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Nonlinear Cyclotron absorption of a hole doppleron in cadmium

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We investigated experimentally the nonlinear behavior of the impedance of a cadmium plate in the region of existence of the hole doppleron. It is shown theoretically that this phenomenon can be attributed to nonlinear cyclotron absorption of the wave in the metal. A theory of nonlinear cyclotron absorption of a hole doppleron in cadmium is constructed. The nonlinearity is due to the influence of the wave magnetic field H that alters the trajectories of the resonant electrons responsible for the cyclotron asorption. The Lorentz force connected with the field H modulates the particle velocity along the magnetic field at a characteristic frequency ω_0 proportional to the square root of the wave amplitude. The modulation of the longitudinal particle velocity leads to violation of the condition of their resonant interaction with the wave, as a result of which the absorption coefficient decreases. The nonlinearity is significant when the frequency ω_0 is large compared with the electron-collision frequency. A decrease of the cyclotron absorption changes radically the picture of the surface-impedance oscillations of the plate in the magnetic field. We studied in the experiment the influence of the temperature, of the angle of inclination of the magnetic field, and of the frequency on the nonlinear-effect threshold field that separates the regions of linear and nonlinear behavior of the sample impedance. The measurement results are in qualitative agreement with the conclusions of the theory.

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1. INTRODUCTION

We have previously reported[1] observation of nonlinear effects in the propagation of a large-amplitude hole doppleron in cadmium. A change in the picture of the oscillations of the surface resistance R and of its derivative with respect to the static magnetic field was observed with increasing amplitude of the exciting field. The waves propagated in the plate along a magnetic field parallel to the [0001] hexagonal axis. No nonlinear effects were observed when an electron doppleron propagated in the cadmium. It is known that a hole doppleron exists in cadmium near the boundary of the doppler-shifted cyclotron resonance (DSCR) of the holes of the monster, and is subject to cyclotron damping by the electrons of the lens.[2] There is no cyclotron damping in the region of the existence of an electron doppleron due to the DSCR of electrons with maximum displacement. Since the nonlinear growth of the oscillation amplitudes was observed only for the propagation of a hole doppleron, it is natural to assume that this nonlinear effect is connected with a decrease of collisionless cyclotron absorption. We construct in this paper a theory of the nonlinear effect and present results of an experimental study of the peculiarities of its manifestation at various temperatures and various angles between the magnetic field and the [0001] axis. Good agreement was observed between the deductions of the theory and the experimental data.

We examine first the physical picture of nonlinear cyclotron absorption. In the case when the wave propagates along a constant magnetic field $\mathbf{H}_0$, the cyclotron absorption is due to resonant interaction with those electrons which revolve on the Larmor orbit together with the electric field of the waves. For these electrons, the angle between the velocity $\mathbf{v}_\perp$ in the plane of revolution and the electric field, which also lies in this field, remains constant and the particle effectively draws (or gives up) energy from (to) the wave. Since the electrons and the hole-doppleron electric field revolve in opposite directions, the resonant electrons are those that overtake the wave and travel along $\mathbf{k} \parallel \mathbf{H}_0 \parallel z$ with a velocity

$$v_z = (\omega_c + \omega)/k. \tag{1}$$

Here $\mathbf{k}$ and ω are the wave vector and frequency of the doppleron, and ω_c is the cyclotron frequency of the resonant electrons. In plasma physics it is customary to call the energy absorption by particles that satisfy condition (1) "anomalous cyclotron absorption." From the equations of motion of a particle in a constant field $\mathbf{H}_0$ it is easy to obtain, in the approximation linear in the electric and magnetic fields $\mathbf{E}$ and $\mathbf{H}$ of the wave, the rate of change of the energies of the motions that are longitudinal and transverse relative to $\mathbf{H}_0$. It turns out here that in the case of anomalous cyclotron absorption, the energy of the transverse motion decreases under the influence of the Lorentz force $(e/c)\mathbf{v}\times\mathbf{H}$, and goes over, together with the energy acquired from the electric field of the wave, into translational-motion energy. This result follows also from quantum concepts. In fact, when the doppleron quantum is absorbed the electron's translational-motion energy is changed by an amount $\hbar k v_z$, which is equal to $\hbar(\omega+\omega_c)$ when (1) is taken into account, whereas the total energy of the particle changes by an amount $\hbar\omega$. This means that the transverse-rotation energy decreases by $\hbar\omega_c$ when the doppleron quantum is absorbed.

The linear theory disregards the influence of the wave field on the particle trajectories. If the wave amplitude is large enough, however (an appropriate criterion will be given below), the effect of the wave on the particle motion turns out to be appreciable. The Lorentz force due to the magnetic field of the wave alters the longitudinal-motion velocity v_z. The resonance condition (1) is therefore violated and the effectiveness of the interaction of the particles with the wave decreases. To describe the particle motion in the wave field it is convenient to introduce the angle between the electric (or magnetic) field of the wave and the velocity vector $\mathbf{v}_\perp$, and then the particles are divided into "trapped" and "untrapped." The former execute finite motion in this angle, while the latter execute infinite motion. As will be shown below, from the mathematical point of view this motion is described in the same manner as the motion of electrons in the field of a longitudinal acoustic wave[3] or in the field of a helicon.[4]

Consider the character of the oscillations of the trapped particles. Assume that at the initial instant the particle had a velocity (1), and the angle between the force $e\mathbf{E}$ and the velocity $\mathbf{v}_\perp$ was less than $\pi/2$. The particle then absorbed energy and its longitudinal velocity increased. As a result, the angle between $e\mathbf{E}$ and $\mathbf{v}_\perp$ also increased and reached $\pi/2$, after which the field started to decrease the velocity v_z. This continued until the angle between $e\mathbf{E}$ and $\mathbf{v}_\perp$ again became smaller than $\pi/2$. If the characteristic frequency ω_0 of the oscillations is higher than the collision frequency τ^{-1}, i.e.,

$$\omega_0\tau\gg 1, \tag{2}$$

then the trapping is effective and the absorption coefficient decreases. But if $\omega_0\tau\ll 1$, the influence of the wave field on the particle motion can be neglected and the linear theory is valid. The decrease of the wave absorption in the nonlinear regime leads to an increase of the amplitude of the oscillations of the surface-impedance $Z(H_0)$ of the plate, as is indeed observed in experiment.

2. EXPERIMENT

We investigated in the experiment the surface impedance of cadmium in a magnetic field at different values of the exciting-field intensity. The measurements were made on cadmium samples with a resistance ratio $\rho_{300}/\rho_{4.2\mathrm{K}}\approx 3\cdot 10^4$, in the form of single-crystal plates 0.57 and 0.6 mm thick, with a surface normal to the direction [0001] of the hexagonal axis.

An RF field circularly polarized in the plane of the sample was produced with the aid of crossed coils.[5] The electromagnetic field intensity was regulated by varying the amplitude of the alternating voltage on both exciting coils. The surface resistance and its derivatives with respect to the magnetic field were measured with the amplitude bridge described in[6]. The use of broadband vacuum-tube amplifiers made it possible to vary the voltage on the tank circuit in the range 0.1–50 V and in a wide frequency band.

The singularities of the surface reactance were investigated with a special measuring oscillator in whose tank-circuit coil the sample was placed. The major difference between the employed system and the traditional oscillator circuits is that the tank circuit, which sets the oscillation frequency, is connected to the oscillator tube through an isolating cathode follower having a high input resistance. This has made it possible to exclude the influence of the variation of the regime of the oscillator tube on the tank-circuit oscillation frequency when the oscillation amplitude was regulated. The amplitude of the oscillations of the tank circuit could be varied by changing the anode voltage and the bias on one of the control grids of the oscillator tube. The use of high-power tubes of the GU-50 type made it possible to obtain voltages up to 70 V across the tank circuit.

To stabilize the oscillation frequency against variation of the tank-circuit Q by the magnetic field (in view of the dependence of the sample surface resistance on the field), negative feedback was used to maintain the tank-circuit Q and the oscillation amplitude constant.[6] Modulation of the constant magnetic field produces in the general case both frequency and amplitude modulation of the oscillations in the tank circuit of the measurement oscillator. If the inertia of the Q-stabilization system is large enough, the amplitude of the tank-circuit voltage modulation is proportional to dR/dH_0 and the depth of the frequency modulation is proportional to the derivative of the reactance, dX/dH_0. This makes it possible to investigate simultaneously the reactance $X(H_0)$ and its derivatives with respect to the magnetic field, as well as the derivatives of the surface resistance $R(H_0)$. The operating bandwidth of the oscillator was 0.03–30 MHz. The frequency drift with changing tank-circuit voltage did not exceed 0.03%. Preliminary measurements have shown that the employed oscillator had high sensitivity and a low intrinsic noise level.

The surface impedance of the cadmium was investigated in the temperature interval 1.6–4.2 K and at frequencies 0.01–2 MHz in a constant magnetic field up to 18 kOe oriented near the direction of the [0001] hexagonal axis of the crystal. The intensity of the alternating magnetic field reached 60 Oe. It was determined from the current in the exciting coils and from the known values of the coil constants.

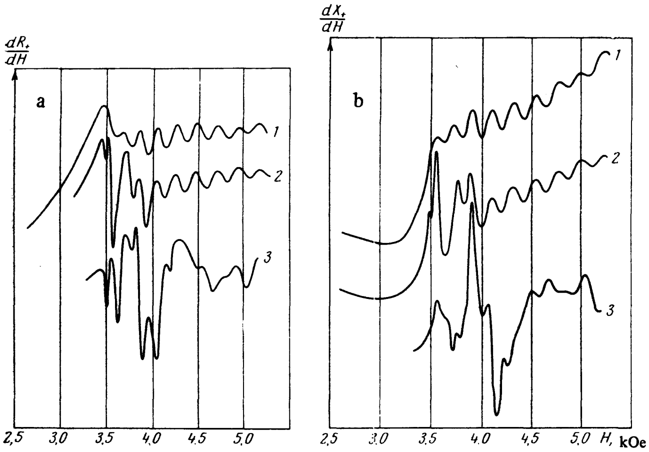

FIG. 1. Plots of the derivatives dR_+/dH_0 (a) and dX_+/dH_0 (b) vs the magnetic field H_0. Curves 1, 2, and 3 were obtained respectively for exciting-field intensities $H = 2.5$, 5, and 17 Oe. Curves 1 and 2 are drawn to the same ordinate scale, but the scale of curve 3 is decreased by one-half. Sample thickness $d = 0.60$ mm, $f = 430$ kHz, $T = 1.6$ K.

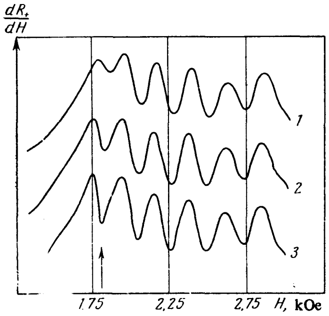

FIG. 2. Plots of dR_+/dH_0 for different values of the exciting field: 1–$H = 2.5$ Oe, 2–3.2Oe, 3–3.7 Oe, $d = 0.60$ mm, $f = 55$ kHz, $T = 1.6$ K.

The experimental part of our study was devoted to a detailed investigation of the manifestation of the nonlinear effect in the real and imaginary parts of the surface impedance. A detailed study was made of the influence of the temperature, of the inclination angle of the constant magnetic field, and of the frequency of the exciting field on its threshold value H_{thr}, which separates the regions of the linear and nonlinear behaviors of the surface impedance $Z_+(H)$ of the sample.

Figure 1 shows typical plots of the surface-resistance derivatives dR_+/DH_0 of the resistance and reactance dX_+/dH_0 as functions of H_0. These plots were obtained with the aid of a measuring oscillator at three values of the exciting field. The curves dR_+/dH_0 and $dX_+/0$ were recorded simultaneously with a two-channel automatic recording potentiometer. In a sufficiently low RF field H, the surface impedance $Z_+(H_0)$ does not depend on H. In the linear regime, the dependence of the derivatives dR_+/dH_0 and dX_+/dH_0 on the constant magnetic field is illustrated by curves 1. The oscillations of the hole doppleron are preceded by a smooth impedance singularity due to the wave threshold. The derivative of the resistance with respect to the magnetic field has a maximum in the vicinity of the doppleron threshold, and the derivative of the reactance increases sharply. Above the threshold, doppleron oscillations are observed, whose characteristic singularities are the same in both the real and the imaginary parts of the surface impedance. From a comparison of curves 1 it is seen that the positions of the extrema of the derivative dR_+/dH_0 correspond to regions of the most abrupt variation of the reactance derivative dX_+/dH_0. Thus, the real and imaginary parts of the surface impedance in a magnetic field are connected by a relation similar to the Kramers–Kronig relations for the frequency dependence of the impedance of a metal.

In the nonlinear regime the shapes of the dR_+/dH_0 and dX_+/dH_0 curves become strongly dependent on the exciting-field intensity. Curves 2 and 3 illustrate cases of a weakly and strongly pronounced nonlinear effect.

The most common feature of the effect with increasing field H is the enhancement of the extrema in both the real and the imaginary parts of the surface impedance. The enhancement takes place primarily near the threshold field of the doppleron (curves 2 on Fig. 1). With further increase of the exciting field the amplitude of the singularities increases, and the region of the nonlinearity propagates towards stronger magnetic fields. In the regime where the effect is strongly nonlinear, the dR_+/dH_0 and dX_+/dH_0 curves differ noticeably in form, but their strongest singularities are observed in practice in identical magnetic fields (curves 3 on Fig. 1).

In view of the complexity of the picture of the appearance of the effect, it was convenient to introduce a parameter that characterizes the degree of nonlinearity. The most convenient characteristic of the nonlinear effect turned out to be the threshold value H_{thr} of the exciting field. To determine the threshold field, the derivative dR_+/dH_0 was plotted as a function of H_0 at various fixed values of the field H. On going from one plot to the other, the coil voltages was increased in steps of 5%. Figure 2 illustrates the variation of the dR_+/dH_0 curves near the boundary of the linear region. With increasing field H, a strong deepening of the first minimum of the derivative was observed (it is marked on the figure by an arrow). Similar measurements have made it possible to determine the threshold field accurate to ~10%. At the experimental conditions under which the curves shown in Fig. 2 were obtained, the value of H_{thr} was approximately 2.7 Oe.

With increasing temperature, the amplitude of the doppleron oscillations decreases rapidly. Raising the temperature from 1.6 to 3.6 K leads to a decrease of the oscillation amplitude by almost two orders, and they become practically indistinguishable against the background of the smoothly varying part of the derivative at $T = 3.8$ K. The threshold field also increases with temperature, but even at $T = 3.8$ K, when the oscillations are indiscernible, the nonlinear effect manifests itself quite distinctly in a splitting due to the threshold of the wave, of the maximum of the derivative dR_+/dH_0, (Fig. 3). Figure 4 shows the temperature dependence of the threshold field intensity H_{thr}. As seen from this figure, the value of the threshold field increases rapidly with increasing temperature.

When the field is inclined to the direction of the hexagonal axis, the nonlinear effect is observed at higher intensities of the exciting field. The dependence of H_{thr}

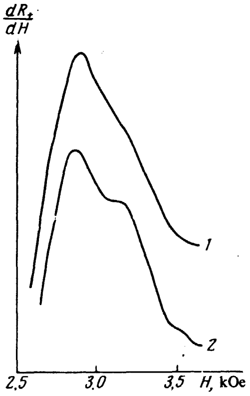

FIG. 3. Singularities of the derivative dR_+/dH_0 on the wave threshold at $T=2.8$ K. Curves: 1–$H=15$ Oe, 2–27 Oe, $d=0.60$ mm, $f=240$ kHz.

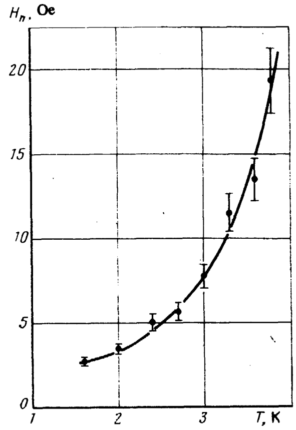

FIG. 4. Plot of $H_{\text{thr}}(T)$ at $f=240$ kHz for a sample 0.60 mm thick.

on the angle θ between the magnetic field and the crystal axis [0001] is shown in Fig. 5. If the angle between the wave propagation direction and the magnetic field is different from zero, the amplitude of the doppleron oscillations decreases as a result of the magnetic Landau damping. This decrease is particularly strongly pronounced near the threshold of the wave. Thus, at $\theta=4°$ and $T=1.6$ K the amplitude of the first oscillation extremum of dR_+/dH_0 is smaller by almost one order of magnitude than at $\theta=0$. The general regularity governing the development of the nonlinear distortions of the $dR_+/dH_0=f(H_0)$ curve in an oblique field has a somewhat different character than in the case $\mathbf{H}_0 \parallel [0001]$. This manifests itself in the fact that at nonzero angles θ the nonlinearity propagates much more rapidly into the region of strong magnetic fields with increasing field H.

We have attempted to investigate the dependence of the threshold field on the frequency. In view of the strong dependence of the nonlinear effect on the temperature and on the orientation of the constant magnetic field, the frequency dependence of the threshold field H_{thr} was measured in a single experiment. The field values measured in the frequency interval 23–730 kHz ranged from 2.7 to 3.3 Oe. Taking the measurement error into account, this indicates that the threshold field of the nonlinear effect is practically independent of the frequency of the exciting field.

3. THEORY

In this section we develop a nonlinear theory of cyclotron absorption of a holde doppleron in cadmium in the case $\mathbf{k} \parallel \mathbf{H}_0 \parallel [0001]$. We shall disregard in the analysis the renormalization of the doppleron spectrum in the nonlinear regime, since the wave distorts appreciably the electron distribution function in a narrow momentum interval, and the doppleron spectrum is determined by all the carriers. It is convenient to seek the electron distribution in a coordinate frame that moves with the wave phase velocity $v_{\text{ph}}=\omega/k$ along the magnetic field. In this system, the electric field of the wave is zero, and the magnetic field is equal, accurate to $(v_{\text{ph}}/c)^2 \ll 1$, to the field in the laboratory coordinate frame. We write down the magnetic field of the wave in the moving coordinate system in the form

$$H_x=H\cos kz, \quad H_y=H\sin kz. \tag{3}$$

We shall solve the kinetic equation by the method of characteristics. The latter are the particle trajectories, and we therefore consider first the equations of the trajectories.

In the new coordinate system we have

$$d\mathbf{p}/dt=(e/c)[\mathbf{v}(\mathbf{p})(\mathbf{H}_0+\mathbf{H})], \tag{4}$$

where $\mathbf{v}(\mathbf{p})=\partial\varepsilon/\partial\mathbf{p}$ and $\varepsilon(\mathbf{p})$ is the dispersion law. From (4) we get the energy integral $\varepsilon(\mathbf{p})=\varepsilon=\text{const}$. Since $v_{\text{ph}} \ll v_F$ (v_F is the Fermi velocity), we shall neglect the difference between the Fermi surfaces in the new and old coordinate systems. In terms of the variables ε, p_z, and $\alpha=kz-\Phi$, where Φ is the polar angle of the vector $\mathbf{p}$, such that

$$p_x=p_\perp(\varepsilon,p_z)\cos\Phi, \quad p_y=p_\perp(\varepsilon,p_z)\sin\Phi,$$

we obtain from (4) and from the equation $dz/dt=v_z(\varepsilon,p_z)$

$$\frac{dp_z}{dt}=-h\frac{|e|}{c}H_0v_\perp(\varepsilon,p_z)\sin\alpha,$$
$$\frac{d\alpha}{dt}=kv_z(\varepsilon,p_z)-\frac{|e|}{c}H_0\left(\frac{v_\perp(\varepsilon,p_z)}{p_\perp(\varepsilon,p_z)}-\frac{hv_z(\varepsilon,p_z)\cos\alpha}{p_\perp(\varepsilon,p_z)}\right). \tag{5}$$

Here $h=H/H_0$ and α is the angle between $\mathbf{v}_\perp$ and $\mathbf{H}$. Owing to the axial symmetry of the electron lens we have

$$p_\perp(\varepsilon,p_z)=m(\varepsilon,p_z)v_\perp(\varepsilon,p_z),$$

where $m(\varepsilon,p_z)$ is the cyclotron mass.

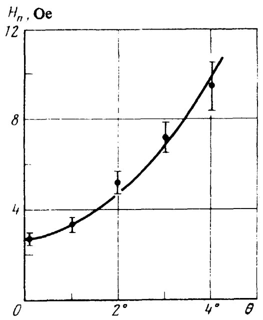

FIG. 5. Angular dependence of the threshold field of the nonlinear effect; $d=0.60$ mm, $f=240$ kHz, $T=1.6$ K.

Neglecting the influence of the wave field on the particle trajectory ($h=0$), the cyclotron-resonance condition $d\alpha/dt=0$ yields an equation for the momentum $p_z^0(\varepsilon)$ of the resonant electrons

$$kv_z(\varepsilon, p_z^0)=|e|H_0/cm(\varepsilon, p_z^0)\equiv\omega_c(\varepsilon, p_z^0). \tag{6}$$

On the Fermi surface ($\varepsilon=\varepsilon_F$) the relation (6) is equivalent to the condition (1) written in a moving coordinate system. At $h\neq 0$ we put $p_z=p_z^0(\varepsilon)+\delta p_z$ for the electrons that are close to resonance. In the right-hand side of the first equation of (5) we neglect δp_z; in the second equation of (5) we discard the second member of the second component, and in the remaining component we expand up to terms of first order in δp_z inclusive (the justification for these approximation is given below). We recognize that

$$mv_z=-\frac{1}{2\pi}\frac{\partial S(\varepsilon,p_z)}{\partial p_z},$$
$$\frac{v_\perp}{v_z}=-\left[\frac{\partial p_\perp(\varepsilon,p_z)}{\partial p_z}\right]^{-1}=-2[\pi S(\varepsilon,p_z)]^{1/2}\left[\frac{\partial S(\varepsilon,p_z)}{\partial p_z}\right]^{-1}, \tag{7}$$

where $S(\varepsilon, p_z)=\pi p_\perp^2$ is the area of the intersection of the surface $\varepsilon(\mathbf{p})=\varepsilon$ with the plane $p_z=$ const. We introduce the dimensionless variable

$$s=\delta p_z/\tilde{p}_z,$$
$$\tilde{p}_z=p_\perp(\varepsilon,p_z^0)\left[h\pi^{1/2}\frac{\partial S}{\partial p_z}\left(S^{1/2}\frac{\partial^2 S}{\partial p_z^2}\right)^{-1}\right]^{1/2}\Bigg|_{p_z=p_z^0(\varepsilon)}. \tag{8}$$

As a result we obtain from (5)–(8)

$$ds/dt=-\omega_0(\varepsilon)\sin\alpha,\quad d\alpha/dt=\omega_0(\varepsilon)s, \tag{9}$$
$$\omega_0(\varepsilon)=\omega_c(\varepsilon,p_z^0)\left[h\frac{S^{1/2}}{\pi^{1/2}}\frac{\partial^2 S}{\partial p_z^2}\left(\frac{\partial S}{\partial p_z}\right)^{-1}\right]^{1/2}\Bigg|_{p_z=p_z^0(\varepsilon)}. \tag{10}$$

It is seen from (10) that on the Fermi surface the ratio $\omega_0(\varepsilon_F)/\omega_c(\varepsilon_F, p_z^0)$ is fully expressed in terms of the function $S(\varepsilon_F, p_z)$. An analytic expression for $S(\varepsilon_F, p_z)$ of cadmium, which agrees well with experiment, was obtained in[2]:

$$S(\varepsilon_F,p_z)=2\pi p_0p_e\left[-\frac{|p_z|}{p_e}-\frac{1}{3}\left(1-\frac{|p_z|}{p_e}\right)^3+1\right], \tag{11}$$

where $p_0/\hbar=1.5$ Å^{-1}, $p_e/\hbar=0.28$ Å^{-1}. We shall use this expression for the actual estimates that follow.

Equations (9) are obtained from (5) if the following inequalities are satisfied:

$$v_\perp(\varepsilon,p_z^0)\gg\left|\delta p_z\frac{\partial v_\perp(\varepsilon,p_z^0)}{\partial p_z^0}\right|,$$
$$\left|\delta p_z\frac{\partial}{\partial p_z^0}[kv_z(\varepsilon,p_z^0)-\omega_c(\varepsilon,p_z^0)]\right|\gg h\omega_c(\varepsilon,p_z^0)\frac{v_z(\varepsilon,p_z^0)}{v_\perp(\varepsilon,p_z^0)}. \tag{12}$$

We reduce these inequalities to the form

$$p_\perp\gg\left|\frac{v_z}{v_\perp}+v_\perp\frac{\partial m}{\partial p_z}\right||\delta p_z|,\quad |\delta p_z|\gg\frac{\tilde{p}_z^2v_z}{v_\perp p_\perp}. \tag{13}$$

In the linear regime ($\omega_0\tau\ll 1$), when the width of the electron strip that takes part in the absorption is determined by the collision and is equal to $\delta p_z\sim\tilde{p}_z/\omega_0\tau$, these inequalities are certainly satisfied for the lens electrons. In the case of strong nonlinearity ($\varepsilon_0\tau\gg 1$) the conditions (13) are limitations on the amplitude, since the characteristic width of the strip is $\delta p_z\sim\tilde{p}_z$ in this case. Estimates show that for resonant electrons in cadmium we have $v_\perp\partial m/\partial p_z\lesssim v_z/v_\perp$, and therefore the two inequalities in (13) reduce in the nonlinear regime to the condition

$$\tilde{p}_z\ll v_\perp p_\perp/v_z,$$

which takes, when (8) is allowed for, the form

$$[h(\partial S/\partial p_z)^3(4\pi^{1/2}S^{3/2}\partial^2 S/\partial p_z^2)^{-1}]^{1/2}_{p_z=p_z^0(\varepsilon)}\ll 1. \tag{14}$$

A numerical estimate shows that the inequality (14) was certainly satisfied under the experimental conditions.

We return to the system (9), which determines the electron trajectories. It has as its integral

$$\mathscr{H}=s^2/2-\cos\alpha, \tag{15}$$

from which it is seen that the particles are divided into untrapped ($\mathscr{H}>1$) and trapped ($\mathscr{H}<1$), as already mentioned in the Introduction. The solutions of (9) are expressed in terms of elliptic Jacobi functions (see, e.g., [4]).

We proceed to consider the kinetic equation. In a moving coordinate system it takes the form

$$v_z\frac{\partial f}{\partial z}+\frac{e}{c}[\mathbf{v}\times(\mathbf{H}_0+\mathbf{H})]\frac{\partial f}{\partial\mathbf{p}}+\hat{I}\{f\}=0. \tag{16}$$

Here $\hat{I}\{f\}$ is the collision integral. We seek the solution of (16) in the form $f=F_0[\varepsilon(\mathbf{p})+\mathbf{p}\mathbf{v}_{ph}]+g$, where F_0 is the equilibrium distribution function. The collision integral is written in the form $\hat{I}\{g\}=g/\tau$. This is justified for the narrow group of electrons responsible for the wave absorption, if τ is taken to mean the time of departure from the effective-interaction strip. Writing down the equation for g in terms of the variables ε, p_z, z and Φ we can easily verify that it is invariant to the substitutions $z\to z+z_0$ and $\Phi\to\Phi+z_0/k$, where z_0 is an arbitrary constant. Consequently the function g depends on the variables ε, p_z, and $\alpha=kz-\Phi$. In terms of these variables the equation for the distribution function is

$$\frac{\partial g}{\partial\alpha}\dot{\alpha}+\frac{\partial g}{\partial p_z}\dot{p}_z+\frac{g}{\tau}=\frac{|e|}{c}hH_0v_{ph}v_\perp(\varepsilon,p_z)F_0'(\varepsilon+p_zv_{ph})\sin\alpha, \tag{17}$$

where $\dot{\alpha}$ and $\dot{p}_z$ are specified by the system (5), and $F_0'=\partial F_0/\partial\varepsilon$. The distribution function at fixed z should be periodic in Φ with a period 2π. This means that it is also periodic in the variable α with the same period 2π. When solving (17) by the method of characteristics it is convenient to introduce the time of motion along the trajectory, $dt=d\alpha/\dot{\alpha}$. The solution periodic in α then takes the form

$$g(\varepsilon,p_z,\alpha)=\int_{-\infty}^{t}dt'\exp\left[-\frac{t-t'}{\tau}\right]\frac{|e|}{c}hH_0v_{ph}v_\perp(\varepsilon,p_z(t'))\cdot$$
$$\times F_0'(\varepsilon+p_z(t')v_{ph})\sin\alpha(t'), \tag{18}$$

where $\alpha(t')$ and $p_z(t')$ describe the trajectory of a particle that reaches the point (α, p_z) at $t' = t$. The particle trajectories are determined by the system. To match the accuracy with which this system is written, it is necessary to put under the integral sign in (18) $v_\perp(\varepsilon, p_z(t')) \approx v_\perp(\varepsilon, p_z^0)$. In the lowest order in $\delta p_z v_{ph}/\varepsilon_F$, when $p_z^0 v_{ph}/\varepsilon_F \ll 1$, we can replace the argument of F_0' by ε. The integral in (18) can then be evaluated in analogy with the similar integral of[4]. The result is

$$g_{ut} = \frac{h|e|H_0 v_{ph} v_\perp(\varepsilon, p_z^0)}{c\omega_0} F_0'(\varepsilon) \left(\frac{2\pi}{\varkappa K(\varkappa)}\right)^2 \sum_{n=1} \frac{n q^n(\varkappa)}{1+q^{2n}(\varkappa)} \times \left[\frac{1}{\omega_0\tau} \sin\frac{\pi n F(\alpha/2, \varkappa)}{K(\varkappa)} - \frac{\pi n}{\varkappa K(\varkappa)} \cos\frac{\pi n F(\alpha/2,\varkappa)}{K(\varkappa)}\right] \times \left[\frac{1}{(\omega_0\tau)^2} + \left(\frac{\pi n}{\varkappa K(\varkappa)}\right)^2\right]^{-1},$$

$$g_t = \frac{h|e|H_0 v_{ph} v_\perp(\varepsilon, p_z^0)}{c\omega_0} F_0'(\varepsilon) \left(\frac{2\pi}{K(\varkappa^{-1})}\right)^2 \sum_{n=1}^{\infty} \frac{(n-{}^1/_2) q^{n-{}^1/_2}(\varkappa^{-1})}{1+q^{2n-1}(\varkappa^{-1})} \times \left[\frac{1}{\omega_0\tau} \sin\frac{\pi(n-{}^1/_2)|\varkappa| F(\alpha/2,\varkappa)}{K(\varkappa^{-1})} - \frac{\pi(n-{}^1/_2)\,\mathrm{sign}\,\varkappa}{K(\varkappa^{-1})} \times \cos\frac{\pi(n-{}^1/_2)|\varkappa| F(\alpha/2,\varkappa)}{K(\varkappa^{-1})}\right] \left[\frac{1}{(\omega_0\tau)^2} + \left(\frac{\pi(n-{}^1/_2)}{K(\varkappa^{-1})}\right)^2\right]^{-1}. \quad (19)$$

Here $\varkappa = [2/(\mathscr{H}+1)]^{1/2}$ sign s, while g_t and g_{ut} are respectively the distribution functions of the trapped ($|\varkappa| > 1$) and untrapped ($|\varkappa| < 1$) particles, $F(\alpha/2, \varkappa)$ is an elliptic integral of the first kind, $K(\varkappa)$ is a complete elliptic integral of the first kind, and

$$q(\varkappa) = \exp\left[-\pi K(\sqrt{1-\varkappa^2})/K(\varkappa)\right].$$

We define the nonlinear absorption coefficient as

$$\Gamma = \langle \mathbf{j}\mathbf{E}\rangle / 2\langle w\rangle |v_g|, \quad (20)$$

where $\mathbf{j}$ is the current density, $\langle w\rangle$ is the average energy density of the wave, and v_g is the doppleron group velocity. Expressing the electric field of the wave in terms of the magnetic field and changing from integration with respect to Φ to integration with respect to $\alpha = kz - \omega t - \Phi$, we obtain the work performed by the wave field on the particles:

$$\mathbf{j}\mathbf{E} = \frac{2}{(2\pi\hbar)^3} \int_0^\infty d\varepsilon \int_{p_{z\,min}(\varepsilon)}^{p_{z\,max}(\varepsilon)} dp_z \int_{-\pi}^{\pi} d\alpha \frac{e}{c} H v_{ph} v_\perp(\varepsilon, p_z) g(\varepsilon, p_z, \alpha) \sin\alpha.$$

We change from the variables p_z and α to $\varkappa$ and α. After integrating we obtain

$$\Gamma = \Gamma_{lin}\{\gamma_{ut}(\omega_0(\varepsilon_F)\tau) + \gamma_t(\omega_0(\varepsilon_F)\tau)\}, \quad (21)$$

$$\Gamma_{lin} = 2\pi e^2 \omega^2 p_\perp^2(\varepsilon_F, p_z^0(\varepsilon_F)) \left[\hbar^3 k^3 c^2 |v_g| \left|\frac{\partial^2 S(\varepsilon_F, p_z)}{\partial p_z^2}\right|_{p_z = p_z^0(\varepsilon_F)}\right]^{-1}. \quad (22)$$

Here Γ_{lin} is the linear coefficient of cyclotron absorption, and $\gamma_{t,ut}(\omega_0\tau)$ are functions that characterize the contributions of the trapped and untrapped particles to the absorption. Formulas and plots for γ_{ut}, γ_t, $\gamma_{ut} + \gamma_t$ as functions of $(\omega_0\tau)^{-1}$ are given in[4]. In the case of strong nonlinearity ($\omega_0\tau \gg 1$) we have

$$\Gamma \approx 2\Gamma_{lin}/\omega_0\tau. \quad (23)$$

Substituting (11) in (22) we get

$$\Gamma_{lin} = \left[1 - \frac{1}{3}\left(1 - \frac{1}{q'}\right)\right] \frac{\omega^2 p_e^2 e^2}{\hbar^3 c^2 k^3 |v_g|}, \quad (22a)$$

where $q' \approx 3.77$ is the ratio of the displacement of the electrons in the limiting point of the lens to the doppleron wavelength. It is easy to verify that (22a) coincides with the result obtained in[2]. Using (11), we find that $p_z^0(\varepsilon_F) \approx 0.14 p_e$ for cadmium. We then obtain from (10)

$$\omega_0 \approx \omega_c (17h)^{1/2}. \quad (24)$$

We consider now wave propagation in an oblique magnetic field. In this case the velocity projection on the direction of the wave vector is modulated at the cyclotron frequency. As a result, the resonance takes place only on the average over the cyclotron period, and the effectiveness of the interaction is decreased. For this reason, the characteristic frequency ω_0 decreases. As shown in the Appendix,

$$\omega_0(\theta) \approx \omega_0(\theta = 0) |J_0(p_\perp \theta/\bar{p}_z^0)|^{1/2}, \quad (25)$$

where J_0 is a Bessel function, and the meanings of $p_\perp$ and $\bar{p}_z^0$ are explained in the Appendix. We note that for the same reason, i.e., on account of the modulation of the velocity projection on the wave-vector direction, the linear cyclotron absorption is also decreased.

4. DISCUSSION OF RESULTS

Let us discuss the character of the variation of the picture of the impedance oscillations in the nonlinear regime. It was noted above that the hole doppleron in cadmium exists under conditions of strong cyclotron absorption of the wave by the lens electrons. The absorption is maximal in the vicinity of the wave threshold. It is precisely in this region that the distortions of the experimental curves are most likely to appear and the amplitude of the oscillations increases strongly. As a result, the maximum of the envelope of the oscillations shifts towards weaker fields, in analogy with the situation when the collision damping is decreased.[7] This allows us to assume that the nonlinear effect is due to the decrease of the cyclotron absorption of the hole doppleron.

A quantitative comparison of the deductions of the theory with the experimental data is difficult. For such a comparison it is necessary to study theoretically the influence of the nonlinear cyclotron absorption on the surface impedance of the cadmium plate. However, the calculation of the cadmium impedance for the model Fermi surface proposed in[2] can be carried out only numerically even in the linear theory. In the nonlinear regime, when the absorption coefficient depends on the wave amplitude, the problem becomes even more complicated, since the Fourier-transformation method cannot be used. We note that in the nonlinear regime the character of the distribution of the wave field in the method is substantially altered. Owing to the dependence of the nonlinear absorption coefficient Γ on the wave intensity, the field distribution near the metal surface turns out to be quadratic:

$$H(z) = H(0)\left[1 - {}^1/_2 z \Gamma(H(0))\right]^2. \quad (26)$$

Despite the complexity of the problem, it can be stated that the necessary condition for the existence of a nonlinear cyclotron absorption is the satisfaction of the inequality (2). Substituting in (24) the experimental value of the threshold field of the nonlinear effect, $H_{thr}=2.7$ Oe and the magnetic field $H_0=3.7$ kOe corresponding to the doppleron threshold at $f=0.43$ MHz (see Fig. 1), and assuming the cyclotron mass of the carriers to be equal to the mass of the free electron, we obtain $\omega_0\approx 7\times 10^9$ sec^{-1} on the sample surface. In accordance with the data of[7], the characteristic time τ in the employed samples of cadmium is of the order of 10^{-9} sec. As a result we have the product $\omega_0\tau\approx 7$ and the inequality (2) is satisfied. This inequality was satisfied also in all other cases when the nonlinear effect was observed. From the experimental data, and also from the estimates (24) and (2), it follows that the nonlinear regime in cadmium takes place in weak RF fields of the order of several oersteds. This makes cadmium a very favorable object for the study of nonlinear absorption of waves.

Within the framework of the developed theory it is easy to explain the strong dependence of the threshold field H_{thr} on the temperature. With increasing temperature, the relaxation time τ of the electrons decreases and the condition (2) which is necessary for the existence of the nonlinear effect is violated. Therefore the nonlinearity is observed at large values of the RF field H. It follows from (2) that $H_{thr}^{1/2}$ is proportional to the carrier collision frequency. One can expect $H_{thr}^{1/2}$ to be a power-law function of the temperature. Figure 6 shows a plot of $H_{thr}^{1/2}(T^3)$. It is seen that within the limits of the measurement errors this dependence is linear, i.e., $H_{thr}^{1/2}\propto T^3$. Such a dependence is natural, since at $T<4.2$ K the momentum of the thermal phonon is comparable with the width $\tilde{p}_z$ of the strip of the lens electrons responsible for the cyclotron absorption. This means that the electron-phonon scattering is effective and its contribution to the reciprocal departure time is proportional to T^3. We note that in strong alternating fields, when the width $\tilde{p}_z$ of the strip is larger than the phonon momentum, the reciprocal time of departure from the resonance region is proportional to T^n, where $n>3$.

We discuss now the experimentally observed strong dependence of the threshold field H_{thr} on the angle of inclination of the magnetic field to the hexagonal axis of the crystal (Fig. 5). It can be assumed that the condition under which the nonlinearity becomes noticeable in the experiment is the relation

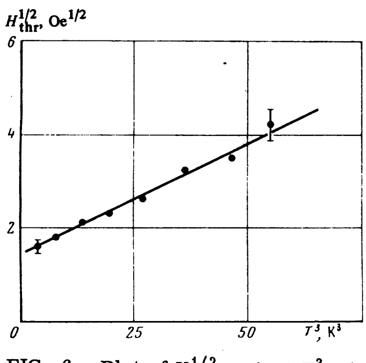

FIG. 6. Plot of $H_{thr}^{1/2}$ against T^3; $d=0.60$ mm, $f=240$ kHz.

$$\delta\Delta\Gamma\geqslant C, \quad (27)$$

where $\Delta\Gamma$ is the nonlinear decrease of the cyclotron absorption, δ is the thickness of the region of the nonlinear cyclotron absorption region in which the condition $\tau\omega_0(H(z))\gg 1$ is satisfied, and C is a certain constant that depends on the sensitivity of the experimental setup. In an oblique field the doppleron absorption increases strongly because of the magnetic Landau damping $\Gamma_L(\theta)$. In addition, the characteristic frequency $\omega_0(\theta)$ and the linear cyclotron absorption $\Gamma_{lin}(\theta)$ decrease (see the Appendix). At small angles θ, however, the changes of $\omega_0(\theta)$ and of $\Gamma_{lin}(\theta)$ are small. The Landau damping leads to a decrease of the thickness δ of the region of the nonlinear absorption. To satisfy the inequality (27) it is therefore necessary to increase the intensity of the exciting field. For a rough estimate of the angular dependence of the thickness δ we can use the formula

$$\delta(\theta)\approx[\Gamma+\Gamma_L(\theta)]^{-1}.$$

Assuming that $\Gamma_L(\theta)\propto\theta^2$ and substituting $\delta(\theta)$ in (27), it is easy to verify that the threshold field increases in proportion to θ^2, in qualitative agreement with experiment.

The influence of the magnetic Landau damping is apparently also the cause of the rapid propagation of the nonlinearity towards stronger fields at $\theta\neq 0$. The point is that above the doppleron threshold the Landau damping decreases with increasing constant magnetic field[8] and the inequality $\omega_0\tau\gg 1$ can be satisfied in a wider field region.

Unfortunately, we are unable to explain the independence of the nonlinearity threshold field H_{thr} of the frequency. In addordance with (24), H_{thr} should be proportional to $\omega^{-1/2}$. To ascertain the cause of this disparity it is apparently necessary to have analytic expressions for the impedance of the metal in the nonlinear regime, and to perform additional experimental investigations.

APPENDIX

Let us obtain the characteristic frequency $\omega_0(\theta)$ of the oscillations in the case when the wave propagates along a hexagonal axis and the magnetic field makes an angle θ with this axis. Since experiments show that even at small θ (on the order of several degrees) the nonlinear effects are much weaker than at $\theta=0$, we confine ourselves to the case $\theta\ll 1$. We shall use two coordinate systems: x, y, z and x, η, ζ, with $\mathbf{H}_0\parallel z$, $k\parallel\zeta$, and $\mathbf{k}$ lying in the yz plane. Owing to the large conductivity along the magnetic field, the doppleron electric field is polarized in the xy plane. To exclude this field, we convert to a coordinate frame that moves along the z axis with velocity $v_0=v_{ph}/\cos\theta$. In view of the smallness of $(v_{ph}/c)^2$, the magnetic field of the wave is not altered in this transition. It is circularly polarized in the $x\eta$ plane and can be expressed in the form

$$H_x=H\cos k\zeta,\quad H_\eta=H\sin k\zeta. \quad (A.1)$$

At small θ, the angle between the projections of the momentum $p_\perp$ and of the velocity $v_\perp$ on the xy is small (of the order of θ), and will henceforth be assumed equal to zero. Inasmuch as for small θ the trajectories of the resonant particles in momentum space lie below the equator of the lens, we neglect the small modulations of the momentum $p_\perp$, of the cyclotron mass $m(\varepsilon, p_\zeta) \approx m(\varepsilon, 0)$, and with them also the modulation of $v_\perp$. Ignoring, as in Sec. 3, the influence of the wave field on the rate of the cyclotron revolution of the particle, we obtain from the equations of motion (4), in terms of the variables p_z, Φ, and $\varepsilon = \text{const}$:

$$\dot{p}_z = -h(|e|/c)H_0 v_\perp \sin(k\zeta - \Phi), \tag{A. 2}$$

$$\dot{\Phi} = |e|H_0/cm(\varepsilon, 0) \equiv \omega_c. \tag{A. 3}$$

In addition to (A. 3), we shall consider the equation

$$\frac{d}{dt}(k\zeta - \Phi) = kv_\zeta(\varepsilon, p_z, \Phi) - \omega_c. \tag{A. 4}$$

Since $\dot{p}_z$ contains the small parameter h we seek, neglecting the small oscillations of p_z, the solution of (A. 2) and (A. 4) in the form

$$p_z = \bar{p}_z, \quad k\zeta - \Phi = \alpha + \Delta,$$

where $\bar{p}_z$ and $\alpha = \overline{k\zeta - \Phi}$ are quantities that vary slowly over the cyclotron period. Averaging over the fast cyclotron rotations, we obtain

$$\dot{\bar{p}}_z = -h(|e|/c)H_0 v_\perp \overline{\sin(\alpha + \Delta)}, \tag{A. 5}$$

$$\dot{\alpha} = k\bar{v}_\zeta - \omega_c, \tag{A. 6}$$

$$\dot{\Delta} = k(v_\zeta - \bar{v}_\zeta). \tag{A. 7}$$

It is easy to show that Δ can be chosen that $\overline{\sin\Delta} = 0$. Recognizing that near the equator of the lens, where p_ζ is small, we have $v_\zeta \propto p$ (in cadmium on the Fermi surface we have $v_\zeta \approx v_\perp(3p_0/p_e)^{1/2}p_\zeta/p_e$) and $p_\zeta \approx p_z + \theta p_\perp \sin\Phi$, we obtain from (A. 3) and (A. 7)

$$\Delta = -p_\perp\theta \cos\Phi/\bar{p}_z^0(\varepsilon), \tag{A. 8}$$

where $\bar{p}_z^0(\varepsilon)$ is the momentum of the resonant particles, for which $k\bar{v}_\zeta(\varepsilon, \bar{p}_z^0) = \omega_c$. Substituting (A. 8) in (A. 5) and using the integral representation of the Bessel function, we write

$$\dot{\bar{p}}_z = -\frac{h|e|}{c}H_0 v_\perp \sin\alpha J_0\left(\frac{p_\perp\theta}{\bar{p}_z^0(\varepsilon)}\right). \tag{A. 9}$$

In terms of the variables

$$\alpha, \quad s = [\bar{p}_z - \bar{p}_z^0(\varepsilon)]/\bar{p}_z |J_0(p_\perp\theta/\bar{p}_z^0(\varepsilon))|^{1/2},$$

where $\bar{p}_z = \bar{p}_z(\varepsilon, \bar{p}_z^0(\varepsilon))$ is defined in Sec. 3, the system (A. 9), (A. 6) reduces to the form (9). The frequency $\omega_0(\varepsilon, \theta)$ is then expressed by formula (25).

A calculation analogous to that of Sec. 3 shows that the nonlinear coefficient of the cyclotron absorption is equal to

$$\Gamma(\theta) = \Gamma_{\text{lin}}(\theta)[\gamma_{ut}(\omega_0(\varepsilon_F, \theta)\tau) + \gamma_t(\omega_0(\varepsilon_F, \theta)\tau)], \tag{A. 10}$$

$$\Gamma_{\text{lin}}(\theta) \approx \Gamma_{\text{lin}}(\theta = 0)J_0^2(p_\perp\theta/\bar{p}_z^0(\varepsilon_F)), \tag{A. 11}$$

where $\Gamma_{\text{lin}}(\theta)$ is the linear coefficient of cyclotron absorption, and the functions $\gamma_{ut,t}$ are the same as in Sec. 3. We note that in the formulas for $\omega_0(\varepsilon, \theta)$ and $\Gamma(\theta)$ the dependence on θ is preserved in fact only where θ enters with the large factor $p_\perp/\bar{p}_z^0(\varepsilon)$.

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