sim k^4$, which leads in its turn to a decrease in the probability in the small k limit.

4. In computing the probability for single-nucleon absorption of slow pions, we should take into account the processes in which the Δ isobar is created in the intermediate states in the diagrams (12). This increases the probability roughly by a factor of one and a half. Such processes have been taken into account in that in the specific calculation (see Table I) we used the experimental π-N scattering lengths (volumes). In this case, since the momentum of the condensate pion is high ($k_0 \gtrsim p_F = 2$), it is necessary to take the form factor $(1 + 0.23k_0^2)^{-1/2}$[3] into account in the π-$\bar{N}$-Δ amplitude. Allowance for the effect of the s-wave scattering in the computation of the partial widths, Γ_1, of the π-atom levels insignificantly changes the magnitudes of Γ_1.

4. CONCLUSIONS

The main result of the work consists in the following. The existence in nuclei of a π condensate leads to a substantial increase in the single-nucleon absorption probability (~100-fold increase in the case of heavy nuclei; see Table I). This allows us to hope that slow-pion absorption experiments will provide an answer to the question of the existence of a condensate in atomic nuclei.

At present we have in the literature several papers on the measurement of stopped-pion absorption, from the results of which we can estimate the probability for one-nucleon emission.[16-18] In Ref. 16 Anderson *et al.* report the measurement of the energy spectrum of neutrons up to an energy of 150 MeV. From the results of this work we can estimate the probability for one-nucleon emission in Pb and U nuclei, which turns out to be $\sim 10^{-3}$. In the work by Dey *et al.*[17] the energy spectrum of the neutrons released in the reaction ^{165}Ho(π^-, xn) was measured. The probability for the emission of a fast neutron in this case is also $\sim 10^{-3}$. This allows a rough estimation of the upper bound of the condensate-field amplitude. For $k_0 = 2$ the estimation yields $a^2 \lesssim 5 \times 10^{-3}$, while for $k_0 = 2.5$ we have $a^2 \lesssim 4 \times 10^{-4}$. Let us mention V. Butsev and D. Chultém's work,[18] in which the probability for the reaction

$$\mathrm{Ta}^{181}(\pi^- n) \rightarrow \mathrm{Hf}^{180m}(T_{1/2} = 5.5h,\ I = 8^-),$$

was measured. The probability turned out to be less than 10^{-5}. Since the estimate obtained from the results of Refs. 16 and 17 for the one-nucleon emission probabilities is a fairly rough estimate, a definitive conclusion concerning the absence of a condensate in nuclei cannot be drawn. And from the results of Ref. 18 we cannot infer the nonexistence of a condensate, since the probability for such a reaction is determined by purely surface phenomena, i. e., by that region of the nucleus where the condensate does not exist. For a definitive answer to the posed question we need further experimental investigations.

In conclusion, we thank A. B. Migdal, B. T. Geĭlikman, V. A. Karnaukhov, É. E. Sapershteĭn, V. A. Khodel', E. V. Boyarinov, and I. N. Polosukhin for useful discussions and for interest in the work.

APPENDIX

It is convenient to compute the particle-hole propagator in a finite system, using the coordinate representation:

$$-\frac{p_F M}{\pi^2}\Phi(\omega;\mathbf{r},\mathbf{r}')=\sum_\lambda n_\lambda\varphi_\lambda(\mathbf{r})\varphi_\lambda^*(\mathbf{r}')G(\varepsilon_\lambda+\omega;\mathbf{r},\mathbf{r}')$$

$$+\sum_\lambda n_\lambda\varphi_\lambda(\mathbf{r})\varphi_\lambda^*(\mathbf{r}')G(\varepsilon_\lambda-\omega;\mathbf{r},\mathbf{r}'), \qquad (A.1)$$

where the n_λ and $\varphi_\lambda(\mathbf{r})$ are the occupation numbers and the single-particle wave function, computed in the Woods–Saxon potential, of the state $\lambda=n, j, l, m$. The Green function $G(\varepsilon;\mathbf{r},\mathbf{r}')$ satisfies the equation

$$(\varepsilon-\hat{H})G(\varepsilon;\ \mathbf{r},\ \mathbf{r}')=\delta(\mathbf{r}-\mathbf{r}'), \qquad (A.2)$$

where $\hat{H}$ is the Hamiltonian of a particle with energy $\varepsilon\sim 140$ MeV. The solution to Eq. (A.2) is sought in the form

$$G(\varepsilon;\mathbf{r},\mathbf{r}')=\sum_{jlm}\Phi_{jlm}^*(\mathbf{n})\Phi_{jlm}(\mathbf{n}')G_{jl}(\varepsilon;r,r'), \qquad (A.3)$$

where

$$\Phi_{jlm}(\mathbf{n})=\sum_{\mu\nu}C_{l\mu^1/_2\nu}^{jm}Y_{lm}(\mathbf{n})\chi_\nu^{1/2},$$

$G_{jl}(\varepsilon;\mathbf{r},\mathbf{r}')$ satisfies the equation

$$(\varepsilon-H_{jl})G_{jl}(\varepsilon;\ r,\ r')=\delta(r-r')/rr', \qquad (A.4)$$

$$H_{jl}=-\frac{\hbar^2}{2M}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)-\frac{l(l+1)}{r^2}\right]+U_1(r)$$

$$+U_2(r)(\boldsymbol{\sigma}\mathbf{l})_{jl}+U_Q(1+\tau_z)/2, \qquad (A.5)$$

where

$$U_1(r)=U_0\left[1+\exp\left(\frac{r-R}{a}\right)\right]^{-1}$$

is the Woods–Saxon potential

$$U_2(r)=\left(\frac{\hbar}{m_\pi c}\right)^2U_{sl}\frac{1}{r}\frac{\partial}{\partial r}\left[1+\exp\left(\frac{r-R}{a}\right)\right]^{-1},$$

U_{sl} is the spin-orbit splitting constant; U_Q is the Coulomb potential of a uniformly charged sphere of radius R, R is the radius of the nucleus, and a is the diffusivity. The solution to Eq. (A.5) can be represented in the form

$$G_{jl}(\varepsilon;r,r')=\frac{2M}{\hbar^2rr'}\begin{cases}y^{(1)}(\varepsilon,r)y^{(2)}(\varepsilon,r'), & r\leqslant r'\\ y^{(2)}(\varepsilon,r)y^{(1)}(\varepsilon,r'), & r>r'\end{cases}, \qquad (A.6)$$

where the Wronskian of the equation, $W(\varepsilon)=y^{(1)}y^{(2)\prime}-y^{(2)(1)\prime}$ has been reduced to unity.

The functions $y^{(1)}$ and $y^{(2)}$ satisfy the following boundary conditions: $y^{(1)}(0)=0$ and $y^{(2)}(r\to\infty)$ is exponentially damped in the upper $\varepsilon^{1/2}$ half-plane. It can easily be shown that Γ_1 is determined by the quantity

$$G_{jl}^I(\varepsilon;r,r')=\frac{2M}{\hbar^2rr'}\begin{cases}y^{(1)}(\varepsilon,r)y^{(1)}(\varepsilon,r'), & \varepsilon>0\\ 0, & \varepsilon\leqslant 0\end{cases}. \qquad (A.7)$$

This formula allows us to compute $\mathrm{Im}\Phi(\omega;\mathbf{r},\mathbf{r}')$, which determines the partial width (11). The constants U_0 and U_{sl} in (A.5) were chosen as follows ($\omega\sim 140$ MeV):

$$U_0=-(15+20i)\ \text{MeV},\quad U_{sl}=(1.5-1i)\ \text{MeV}. \qquad (A.8)$$

Let us give the final expression for the partial width Γ_1, which determines the single-nucleon absorption probability:

$$\Gamma_1=\frac{4Mf^2}{\hbar^2m_\pi^2}\sum_{\substack{njl\\ j'l'}}n_{njl}I_{njl}^{j'l'}, \qquad (A.9)$$

$$I_{njl}^{j'l'}=\frac{\exp(-2\eta_{njl})}{2L+1}\left|\int y_{j'l'}^{(1)}(\varepsilon_{njl}+\omega,r)F_{njl}^{j'l'}(r)r\,dr\right|^2, \qquad (A.10)$$

where η_{njl} is the imaginary part of the phase shift in the wave function of the emitted nucleon,

$$F_{njl}^{j'l'}(r)=c_1(r)-\frac{m_\pi}{M}c_2(r),$$

$$c_1(r)=-\sum_{\Lambda=L\pm1}(T_{L\Lambda 1})_{jl}^{j'l'}\frac{R_{njl}(r)}{(2L+1)^{1/2}}\Big(\langle\Lambda\|\nabla_n\|L\rangle\frac{R_{NL}^\pi(r)}{r}$$

$$+\langle\Lambda\|\mathbf{n}\|L\rangle\frac{dR_{NL}^\pi(r)}{dr}\Big), \qquad (A.11)$$

$$c_2(r)=(-1)^{l+j+1/2}\begin{Bmatrix}2j-l & 1/2 & j\\ 1/2 & l & 1\end{Bmatrix}(T_{LL0})_{j\,2j-l}^{j'l'}R_{NL}^\pi(r)$$

$$\times\left(\langle 2j-l\|\nabla_n\|l\rangle\frac{R_{njl}(r)}{r}+\langle 2j-l\|\mathbf{n}\|l\rangle\frac{dR_{njl}(r)}{dr}\right);$$

$R_{njl}(r)$ is the radial part of the nucleon wave function in the Woods–Saxon well, $R_{NL}^\pi(r)$ is the radial part of the π-meson wave function in a π-atom, and

$$(T_{JLS})_{j_1l_1}^{j_2l_2}=(-1)^{l_2}\Big[2(2l_1+1)(2l_2+1)(2j_1+1)(2j_2+1)(2J+1)$$

$$\times(2L+1)\frac{2S+1}{4\pi}\Big]^{1/2}\begin{pmatrix}l_1 & l_2 & L\\ 0 & 0 & 0\end{pmatrix}\begin{Bmatrix}l_2 & 1/2 & j_2\\ l_1 & 1/2 & j_1\\ L & S & J\end{Bmatrix}. \qquad (A.12)$$

The reduced matrix elements are equal to

$$\langle L+1\|\nabla_n\|L\rangle=-L(L+1)^{1/2};\quad \langle L-1\|\nabla_n\|L\rangle=-(L+1)L^{1/2},$$
$$\langle L+1\|\mathbf{n}\|L\rangle=(L+1)^{1/2},\quad \langle L-1\|\mathbf{n}\|L\rangle=-L^{1/2}. \qquad (A.13)$$

[1]It is assumed that $\hbar=m_\pi=c=1$.

[2]Notice that the nonconservation of momentum in a finite system leads to the result that the terms $\sim a^2$ will be nonzero for $k_0=2$, which will give rise to some increase in the probability for the absorption caused by the condensate.

[3]See also the paper by Dmitriev.[15]

1A. B. Migdal, Zh. Eksp. Teor. Fiz. **61**, 2209 (1971) [Sov. Phys. JETP **34**, 1184 (1972)].

2A. B. Migdal, Zh. Eksp. Teor. Fiz. **63**, 1993 (1972) [Sov. Phys. JETP **36**, 1052 (1973)]; Phys. Rev. Lett. **31**, 257 (1973).

3A. B. Migdal, O. A. Markin, and I. N. Mishustin, Zh. Eksp. Teor. Fiz. **66**, 443 (1974); **70**, 1592 (1976) [Sov. Phys. JETP **39**, 212 (1974); **43**, 830 (1976)].

4A. B. Migdal, Fermiony i bozony v sil'nykh polyakh (Fermions and Bosons in Strong Fields), Nauka, 1977.

5A. B. Migdal, Pis'ma Zh. Eksp. Teor. Fiz. **19**, 539 (1974 [JETP Lett. **19**, 284 (1974)].

6É. E. Sapershteĭn and M. A. Troitskiĭ, Yad. Fiz. **22**, 257 (1975) [Sov. J. Nucl. Phys. **22**, 132 (1975)].

7M. A. Troitskiĭ, É. E. Sapershteĭn, O. A. Markin, and I. N. Mishustin, Pis'ma Zh. Eksp. Teor. Fiz. **21**, 96 (1975) [JETP Lett. **21**, 44 (1975)].

8M. A. Troitskiĭ and N. I. Chekunaev, Yad. Fiz. **24**, 52, 1039 (1976) [Sov. J. Nucl. Phys. **24**, 26, 544 (1976)].

9M. A. Troitskiĭ, M. V. Koldaev, and N. I. Chekunaev, Pis'ma Zh. Eksp. Teor. Fiz. **25**, 136 (1977) [JETP Lett. **25**, 123 (1977)].

[10]G. Backenstoss, Annu. Rev. Nucl. Sci. 20, 467 (1970) (Russ. Transl in: Usp. Fiz. Nauk 107, 405 (1972).
[11]A. B. Migdal, Teoriya konechnykh fermi-sistem i svoǐstva atomnykh yader (The Theory of Finite Fermi Systems and the Properties of Atomic Nuclei), Nauka, 1965 (Eng. Transl., Wiley, New York, 1967).
[12]É. E. Sapershteǐn, S. V. Tolokonnikov, and S. A. Fayans, Preprint IAÉ-2571, 1976.
[13]M. Preston, Physics of the Nucleus, Addison-Wesley, Reading, Mass., 1962 (Russ. Transl., Mir, 1964); A. Iohansson, U. Svanberg, and P. E. Hodgson, Ark. Fys. 19, 541 (1961).
[14]V. Osadchiev and M. Troitsky, Phys. Lett. 26B, 421 (1968); Yad. Fiz. 6, 961 (1967) [Sov. J. Nucl. Phys. 6, 700 (1967)].
[15]V. F. Dmitriev, Yad. Fiz. 24, 913 (1976) [Sov. J. Nucl. Phys. 24, 477 (1976)].
[16]H. L. Anderson, E. P. Hincks, C. S. Johnson, C. Rey, and A. M. Segar, Phys. Rev. 133B, 392 (1964).
[17]W. Dey, H. P. Isaak, H. K. Walter, R. Engfer *et al.*, Helv. Phys. Acta 49, 778 (1976).
[18]V. S. Butsev and D. Chultem, Phys. Lett. 67B, 33 (1977).

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Given-intensity approximation in the theory of nonlinear waves

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A large class of problems in the theory of linear waves can be solved only in the given-field (GF) approximation. In the present paper a new approximation, that of given intensity (GI), is developed, and takes into account the reaction to the phase of an intense wave. Physically this approximation is justified by the fact that the scales over which significant changes of the phase relation and transfer of the energy of the intense wave can take place can differ greatly in the presence of a mechanism that mismatches the phase relations between the interacting waves. It is shown that even in the absence of a phase mismatch the region in which the GI approximation is valid is larger than that of the GF approximation. The GI approximation is used to analyze the stationary interaction of waves in inhomogeneous nonlinear media and the nonstationary interaction of waves in homogeneous media. Expressions are obtained for the intensities and spectra of the excited or amplified waves. A number of effects that do not appear in the GF approximation are observed, particularly the influence on the parametric amplification of the intensity at the supplementary frequency and the dependence of the structure of the harmonic spectrum under nonstationary excitation conditions on the shape of the main beam.

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INTRODUCTION

The given-field (GF) approximation is widely used in the theory of nonlinear interaction of waves in dispersive media.[1–3] In this approximation, the complex amplitude of the intense initial wave is assumed to be given, as a result of which the nonlinear equations (which are partial differential equations in the general case) become simply coupled equations, and this facilitates greatly their solution for real wave beams and real nonlinear media. The GF approximation, however, describes correctly only the initial stage of the nonlinear wave interaction, so long as the reaction of the excited or amplified waves on the intense wave can be neglected. If the reaction is taken into account, however, the nonlinear wave equations can be solved exactly, even for homogeneous systems, only in a limited number of cases: for the interaction of plane waves or narrow wave packets (the so-called quasistatic approximation[4]) or for special cases of nonstationary wave interaction.[5–8] Recently, the method of the inverse scattering problem[9] has been applied to the analysis of nonlinear wave interactions, although this method imposes no limitation on the wave coupling coefficient, but its complexity is such that only asymptotic solutions are obtained. Numerical methods have recently been used to solve problems of nonlinear interaction of focused beams[10] and of the interaction of waves in inhomogeneous nonlinear media.[11] At the same time, of considerable interest in the theory of nonlinear waves is the development of analytic methods that make it possible to go beyond the framework of the GF approximation and at the same time produce results that can be easily interpreted.

In the present paper we develop, for the analysis of the interaction of waves in nonlinear dispersive media, the given-intensity (GI) approximation, in which, in contrast to the GF approximation, the reaction on the phase of the exciting wave is taken into account. The physical basis of the proposed approximation is the difference between the rates of change of the amplitudes and phases of the interacting waves. Therefore the GI approximation is effective in those cases where there is a mechanism of mismatching the phases of the interacting waves, such as wave detuning or group-velocity mismatch. However, even where there is no such mechanism, the accuracy of the solutions obtained by the GI

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