Using (7), we have for systems with $\tau \ll T_2$

$$S(t)=\varkappa N_2(t)+\frac{N^2}{4}\left(1-\frac{\tau_c}{T_2}\right)^2-\frac{1}{4}\left(R(t)-\frac{N\tau_c}{T_2}\right)^2,$$
$$R(t)=N\alpha+N\beta\frac{(1-\alpha)\operatorname{ch}\varphi-\beta\operatorname{sh}\varphi}{\beta\operatorname{ch}\varphi-(1-\alpha)\operatorname{sh}\varphi} \quad (8)$$

where

$$\alpha=\frac{\tau_c}{T_2}+\frac{\tau_c}{T_1},\quad \beta=\left[\left(1-\frac{\tau_c}{T_2}+\frac{\tau_c}{T_1}\right)^2+\frac{4\tau_c^2}{T_2T_1}\right]^{1/2},\quad \varphi=\frac{\beta t}{2\tau_c}.$$

The maximum radiation intensity is

$$I=\frac{N}{2T_1}\left(1+\frac{\tau_c}{T_2}+\frac{\tau_c}{T_1}\right)+\frac{N}{4\tau_c}\left[\left(1-\frac{\tau_c}{T_2}\right)^2-\left(\frac{\tau_c}{T_1}\right)^2\right], \quad (9)$$

where the first and second terms describe the intensity of the incoherent and coherent spontaneous decay, respectively. The intensity (9) is reached at the instant of time

$$t_0=\frac{\tau_c}{\beta}\ln\frac{\beta+(1-\alpha)}{\beta-(1-\alpha)}. \quad (10)$$

The coherence for the existence of a coherent component is

$$\mu_0L>1+1/\Gamma T_1. \quad (11)$$

Thus, for natural-width lines the conditions for the onset of coherence decay become twice as stringent.

Let $\Gamma T_1\approx 1$ and $\mu_0 L=2(1+\varepsilon)$, where $\varepsilon\ll 1$, then

$$I_{\rm coh}=N\varepsilon/4\tau_c,\quad t_0=\tau_c\varepsilon,$$

where $I_{\rm coh}$ is the intensity of the coherent component of the radiation. Since $T_1\approx 2\tau_c$ it follows that in such systems the intensity of the coherent component will be smaller than the integrated intensity of the spontaneous decay. Consequently, in needle-shaped crystals, spikes of directed coherent radiations will appear against the background of the incoherent spontaneous emission, with an intensity comparable with the intensity of the latter.

Thus, the relative narrowness of the Mössbauer transition lines, in conjunction with the large values of the photoabsorption cross sections for γ quanta in the Mössbauer energy region causes the entire volume of the sample in which population inversion was produced will radiate like a single quantum-mechanical ensemble of emitters. The conditions for the onset of this collective emission coincides in form with the conditions for the amplification with the aid of stimulated emission, which were discussed earlier in connection with the γ-laser problem. Consequently, the problem of attaining amplification in the γ band, posed above, as well as the methods for its solution, remains as important as ever.

[1] V. A. Bushuev and R. N. Kuz'min, Usp. Fiz. Nauk **114**, 677 (1974) [Sov. Phys. Usp. **17**, 942 (1975)].
[2] G. S. Baldwin and R. V. Khokhlov, Phys. Today **28**, 33 (1975).
[3] Yu. A. Il'inskiĭ and R. V. Khokhlov, Izv. Vyssh. Ucheb. Zaved., Radiofizika **19**, 792 (1976).
[4] A. V. Andreev, Zh. Eksp. Teor. Fiz. **72**, 1397 (1977) [Sov. Phys. JETP **45**, 734 (1977)].
[5] R. H. Dicke, Phys. Rev. **93**, 99 (1954).
[6] S. Stenholm, Phys. Reports 6C, 1 (1973).
[7] R. Bonifacio and L. A. Lugiato, Phys. Rev. **11A**, 1507 (1975).
[8] A. M. Afanas'ev and Yu. Kagan, Pis'ma Zh. Eksp. Teor. Fiz. **2**, 130 (1965) [JETP Lett. **2**, 81 (1965)].
[9] V. F. Dmitriev and E. V. Shuryak, Zh. Eksp. Teor. Fiz. **67**, 494 (1974) [Sov. Phys. JETP **40**, 244 (1975)].
[10] G. T. Trammell and J. P. Hannon, Opt. Comm. **15**, 325 (1975).
[11] V. I. Emel'yanov and Yu. L. Klimontovich, Opt. Spectrosk. **41**, 913 (1976).

Translated by J. G. Adashko

Propagation of photons in a magnetic field

V. V. Skobelev

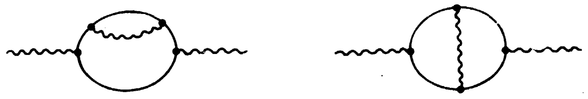
(Submitted April 19, 1977)
Zh. Eksp. Teor. Fiz. **73**, 1301–1305 (October 1977)

A radiative correction to the polarization operator of a photon in a strong magnetic field is obtained which corresponds to the contribution of the "mass" and the "vertex" diagrams. Arguments are given in support of the contention that the expansion parameter for the polarization operator for $|q^2|\ll m^2$ is the quantity $\alpha\ln^2(B/B_0)$, with $B_0=m^2/e=4.41\times10^{13}$ G.

PACS numbers: 12.20.Ds

Possible astrophysical applications have stimulated in recent times the appearance of different approximate methods of calculating electrodynamical processes in strong magnetic fields. One of the most popular ones is the crossed field approximation the idea of which is due to Ritus and Nikishov.[1]

If one has in mind purely magnetic fields (say, of the order of $B_0=m^2/e=4.41\times10^{13}$ G), then in making calculations of vacuum diagrams with external photon lines this approximation is correct in the domain of high photon energies $\omega\gg m$ (ω is the electron mass). In invariant form of recording this means that the parameters



a b

FIG. 1.

$F_{\mu\nu}F^{\mu\nu}$ and $F^*_{\mu\nu}F^{\mu\nu}$ must be small compared to $(F_{\mu\nu}q^\nu)^2/m^2$ ($F^*_{\mu\nu}$ is a dual tensor, q is the four-momentum of the photon). The two-dimensional approximation of quantum electrodynamics proposed by us[2,3] is in a certain sense complementary to the crossed field method. Its idea is based on the fact that in fields $B \gg B_0$ and for sufficiently small momenta of the external lines of the diagram ($\omega^2 \ll |eB|$) the phase space of vacuum electrons degenerates into a two-dimensional space since their transverse degrees of freedom are suppressed. In invariant formulation the conditions for the applicability of the method in the case of loop diagrams can be written in the form

$$F_{\mu\nu}F^{\mu\nu} \gg |F_{\mu\nu}{}^*F^{\mu\nu}|,\ B_0^2,$$
$$(e^2(F_{\mu\nu}F^{\mu\nu}))^{1/2} \gg |q^2|,\ (F_{\mu\nu}q^\nu)^2/(F_{\mu\nu}F^{\mu\nu}).$$

From this it can be seen that in the case of a magnetic field with magnetic induction $B \gg B_0$ and in the range of photon energy $m \ll \omega \ll eB$ both methods are applicable and give identical results.[2] For values $\omega \lesssim m$ in the case $B \gg B_0$ only the two-dimensional approximation is applicable.[2,3] In a recent paper by Morozov and Narozhnyĭ[4] a radiative correction to the polarization operator for the photon in a crossed field was obtained with the authors having considered only the contribution of the mass diagram a (see Fig. 1). On the basis of physical considerations concerning the smallness of exchange effects in a strong field and for high photon energies the authors do not take into account the contribution of Fig. 1b to the polarization operator, however, mentioning the fact that only the contribution of both diagrams is gauge-invariant.

In the present note we calculate the gauge-invariant contribution of both diagrams in the two-dimensional approximation and, in particular, we investigate in detail the case $q^2 (\equiv q_0^2 - q_3^2) \ll m^2$ where the crossed field approximation is inapplicable. Concrete calculations have shown that in this case in the formal limit $B \to \infty$ the contribution of Fig. 1a differs from the contribution of Fig. 1b by the factor $\ln(B/B_0)/3$, so that in fact over a large range of fields the contribution of both diagrams is quite comparable. Further we have shown that the parameter for the expansion of the polarization operator into a perturbation theory series is the quantity $\alpha \ln^2(B/B_0)$ (for $|q^2| \ll m^2$). Thus, the series diverges in the case of fields $B \sim 10^{17-18}$ G.

Using Feynman rules for constructing matrix elements in the two-dimensional approximation[3] we can obtain the following expressions for the contributions of Figs. 1a and 1b to the polarization tensor of the photon:

$$P_{\mu\nu}^{(a,b)} = \frac{2\alpha^2 \tilde{B} m^6}{\pi^4} \exp\left\{-\frac{q_\perp^2}{2\gamma}\right\} \int_0^\infty dx \exp\left\{-\frac{x}{2\tilde{B}}\right\} I_{\mu\nu}^{(a,b)}(x), \quad \text{(1a)}$$

$$I_{\mu\nu}^{(a)} = \frac{1}{2}\int d^2p \int d^2k \frac{g_{\mu\nu}(m^2-k^2+2kq)+2k_\mu(k-q)_\nu+2k_\nu(k-q)_\mu}{(p^2-m^2)[(p-k)^2-m^2x](k^2-m^2)^2[(k-q)^2-m^2]} \quad \text{(1b)}$$

$$I_{\mu\nu}^{(b)} = \frac{1}{4}\int d^2p \int d^2k$$
$$\times \frac{4p_\mu k_\nu - 2p_\mu q_\nu - 2k_\nu q_\mu + 2q_\mu q_\nu - g_{\mu\nu}q^2}{[(p-q)^2-m^2](p^2-m^2)[(p-k)^2-m^2x][(k-q)^2-m^2](k^2-m^2)}, \quad \text{(1c)}$$

where all the scalar products are two-dimensional (0, 3), $\gamma = |eB|$, $\tilde{B} = B/B_0$, and the tensor differs from zero for $\mu, \nu = 0, 3$. In order to prove gradient invariance we note that

$$I_{\mu\nu}^{(a)} q^\nu = \frac{1}{2}\int d^2p \int d^2k \left\{\frac{2k_\mu - q_\mu}{(p^2-m^2)[(p-k)^2-m^2x](k^2-m^2)[(k-q)^2-m^2]} - \frac{2k_\mu}{(p^2-m^2)[(p-k)^2-m^2x](k^2-m^2)^2}\right\}.$$

The second term disappears, and the first after obvious transformations is equal to the negative of the contraction $I_{\mu\nu}^{(b)} q^\nu$. In virtue of this the tensor has the form

$$P_{\mu\nu} = P(q^2)\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right), \quad \text{(2a)}$$

so that the polarization operator is given by

$$P(q^2) = P_\mu^{(a)\mu} + P_\mu^{(b)\mu}. \quad \text{(2b)}$$

Evaluation of the integrals over d^2p and d^2k and a further analysis of the formulas can be more conveniently carried out by going over to the α-representation for the propagators:

$$\frac{1}{p^2-m^2} = -i\int_0^\infty d\alpha \exp[i\alpha(p^2-m^2)-\alpha\varepsilon], \quad \text{(3)}$$

as a result of which the contractions $I_\mu^{(a)\mu}$ and $I_\mu^{(b)\mu}$ can be represented in the form

$$I_\mu^{(a)\mu} = \frac{\pi^2}{m^4}\int_0 d\alpha_1 \int_0 d\alpha_2 \int_0 \frac{d\alpha_3}{(1+\alpha_3)(\alpha_1+\alpha_2)+\alpha_1\alpha_2} \frac{1}{A^2}\left[\frac{4\alpha_3}{A} - 1\right],$$
$$A = 1+\alpha_1+\alpha_3+\alpha_2 x - \tilde{q}^2 \frac{\alpha_3(\alpha_1+\alpha_2)+\alpha_1\alpha_2}{(1+\alpha_3)(\alpha_1+\alpha_2)+\alpha_1\alpha_2}, \quad \tilde{q}^2 = q^2/m^2; \quad \text{(4)}$$

$$I_\mu^{(b)\mu} = -\frac{\pi^2}{m^4}\prod_{i=1}^4\left(\int_0^\infty d\alpha_i\right)\frac{1}{C^2}[(\alpha_1+\alpha_2+\alpha_3)(1+\alpha_2+\alpha_4)-\alpha_2^2]^{-2}$$
$$\times\left\{\alpha_2 + \tilde{q}^2 \frac{2}{C}\frac{[\alpha_1(1+\alpha_2+\alpha_4)+\alpha_2\alpha_4][\alpha_1+\alpha_2+\alpha_3+\alpha_2\alpha_3]}{(\alpha_1+\alpha_2+\alpha_3)(1+\alpha_2+\alpha_4)-\alpha_2^2}\right\}$$
$$C = 1+\alpha_1+\alpha_3+\alpha_4+\alpha_2 x + \frac{\tilde{q}^2}{\alpha_1+\alpha_2+\alpha_3} \quad \text{(5)}$$
$$\times\left\{\frac{[\alpha_4(\alpha_1+\alpha_2+\alpha_3)+\alpha_1\alpha_2]^2}{(\alpha_1+\alpha_2+\alpha_3)(1+\alpha_2+\alpha_4)-\alpha_2^2} - (\alpha_2+\alpha_3)(\alpha_1+\alpha_4)-\alpha_1\alpha_4\right\}.$$

We note that for $\tilde{q}^2 \geq 4$ the quantities $I_\mu^{(a,b)\mu}$, as expected, acquire an imaginary part, since A and C vanish within the region of integration. Expressions (4) and (5) must be "regularized":

$$[I_\mu^{(a,b)\mu}]_R = I_\mu^{(a,b)\mu} - I_\mu^{(a,b)\mu}|_{\tilde{q}^2=0}, \quad \text{(6)}$$

although they contain no divergences corresponding to large virtual momenta. Both expressions also do not contain an infra-red divergence and do not depend on the photon mass, but this circumstance requires explanation. The contribution of the diagram of Fig. 1b does not contain the photon mass in the principal order in the field $\tilde{B}$, while concerning the contribution of the diagram of Fig. 1a one can only assert that the photon mass does not appear in the leading and in the next order in powers of $\ln\tilde{B}$. This is explained by the fact that in the integral over d^2p in (1c) for $\varkappa \sim \tilde{B}$ the contribution $|p^2| \sim \gamma$ is also contained, and this does not correspond to the condition of applicability of the "two-dimensional" representation of the electron Green's function.[2,3] But the corresponding contributions are of a logarithmic character, and formula (1c) in any case correctly gives the leading and the next highest power of $\ln\tilde{B}$.

For a more detailed analysis we consider the case $|\tilde{q}|^2 \ll 1$ (no limitations are imposed in this case on the value of the transverse momentum). If we expand (4) in a series in terms of $\tilde{q}^2$ and take only the "regularized" part linear in $\tilde{q}^2$, then it can be easily noted that the integral over α_3 can be factorized by means of a replacement $\alpha_i \to \alpha_i/(1+\alpha_3)$ $(i=1, 2)$, after which the integral over α_2 can be evaluated in an elementary manner. As a result of this we obtain

$$[I_\mu^{(a)\mu}]_R \approx \frac{\pi^2}{m^4}\tilde{q}^2 \int_0^\infty \frac{d\alpha_1}{\Delta}\left\{\frac{1}{6} - \frac{\alpha_1}{3\Delta} - \frac{2\alpha_1^2}{\Delta^2} - \frac{4\alpha_1^3}{\Delta^3} + \frac{\alpha_1^2}{\Delta^2}\left(1 + \frac{4\alpha_1}{\Delta} + \frac{4\alpha_1^2}{\Delta^2}\right)\ln\left[\frac{(1+\alpha_1)(\alpha_1+x)}{\alpha_1}\right]\right\}, \quad \Delta = x(1+\alpha_1)+\alpha_1^2. \tag{7}$$

In the principal order in $\tilde{B}$ the main term is the first term (7). The corresponding contribution to the polarization operator can be transformed to the form

$$[P_\mu^{(a)\mu}]_R \approx \frac{\alpha^2}{3\pi^2}\tilde{B}\tilde{q}^2 m^2 \exp\left\{-\frac{q_\perp^2}{2\gamma}\right\} \cdot \left[2\int_0^4 \frac{dx\, e^{-x/2\tilde{B}}}{(x(4-x))^{1/2}} \operatorname{arctg}\left(\frac{4}{x}-1\right)^{1/2} + \frac{1}{4\tilde{B}}\int_4^\infty dx \exp\left\{-\frac{x}{2\tilde{B}}\right\}\left(\ln\frac{x+(x(x-4))^{1/2}}{x-(x(x-4))^{1/2}}\right)^2\right]. \tag{8}$$

It is not possible to carry out the integration in (8) exactly, but we can pick out the principal power of $\ln\tilde{B}$. For $\tilde{B}\to\infty$ the integral of the first term tends to a constant, while in the second term the argument of the logarithm can be replaced by x, after which the integral is available from tables. This yields

$$[P_\mu^{(a)\mu}]_R \approx \frac{\alpha^2}{6\pi^2} m^2\tilde{q}^2\tilde{B}\ln^2(\tilde{B})\exp\left\{-\frac{q_\perp^2}{2\gamma}\right\}. \tag{9}$$

The quadratic dependence on $\ln\tilde{B}$ of the contribution of the "mass" diagram is quite natural, since the self-energy correction to the electron mass in a strong magnetic field is also proportional to $\ln^2\tilde{B}$.[5]

In a similar manner, it is possible to separate out the term linear in $\tilde{q}^2$ in formula (5) with the integrals over α_4 and α_3 becoming factorized with the aid of the change of variables introduced above. In the principal order in $\tilde{B}$ the result has the form

$$[P_\mu^{(b)\mu}]_R = -\frac{\alpha^2}{2\pi^2}\tilde{B}\tilde{q}^2 m^2 \exp\left\{-\frac{q_\perp^2}{2\gamma}\right\} \times\left[\int_0^4 \frac{dx\, e^{-x/2\tilde{B}}}{x-4}\left[1 - \frac{4}{(x(4-x))^{1/2}}\operatorname{arc\,tg}\left(\frac{4}{x}-1\right)^{1/2}\right] + \int_4^\infty \frac{dx\, e^{-x/2B}}{x-4}\left[1 - \frac{2}{(x(x-4))^{1/2}}\ln\frac{x+(x(x-4))^{1/2}}{x-(x(x-4))^{1/2}}\right]\right]. \tag{10}$$

For $\tilde{B}\to\infty$ we obtain

$$[P_\mu^{(b)\mu}]_R \approx -\frac{\alpha^2}{2\pi^2}\tilde{q}^2 m^2\tilde{B}\ln(\tilde{B})\exp\left\{-\frac{q_\perp^2}{2\gamma}\right\}, \tag{11}$$

i.e., in this limit the contribution of the "vertex" diagram is smaller than the contribution of the "mass" diagram by factor of $\ln\tilde{B}$, with the contributions being of opposite sign, and for $\tilde{B}\approx 20$ approximately cancelling each other. We could have expected the appearance of the first power of $\ln\tilde{B}$ in (11) since the vertex function in the two-dimensional approximation is also proportional to $\ln\tilde{B}$.[3]

It is well known that the contribution of a simple loop in the approximation under consideration is proportional to the first power of the field,[6] i.e., in the next order of perturbation theory in terms of α there appears an additional factor $\alpha\ln^2\tilde{B}$. This enables us to suppose that the expansion parameter for the polarization operator for $|q^2| \ll 1$ is indeed $\alpha\ln^2\tilde{B}$ from which one obtains the aforementioned limitation on the magnitude of the field.

The author considers it his pleasant duty to thank V. I. Ritus for a number of explanations he has provided on the subject under consideration.

[1]A. I. Nikishov and V. I. Ritus, Zh. Eksp. Teor. Fiz. 52, 1707 (1967) [Sov. Phys. JETP 25, 1135 (1967)].

[2]V. V. Skobelev, Zh. Eksp. Teor. Fiz. 71, 1263 (1976) [Sov. Phys. JETP 44, 660 (1976)].

[3]V. V. Skobelev, Zh. Eksp. Teor. Fiz. 72, 1298 (1977) [Sov. Phys. JETP 45, 682 (1977)].

[4]D. A. Morozov and N. B. Narozhnyĭ, Zh. Eksp. Teor. Fiz. 72, 44 (1977) [Sov. Phys. JETP 45, 23 (1977)].

[5]B. Jancovici, Phys. Rev. 187, 2275 (1965).

[6]V. V. Skobelev, Izv. Vyssh. Uchebn. Zaved. Fiz. 10, 142 (1975) Yu. M. Loskutov and V. V. Skobelev, Phys. Lett. 56A, 151 (1975).

Translated by G. Volkoff