On the problem of the decay instability of electromagnetic waves in a magnetoactive plasma

V. A. Roslyakov and A. N. Starostin

Nuclear Physics Research Institute of the Moscow State University
(Submitted 31 March 1977)
Zh. Eksp. Teor. Fiz. 73, 1747–1756 (November 1977)

The processes of decay of transverse electromagnetic waves propagating across a constant magnetic field are considered. The possibility of using these decays for tuning the frequency of the stimulated radiation in the magnetoactive plasma of semiconductors, as well as for the diagnostics of the magnetic fields that spontaneously develop in a laser plasma is discussed.

PACS numbers: 52.35.Hr, 52.35.Py, 72.30.+q, 52.40.Db

It is well known that the nonlinear properties of a plasma lead to numerous effects, in particular, to the existence of decay (parametric) instabilities of high-power electromagnetic waves.[1] The number of possible decay channels increases substantially for a plasma located in a magnetic field. Some of these processes have been studied in a number of papers.[2–5] In the present paper we analyze the possibility of using the decay instabilities of electromagnetic waves propagating perpendicularly to the external magnetic field $\mathbf{H}_0$ for the continuous tuning of the coherent-radiation frequency in the far infrared region in the magnetoactive plasma of semiconductors,[6] as well as for the diagnostics of the magnetic fields that spontaneously develop in a laser plasma.[7]

The analysis of the case of the propagation of the waves along $\mathbf{H}_0$ is less interesting from the standpoint of the continuous tuning of the infrared-radiation frequency in a sufficiently broad band.

It should also be noted that, because of the conservation of the photon momentum, there arise additional—with respect to the case of transverse propagation—selection rules for the possible decays. Some decay cases for waves propagating along the magnetic field are considered in Refs. 4 and 5.

For high-frequency electromagnetic waves propagating perpendicularly to the external magnetic field $\mathbf{H}_0$, there exist the following types of oscillations[8]:

1. An ordinary wave (O-wave) with the electric-field vector, $\mathbf{E}$, parallel to $\mathbf{H}_0$ ($\mathbf{H}_0$ is directed along the z axis), whose dispersion equation has the form

$$k^2c^2/\omega^2=\varepsilon_{zz}\equiv\eta, \tag{1}$$

where $\eta=\varepsilon_\infty(1-\omega_p^2/\omega^2)$, k is the wave number, ω the wave frequency, c the velocity of light *in vacuo*, ε_∞ the high-frequency permittivity of the medium, $\omega_p=(4\pi e^2n/\varepsilon_\infty m^*)^{1/2}$ the plasma frequency, n the electron concentration (for semiconductors, in the conduction band), and m^* the electron effective mass.

2. An extraordinary wave (E-wave) with electric vector, $\mathbf{E}$, perpendicular to $\mathbf{H}_0$. For the case when the vector $\mathbf{k}$ is directed parallel to the x axis, the dispersion equation has the form

$$\frac{k^2c^2}{\omega^2}=\varepsilon_{xx}-\frac{\varepsilon_{xy}\varepsilon_{yx}}{\varepsilon_{yy}}\equiv\varepsilon_t, \tag{2}$$

where

$$\varepsilon_{xx}\equiv\varepsilon_1=\varepsilon_{yy}=\varepsilon_2=\varepsilon_\infty(1-\omega_p^2/(\omega^2-\Omega^2)),$$
$$\varepsilon_{xy}=-\varepsilon_{yx}\equiv ig,\quad g=\varepsilon_\infty\omega_p^2\Omega/\omega(\omega^2-\Omega^2),$$

$\Omega=eH_0/m^*c$ is the electron cyclotron frequency,

$$\varepsilon_t=1-\varepsilon_\infty\omega_p^2(\omega^2-\Omega^2)/\omega^2(\omega^2-\Omega_h^2),\quad \Omega_h=(\Omega^2+\omega_p^2)^{1/2}$$

is the "upper" hybrid frequency.

3. A Bernstein mode (B-wave), an almost longitudinal ($\mathbf{E}\parallel\mathbf{k}\parallel x$) wave whose dispersion equation has the form

$$\varepsilon_{yy}=0. \tag{3}$$

Here we should make allowance for the resonance at the second cyclotron harmonic in the expression for ε_{yy}. The expression for ε_{yy} for a semiconductor plasma in a quantizing magnetic field, for example, has the form[9]

$$\varepsilon_{yy}=\frac{\varepsilon_\infty}{\omega^2-\Omega^2}\left(\omega^2-\Omega_h^2-\frac{3}{2}\frac{\hbar k^2\omega_p^2\Omega}{m^*(\omega^2-4\Omega^2)}\right). \tag{4}$$

In Figs. 1–3 we show the dispersion curves for the indicated waves for the case when $\omega_p\ll\Omega$. Let us consider those possible decays for these waves that satisfy the phase-synchronism conditions: $\mathbf{k}_1=\mathbf{k}_2+\mathbf{k}_3$, $\omega_1=\omega_2+\omega_3$. Here $\mathbf{k}_1$ and ω_1 are the wave vector and frequency of the incident laser radiation, $\mathbf{k}_2$, $\mathbf{k}_3$ and ω_2, ω_3 are the wave vectors and frequencies of the quanta generated as a result of the decay. When the condition $\frac{4}{3}\Omega^2+\omega_p^2>\omega_1^2$ is fulfilled, a phase-synchronous decay of the E-wave into two O-waves is possible, which allows the realization of frequency conversion with continuous tuning near $\omega_1/2$. The decays $O_1\to O_2+E_3$, as well as the decays

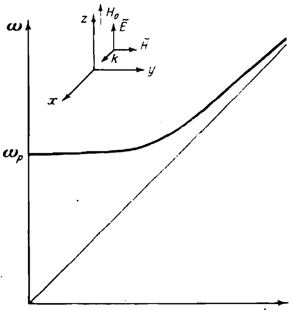


FIG. 1. Dispersion curve of an ordinary electromagnetic wave.

 0038-5646/77/4605-0917$02.40

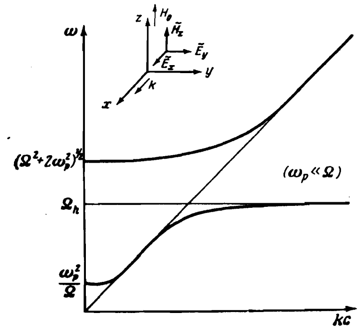


FIG. 2. Dispersion curves of an extraordinary electromagnetic wave.

$E_1 \to E_2 + E_3$ and $E_1 \to E_2 + B_3$, in the plasma of semiconductors allow the achievement of frequency tuning in the far infrared region ($\omega_3 \sim \Omega_h$ or $\omega_3 \sim \Omega$).

1. Let us consider in greater detail the indicated decay processes for the plasma of semiconductors. The computation of the nonlinear properties of a magnetoactive plasma can be carried out in the hydrodynamic approximation. The thermal corrections in the case under consideration are small in the parameter $kv_T/\Omega \ll 1$ (v_T is the thermal velocity of the electrons), and can be neglected. The polarizability, **P**, of a magnetoactive plasma and the current connected with it through the relation $\mathbf{j} = \partial \mathbf{P}/\partial t$ can be computed by expanding the electromagnetic waves participating in the interaction into power series in the amplitudes:

$$\mathbf{j} = ne\mathbf{v} = \mathbf{j}_l + \mathbf{j}_n + \ .$$

Here $\mathbf{j}_l$ is the linear—in the field—part of the total current and $j_n = \alpha_{ijk} E_{1k} E_{2k}^*$ is the nonlinear current, proportional to the product of the amplitudes of the fields $\mathbf{E}_1$ and $\mathbf{E}_2^*$.

As an example, let us compute the coefficient α_{yzz} in the nonlinear current at the frequency ω_3 in the decay $O_1 \to O_2 + E_3$. It follows from the continuity equation

$$\partial n/\partial t + \operatorname{div}(n\mathbf{v}) = 0$$

that the transverse electromagnetic waves O_1 and O_2 do not perturb the equilibrium plasma density n_0, and, for the computation of $j_n^{\omega_3}$, it is necessary to compute the components of the velocity $\mathbf{v}_2$ that are proportional to the product of the amplitudes of $\mathbf{E}_1$ and $\mathbf{E}_2^*$:

$$\frac{\partial v_{2x}^{\omega 3}}{\partial t} = \frac{e}{m^* c}[v_{1z}^{\omega 1}(H_{2y}^{\omega 2})^* + (v_{1z}^{\omega 2})^* H_{1y}^{\omega 1}] - \frac{e}{m^* c} H_0 v_{2y}^{\omega 3},$$
$$\frac{\partial v_{2y}^{\omega 3}}{\partial t} = \frac{e}{m^* c} H_0 v_{2x}^{\omega 3}. \qquad (5)$$

(There are no nonlinear terms of the type $(\mathbf{v}_1 \cdot \nabla)\mathbf{v}_1$ in the equation of motion because of the transverse character of the O_1 and O_2 waves.) In Eqs. (5), the $\mathbf{v}_1$ are the linear—in the field—velocities, determinable from the equation $\partial \mathbf{v}_1/\partial t = e\mathbf{E}_1/m^*$; $\mathbf{H}_1$ and $\mathbf{H}_2$ are the magnetic fields of the waves with frequencies ω_1 and ω_2. Solving Eq. (5) for $v_{2y}^{\omega 3}$, and expressing $v_{1z}^{\omega 1}$, $v_{1z}^{\omega 2}$, $H_{1y}^{\omega 1}$, and $H_{2y}^{\omega 2}$ in terms of the components $E_{1z}^{\omega 1}$ and $E_{2z}^{\omega 2}$ of the fields with frequencies ω_1 and ω_2, we obtain

$$v_{2y}^{\omega 3} = \frac{i\Omega e^2 k_3}{m^{*2}\omega_1\omega_2(\omega_3^2 - \Omega^2)} E_1 E_2^*.$$

The nonlinear coefficient α_{yzz} for the $O_1 \to O_2 + E_3$ decay turns out to be equal to

$$\alpha_{yzz} = \frac{i\varepsilon_\infty \omega_p^2 e k_3 \Omega}{4\pi m^* \omega_1 \omega_2 (\omega_3^2 - \Omega^2)}.$$

Below we give the values, computed by us, of the quantities for the above-enumerated decays.

The $E_1 \to O_2 + O_3$ decay:

$$\alpha_{zyz} = -\frac{i\varepsilon_\infty \omega_p^2 e k_1 \Omega}{4\pi m^* \omega_2 \omega_1 (\omega_1^2 - \Omega_h^2)} \qquad (6a)$$

the $O_1 \to O_2 + E_3$ decay:

$$\alpha_{yzz} = \frac{i\varepsilon_\infty \omega_p^2 e k_3 \Omega}{4\pi m^* \omega_2 \omega_1 (\omega_3^2 - \Omega^2)}, \qquad (6b)$$

the $E_1 \to E_2 + E_3$ decay:

$$\alpha_{yyy} = -\frac{i\varepsilon_\infty \omega_p^2 e k_3 \Omega}{4\pi m^* \omega_2 \omega_1 (\omega_3^2 - \Omega^2)}, \qquad (6c)$$

the $E_1 \to E_2 + B_3$ decay:

$$\alpha_{xyy} = i\alpha_{yyy} \quad (\omega_1, \omega_2 \gg \Omega, \omega_p). \qquad (6d)$$

Let us note that, for electron-concentration values in the conduction band $n \sim 10^{16}$ cm^{-3} and $H_0 \sim 10^5$ Oe, in the $E_1 \to O_2 + O_3$ decay for InSb the coefficient $\alpha \sim 10^8$ cgs units, which corresponds to a nonlinear polarizability coefficient[10] $\chi = \alpha/\omega \sim 10^{-6}$ cgs units. For the other decays the numerical values of α lie within the limits 10^7–10^8 cgs units. Thus, it can be seen that, by its nonlinear properties, the magnetoactive plasma of semiconductors can successfully compete with the crystals widely used in nonlinear optics.[11]

The $E_1 \to E_2 + E_3$ decay process was observed experimentally in InSb crystals in fields of $H_0 \sim 10^4$ Oe when $n \sim 2 \times 10^{15}$ cm^{-3} and $T \sim 70$ K.[12] The estimation of the magnitude of the nonlinearity for these conditions yields $\chi \sim \alpha/\omega_3 \approx 4.5 \times 10^{-7}$ cgs units, which is close to the experimental value of $\chi = 2d_{14} \approx (3.5 \pm 1) \times 10^{-7}$ cgs units, found from the magnitude of the output power at the frequency $\omega_3 \approx 10^2$ cm^{-1}. Notice that in Van Tran and Patel's paper[12] the corresponding nonlinear susceptibility was attributed to the crystal, the influence of the plasma amounting, in the opinion of the authors, to the securing of the phase synchronism. As follows from the above-given estimate, the effect of the presence of the plasma is not only to change the permittivity of the

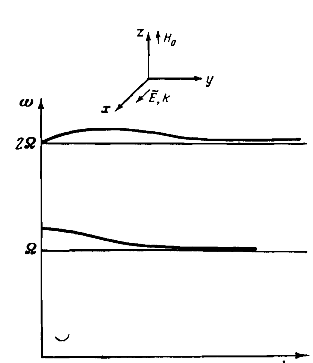


FIG. 3. Dispersion curves of the Bernstein mode.

crystal in a magnetic field (in Ref. 12 a not exactly correct expression is, moreover, given for $\varepsilon(\omega_3)$), but also to increase in resonant fashion the nonlinear susceptibility as the magnetic-field strength is varied.

In semiconductors, under experimental conditions, the plasma is often degenerate ($\varepsilon_F \gtrsim T$, where ε_F is the Fermi energy of the electrons), and the magnetic fields are of quantizing intensities, $\hbar\Omega \gg T$. Also realizable is the case when the electron gas is highly degenerate, and all the electrons are in the zero Landau level, with the spin oriented in the direction opposite to that of the magnetic field; in this case the Fermi energy is equal to[13]

$$\varepsilon_F = {}^1/_2\hbar\Omega - {}^1/_2 g\mu H_0 + 2\pi^4\hbar^4 c^2 n^2/m^* H_0 e^2 \leqslant \hbar\Omega.$$

Here $g\mu$ is the effective Bohr magneton, n is the carrier concentration (in a gaseous plasma $\hbar\Omega = g\mu H_0$, $m^* = m_e$, and $g=2$). Under the indicated conditions a quantum-mechanical treatment is necessary for the computation of the nonlinear currents. The application of the methods of quantum field theory[14] with the use of the temperature diagram technique is convenient. The corresponding computations, using as an example, the $E_1 \to O_2 + O_3$ decay, are presented in the Appendix. If the wavelengths $\lambda \gg (\hbar c/eH_0)^{1/2}$, the Landau length for electrons in a quantizing magnetic field, then the answers for the nonlinear susceptibilities coincide with the answers given above. This circumstance is quite evident if we take into account the fact that the quantization primarily affects the effects connected with pressure and that it determines the spatial dispersion of the plasma.[9] If $v_T/v_{ph} \ll 1$ (v_T is the "thermal" velocity, determined by the quantities $(\hbar\Omega/m^*)^{1/2}$, $(\varepsilon_F/m^*)^{1/2}$), then the dynamics of the electrons is determined to a high degree of accuracy by the equations of motion in which the terms containing the strain tensor and carrying information about the equation of state of the plasma, have been dropped. The remaining corrections are small in the parameter $(\hbar c/eH_0)^{1/2}/\lambda$.

The presence of the nonlinear currents at the frequencies ω_2 and ω_3 leads to a parametric coupling between the waves 2 and 3, and, if the intensity of the pumping wave exceeds a certain value (threshold), there develops an exponential growth of these waves in time, or in space, depending on the conditions of the problem.

In the decay of the $(\omega_1, \mathbf{k}_1)$ extraordinary wave into two extraordinary waves $(\omega_2, \mathbf{k}_2)$ and $(\omega_3, \mathbf{k}_3)$ (the other processes can be considered in much the same way), the reduced Maxwell equations for the slowly varying—in space—amplitudes have the form

$$-k_l^2 E_{ly} + \frac{\omega_l^2}{c^2}[\varepsilon_2(\omega_l)E_{ly} - ig(\omega_l)E_{lx}] + 2ik_l\frac{\partial E_{ly}}{\partial x} = -\frac{4\pi i\omega_l}{c^2} j_{Hy}^{\omega l}, \tag{7}$$

$$-\frac{\omega_l^2}{c^2}[\varepsilon_1(\omega_l)E_{lx} + ig(\omega_l)E_{ly}] = -\frac{4\pi i\omega_l}{c^2} j_{Hx}^{\omega l}. \tag{8}$$

Here $l=2, 3$. The attenuation of the pumping wave is neglected.

Eliminating the component E_{lx} from (8), we obtain

$$\frac{\partial E_{2y}}{\partial x} + \gamma_2 E_{2y} = -\frac{2\pi\omega_2}{k_2 c^2}\alpha_2 E_{1y} E_{3y}^*,$$

$$\frac{\partial E_{3y}^*}{\partial x} + \gamma_3 E_{3y}^* = -\frac{2\pi\omega_3}{k_3 c^2}\alpha_3^* E_{1y}^* E_{2y}, \tag{9}$$

where

$$\gamma_l = \frac{\omega_l^2}{2c^2 k_l}\operatorname{Im}\varepsilon_l(\omega_l), \quad \alpha_l = \alpha_{yyy}^l - i\frac{g(\omega_l)}{\varepsilon_1(\omega_l)}\alpha_{xyy}^l.$$

The imaginary part of ε_t arises upon allowance for collisions or other wave-attenuation mechanisms.

The wave number k_l is connected with ω_l through the relation

$$k_l^2 c^2/\omega_l^2 = \operatorname{Re}\varepsilon_t(\omega_l).$$

The solution to the system (9) has the form of exponential functions with exponents $P_{1,2}$ equal to

$$P_{1,2} = \frac{-\gamma_2-\gamma_3}{2} \pm \left(\frac{(\gamma_2+\gamma_3)^2}{2} + \gamma_L^2 - \gamma_2\gamma_3\right)^{1/2}, \tag{10}$$

where

$$\gamma_L^2 = \frac{4\pi^2}{c^2}\frac{\omega_3}{k_3 c}\frac{\omega_2}{k_2 c}\alpha_2\alpha_3^*|E_{1y}|^2.$$

As follows from (10), when the threshold condition $\gamma_L^2 > \gamma_2\gamma_3$ is fulfilled, one of the solutions increases exponentially in space. In a semiconductor plasma with $n \sim 10^{16}$–10^{17} cm^{-3} and $T \sim 20$–70 K, the attenuation of the electromagnetic waves is fairly high: $\gamma \sim 1$–10 cm^{-1}. This corresponds to a threshold pump intensity $I_{thr} \sim 10^6$–10^7 W/cm^2. Although the introduction of such a power into semiconducting crystals is a complicated problem because of a possible optical breakdown, it is, apparently, practicable.

Thus, the use of the magnetoactive plasma of semiconductors for the tuning of radiation in the far infrared region is attractive, since the plasma is a high-performance nonlinear element, and an external magnetic field enables us to resonantly increase the nonlinear susceptibility of the plasma (see (6)), as well as to continuously tune the scattered-radiation frequency in a broad band.

2. The above-indicated examples of decay instability may be important in a laser plasma in the presence of magnetic fields spontaneously developing in it.[7] Since the appearance of such fields can affect the symmetrical compression and heating of the laser target, it is important to have reliable information about the intensity magnitudes and the configuration of the lines of force of the magnetic fields. There are at present no reliable measurements and theoretical computations of these fields.

In view of this, the question of the influence of the magnetic fields in a laser plasma on the processes of the nonlinear conversion of the laser radiation is of interest. Let us consider, for example, the transformation of an electromagnetic wave in the $\frac{1}{4}n_{cr}$ region ($n_{cr} = m\omega_1^2/4\pi e^2$).

In the absence of a magnetic field, the process of decay of a transverse wave into two plasmons ($t \to l + l'$) is widely enlisted in the explanation of the observed $\frac{1}{2}\omega_1$

and $\frac{3}{2}\omega_1$ laser-radiation harmonics.[15,16] In a magnetic field, which, according to estimates, can be of strength of up to $\sim 10^6$ Oe, a collinear decay of the extraordinary wave into two extraordinary waves is possible, it being necessary for the fulfillment of the synchronism conditions, $\mathbf{k}_1 = \mathbf{k}_2 + \mathbf{k}_3$, that for one of the waves $(\omega_2, \mathbf{k}_2)$ the relations $k_2 \approx k_1 \approx \omega_1/c$, $\omega_2 \approx \Omega_h \approx \omega_p$ be fulfilled, while for the other $(\omega_3, \mathbf{k}_3)$ the relations $k_3 \sim \Omega/c \ll k_2$, $\omega_3 \approx \omega_p \pm \Omega/2$, correspondingly. (The analysis of the case of noncollinear propagation of the waves is substantially more tedious, although it may be necessary in a detailed analysis of the experimental data.) In this case the following characteristic values of the parameters were used: $\omega_p \approx \omega_1/2 \sim 10^{15}$ sec^{-1} (for a neodymium laser), $\Omega \sim 10^{13}$ sec^{-1} (for fields $H \sim 10^6$ Oe).

In the decay $E_1 \to E_2 + E_3$, the generated photons can be observed experimentally, and manifest themselves in the form of $\omega_1/2$ and $\omega_1/2 + \Omega/4$ harmonics of the reflected radiation. The presently accepted explanation of the observed harmonics makes use (in the absence of a magnetic field) of the incident-radiation scattering processes on the plasmons, $t + l \to t'$ and $3l \to t'$. The first of these processes occurs when allowance is made for the scattering of the plasmons produced in the $t \to l + l'$ decay on the ions, since in the opposite case it is not possible to guarantee the fulfillment of the law of conservation of momentum $(\mathbf{k}_2 + \mathbf{k}_3 = \mathbf{k}_1)$.

In a magnetic field the direct conversion of the incident radiation into the $\omega_1/2$ harmonic on account of the $E_1 \to E_2 + E_3$ process is possible, and the value of the relative shift, $\Delta\omega/\omega \sim \Omega/2\omega$, of the harmonic toward the blue region allows us to estimate the strength of the magnetic field. The $\omega_1/2$ harmonics that are shifted toward the red region apparently turn out to be forbidden. In a magnetic field the $\frac{3}{2}\omega_1$ harmonics can be generated upon the merging of three extraordinary waves, and lead to shifts, $\sim \Omega$, toward the red and blue regions. The normally observed spectrum of the $\frac{3}{2}\omega_1$ harmonics[15,16] contains a red and a blue component, the relative magnitude ($\lesssim 10^{-2}$) of the shift being accounted for by the thermal corrections to the plasmon dispersion: $\Delta\omega/\omega \sim (v_{Te}/c)^2$. A frequency shift of the same order of magnitude can be due to magnetic fields of intensity 1 MOe. To distinguish one harmonic-shift mechanism from the other, we need a careful independent measurement of the electron temperature.

Let us give estimates for the threshold power and the increments of the decay instability $E_1 \to E_2 + E_3$ for the conventional laser plasma. The expression for the nonlinear coefficient α in the case when $\omega_1 = 2\omega_2 \approx 2\omega_p \gg \Omega$ has the form

$$\alpha_3 = \frac{ie^3 n\Omega k_2}{m^2\omega_1\omega_2[(\omega_2+i\nu)^2 - \Omega^2 - (\omega_p^2 + k^2 v_{Te}^2)(1+i\nu/\omega_2)]} \approx \frac{e\Omega}{4\pi mc\nu}. \quad (11)$$

Here ν is the electron-ion collision rate. It follows from the expressions (10) and (11) that the quantity γ_L, which determines the spatial increment, is equal to

$$\gamma_L = \left(\frac{1}{16}\frac{\omega_p}{\Omega}\left(\frac{\Omega}{\nu}\right)^2 \frac{I}{nmc^3}\left(\frac{\omega_p}{c}\right)^2\right)^{1/2}, \quad (12)$$

where I is the intensity of the pumping wave.

The threshold intensity of the laser radiation is determined by the expression

$$I_{\text{thr}} \approx 2(\nu/\Omega)^2 nmc^3 (\nu/\omega_p)^2. \quad (13)$$

For the conditions of a laser plasma produced by a neodymium laser ($n \sim n_{\text{cr}}/4 \approx 2.5\times10^{20}$ cm^{-3}, $\nu/\omega_p \leqslant 10^{-2}$, $\nu/\Omega \leqslant 1$), we obtain from (13) that $I_{\text{thr}} \lesssim 10^{14}$ W/cm^2. Because of the critical dependence of I_{thr} on the density, the above-described processes for a plasma produced by a CO_2 laser set in at $I_{\text{thr}} \lesssim 10^{10}$ W/cm^2.

Taking account of the expressions for the decrements, γ_2 and γ_3, we obtain for the nonlinear increment from (10) the value

$$\gamma_H \approx \gamma_L^2/\gamma_3 \sim 10^3 \text{ cm}^{-1}.$$

The temporal increment, γ_τ, corresponding to this case is equal to

$$\gamma_\tau = \gamma_H (v_2 v_3)^{1/2} \sim 10^{11} \text{ sec}^{-1} \gg 1/\tau,$$

where $v_2 \approx c(v_{Te}/c)^2$ is the group velocity of the $(\omega_2, \mathbf{k}_2)$ wave, $v_3 \approx c\Omega/2\omega_p$ is that of the $(\omega_3, \mathbf{k}_3)$ wave, and $\tau \sim 1$ nsec is the duration of the laser pulse.

The estimate for the effect of the spatial inhomogeneity of the magnetic field over dimensions $\sim 10^{-2}$ cm does not, according to Ref. 17, change the magnitude of the spatial increment.

In conclusion, we thank L. A. Bol'shov, A. A. Vedenov, and A. M. Dykhne for interest in the work and for useful discussions.

APPENDIX

The nonlinear currents in a plasma can be expressed with the aid of the Green function for an electron located in external electromagnetic fields[14] (for simplicity we shall neglect the electron spin):

$$j_i = \left[-\frac{ie\hbar}{2m}\left(\frac{\partial}{\partial r_i} - \frac{\partial}{\partial r_i'}\right) - \frac{e^2 A_i}{mc}\right] G(r, r', \tau, \tau')|_{r'\to r,\, \tau'\to\tau+0} \quad (A.1)$$

$(i = x, y, z)$. The quantity $G(\mathbf{r}, \mathbf{r}', \tau, \tau')$ is found in the form of a series in the vector potential $\mathbf{A}$:

$$G(\mathbf{r}, \mathbf{r}', \tau, \tau') = G_0(\mathbf{r}, \mathbf{r}', \tau, \tau') + \int d^4x_1\, G_0(x, x_1)\Sigma(x_1)G_0(x_1, x')$$
$$+ \int d^4x_1\, d^4x_2\, G_0(x, x_1)\Sigma(x_1)G_0(x_1, x_2)\Sigma(x_2)G_0(x_2, x') + \ldots, \quad (A.2)$$

where

$$G_0(\mathbf{r}, \mathbf{r}', \tau, \tau') = -\langle T(\psi(r, \tau)\psi^+(r', \tau'))\rangle, \quad (A.3)$$

the ψ being Heisenberg field operators. In an external magnetic field $\mathbf{H}_0 \parallel \mathbf{z}$ with gauge $A_0 = (0, H_0 x, 0)$, the Fourier component of the zeroth-order—in the "temperatures" $(\tau - \tau')$—Green function has the form[9]

$$G_0(\mathbf{r}, \mathbf{r}', \omega_m) = \frac{\eta}{2\pi^2\hbar}\exp\left[-i\eta\frac{(x+x')(y-y')}{2}\right]\sum_n \int_{-\infty}^{\infty} dp_z \frac{1}{i\omega_m - \xi(n, p_z)}$$
$$\times \exp\left[ip_z(z-z') - \eta\frac{(x-x')^2 + (y-y')^2}{4}\right] L_n\left(\eta\frac{(x-x')^2 + (y-y')^2}{2}\right), \quad (A.4)$$

where $\eta = |e|H/\hbar c$ is the square of the inverse magnetic length, $L_n(x)$ is a Laguerre polynomial, $\omega_m = (2m+1)T$ (T is the temperature of the system),

$$\xi(n, p_z) = \varepsilon_n(p_z) - \mu,$$

$\varepsilon_n(p_z)$ is the electron energy in the magnetic field,

$$\varepsilon_n(p_z) = \hbar\Omega(n + {}^1/_2) + \hbar^2 p_z^2/2m,$$

and μ is the chemical potential. The quantity $\hat{\Sigma}$ is equal to

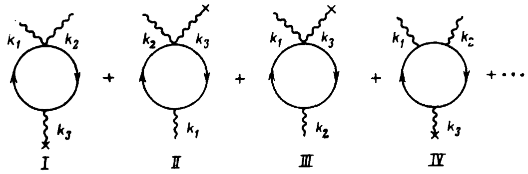
$$\hat{\Sigma}^l = -\frac{e\hbar}{mc}\left(\mathbf{A}_{il}\hat{\mathbf{p}} + \frac{\mathbf{k}_l}{2}\mathbf{A}_l\right) + \frac{e^2\mathbf{A}_0\mathbf{A}_l}{mc^2} + \frac{e^2\mathbf{A}^2}{2mc^2}. \qquad (\mathrm{A.5})$$

The vector potential of the wave with wave vector k_l and frequency ω_m^l has the form

$$A_l(x) = A^l \exp(ik_l r - i\omega_m{}^l\tau),$$

and $\hat{\mathbf{p}} = -i\hbar\nabla$ is the particle-momentum operator.

If we are interested in the nonlinear current at the frequency ω_3, which is proportional to the product of the $\mathbf{A}^{(1)}$ and $\mathbf{A}^{(2)}$ fields, then we should retain precisely such terms in the last term in the expression for $\hat{\Sigma}$. We select in much the same way from (A.2) the other terms containing the product of the $\mathbf{A}^{(1)}$ and $\mathbf{A}^{(2)}$ fields. It is convenient to depict diagrammatically the corresponding terms of the nonlinear current:



(A.6)

Here a wavy line represents the vector potential; a line ending in a cross, the current vertex. The continuous lines represent the Green function G_0.

Let us, for example, consider the contribution from the first three diagrams for a plasma without a magnetic field:

$$\mathbf{j}_{\mathrm{I}}^{(3)} = \frac{e^3\hbar}{m^2c^2} T \sum_{\omega_p} \int \frac{d\mathbf{p}}{(2\pi^3)} G_0(\mathbf{p})\left(\mathbf{p} + \frac{\mathbf{k}_3}{2}\right) G_0(\mathbf{p}+\mathbf{k}_3)(\mathbf{A}_2\mathbf{A}_1),$$

$$\mathbf{j}_{\mathrm{II}}^{(3)} = \frac{e^3\hbar}{m^2c^2} T\mathbf{A}_2 \sum_{\omega_p} \int \frac{d\mathbf{p}}{(2\pi)^3} G_0(\mathbf{p})\left(\mathbf{p} - \frac{\mathbf{k}_1}{2}, \mathbf{A}_1\right) G_0(\mathbf{p}-\mathbf{k}_1), \qquad (\mathrm{A.7})$$

$$\mathbf{j}_{\mathrm{III}}^{(3)} = \frac{e^3\hbar}{m^2c^2} T\mathbf{A}_1 \sum_{\omega_p} \int \frac{d\mathbf{p}}{(2\pi)^3} G_0(\mathbf{p})\left(\mathbf{p} - \frac{\mathbf{k}_2}{2}, \mathbf{A}_2\right) G_0(\mathbf{p}-\mathbf{k}_2).$$

Summing over the frequencies ω_p, we obtain[14]

$$T \sum_{\omega_p} G(\mathbf{p}) G(\mathbf{p}+\mathbf{k}) = -\frac{n_\mathbf{p} - n_{\mathbf{p}+\mathbf{k}}}{i\omega_k - \varepsilon_{\mathbf{p}+\mathbf{k}} + \varepsilon_\mathbf{p}} \qquad (\mathrm{A.8})$$

where $n_\mathbf{p}$ is the momentum distribution function of the electrons. After performing the summation over the frequencies, we can go over to the retarded response: $i\omega_m^l \to \hbar\omega^l + i\varepsilon$ ($\varepsilon \to 0$). Analyzing the obtained expressions for $k_l \ll p$ and $\omega \gg kv_T$, we obtain

$$\mathbf{j}_{\mathrm{I}}^{(3)} = -\frac{e^3(\mathbf{A}_2\mathbf{A}_1)}{m^2c^2}\mathbf{k}_3 \int \frac{d\mathbf{p}}{(2\pi)^3} \frac{\mathbf{p}\,\partial n/\partial \mathbf{p}}{\omega_1} = \frac{e^3(\mathbf{A}_2\mathbf{A}_1)}{m^2c^3\omega_1}\mathbf{k}_3 n, \qquad (\mathrm{A.9})$$

where n is the electron concentration.

Similarly, computing the other terms and going over from the vector potentials to the intensities $A_i = -(ic/\omega_i) \times E_i$, we obtain

$$\mathbf{j}^{(3)} = -\frac{e^3 n}{\omega_1\omega_2 m^2}\left[\frac{\mathbf{k}_3}{\omega_3}(\mathbf{E}_2\mathbf{E}_1) + \frac{(\mathbf{k}_1\mathbf{E}_1)}{\omega_1}\mathbf{E}_2 + \frac{(\mathbf{k}_2\mathbf{E}_2)}{\omega_2}\mathbf{E}_3\right]. \qquad (\mathrm{A.10})$$

This expression coincides with the result of the computation of the nonlinear current in the cold-hydrodynamics approximation[18]: the first term corresponds to the Miller force (from the $(e/mc)\mathbf{v}\cdot\mathbf{H}$ and $(\mathbf{v}_1\cdot\nabla)\mathbf{v}_1$ terms), while the last two are obtained from the consideration of the perturbation of the density and the velocity in first order in the fields. The contribution of the remaining diagrams (IV, etc.) is of the same order of magnitude as the discarded terms, which are $\sim kv/\omega$, k/p.

Let us consider the $E_1 \to O_2 + O_3$ decay in a magnetized plasma. Taking interest in the current at the frequency ω_3, which generates an extraordinary wave with $\mathbf{j} \parallel \mathbf{z}$, we can easily see that $\mathbf{A}_2 \cdot \mathbf{A}_1 = 0$ (and the contribution from the diagram I vanishes accordingly), where $\mathbf{A}_1$, with components $(A_x^{(1)}, A_y^{(1)}, 0)$, is the vector potential of the extraordinary wave. Similarly, the diagram II vanishes, since $k_{2z} = 0$ (the wave propagates across the magnetic field parallel to the x axis), and the integration over p_z ($\mathbf{p}\cdot\mathbf{A}_2 = p_z A_z^{(2)}$) yields zero because of the oddness of the integrand as a function of p_z.

There thus remains the diagram III, the expression for which has the form

$$\mathbf{j}_{\mathrm{III}}^{(3)} = -\frac{e^2}{mc^2}\mathbf{A}_2^{\boldsymbol{\cdot}} \iint \frac{dx\,dx_1}{V} G_0(x, x_1)\Sigma_1(x) G_0(x, x_1).$$

Here

$$\Sigma_1(x_1) = -\frac{e\hbar}{mc}\left(\mathbf{A}^{(1)}\hat{\mathbf{p}} + \frac{\mathbf{k}_1}{2}\mathbf{A}^{(1)}\right) + \frac{e^2\mathbf{A}_0\mathbf{A}^{(1)}}{mc^2}.$$

Using the expression (A.4) for G_0, summing over the frequencies with the aid of (A.2), and going over to the variables $\rho = (x^2 + y^2)^{1/2}$ and φ, we obtain

$$\begin{aligned} j_z^{(3)} = &-\frac{e^2}{mc^2} A_{2z}^{\boldsymbol{\cdot}} \sum_{n_1 n_2} \int_{-\infty}^{\infty} \frac{dp_z}{(2\pi)^3}\left(-\frac{\eta^2}{\hbar^2}\right) \int_0^\infty \rho\,d\rho \int_0^{2\pi} d\varphi \exp\left(-\frac{\eta\rho^2}{2} + ik_1\rho\cos\varphi\right) \\ &\times L_{n1}\left(\frac{\eta\rho^2}{2}\right)\left[\frac{ie\eta}{2mc} A_x^{(1)} e^{i\varphi}\rho L_{n2}\left(\frac{\eta\rho^2}{2}\right) + \frac{e\eta}{2mc} A_y^{(1)} \rho e^{i\varphi} L_{n2}\left(\frac{\eta\rho^2}{2}\right)\right. \\ &\left. - \frac{e\hbar}{2mc} A_x^{(1)} k_1 L_n\left(\frac{\eta\rho^2}{2}\right) - \frac{i\eta}{mc}\rho \frac{\partial L_{n2}(\xi)}{\partial \xi} \times (A_x^{(1)}\cos\varphi + A_y^{(1)}\sin\varphi)\right] \frac{n(\varepsilon_{p_z}{}^{n1}) - n(\varepsilon_{p_z}{}^{n2})}{\hbar\omega_1 - \varepsilon_{p_z}{}^{n1} + \varepsilon_{p_z}{}^{n2}} \end{aligned} \qquad (\mathrm{A.11})$$

($\xi = \eta\rho^2/2$). Expanding in (A.11) the function $\exp(ik_2\rho \times \cos\varphi)$ in a series ($\lambda_2 \gg \eta^{-1/2}$), and limiting ourselves to the terms linear in k_2, we obtain after integrating over φ and ρ the expression

$$j_z^{(3)} = \frac{e^3 E_z^{(2)*}}{m^2\omega_2\omega_1} k_1 \left[E_x^{(1)} \frac{\omega_1}{\omega_1^2 - \Omega^2} - iE_y^{(1)} \frac{\Omega}{\omega_1^2 - \Omega^2}\right] \cdot \times \frac{\eta}{4\pi^2} \int_{-\infty}^{\infty} dp_z \sum_n (n+1)[n(\varepsilon_{p_z}^{n+1}) - n(\varepsilon_{p_z}{}^n)].$$

The last expression with allowance for the relation, $E_x^{(1)} = -igE_y^{(1)}/\varepsilon_1$, between the components of the extraordinary wave, where $\varepsilon_1 = 1 - \omega_p^2/(\omega^2 - \Omega^2)$ and $g = \Omega\omega_p^2/\omega(\omega^2 - \Omega^2)$, reduces to the form

$$j_z^{(3)} = \frac{i\omega_p^2 E_z^{(2)*} E_y^{(1)} e k_1 \Omega}{4\pi m \omega_1 \omega_2 (\omega_1^2 - \Omega^2 - \omega_p^2)}$$

(cf. (6)) irrespective of the degeneracy and the strength of the magnetic field. The remaining corrections are then $\sim 1/\eta^{1/2}\lambda \ll 1$.

[1]V. N. Oraevskiĭ and R. Z. Sagdeev, Zh. Tekh. Fiz. 32, 1291 (1962) [Sov. Phys. Tech. Phys. 7, 955 (1963)].

[2]V. N. Tsytovich and A. B. Shvartsburg, Zh. Eksp. Teor. Fiz. 49, 797 (1965) [Sov. Phys. JETP 22, 554 (1966)].

[3]V. P. Silin, Parametricheskoe vozdeĭctvie izlucheniya bol'shoĭ moshchnosti na plazmu (Parametric Action of High-Power Radiation on a Plasma), Nauka, 1973.

[4]Akira Hasegawa, Phys. Rev. Lett. 32, 817 (1974).

[5]J. Larson and L. Stenflo, Beitrage aus der Plasmaphys. 16, 79 (1976); 13, 169 (1973).

[6]A. A. Vedenov, G. D. Myl'nikov, V. A. Roslyakov, D. N. Sobolenko, and A. N. Starostin, Tezisy dokl. VII Vsesoyuznoĭ konf. po kogerentnoĭ i nelineĭnoĭ optike (Abstracts of Papers presented at the 7th All-Union Conference on Coherent and Nonlinear Optics), Tashkent, 1974, p. 453.

[7]L. A. Bol'shov, Yu. A. Dreĭzin, and A. M. Dykhne, Pis'ma Zh. Eksp. Teor. Fiz. 19, 288 (1974) [JETP Lett. 19, 168 (1974)].

[8]N. A. Krall and A. W. Trivelpiece, Principles of Plasma Physics, McGraw Hill, New York, 1972 (Russ. Transl., Mir, 1975).

[9]V. V. Korneev and A. N. Starostin, Zh. Eksp. Teor. Fiz. 63, 930 (1972) [Sov. Phys. JETP 36, 487 (1973)].

[10]N. Bloembergen, Nonlinear Optics, W. A. Benjamin, New York, 1965 (Russ. Transl., Mir, 1966).

[11]D. N. Nikogosyan, Kvantovaya Elektron. (Moscow) 4, 5 (1977) [Sov. J. Quantum Elec. 7, 1 (1977)].

[12]H. Van Tran and C. K. N. Patel, Phys. Rev. Lett. 22, 463 (1969).

[13]P. M. Platzman and P. A. Wolf, Waves and Interactions in Solid State Plasmas, Academic Press, New York, 1972 (Russ. Transl., Mir, 1975).

[14]A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskiĭ, Metody kvantovoĭ teorii polya v statisticheskoĭ fizike (Methods of Quantum Field Theory in Statistical Physics), Fizmatgiz, 1962 (Eng. Transl., Prentice-Hall, Englewood Cliffs, New Jersey, 1963).

[15]V. V. Aleksandrov, S. I. Anisimov, M. V. Brenner, E. P. Velikhov, V. D. Vikharev, V. P. Zotov, N. G. Koval'skiĭ, M. I. Pergament, and A. N. Yaroslavskiĭ, Zh. Eksp. Teor. Fiz. 71, 1826 (1976) [Sov. Phys. JETP 44, 958 (1976)].

[16]A. I. Avrov, V. D. Bychenkov, O. N. Krokhin, V. V. Pustovalov, A. A. Rupasov, V. P. Silin, G. V. Sklizkov, V. T. Tikhonchuk, and A. S. Shikanov, Zh. Eksp. Teor. Fiz. 72, 970 (1977) [Sov. Phys. JETP 45, 507 (1977)].

[17]A. A. Galeev and R. Z. Sagdeev, Nelineĭnaya teoriya plazmy, vyp. 6. Voprosy teorii plazmy (Nonlinear Plasma Theory, No. 6 in the series: Problems of Plasma Theory), edited by M. A. Leontovich, Atomizdat, 1973.

[18]V. N. Tsytovich, Nelineĭnye éffekty v plazme (Nonlinear Effects in a Plasma), Nauka, 1967 (Eng. Transl., Plenum, New York, 1970).

Translated by A. K. Agyei

Nonlinear theory of the low-frequency oscillations in a weakly turbulent plasma

A. I. Akhiezer, V. F. Aleksin, and V. D. Khodusov

Physico-technical Institute, Ukrainian Academy of Sciences

(Submitted 19 May 1977)
Zh. Eksp. Teor. Fiz. 73, 1757–1766 (November 1977)

A theory is developed for determining the frequency shift of electromagnetic waves in a weakly turbulent plasma as a function of the level of the turbulent pulsations. The case of magnetohydrodynamic waves is considered. It is shown that the dispersion laws for Alfvén and slow magnetosonic waves change markedly at low values of the longitudinal (parallel to the magnetic field) component of the wave vector. Modified dispersion laws are obtained for them and these are taken into account in a study of relaxation processes of excitations in the wave spectra.

PACS numbers: 52.35.Bj, 52.35.Dm, 52.35.Mw, 52.35.Ra

1. INTRODUCTION

As is well known, the interaction between particles or quasiparticles leads as a rule to a shift in their energies relative to the values of the energy corresponding to the free states of the particles. For example, the interaction of an atomic electron with the zero-point oscillations of an electromagnetic or electron-positron field leads to a shift in the energy levels of the atomic electron.[1]

A similar situation exists also for the energy spectra of electrons, photons and magnons in a solid (see, for example, Refs. 2 and 3) and for spectra of the natural oscillations in a plasma. In the last case, we need to take into account both the nonlinear interaction between

0038-5646/77/4605-0922$02.40