tal value

$$\varepsilon = H_{ph}^2 / L\ {}^1H_L^2. \quad (9)$$

From (9) we obtained the ratio N_S/N_I of the nuclei. The satisfaction of the condition $\tau \gg T_{IS}$ was monitored against the ratio ε/H_{1ph}^3, which should be independent of τ and H_{1ph} if this inequality holds. The experimental value of this quantity for Na nuclei in tetragonally distorted positions (transition $f = 6.175$ MHz at $\mathbf{H}_0 \parallel [100]$) is equal to $(74 \pm 10) \times 10^{-6}$ G^{-2}. The ratio of the number of Na nuclei in the tetragonally distorted positions to the number of F nuclei in the sample is therefore ~ 0.6%, or (0.1 ± 0.02)% when converted to the number of the added impurities. The concentration of ^{7}Li in the sample was measured at $\mathbf{H}_0 \parallel [110]$, $\tau = 10.4$ msec, and $H_{1ph} = 2.7$ G. The value of ε/H_{1ph}^2 was found to be $(168 \pm 20) \times 10^{-6}$ G^{-2}. Consequently the ratio of the number of ^{7}Li atoms to the number of F atoms is (0.062 ± 0.008)%, in agreement with the reported measurement results.

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Radiation polarization and ferromagnetic ordering in a semiconductor laser

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By pumping the electrons and holes in a semiconductor by an external source it is possible to establish a quasi-Fermi distribution of the nonequilibrium carriers. At temperatures below a certain critical value T_g this produces a simultaneous Bose condensation of electron-hole pairs and photons, meaning the onset of a lasing regime in the system. If the electron and hole densities are unequal (because of doping), singlet and triplet electron-hole pairings can coexist at $T \leq T_C (T_C \leq T_g)$, thus attesting to the appearance of ferromagnetic ordering. Spin-orbit interaction of the electrons with the radiation, by causing spin flip of the electrons in interband transitions, produces at $T \leq T_C$ in the laser radiation a circular polarization proportional to the degree of the ferromagnetic order.

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1. It is known that metals with congruent sections of the Fermi surface are unstable to electron-hole pairing (exciton instability[1]) and are subject also to phonon instability.[2] Consequently, either electron and ion charge-density waves (CDW) or an electron spin density wave (SDW) is produced in the stable state.

A similar situation can obtain in semiconductors under nonequilibrium conditions, when the external source produces an excess of electron density in the conduction band or of hole density in the valence band. If the energy relaxation time τ_E within each band is much shorter than the interband recombination time τ_R, then the electron and hole distribution functions are of quasi-Fermi form. At sufficiently high pump intensity, the Fermi quasi-levels μ_e of the electrons and μ_h of the holes are located respectively inside the conduction and valence bands, i.e., population inversion takes place. A situation is then possible wherein the constant-energy surfaces correspond to the Fermi quasi-levels of the electrons and holes are almost congruent.

In this case, exciton instability can take place and charge- or spin-density waves can be produced if the

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condition $\tau_R\Delta_0 > 1$ is satisfied ($\Delta_{0s,t}$ is the order parameter corresponding to the spin or density waves at $\tau_R = \infty$).

There are also substantial deviations from the equilibrium case.[1] First, the phases of the order parameters Δ_s and Δ_t are not fixed, in contrast to the equilibrium case[3] wherein the Fermi levels of the electrons and holes coincide. Therefore the transition to the CDS or SDW state is of second order, and superfluidity due to excitation-energy transfer is possible in the system.[4]

Second, the interband electron-phonon interaction does not lead to a singularity, so that no ion charge-density waves are produced, i.e., the crystal symmetry does not change. The CDS and SDW oscillate then with a frequency $\Omega = \mu_e - \mu_h$. In the case of doped semiconductors, coexistence of CDW and SDW is possible and leads to a time-independent magnetization.[5]

If the CDW or the SDW exist in two semiconductors separated by a tunnel junction, then an alternating current analogous to the Josephson current and due to the beats is produced in the system; the frequency ω of the current is proportional to the difference between the values of $\mu_e - \mu_h$ for the contacting semiconductors.[6] But if a CDW exists in one of the semiconductors, and an SDW in the other, then the beats at the frequency ω correspond to an alternating magnetization, depending on the tunneling probability.[7]

If the dipole moment corresponding to the interband transition differs from zero, then under nonequilibrium conditions and when the extrema of the conduction and valence bands coincide in momentum space, photon instability sets in[8] at a frequency $\Omega = \mu_e - \mu_h$ instead of the phonon instability obtained at zero frequency under equilibrium conditions. In the stable state there is produced together with CDW a Bose condensate of the photons, and corresponds to a transition of the system into the laser regime, instead of the restructuring of the crystal under equilibrium conditions.

The first to raise the question of the onset of an electromagnetic wave together with a CDW was Keldysh.[8] Whereas under equilibrium conditions the realignment of the crystal structure (i.e., the onset of a static ion CDS) fixed the CDW phase, Bose condensation of the photons does not lift the phase degeneracy, since the Bose condensation takes place at a finite frequency Ω (see Sec. 2). For the same reason, however, the photon instability is not completely eliminated by introducing solely the Bose condensate of the photons,[8] if no account is taken of the finite damping of the electrons and the electromagnetic waves in the system. The presence of the photon Bose condensate leads to one more singularity, namely the spatial period of the electron CDW and the condensate-photon wavelength[8] turn out to be generally speaking incommensurate with the crystal period. The purpose of the present paper is to study the possibility of polarizing the radiation of a semiconductor laser (a photon Bose condensate) via magnetization of the electronic subsystem, when CDW and SDW coexist.[5] It will be shown in Sec. 2 that the radiation polarization turns out in this case to be different from zero only to the extent that spin-orbit interaction is present.

In Sec. 3 we investigate the spectrum of the photons in the laser regime, for the case considered in Sec. 2. It is shown that a consistent allowance for the Coulomb interaction and for the ferromagnetic ordering, which were disregarded in[8], does not eliminate the photon instability. Therefore, strictly speaking the results of Sec. 2 remain valid only qualitatively when account is taken of the finite damping in the electronic and photon subsystems.

2. We consider a simple two-band model of a semiconductor, with a large forbidden gap E_g such that there is no exciton instability[9] at equilibrium. We assume that the recombination time τ_R of the conduction-band electrons $\varepsilon_c(\mathbf{p})$ with the valence-band holes $\varepsilon_v(\mathbf{p})$ is much longer than the intraband energy relaxation time τ_E. Under these conditions, at a sufficiently high pump intensity, the electrons and holes have a quasi-Fermi distribution with quasi-Fermi levels μ_c and μ_v for the electrons and holes in the corresponding bands. We assume also that near the Fermi quasilevels the dispersion laws in the bands satisfy the condition $\varepsilon_c(\mathbf{p}) = -\varepsilon_v(\mathbf{p}) \equiv \varepsilon(\mathbf{p})$, if the energy in each band is reckoned from the corresponding Fermi level.

After the unitary transformation

$$U = \exp\left\{\frac{i\Omega t}{2}\sum_{\mathbf{p},\alpha}(a^+_{c\mathbf{p}\alpha}a_{c\mathbf{p}\alpha} - a^+_{v\mathbf{p}\alpha}a_{v\mathbf{p}\alpha}) + i\Omega t\sum_{\mathbf{k}}(c^+_{\mathbf{k}+}c_{\mathbf{k}+} + c^+_{\mathbf{k}-}c_{\mathbf{k}-})\right\}$$

with allowance for the spin structure and the two circular polarizations of the photons, the Hamiltonian of the system takes the form

$$\begin{aligned} H = &\sum_{\mathbf{p},\alpha}\xi(\mathbf{p})\,(a^+_{c\mathbf{p}\alpha}a_{c\mathbf{p}\alpha} - a^+_{v\mathbf{p}\alpha}a_{v\mathbf{p}\alpha}) + \Omega_{\mathbf{k}_0}(c^+_{\mathbf{k}_0+}c_{\mathbf{k}_0+} + c^+_{\mathbf{k}_0-}c_{\mathbf{k}_0-}) \\ &+ \sum_{\substack{\mathbf{p},\mathbf{p}',\mathbf{q}\\ \alpha,\beta}} g a^+_{c\mathbf{p}\alpha}a^+_{v\mathbf{p}'\beta}a_{v\mathbf{p}'-\mathbf{q}\beta}a_{c\mathbf{p}+\mathbf{q}\alpha} + \Big\{\sum_{\mathbf{p},\alpha}\frac{M}{2}a^+_{c\mathbf{p}\alpha}a_{v\mathbf{p}-\mathbf{k}_0\alpha}(c_{\mathbf{k}_0-} + c_{\mathbf{k}_0+}) \\ &+ \sum_{\mathbf{p}} L(a^+_{c\mathbf{p}\uparrow}a_{v\mathbf{p}-\mathbf{k}_0\downarrow}c_{\mathbf{k}_0+} - a^+_{c\mathbf{p}\downarrow}a_{v\mathbf{p}-\mathbf{k}_0\uparrow}c_{\mathbf{k}_0-}) + \text{c.c.}\Big\} \\ &- \delta\mu\sum_{\mathbf{p},\alpha}(a^+_{c\mathbf{p}\alpha}a_{c\mathbf{p}\alpha} + a^+_{v\mathbf{p}\alpha}a_{v\mathbf{p}\alpha}). \end{aligned} \tag{1}$$

Here

$$\Omega = \mu_c - \mu_v, \quad \Omega_{\mathbf{k}_0} = k_0 c - \Omega, \quad \xi(\mathbf{p}) = p^2/2m - (\Omega - E_g)/2,$$

$a_{c\mathbf{p}\alpha}$ and $a_{v\mathbf{p}\alpha}$ are the annihilation operators in the conduction and valence bands for electrons with spin α, $c_{\mathbf{k}_0+}$ and $c_{\mathbf{k}_0-}$ are the annihlation operators for photons with right and left polarizations, $\delta\mu$ is the shift of the Fermi quasilevels on account of the doping, g is the effective constant of the interband Coulomb interaction, of the density–density type,[1] between the electrons and the holes, and M and L are the matrix elements of the interband transitions with photon absorption without and with spin flip, respectively. The Hamiltonian (1) contains only interaction terms that lead to singularities in the corresponding scattering amplitudes even in the case of weak interaction $na_B^3 \gg 1$[1] to which we confine our-

selves here (n is the concentration of the nonequilibrium electrons and holes, and a_B is the exciton Bohr radius).

The terms in (1) corresponding to the electron-photon interaction are analogous to the electron-phonon interaction terms in the equilibrium case. There is also a substantial difference, in that the phonon operators b_k, since the ion displacements are real, enter in the similar terms in the form of the combination $b_k + b_{-k}^+$. Therefore the corresponding equations of motion contain the order parameters Δ_s and Δ_t in combination with their conjugates, and this lifts the phase degeneracy.[3] In the present case, however, no such combination occurs (see Eqs. (3) below).

Besides the instability to electron-hole pairing in the singlet and triplet channels, the system with the Hamiltonian (1) has photon instability.[8] The analysis of the question of coexistence of singlet and triplet electron-hole pairings, i.e., the question of ferromagnetic order, is similar to that used in the equilibrium case,[10] with the phonon Bose condensate replaced by the photon condensate, for which the two possible polarizations will be taken into account. We introduce in analogy with[10] the following Green's functions:

$$G_c^{\alpha\beta}=-i\langle Ta_{cp\alpha}(t)a_{cp\beta}^+(t')\rangle,\quad G_{vc}^{\alpha\beta}=-i\langle Ta_{vp\alpha}(t)a_{cp\beta}^+(t')\rangle, \tag{2}$$

which form matrices $\mathbf{G}_c$ and $\mathbf{G}_{vc}$ in the indices α and β. We can write for these matrices a system of equation by taking the Fourier transform with respect to time:

$$(\omega-\xi(\mathbf{p}))\mathbf{G}_c=\mathbf{I}+\Delta\mathbf{G}_{vc},\quad (\omega+\xi(\mathbf{p}))\mathbf{G}_{vc}=\Delta^+\mathbf{G}_c. \tag{3}$$

By using the standard procedure[10] we obtain for the components of the order-parameter matrix Δ the following system of consistency equations

$$\begin{aligned}
\Delta_{\uparrow\uparrow}-\Delta_{\downarrow\downarrow}&=gA(\Delta_{\uparrow\uparrow}-\Delta_{\downarrow\downarrow}),\\
\Delta_{\uparrow\uparrow}+\Delta_{\downarrow\downarrow}&=\left(g+\frac{M^2}{\Omega_{\mathbf{k}_0}}\right)B(\Delta_{\uparrow\uparrow}+\Delta_{\downarrow\downarrow})+\frac{ML}{\Omega_{\mathbf{k}_0}}B(\Delta_{\uparrow\downarrow}-\Delta_{\downarrow\uparrow}),\\
\Delta_{\uparrow\downarrow}+\Delta_{\downarrow\uparrow}&=\left(g+\frac{L^2}{\Omega_{\mathbf{k}_0}}\right)A(\Delta_{\uparrow\uparrow}+\Delta_{\downarrow\downarrow}),\\
\Delta_{\uparrow\downarrow}-\Delta_{\downarrow\uparrow}&=\left(g+\frac{L^2}{\Omega_{\mathbf{k}_0}}\right)B(\Delta_{\uparrow\downarrow}-\Delta_{\downarrow\uparrow})+\frac{ML}{\Omega_{\mathbf{k}_0}}B(\Delta_{\uparrow\uparrow}+\Delta_{\downarrow\downarrow});
\end{aligned} \tag{4}$$

$$A=(-i)\sum_{p,\omega}\frac{\omega^2-\xi^2(\mathbf{p})+\Delta_{\uparrow\uparrow}\Delta_{\downarrow\downarrow}-\Delta_{\uparrow\downarrow}\Delta_{\downarrow\uparrow}}{\mathrm{Det}},$$

$$B=(-i)\sum_{p,\omega}\frac{\omega^2-\xi^2(\mathbf{p})-\Delta_{\uparrow\uparrow}\Delta_{\downarrow\downarrow}+\Delta_{\uparrow\uparrow}\Delta_{\downarrow\uparrow}}{\mathrm{Det}};$$

$\mathbf{k}_0$ is the momentum of the Bose condensate of the photons and electron-hole pairs.[8] The symbol Det stands for the expression that determines the spectrum of the single-particle excitations of the nonequilibrium system:

$$\begin{aligned}
\mathrm{Det}=&(\omega^2-\xi^2(\mathbf{p})-\Delta_{\uparrow\uparrow}{}^2-\Delta_{\downarrow\uparrow}{}^2)(\omega^2-\xi^2(\mathbf{p})-\Delta_{\downarrow\downarrow}{}^2-\Delta_{\uparrow\downarrow}{}^2)\\
&-(\Delta_{\uparrow\uparrow}\Delta_{\uparrow\downarrow}+\Delta_{\downarrow\downarrow}\Delta_{\downarrow\uparrow})^2.
\end{aligned} \tag{5}$$

The system (4) can be reduced to two equations relative to the new unknowns Δ_s and Δ_t defined by the relations

$$\frac{\Delta_{\uparrow\uparrow}+\Delta_{\downarrow\downarrow}}{2}\left(1+\frac{L^2}{M^2}\right)^{1/2}=\Delta_s,\quad \frac{\Delta_{\uparrow\downarrow}+\Delta_{\downarrow\uparrow}}{2}=\Delta_t. \tag{6}$$

Under the supplementary condition that follows from (4):

$$\Delta_{\uparrow\downarrow}-\Delta_{\downarrow\uparrow}=\frac{2L}{M}\left(\frac{\Delta_{\uparrow\uparrow}+\Delta_{\downarrow\downarrow}}{2}\right),\quad \Delta_{\uparrow\uparrow}=\Delta_{\downarrow\downarrow}, \tag{7}$$

we obtain for Δ_s and Δ_t the same system of equation as for the equilibrium case. The singlet and triplet coupling constants are then determined by

$$g_s=g+\frac{L^2}{\Omega_{\mathbf{k}_0}}+\frac{M^2}{\Omega_{\mathbf{k}_0}},\quad g_t=g+\frac{L^2}{\Omega_{\mathbf{k}_0}}. \tag{8}$$

Thus, allowance for the spin-orbit interaction leads to a renormalization of the coupling constants. In all other respects the construction of the phase diagram is perfectly analogous to the equilibrium case.[10] In particular, the coexistence of the singlet and triplet parameters Δ_s and Δ_t, i.e., ferromagnetic ordering, is possible only for a doped semiconductor.

We examine now how the ferromagnetic ordering in the electronic system affects the electromagnetic radiation of the laser, i.e., we calculate the degree of polarization P of the generated radiation. This polarization is defined as

$$P=(N_+^{1/2}-N_-^{1/2})/(N_+^{1/2}+N_-^{1/2}), \tag{9}$$

where $N_\pm$ is the concentration of the Bose-condensate photons with right and left polarizations ($N_\pm=\langle c_{\mathbf{k}0\pm}\rangle^2$). Writing down expressions for the mean values of $c_{\mathbf{k}0+}$ and $c_{\mathbf{k}-}$ with the Hamiltonian (1), in analogy with[8], and taking (6) into account, we obtain the relations

$$N_+^{1/2}-N_-^{1/2}=\frac{2L\Delta_t}{\Omega_{\mathbf{k}_0}g_t},\quad N_+^{1/2}+N_-^{1/2}=\frac{2M\Delta_s(1+L^2/M^2)^{1/2}}{\Omega_{\mathbf{k}_0}g_s}. \tag{10}$$

This yields in the first-order approximation in L

$$P=\frac{L}{M}\left(1+\frac{M^2}{g\Omega_{\mathbf{k}_0}}\right)\frac{\Delta_t}{\Delta_s}. \tag{11}$$

Thus, the presence of spin-orbit interaction and of ferromagnetic ordering ($\Delta_s\neq 0$, $\Delta_t\neq 0$) leads to polarization of the emission of a semiconductor laser.

The foregoing results are formally valid in the absence of spin-orbit splitting of the bands of the semiconductor prior to its interaction with the electromagnetic field. The degree of polarization calculated from (11) is quite small ($\sim E_g/mc^2$). We shall show, however, that if the semiconductor has initially a large enough spin-orbit interaction, which entangles the states with equal spin projections, the degree of polarization can reach values close to unity.

Consider, for example, crystals such as diamond and zinc blende, in which the valence band is known[11] to belong to the representation Γ_8, and the bottom of the conduction band to the representation Γ_6. The basis functions of the valence and conduction bands are the functions $|j\mu\rangle$ and $|s\rangle$, the explicit form of which is given in[11]. For simplicity we confine ourselves to the model of a semiconductor with an anomalous sign of the spin-orbit splitting Λ of the valence band.[12] At $(\Omega-E_g)/2<|\Lambda|$ the optical transitions of the electrons take place

between the conduction band and a split-off subband of the valence band. Neglecting the dependence of the matrix elements of the dipole moment on the quasimomentum of the electron and using the selection rules for dipole transitions, we readily reduce the Hamiltonian of the system to the form (1), but now $M=0$ and the indices α and β denote the projections $\mu=\pm\frac{1}{2}$ of the total angular momentum $j=\frac{1}{2}$ on the z axis. The degree of polarization of the radiation, according to (10), will be

$$P=\Delta_t/\Delta_s, \tag{12}$$

i.e., it can be of the order of unity. In real structures the degree of polarization will be less than in the considered model problem, since the eigenfunctions are superpositions of the functions $|j\mu\rangle$ and $|s\rangle$. [11]

To be able to separate the considered polarization mechanism of laser emission from the others[13] it is necessary to investigate the temperature dependence of the polarization. The critical temperature of the onset of generation, T_g, is defined by the condition for the initiation of the CDW (i.e., the appearance of the parameter Δ_s) simultaneously with the photon condensate $\langle c_{\mathbf{k}_0}\rangle$. Just as in the equilibrium case, [1,10] T_g is connected with the singlet parameter at zero temperature $\Delta_s(0)$ by the relation

$$T_g=0.577\Delta_s(0)=1.15\omega_p\exp(-1/g_sN(0)), \tag{13}$$

where ω_p is of the order of plasma frequency of the nonequilibrium carriers, and $N(0)$ is the density of states on the Fermi quasilevels.

With further lowering of the temperature, the triplet order parameter Δ_t appears at the Curie point T_C[10] and ferromagnetic order sets in. It is precisely at this point that the radiation acquires a polarization whose temperature dependence is determined by the temperature dependences of the parameters Δ_s and Δ_t. [10] In particular, a nonmonotonic temperature dependence of the degree of polarization is possible. [10]

3. We consider now the photon spectrum in the laser regime, i.e., in the presence of a photon Bose condensate. We recall that in the absence of a Bose condensate and of CDW the photon Green's function D acquires an imaginary pole, the modulus of which coincides with Δ_{s0} (see (13)) and which indicates the need for introducing a Bose condensate of photons with momentum $\mathbf{k}_0$. [8]

Under equilibrium conditions, the system with the Bose condensate is stable. On the other hand, at disequilibrium, as shown in[8], when only the electron-photon interaction is taken into account and there is no doping, the instability in the photon system is preserved, although the magnitude of the imaginary pole in the D function is much smaller than the initial value (13).

We now investigate this question with account taken of the Coulomb interaction and of the ferromagnetic ordering in a doped semiconductor. To describe the photons in excess of the condensate we introduce, following[8], the normal and anomalous Green's functions

$$D(\mathbf{k},\ t-t')=-i\langle Tc_{\mathbf{k}}'(t)c_{\mathbf{k}}'^{+}(t')\rangle,\ \tilde{D}(\mathbf{k}',\ \mathbf{k}'',\ t-t')$$
$$=-i\langle Tc_{\mathbf{k}'}^{+}(t)c_{\mathbf{k}''}'^{+}(t')\rangle \tag{14}$$

For the Fourier components of these functions we can obtain the following system of equations, neglecting the renormalization of the vertex parts with the electron-photon interaction:

$$(\omega-\Omega_k-\Sigma_{11}(\mathbf{k},-\omega))D(\mathbf{k},\omega)-\Sigma_{20}(\mathbf{k},-\omega)\tilde{D}(2\mathbf{k}_0-\mathbf{k},\mathbf{k},\omega)=1,$$
$$(\omega+\Omega_{2\mathbf{k}_0-\mathbf{k}}+\Sigma_{11}(2\mathbf{k}_0-\mathbf{k},\omega))\tilde{D}(2\mathbf{k}_0-\mathbf{k},\mathbf{k},\omega)+\Sigma_{02}(-\mathbf{k},\omega)D(\mathbf{k},\omega)=0,$$
$$\Sigma_{02}(-\mathbf{k},\omega)=\Sigma_{20}(2\mathbf{k}_0-\mathbf{k},\omega), \tag{15}$$

Σ_{11}, Σ_{20}, Σ_{02} are effective potentials, in which the entire "ladder" of diagrams with the Coulomb interaction is summed:

$$\Sigma_{20}(\mathbf{k},-\omega)=\Sigma_{20}^{+}(\mathbf{k},-\omega)+\Sigma_{20}^{-}(\mathbf{k},-\omega),$$
$$\Sigma_{11}(\mathbf{k},-\omega)=\Sigma_{11}^{+}(\mathbf{k},-\omega)+\Sigma_{11}^{-}(\mathbf{k},-\omega). \tag{16}$$

The $\pm$ signs coincide with the signs of the spin projection on the z axis.

Summing the ladder diagrams we obtain, neglecting the spin-orbit interaction and taking into account the system of equations for Δ_s and Δ_t, [10] the following relations:

$$\Sigma_{20}^{\pm}-\Sigma_{11}^{\pm}=\frac{1}{2}\left[1\pm\frac{g_s}{g_t}\frac{\Delta_t}{\Delta_s}\right]\Omega_{\mathbf{k}_0},$$
$$\Sigma_{20}^{\pm}+\Sigma_{11}^{\pm}=\frac{1}{2}\left(\frac{\Sigma_{20}^{\pm}+\Sigma_{11}^{\pm}}{1+g(\Sigma_{20}^{\pm}+\Sigma_{11}^{\pm})/M^2}\right). \tag{17}$$

In the limit $\mathbf{k}\approx\mathbf{k}_0$, $\omega\ll\Delta_+$, $\Delta_-(\Delta_\pm=\Delta_s\pm\Delta_t)$, we have ($\Delta_+>\delta\mu>\Delta_-$):

$$\Sigma_{20}^{+}(\mathbf{k},-\omega)=\frac{\Sigma_{20}(\mathbf{k}_0,0)}{2}\left(1+\frac{\omega^2}{6\Delta_+^2}\right),$$
$$\Sigma_{20}^{-}(\mathbf{k},-\omega)=\frac{\Sigma_{20}(\mathbf{k}_0,0)}{2}\left(1-\frac{2n}{(4n^2+\Delta_-^2)^{1/2}}\right)$$
$$+\frac{\Sigma_{20}(\mathbf{k}_0,0)}{2}\frac{\omega^2}{4\Delta_-^2}\left[\frac{2}{3}-\frac{2n}{(4n^2+\Delta_-^2)^{1/2}}+\frac{1}{3}\left(\frac{2n}{(4n^2+\Delta_-^2)^{1/2}}\right)^3\right],$$
$$\Sigma_{11}^{+}(\mathbf{k},-\omega)=\frac{\Sigma_{20}(\mathbf{k}_0,0)}{2}\left(1-\ln\frac{\omega_p}{\Delta_+}\right), \tag{18}$$
$$\Sigma_{11}^{-}(\mathbf{k},-\omega)=\frac{\Sigma_{20}(\mathbf{k}_0,0)}{2}\left(1-\frac{2n}{(4n^2+\Delta_-^2)^{1/2}}-\ln\frac{\omega_p}{2n+(4n^2+\Delta_-^2)^{1/2}}\right)$$
$$-\frac{\Sigma_{20}(\mathbf{k}_0,0)}{2}\frac{\omega^2}{4\Delta_-^2}\left[\frac{4}{3}-\frac{2n}{(4n^2+\Delta_-^2)^{1/2}}-\frac{1}{3}\left(\frac{2n}{(4n^2+\Delta_-^2)^{1/2}}\right)^3\right],$$
$$\Sigma_{20}(\mathbf{k}_0,0)=M^2N(0),\quad \delta\mu^2=4n^2+\Delta_-^2.$$

Using expressions (16)–(18) we can show that the following relation holds:

$$\Sigma_{20}(\mathbf{k}_0,0)-\Sigma_{11}(\mathbf{k}_0,0)=\Omega_{\mathbf{k}_0}. \tag{19}$$

Taking (19) into account, we can obtain from the system (15) the dispersion equation with the terms of higher order in ω neglected:

$$\omega^2(1+b)-2c(k-k_0)\omega+c^2(k-k_0)^2=0,\quad \mathbf{k}\|\mathbf{k}_0,$$
$$b=\frac{\Sigma_{20}^2(\mathbf{k}_0,0)}{2A^2}\left[\frac{1}{\Delta_+^2}+\frac{1}{\Delta_-^2}\left(1-\frac{2n}{(4n^2+\Delta_-^2)^{1/2}}\right)\right], \tag{20}$$
$$A=(1-M^2\Delta_t^2/g\Omega_{\mathbf{k}_0}(\Delta_s^2-\Delta_t^2))\,(1+g\Omega_{\mathbf{k}_0}/M^2)^{-1}.$$

Thus, the spectrum of the "above-the-condensate" photons is of the form

$$\omega=[c(k-k_0)\pm ib^{1/2}c(k-k_0)]/(1+b), \tag{21}$$

i.e., the system has photon instability in the presence

of a photon Bose condensate. To be sure, the maximum value δ_{max} of the imaginary part turns out to be of the order of $b^{1/2}\omega_0$, i.e., much smaller than the value ω_0 (see (13)) of the imaginary part that characterizes the system instability in the absence of the photon Bose condensate. Therefore the state investigated in Sec. 2 can be stable in the presence in the system of a damping ν much smaller than ω_0 but larger than δ_{max}. Then, to the extent that $\nu/\omega_0 \ll 1$ is small, the results of Sec. 2 remain practically unchanged. If, however, $\nu \ll \delta_{max}$, then a multimode generation regime will be realized in the system, and the results of Sec. 2 will be unsuitable in this case.

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Effect of electron-hole drop size in germanium on the absorption and scattering of radiation in the far infrared region of the spectrum

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The principal optical characteristics of an EHD in the long-wave IR region is investigated theoretically and experimentally for drops of increasing size. The spectral dependences of the light-pressure and radiation extinction, absorption, and scattering coefficients, as well as the radiation scattering indicatrices have been calculated with the aid of a computer in the framework of the general Mie theory for radiation in the 40–1200-μ region and EHD of different radii in the 0.5–500-μ range. A shift of the spectra toward the region of longer waves as the EHD size increases and the excitation level is raised is theoretically predicted and experimentally confirmed. On the basis of these data, the values of the EHD radius, r, are determined, and it is concluded that the size distribution of the drops at a temperature of 1.5–2.0 K in germanium is characterized by an $r^{-\nu}$-type function (Young's distribution) truncated at $r \leq r_{min} = 1$ and $r \geq r_{max} = 16$ μ, $\nu = 3$–4. It is shown that larger-sized EHD are formed in germanium doped with shallow impurities. The phenomenon of mutual repulsion between the electron-hole drops and the phonon wind was observed in the experiments at elevated excitation levels and mean nonequilibrium-carrier concentrations higher than 10^{15} cm^{-3}. From these data the averaged characteristics of the phonons effectively interacting with the EHD in germanium are estimated.

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I. INTRODUCTION

The discovery of the phenomenon of resonance absorption of long-wave IR radiation by electron-hole drops (EHD) in germanium[1] marked the beginning of the investigation of exciton condensation and the properties of EHD in the far IR and the submillimeter regions of the spectrum. As a result of the development of the theory of plasma resonance in EHD under the assumption that the EHD dimensions are small compared to the radiation wavelength and with allowance for the intraband and interband electron transitions in the crystal,[2]

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