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Role of collective and induced processes in the generation of Mössbauer γ radiation

A. V. Andreev, Yu. A. Il'inskiĭ, and R. M. Khokhlov

Moscow State University
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It is shown that the principal role in the generation of coherent Mössbauer coherent radiation is played by processes of collective spontaneous radiation. Observation of induced Mössbauer γ radiation in the decay of strongly excited polyatomic systems encounters unsurmountable difficulties.

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The question of the possibility of extending the principle of laser generation of the γ-ray band is constantly discussed in the literature of the last fifteen years (see the reviews[1–3]). It was assumed that the main idea of lasing in the optical band, i.e., amplification of the light with the aid of stimulated emission, could be directly realized also in the γ band. No account was taken, however, of the peculiarities of the electromagnetic waves in the γ band, or of the peculiarities of the Mössbauer γ radiation. The need for using the latter has already been repeatedly emphasized. In the present article we consider in greater detail the kinetics of emission of extended resonatorless systems of two-level emitters that are strongly excited and are uncorrelated at the initial instant, and show that in the Mössbauer energy region the stimulated processes make a negligibly small contribution to the radiation intensity. The principal role is played here by processes of collective spontaneous emission, which replace stimulated emission when it comes to generation of coherent γ photons.

The semiclassical approximation of the quantum equations of field dynamics is of the form (for a detailed derivation see[4]):

$$\frac{dn}{dt}+\frac{n}{\tau}=F,\quad \frac{dF}{dt}+\frac{1}{2}\left(\frac{1}{\tau}+\frac{1}{T_2}\right)F=\frac{1}{T_0^2}(nR+S),$$
$$\frac{dS}{dt}+\frac{S}{T_2}=FR-\varkappa F,\quad \frac{dR}{dt}=-2F, \tag{1}$$

where

$$n=\sum_{\mathbf{k}}\langle a_{\mathbf{k}}{}^+a_{\mathbf{k}}\rangle,\quad R=\sum_{i=1}^{N}\langle\sigma_z^{(i)}\rangle$$

are respectively the number of quanta in the sample volume and the population difference; $\tau=L/c$; $1/T_2$ is the width of the Mössbauer transition line;

$$\frac{1}{T_0^2}=\frac{2\overline{|g_{\mathbf{k}}|^2}}{\hbar^2}=\frac{4\pi f}{V\hbar\omega}\overline{|M|^2};$$

V is the volume of the sample; f is the probability of the Mössbauer radiation; M is the matrix element of the nuclear-transition current density; the bar denotes averaging over the directions of $\mathbf{k}$: σ_+, σ_-, σ_z are Pauli spin operators;

$$F=\frac{i}{\hbar}\sum_{\mathbf{k}}\sum_{i}\left(g_{\mathbf{k}}a_{\mathbf{k}}{}^+\sigma_-^{(i)}\exp(i\mathbf{k}\mathbf{r}_{0i})-\text{H.c.}\right), \tag{2}$$

$$S=\varkappa N_2+\sum_{\mathbf{k}}\sum_{i\neq j}\langle\sigma_+^{(i)}\sigma_-^{(j)}\rangle\exp[-i\mathbf{k}(\mathbf{r}_{0i}-\mathbf{r}_{0j})], \tag{3}$$

$$\varkappa=\sum_{\mathbf{k}}1=8\pi\frac{V\omega^2}{(2\pi c)^3}\Delta\omega=\frac{V\omega^2}{2\pi c^3\tau}. \tag{4}$$

Summation with respect to $\mathbf{k}$ is carried out over a spherical layer of thickness

$$\Delta k=\frac{\pi}{2c}\left(\frac{1}{\tau}+\frac{1}{T_2}\right),$$

and account is taken in (4) of the fact that $\tau\ll T_2$ for the

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case considered here.

The initial conditions are

$$n(0)=0,\ F(0)=0,\ S(0)=\varkappa N,\ R(0)=N.$$

We have assumed for simplicity that at the initial instant all the nuclei are in an excited state. All the formulas obtained below can be readily generalized to the case of arbitrary excitation by simply replacing N in them by $N_2^0-N_1^0$ (where N_2^0 and N_1^0 are the numbers of the nuclei in the excited and ground state at the initial instant of time).

In the right hand side of (3), the first term $S_0=\varkappa N_2$ describe the spontaneous decay of the independent emitters, and the second term S_1 describes the processes of emitter correlation in the course of emission. Let us estimate the contribution made by the last effect in comparison with the effect of stimulated emission. From (1) we have

$$nR+S_1=\int_0^t F(t')\,[R(t)\exp\{-(t-t')/\tau\}+R(t')\exp\{-(t-t')/T_2\}]\,dt'. \quad (5)$$

Consequently, the effects of stimulated emission play the principal role in systems with $\tau \gg T_2$ (in lasers we have $\tau=L/c(1-p)$, where p is the coefficient of reflection from the mirrors). On the other hand in systems with $\tau \ll T_2$ the stimulated effects play a noticeable role only at times $t\sim\tau$. Since the delay time of the emission pulse is $t_0>T_2$, the influence of stimulated emission on the decay intensity in such systems will be negligibly small.

In the Mössbauer γ-quantum energy region, the characteristic time constants are

$$\tau\sim10^{-11}-10^{-10}\ \text{sec},\ T_2\sim10^{-7}-10^{-5}\ \text{sec},\ T_1\sim10^{-7}-10^{-8}\ \text{sec}.$$

Consequently, the realization of γ-ray lasing would make it necessary to increase by four or five orders of magnitude the mean free path of the γ quanta. The principal role in the generation of coherent γ radiation will therefore be played by collective spontaneous emission.

Dicke has shown in his classical paper[5] that the correlation of the emitters in the course of decay of polyatomic strongly excited systems can lead, under certain conditions, to a sharp decrease of the emission time of the system in comparison with the decay time of an individual emitter, and to an N-fold increase of the decay intensity $I_D=N^2/4T_1$. This effect was called superradiance and is observed in systems with $d<\lambda$ and $L<\lambda$ (where d and L are the transverse and longitudinal dimensions of the sample). In a number of later studies (see the review[6] and also[7]) it was shown that cooperative effects can be observed in extended systems with dimensions much larger than the radiation wavelength. The maximum emission intensity of such systems is $I_{SR}=N/4\tau_c$ and is reached at the instant of time $t_0=\tau_c\ln N$, where

$$\tau_c=\left(\frac{\lambda^2}{2\pi}\frac{N}{V}\frac{L}{T_1}\right)^{-1}. \quad (6)$$

Thus, the intensity of the radiation in such systems increases by a $T_1/\tau_c=N\lambda^2L/2\pi V$ times in comparison with the intensity of the isotropic spontaneous decay. In needle-shaped samples, this emission is accompanied by high directivity of the radiation along the sample axis. The condition for the decay of systems with intensity I_{SR} takes the form $T_2\gg\tau_c$ or $\mu_0L>\ln N$ (where μ_0 is the resonant gain), i.e., the effect is observed in strongly amplifying media.

A formula for the emission intensity of extended systems was first obtained in[8], where the possibility of observing the indicated effect in the γ band was discussed. The possibility of using such processes for purposes of generating coherent γ radiation was discussed also in[4,9–11]. However, inasmuch as in the γ band the satisfaction of the condition $\mu_0L>1$ is doubtful, it is clear that observation of the indicated effects is fraught with very great difficulties. It turns out that the cooperative phenomena are possible also in weakly amplifying media ($1<\mu_0L<\ln N$). In this case the maximum radiation intensity is

$$I_0=\frac{N}{4\tau_c}\left(1-\frac{\tau_c}{T_2}\right)^2$$

and is attained at the instant of time

$$t_0=\left(\frac{1}{\tau_c}-\frac{1}{T_2}\right)^{-1}\ln\frac{N}{\varkappa}.$$

In strongly amplifying media, I_0 goes over into I_{SR} and, on the other hand, collective effects disappear completely in media with $\mu_0L=1(T_2=\tau_c)$. Consequently, the condition for coherent decay of polyatomic systems is of the same form for the coherence resulting from processes of stimulated emission as for coherence due to collective processes. This agreement, however, is purely extraneous and is due to the processes of interaction of resonant electromagnetic radiation with quantum objects common to both cases.

All the collective effects mentioned above are observed in systems where the time of isotropic spontaneous decay exceeds τ_c by many orders of magnitude. Thus, for example, in the optical band the ratio $T_1/\tau_c\sim10^{10}$ at $N/V\sim10^{20}$. The role of the spontaneous processes in such systems is important only during the initial stage of emission, when they serve as primers for the cascade-like collective process. In the band, however, $T_1/\tau_c\sim10$ at $N/V\sim10^{20}$, and consequently the isotropic spontaneous processes cannot be neglected in such systems. The small contribution of the spontaneous processes means that $\varkappa\ll N$. Comparing (4) and (6) it is easy to get

$$\varkappa=N\tau_c/T_1. \quad (7)$$

Consequently, the fewer field modes contained in a volume equal to the volume of the sample, the higher the emission intensity. $\varkappa$ is equal to unity in a sample with dimensions smaller than λ, and consequently the radiation intensity increases by a factor N (Dicke superradiance).

Using (7), we have for systems with $\tau \ll T_2$

$$S(t)=\varkappa N_2(t)+\frac{N^2}{4}\left(1-\frac{\tau_c}{T_2}\right)^2-\frac{1}{4}\left(R(t)-\frac{N\tau_c}{T_2}\right)^2,$$
$$R(t)=N\alpha+N\beta\frac{(1-\alpha)\operatorname{ch}\varphi-\beta\operatorname{sh}\varphi}{\beta\operatorname{ch}\varphi-(1-\alpha)\operatorname{sh}\varphi} \quad (8)$$

where

$$\alpha=\frac{\tau_c}{T_2}+\frac{\tau_c}{T_1},\quad \beta=\left[\left(1-\frac{\tau_c}{T_2}+\frac{\tau_c}{T_1}\right)^2+\frac{4\tau_c^2}{T_2T_1}\right]^{1/2},\quad \varphi=\frac{\beta t}{2\tau_c}.$$

The maximum radiation intensity is

$$I=\frac{N}{2T_1}\left(1+\frac{\tau_c}{T_2}+\frac{\tau_c}{T_1}\right)+\frac{N}{4\tau_c}\left[\left(1-\frac{\tau_c}{T_2}\right)^2-\left(\frac{\tau_c}{T_1}\right)^2\right], \quad (9)$$

where the first and second terms describe the intensity of the incoherent and coherent spontaneous decay, respectively. The intensity (9) is reached at the instant of time

$$t_0=\frac{\tau_c}{\beta}\ln\frac{\beta+(1-\alpha)}{\beta-(1-\alpha)}. \quad (10)$$

The coherence for the existence of a coherent component is

$$\mu_0 L>1+1/\Gamma T_1. \quad (11)$$

Thus, for natural-width lines the conditions for the onset of coherence decay become twice as stringent.

Let $\Gamma T_1 \approx 1$ and $\mu_0 L = 2(1+\varepsilon)$, where $\varepsilon \ll 1$, then

$$I_{coh}=N\varepsilon/4\tau_c,\quad t_0=\tau_c\varepsilon,$$

where I_{coh} is the intensity of the coherent component of the radiation. Since $T_1 \approx 2\tau_c$ it follows that in such systems the intensity of the coherent component will be smaller than the integrated intensity of the spontaneous decay. Consequently, in needle-shaped crystals, spikes of directed coherent radiations will appear against the background of the incoherent spontaneous emission, with an intensity comparable with the intensity of the latter.

Thus, the relative narrowness of the Mössbauer transition lines, in conjunction with the large values of the photoabsorption cross sections for γ quanta in the Mössbauer energy region causes the entire volume of the sample in which population inversion was produced will radiate like a single quantum-mechanical ensemble of emitters. The conditions for the onset of this collective emission coincides in form with the conditions for the amplification with the aid of stimulated emission, which were discussed earlier in connection with the γ-laser problem. Consequently, the problem of attaining amplification in the γ band, posed above, as well as the methods for its solution, remains as important as ever.

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Propagation of photons in a magnetic field

V. V. Skobelev

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A radiative correction to the polarization operator of a photon in a strong magnetic field is obtained which corresponds to the contribution of the "mass" and the "vertex" diagrams. Arguments are given in support of the contention that the expansion parameter for the polarization operator for $|q^2|< m^2$ is the quantity $\alpha\ln^2(B/B_0)$, with $B_0=m^2/e=4.41\times10^{13}$ G.

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Possible astrophysical applications have stimulated in recent times the appearance of different approximate methods of calculating electrodynamical processes in strong magnetic fields. One of the most popular ones is the crossed field approximation the idea of which is due to Ritus and Nikishov.[1]

If one has in mind purely magnetic fields (say, of the order of $B_0=m^2/e=4.41\times10^{13}$ G), then in making calculations of vacuum diagrams with external photon lines this approximation is correct in the domain of high photon energies $\omega\gg m$ (ω is the electron mass). In invariant form of recording this means that the parameters

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