emission. Assuming $\varepsilon \cong 5$ MeV and $n_d = 10^{13}$ cm^{-3} and using the values of $\sigma_{e,n}(\varepsilon)$ of[14], we obtain $I_e \cong 10-15$ kA. We note that if we attribute the unusual course of the neutron emission to the (γ, n) reaction on the diaphragm, this would call for an accelerated-electron current of only several amperes.

4. CONCLUSION

1. The stable neutron yield observed in "normal" regimes, amounting to 2×10^9 neutrons per discharge, agrees fully in magnitude and in time dependence with the preliminary estimates of the thermonuclear emission at the given current strength and deuterium density.

2. The giant yield 10^{12}–10^{13} neutrons per discharge, observed in the acceleration regimes, is due to (γ, n) reactions in the restricting diaphragm, and corresponds to a relativistic electron beam current of several dozen kiloampere. A rough model of free acceleration of a detached group of electron in the vortical electric field and of radial drift of the electron trajectory agrees with the time variation of the torus circuit voltage and with the spatial displacement of the plasma pinch.

3. It is plausible to relate the anomalous course of the neutron emission in the concluding stage of certain discharges with the development of an (e, n) reaction in the plasma under the influence of a weakly relativistic beam having an energy lower than the threshold of the (γ, n) reaction on the diaphragm.

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1)The significance of this agreement, of course, must not be overestimated, since the complicated calibration procedure yields the experimental values of $\bar{N}$ only accurate to a numerical factor ~2.

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Stability of high-frequency discharges under strong skin-effect conditions

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The stability of high-frequency discharges under strong skin-effect conditions is investigated with an inductive discharge in a gas stream as an example. An approximate method is developed for the study of the stability of different types of high-frequency discharges under various conditions. An analysis of a criterion is formulated for the stability of high-frequency discharges to one-dimensional perturbations and its analysis shows why discharges in molecular gases become unstable when the thermal-conductivity coefficient decreases as a function of the temperature.

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1. INTRODUCTION

By now there are many papers devoted to various aspects of the stability of a gas discharge and covering the range from a high-temperature plasma to a glow-discharge plasma. There is, however, a patent lack of studies of the stability of stationary high-frequency discharges, which are of great practical importance, notwithstanding the availability of various experimental data attesting to the important effect of the instability on the combustion conditions in discharges of this type. Thus, a discharge in hydrogen was observed[1] to change from

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a diffuse form to a pinch with increasing power imput. A similar effect was noted also in[2], where the transition of the discharge from the diffuse to the pinch form occurred when a small amount of argon was added to the hydrogen. Solution of the stationary problem was used in[3] to attribute the existence of two high-frequency discharge regimes to the fact that the thermal-conductivity coefficient of diatomic gases has a characteristic maximum due to molecule-dissociation processes. It was suggested in[3] that in the temperature interval in which the thermal-conductivity coefficient is a decreasing function of the temperature no discharge can be produced, owing to instability.

In the present paper, using an equilibrium inductive discharge in a gas stream, we investigate in the one-dimensional approximation the question of the stability of high-frequency discharges in the case of strong skin effect. In analogy with the approach used in combustion theory,[4] an approximate method is developed of investigating the stability of high-frequency discharges of various types and under various conditions. A criterion is formulated for the stability of high-frequency discharges under the conditions in question. An analysis of this criterion explains why a discharge in a diatomic gas loses stability in the region where the thermal-conductivity coefficient is a decreasing function of the temperature.

2. INITIAL CONDITIONS AND APPROXIMATE ANALYSIS OF THE STATIONARY REGIME

To describe the process, we use the following system of equations:

$$\rho c_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}\right)=\frac{\partial}{\partial x}\lambda\frac{\partial T}{\partial x}+\frac{1}{2}\sigma|E|^2, \qquad (1)$$

$$\frac{\partial \rho}{\partial t}+\frac{\partial \rho u}{\partial x}=0, \qquad p=\text{const}, \qquad (2)$$

$$-\frac{\partial H}{\partial x}=\frac{4\pi}{c}\sigma E, \qquad \frac{\partial E}{\partial x}=\frac{j\omega}{c}H, \qquad (3)$$

where ρ is the gas density, c_p is the specific heat at constant pressure, T is the temperature, u is the gas velocity, λ is the thermal-conductivity coefficient, E and H are the complex amplitudes of the electric and magnetic field intensities, σ is the plasma conductivity, ω is the circular frequency of the electromagnetic field, c is the speed of light, p is the pressure, and j is the imaginary unit. We neglect in Maxwell's equations the displacement current in comparison with the conduction current.

The stationary problem of a high discharge in a gas stream under strong skin-effect conditions was formulated and solved in[5]. The question of the structure of a stationary discharge in a gas stream was investigated in greater detail in[3].

The main idea used in[5] to describe the stationary process is that the discharge front orients itself to the incident stream in such a way that the normal velocity component of the cold gas is a perfectly defined quantity whose values are set by the conditions for the existence of the stationary regime. The corresponding conditions are formulated as follows[5]:

$$-\lambda\partial T/\partial x=0, \quad E=0 \quad \text{if} \quad x=+\infty, \qquad (4)$$

(i.e., behind the discharge front),

$$T=T_0\approx 0, \quad H=H_0=\frac{4\pi}{c}in \quad \text{if} \quad x=-\infty \qquad (5)$$

(i.e., ahead of the front), where T_0 and H_0 are the cold-gas temperature and the amplitude of the magnetic field intensity ahead of the discharge, *in* is the number of ampere-turns per unit of inductor length.

To determine the square of the modulus of the electric field intensity amplitude the following equation was derived in[6] from the system (3):

$$\frac{\partial}{\partial x}\frac{1}{\sigma}\frac{\partial^3|E|^2}{\partial x^3}-\frac{64\pi^2\omega^2}{c^4}\sigma|E|^2=0. \qquad (6)$$

Equation (6) allows us to solve the problem (1), (2), (6) in the case of practical importance $(I/2kT)\gg 1$ (I is the ionization potential and k is the Boltzmann's constant) by using an approximation wherein the zone where the Joule energy is released is assumed infinitesimally narrow, and the discharge front is regarded as a weak-discontinuity surface. This is precisely the approach assumed and developed in the present paper.

The circumstance that confirms the validity of the employed approach is that in the stationary case the solution obtained in such an approximation leads to the same conclusions concerning the discharge temperature behind the front, and to the same analytic dependence of the normal velocity of the discharge on the final temperature, as in the case of a more detailed study of the structure of the discharge front.[3]

In fact, in the approximation of an infinitesimally narrow Joule energy release zone, we replace the term $\frac{1}{2}\sigma|E|^2$ in (1) and (6) by the expression $S\delta(x-x_0)$, where $\delta(x)$ is a delta function, x_0 is the coordinate that defines the position of the discharge front, and by virtue of the known property of $\delta(x)$ the quantity

$$S=\frac{1}{2}\int_{-\infty}^{\infty}\sigma|E|^2dx,$$

determines the electromagnetic-energy flux absorbed in the plasma.

Placing now the origin at the discharge front, i.e., putting $x_0=0$, and neglecting for the sake of simplicity the temperature dependences of ρ, c_p, and λ, we obtain from (1), taking (4) and (5) into account, the following expression for the temperature profile in the case of the stationary regime:

$$T=T_f\exp(x/\Delta), \quad x<0,$$
$$T=T_f, \quad x>0, \qquad (7)$$

where $\Delta=\lambda/\rho u c_p$ and T_c is the final temperature of the discharge behind the front, and its value is determined in the course of the solution.

Recognizing that in the region of relatively low temperatures $\sigma\sim\exp(-I/2kT)$, and taking into account relation (7) and the second boundary condition in (4), we obtain after integrating (6) the following expression for the square of the modulus of the amplitude of the electric field intensity outside the Joule energy-release band

$$|E|^2=-\frac{128\pi^2\omega^2}{c^4}S\left(\frac{2kT_f}{I}\right)^3\Delta^3\sigma+Ax^2+Bx+C,\quad x<0,$$
$$|E|^2=0,\quad x>0 \tag{8}$$

(the solution is written out accurate to terms $\sim(2kT_f/I)\ll 1$). The integration constants A, B, and C are determined from the condition that $|E|^2$, $d|E|^2/dx$ and $d^2|E|^2/dx^2$ be continuous at $x=0$:

$$A=\frac{64\pi^2\omega^2}{c^4}S\frac{2kT_f}{I}\Delta\sigma_f,$$
$$B=\frac{128\pi^2\omega^2}{c^4}S\left(\frac{2kT_f}{I}\right)^2\Delta^2\sigma_f, \tag{9}$$
$$C=\frac{128\pi^2\omega^2}{c^4}S\left(\frac{2kT_f}{I}\right)^3\Delta^3\sigma_f,$$

where σ_f is the value of σ at $T=T_f$ (the values of the coefficients A, B, and C, just as the solution (8), are accurate to terms $\sim(2kT_f/I)$).

It is seen from the second equation of (3) that the second boundary condition in (5) is equivalent to the relation

$$\left.\frac{d^2|E|^2}{dx^2}\right|_{x=-\infty}=\frac{32\pi^2\omega^2(in)^2}{c^4}. \tag{10}$$

From (1) we have in the stationary case an equation for the connection between the flux S and the final temperature T_f:

$$\rho uc_pT_f=S. \tag{11}$$

With the aid of the condition (10) and relations (8), (9), and (11) we obtain an equation for the final temperature T_f:

$$\lambda T_f\sigma_f\frac{2kT_f}{I}=\frac{(in)^2}{4}. \tag{12}$$

In the static case ($u=0$) there is a known rigorous solution

$$\int_0^{T_f}\lambda\sigma dT=(in)^2/4$$

(see[7]). Recognizing that $\sigma\sim\exp(-I/2kT)$, we obtain accurate to terms $\sim(2kT/I)$ the relation (12). Consequently, when the condition $2kT_f/I\ll 1$ is satisfied the values of the discharge temperature behind the front in the static case and in a gas stream are practically the same. This result agrees fully with a conclusion drawn in[3] via a rigorous solution of the problem.

Let us determine now the normal velocity u. We use for this purpose the heat-balance method, which is widely used in the solution of flame-propagation problems.[8] The idea of this method is based on the concept of an infinitesimally narrow chemical-reaction zone (in our case—the zone of the Joule heat release), which makes it easy to obtain the solution outside this zone (see (7) and (8)). Extrapolating the temperature and the electric field intensity obtained in this manner into the zone of Joule heat release, we can consider approximately the structure of this zone. Then, substituting the values of T and $|E|^2$ from (7) and (8) in the relation

$$\rho uc_pT_f=\frac{1}{2}\int_{-\infty}^{\infty}\sigma|E|^2dx,$$

which is the heat-balance condition and is obtained in the stationary case from (1) we obtain, accurate to terms $\sim(2kT_f/I)$, the following expression for u:

$$u=2^{1/4}(kT_f/I)(\lambda/\rho c_p)\delta_f^{-1},$$

where $\delta_f=c/(8\pi\omega\sigma_f)^{1/2}$ is the thickness of the skin layer. The value of u obtained in this manner differs from the more accurate solution of[3] only by a numerical coefficient of the order of unity, but it has the same analytic dependence on the final temperature T_f as in[3]. The slight error in the determination of the numerical coefficient is due to the approximation used in the extrapolation of the external solution (7), (8) to the zone of Joule energy release.

Thus, with the analysis of the stationary regime of the high-frequency discharge as an example, we see that the approach used in the present study to the study of the structure of the discharge in question, based on the notion that the discharge front is a discontinuity surface, is perfectly acceptable and leads to results close to those obtained from a rigorous solution of the problem. We now extend this approach to the case of the stability investigation, using a method similar to that used in combustion theory.[4]

In the present section, the stationary regime was investigated for the case $d\lambda/dT\approx 0$. Hereafter, however, the dependence of λ on T will be of prime significance for the investigated instability, and in the analysis of the stability we must know the profiles of T and $|E|^2$ at $d\lambda/dT\neq 0$. The solution of Eq. (1) at $\partial T/\partial t=0$, $\frac{1}{2}\sigma|E|^2=S\delta(x)$ in the general case $d\lambda/dT\neq 0$, neglecting the temperature dependences of ρ and c_p, entails no difficulty and can be obtained in quadratures. It will therefore not be presented here. On the other hand the solution (8) for $|E|^2$, obtained in the case $d\lambda/dT\approx 0$ at $I/2kT_f\gg 1$, can be approximately used, as is readily seen, also at $d\lambda/dT\neq 0$, if account is taken of the case frequently encountered in practice, wherein σ changes more rapidly with T than λ. We confine ourselves hereafter to just this approximation.

3. STABILITY OF HIGH FREQUENCY DISCHARGE

In the general case, the investigation of the stability of the solutions of the system (1), (2), and (6) is a rather complicated problem. We confine ourselves therefore to a situation wherein we can neglect the influence of the hydrodynamics on the discharge front, i.e., we disregard the perturbed motion of the gas, assuming the mass velocity ρu to be known from the stationary solution. This approach to the investigated process is justified in the case of small width of the zone of Joule energy release (see below), which was in fact assumed by us.

With an aim at studying the influence exerted on the stability of the discharge by the temperature dependence of the thermal-conductivity coefficient λ, we assume hereafter that λ is a strongly varying function of T in the region adjacent to the discharge front, i.e., $d\ln\lambda/d\ln T \gg 1$.

Then, just as in the analysis of the stationary solution, neglecting the weak temperature dependences of ρ and c_p in comparison with $\lambda(T)$, we obtain after simple transformations, in an approximation linear in the perturbation, the following system of equations for the determination of the fluctuations δT and $\delta|E|^2$ outside the zone of the Joule energy release:

$$\rho c_p\left(\frac{\partial\delta T}{\partial t}+u\frac{\partial\delta T}{\partial x}\right)=\frac{\partial^2}{\partial x^2}(\lambda\delta T), \tag{13}$$

$$\frac{\partial}{\partial x}\left[\frac{1}{\sigma}\left(\frac{128\pi^2\omega^2}{c^4}\rho u c_p T_f\frac{I}{2kT_f}\sigma\frac{\delta T}{T}+\frac{\partial^3\delta|E|^2}{\partial x^3}\right)\right]=0, \tag{14}$$

where the symbols without δ pertain to the stationary solution. Equation (14) describes the spatial distribution of the fluctuation $\delta|E|^2$ ahead of the discharge front. On the other hand in the region behind the discharge front, as seen from the boundary conditions presented below, we have $\delta|E|^2=0$. We seek the solution in the form $\delta T\propto e^{\nu t}$ and $\delta|E|^2\propto e^{\nu t}$. To determine the eigenvalue ν of the problem, boundary conditions must be imposed on the perturbed temperature and field intensity. In the region behind the discharge front ($x=+\infty$), since the electromagnetic field does not penetrate behind the front and is fully absorbed in the discharge plasma, we have $\delta|E|^2(\infty)=0$. Next, in the region behind the discharge front the fluctuation of the heat flux must also vanish, and this is equivalent to the condition $\lambda\partial\delta T(\infty)/\partial x=0$, since we are investigating stability with respect to internal perturbations that are not connected with supply of additional heat (other than the Joule heating) from the outside. Ahead of the discharge front ($x=-\infty$) one of the conditions is easily formulated, viz., $\delta T(-\infty)=0$, since the temperature of the incident gas is assumed given. To determine the second condition that the fluctuation $\delta|E|^2$ must satisfy, we consider the equation for the external circuit

$$U+\frac{j\omega}{c}\Phi=\Omega i,$$

where U is the voltage applied to the inductor in which the discharge is produced, i is the current in the inductor, Ω is the active resistance, and Φ is the magnetic flux in the inductor. This equation is based on the assumption that the characteristic time of instability development greatly exceeds the period of the current oscillations in the inductor. We confine ourselves from now on to the case $\Omega i\ll(\omega/c)\Phi$, which is usually encountered in practice.

Assuming the voltage to be specified (stable generator operation), we see that during the evolution of the instability the magnetic flux remains constant: $\delta\Phi=0$. Obviously, the flux can vary in the course of development of the fluctuations for two reasons, one connected with the displacement of the discharge front, and the other with the change of the current flowing through the discharge and through the inductor. A displacement of the discharge front by an amount ξ changes the flux Φ by an amount $\delta\Phi/\Phi\sim\xi/R$, where R is the characteristic dimension of the region occupied by the discharge. Since R can be regarded as large enough, it is possible during the initial stage of the perturbation development to neglect, with an arbitrarily prescribed accuracy, the change of $\delta\Phi$ due to the displacement of the discharge front. Therefore $\delta\Phi$ can change only as a result of a change of the discharge current which, since the electromagnetic field does not penetrate behind the discharge front (E and H vanish at $x=+\infty$), is equal to the current in the inductor, i.e., $\delta\Phi\propto\delta i$. The condition $\delta\Phi=0$ above is therefore equivalent to the relation $\delta i=0$. This in turn allows us to formulate ahead of the discharge front ($x=-\infty$) the boundary condition $\delta H(-\infty)=0$ or $\partial^2\delta|E|^2/\partial x^2=0$.

We proceed now to solve the just-formulated problem of discharge stability.

In order not to clutter up the solution with secondary details, we confine ourselves to the stability of the discharge near the stability threshold, when the characteristic time of variation of the fluctuations that are produced in the system is much longer than the characteristic time of passage of a gas particle through the gas-heating zone as a result of thermal conductivity: $\lambda/\rho c_p u^2$. It will be shown below that this circumstance allows us to retain in the expressions below terms of order not higher than that of first order in ν, the reciprocal of the characteristic time of instability development. Taking into account the relation $\delta T=\delta T_0\exp(\nu t)$ as well as the boundary conditions $\delta T(-\infty)=0$, $\partial\delta T(\infty)/\partial x=0$, we obtain, accurate to terms quadratic in ν, the following expression for δT_0:

$$\begin{aligned}\delta T_0&=\frac{\tau_1}{\varkappa}\exp\left[-\int\limits_x^0\left(\frac{u}{\varkappa}+\frac{\nu}{u}\right)dx'\right],\quad x<\xi,\\ \delta T_0&=\tau_2\exp\left(-\frac{\nu}{u}x\right),\quad x>\xi,\end{aligned} \tag{15}$$

where $\varkappa=\lambda/\rho c_p$, τ_1 and τ_2 are integration constants, and ξ, as before means the distance over which the discharge front moves during the development of the fluctuations.

Taking into account the relation $\delta|E|^2=\delta|E|_0^2e^{\nu t}$ and neglecting, as before, the temperature dependence of λ compared with that of σ, we obtain, accurate to terms $\sim(2kT_f/I)$, by solving Eq. (14) the following expression for $\delta|E|_0^2$ at $x<\xi$:

$$\delta|E|_0^2=-\frac{128\pi^2\omega^2}{c^4}\rho u c_p T_f\left(\frac{2kT_f}{I}\right)^2\Delta^3\sigma\frac{\delta T_0}{T}+\frac{128\pi^2\omega^2}{c^4}a\left(\frac{2kT_f}{I}\right)^3\Delta^3\sigma+bx^2+cx+d.$$

On the other hand, in the region behind the front ($x>\xi$), as already mentioned, we have $\delta|E|^2=0$.

To determine the integration constants τ_1, τ_2, a, b, c and d as functions of the amplitude ε of the displacement ξ (it is assumed that, as any other fluctuation, $\xi=\varepsilon\times\exp(\nu t)$) we must formulate seven relations.

It follows from the condition $\partial^2\delta|E|^2(-\infty)/\partial x^2=0$ that

$b=0$. From the condition of continuity of the quantity $|E|^2+\delta|E|^2$ on the discharge front (i.e., at $x=\xi$) ($|E|^2$ corresponds to the stationary solution), as well as the continuity of its derivative up to second order inclusive, we have accurate infinitesimals up to second order in the fluctuations

$$\varepsilon\frac{d^{(i+1)}|E|^2(0)}{dx^{(i+1)}}+\frac{d^i\delta|E|_0^2(0)}{dx^i}=0,\qquad(16)$$
$$i=0,1,2;\quad \frac{d^0}{dx^0}=1.$$

Since $d^i|E|^2(0)/dx^i=0$ at $i=1$ and 2 (see above), we have from (16) $d^i\delta|E|_0^2/dx^i=0$ at $i=0$ and 1. Taking (16) into account, neglecting the temperature dependence of λ compared with σ, and extrapolating, just as in the analysis of the stationary regime, the external solution for δT and $\delta|E|^2$ into the zone of Joule energy release, we obtain accurate to terms of order $2kTI$ the change in the flux of the dissipated electromagnetic energy

$$\delta S=\frac{1}{2}\int_{-\infty}^{\infty}\left[|E|^2\frac{I}{2kT}\frac{\delta T}{T}+\delta|E|^2\right]\sigma dx$$

the following expression

$$\delta S/S=(I/2kT_f)(\tilde{\tau}_2+\tilde{\varepsilon})e^{\nu t},$$

where $\tilde{\tau}_2\equiv\tau_2/T_f$, $\tilde{\varepsilon}\equiv\varepsilon/\Delta_f$, $\Delta_f\equiv\lambda_f/\rho c_p u$ (the subscript "f" pertains throughout to the values at $T=T_f$), and S is the flux of the electromagnetic energy dissipated in the plasma in the stationary regime.

From the definition of the integration constant a it is seen that it is connected with δS by the relation $a\exp(\nu t)=-\delta S$, whence

$$\tilde{a}=-(I/2kT_f)(\tilde{\tau}_2+\tilde{\varepsilon}),\quad \tilde{a}\equiv a/S.\qquad(17)$$

From the condition of continuity of the temperature $T+\delta T$ (T corresponds to the stationary regime) we have the sixth relation

$$\varepsilon\frac{dT(-0)}{dx}+\delta T_0(-0)=\delta T_0(+0),$$

so that

$$\tilde{\varepsilon}+\tilde{\tau}_1=\tilde{\tau}_2,\quad \tilde{\tau}_1\equiv\tau_1/\varkappa_f T_f.\qquad(18)$$

The seventh and last condition needed to solve the problem is obtained by integrating Eqs. (1) and (6) over a region arbitrarily close to the zone of the intense Joule energy release (the discharge front). Taking into account the δ-like (in our approximation) character of the heat source in this zone, letting the dimension of the integration region tend to zero, and retaining terms of order not higher than first in the fluctuations, we obtain after eliminating the term that describes the Joule heating

$$\lambda_f\left[\varepsilon\frac{d^2T(-0)}{dx^2}+\frac{d\delta T_0(-0)}{dx}+\left.\frac{d\ln\lambda}{d\ln T}\right|_{T=T_f}\frac{\delta T_0(-0)}{\Delta_f}-\frac{d\delta T_0(+0)}{dx}\right]=-a.$$

From this, using (15) and taking into account the relation $d^2T(-0)/dx^2=(-\hat{\varkappa}_f+1)(u^2T_f/\varkappa_f^2)$, which follows from (1) at ($\hat{\varkappa}\equiv d\ln\varkappa/d\ln T$), we obtain

$$\tilde{\varepsilon}(-\hat{\varkappa}_f+1)+\tilde{\tau}_1(1+\tilde{\nu})+\tilde{\tau}_2\tilde{\nu}=-\tilde{a},\qquad(19)$$
$$\tilde{\nu}\equiv\nu\varkappa_f/u^2.$$

Equations (17)–(19) together with the relation

$$\tilde{\varepsilon}+\tilde{\tau}_1-\frac{2kT_f}{I}\tilde{a}=0,$$

that follows from (16) at $i=2$ and from the expression given above for $\delta|E|_0^2$, in view of the vanishing of the determinant of this system, determine the value of ν:

$$\nu=\frac{u^2}{4\varkappa_f}\left(-1-2\hat{\varkappa}_f-\frac{I}{2kT_f}\right).\qquad(20)$$

If we analyze the origins of the various terms of (20), we note that the first two terms in the parentheses of (20) describe the influence of the thermal conductivity on the discharge stability, and the third term takes into account the role of the electromagnetic field energy dissipated in the discharge. It is interesting to note that the last term is always smaller than zero, owing to the decrease (due to the skin effect) of the energy absorption in the discharge front with increasing discharge temperature. It is seen from (20) that under our conditions the discharge can lose stability ($\nu>0$) only if the thermal diffusivity decreases rapidly enough with increasing T, as is the case, e.g., in a definite temperature interval for diatomic gases. This circumstance has a clear physical meaning. In fact, since the width of the zone of the Joule energy release is assumed to be very narrow, and those terms of (1) which describe the convective and conductive heat outflow from this zone are inversely proportional respectively to the width and square of the width of the considered region, the convective term in this region is much smaller than the conductive one and the role of convection can be neglected. Whether the discharge loses stability or not is equivalent to the question whether the temperature fluctuations on the discharge front will grow or not. A random increase of the temperature in the discharge front leads, on the one hand, to a decrease in the electromagnetic-field energy dissipated in the discharge (see above) and on the other hand, if $\varkappa$ decreases rapidly enough with increasing T, also to a decrease of the heat flux that removes energy from the front region. If the attenuation of the heat flux is stronger than the attenuation of the dissipated electromagnetic-field energy, then the initial temperature fluctuation will start increasing and the discharge loses its stability.

We note that by virtue of the negligible effect of the convection on the energy balance in the Joule heat release zone, where the character of the development of the fluctuations determines in fact the stability of the discharge, we can, as we did, neglect the influence of the perturbed gas motion on the discharge stability.

Neglecting in (20) unity in comparison with $I/2kT_f$, we write down the condition for the discharge instability in the form $-\hat{\varkappa}_f>I/4kT_f$. Recognizing that in diatomic gases, in the region where the coefficient of thermal conductivity decreases, the value of $-\hat{\varkappa}_f$ is close to D/kT_f, where D is the molecule dissociation energy, we can rewrite the instability condition in the form $D>\frac{1}{4}I$. It is easily seen that this relation is satisfied in molecular hydrogen. Therefore the mechanism considered in this paper can lead to loss of stability of a discharge produced in hydrogen at temperature corresponding to the decreasing branch of the thermal-conductivity coefficient.

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Nonlinear variation of the energy gap and the phonon spectrum of indium near the electron transition

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The method of electron tunneling is used to investigate the electron-phonon interaction and the phonon spectrum of indium in a wide range of pressures and tin-impurity concentrations. An anomalous behavior of the lattice-vibrational spectrum is discovered in the region topolgical changes in the Fermi surface. The possible mechanisms leading to such changes in the lattice are considered.

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1. INTRODUCTION

The electron transition, due to changes in the topology of the Fermi surface, leads to anomalies in the thermodynamic and kinetic characteristics of metals (the so-called phase transition of order $2\frac{1}{2}$).[1] Upon the attainment of some critical value by the Fermi energy of the metal, there should arise in the electron density of states a distinctive feature that is reflected in the electronic specific heat and other second derivatives of the thermodynamic potential. The presence of such anomalies in the behavior of the electron spectrum has been confirmed in experiments on specific heat[2a] and the de Haas–van Alphen effect.[2b] The method developed by Lazarev and his students[3–5] for observing the topological anomalies in the electron spectrum through the study of the combined effect of impurities and pressure on the superconducting-transition temperature T_c turned out to be effective. Underlying this method is Makarov and Bar'yakhtar's theoretical model,[4] which establishes a connection between the variation of T_c and the changes in the Fermi surface. From the investigations of the shape of the $dT_c(C, P)/dP$ curves arose the possibility of reestablishing the type of topological transition and of extracting quantitative information about a number of parameters of the electron spectrum.

At the same time, the question of the behavior of the phonon spectrum of a metal in the vicinity of a phase transition of order $2\frac{1}{2}$ was not rigorously analyzed. However, a correlation was observed between the nonlinear variation of the lattice parameters and the anomalies of the electronic characteristics in alloys for the same impurity concentrations.[6] Il'ina, Itskevich, and Titov[7] have shown that, under the action of high pressures, the anomalies in T_c and the critical magnetic field, H_c, are accompanied by irregularities in the lattice parameters.[8] All this points, it seems to us, to the possible appearance of anomalies in the phonon spectrum as well. A detailed critical analysis of the large amount of factual material, obtained by various groups of investigators, on the problem under discussion predetermined the necessity for the performance of a careful investigation of the vibrational spectrum of the lattice of a metal that clearly exhibits changes in the topology of the Fermi surface. In the present work we studied indium. The topological changes were realized through the introduction of a tin impurity, or the application of pressure. In both cases, as follows from the published data, there appear in indium new electron holes in the third Brillouin zone: first at the symmetry points W and then as α tubes in the [101] direction.[1] The choice of the tunneling method of investigation was dictated by its high resolving power and its high information-yielding capacity.

2. EXPERIMENT

1. *Sample preparation.* The Al-Al_2O_3-In tunnel junctions (or the alloys $In_{1-x}Sn_x$, $x=0$–8 at.% Sn) were pre-

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