Nonadiabatic effects in the phonon spectrum of metals in a magnetic field

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A dispersion equation is obtained and the spectrum and damping of the longitudinal oscillations of an electron Fermi liquid in a quantizing magnetic field are investigated. It is shown that coupling results from the interaction of the phonons with quantum waves of the "electron sound" type. The excitations having close frequencies and velocities and due to the close coupling of the phonon with one of the quantum waves are named by us magnetic phonons. We discuss the applicability of the adiabatic approximation to metals in a magnetic field. It is shown that the Frölich model cannot be used to describe the strong nonadiabatic restructuring of the phonon spectrum of the metal. The phonon damping is relatively small even at the maximum of the collisionless absorption. The electron contribution to the speed of sound drops out in this case.

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1. INTRODUCTION

The adiabatic approximation, as is well known, makes it possible to obtain the phonon spectrum of a metal with good accuracy.[1,2] The feasibility of the adiabatic description is based on the fact that the limiting frequency ω_D of the phonons in the crystal is appreciably lower than the characteristic electron energy ε_F, i.e., the dimensionless adiabatic parameter is $\omega_D/\varepsilon_F \sim s/v_F$, where s is the speed of sound and v_F is the Fermi velocity of the conduction electrons. The small value of this parameter is due in final analysis to the large difference between the masses of the electrons and ions ($s/v_F \sim (m/M)^{1/2}$, where m and M are the electron and ion masses). For long-wave phonons, the role of the adiabatic parameter is played by the ratio of the frequency of the acoustic phonon $\omega_q = qs$ to the characteristic frequency of the electronic transition in the interaction with the phonon, $\varepsilon_{p+q} - \varepsilon_p \approx \mathbf{q}\cdot\mathbf{v}$. Since all the electrons on the Fermi surface take part in the formation of the phonon spectrum, the adiabatic parameter has in the long-wave limit the same order of magnitude s/v_F as before. In other words, the existence of a single adiabatic parameter for all phonons is due to the fact that in the electron system there are no collective excitations whose frequency, at a given $\mathbf{q}$, is comparable with the phonon energy ω_q.

To describe the interaction of the electrons with the lattice one uses extensively the Fröhlich Hamiltonian, which is based on the concept of the deformation potential. The Fröhlich model is essentially adiabatic, since the electron energy is assumed in it to be dependent on the instantaneous position of the ions in the vibrating lattice. This model provides a correct qualitative description of the effective short-range interaction between the electrons and the ions in the metal, and makes possible a correct estimate of the small nonadiabatic effects such as the electron contribution to the phonon damping or to the dispersion of their velocity, as well as allowance for the influence of this interaction on the electrons near the Fermi surface.

The applicability of the Fröhlich model is limited primarily by the fact that to write down the Hamiltonian of a bound system of electrons and phonons in a metal one must inevitably introduce "bare" (i.e., non-interacting) electrons and phonons. Whereas for the bulk of the electrons this approximation is valid to some degree, by virtue of Migdal's theorem,[2] for the phonons the electronic renormalization (which is adiabatic in its character) of the spectrum turns out to be of the same scale as the energy of the bare phonon. This shortcoming of the Fröhlich model is well known. Another no less substantial shortcoming of the Fröhlich model is that it is impossible to take into account in it consistently the Coulomb interactions. The model is based on the assumption that the interaction of the conduction electrons with the ions is via a screened potential whose radius is of the order of the interatomic distance. Since the characteristic spatial scales of the electron motion (the mean free path, the radius of the electron orbit in the magnetic field, and so on) greatly exceed the lattice constant, the radius of the interaction potential is made to tend to zero. In other words, in the Fröhlich model it is assumed that $r_D \to 0$, where r_D is the Debye radius of the electrons in the metal. By virtue of this assumption, there are no electron plasma oscillations in this model, and oscillations of the Landau zero-sound type occur in their place. This means that within the framework of the Fröhlich model the electrons in the metal behave in analogy with an uncharged Fermi liquid coupled to the lattice via the deformation potential. Introduction of Coulomb interactions is therefore inconsistent in the Fröhlich model and it is impossible to take correct account of their influence on the phonons in the metal.

Thus, although the adiabatic approximation in metals is valid with high accuracy, its realization in the Fröh-

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lich model encounters objections in a number of respects. Recently Brovman and Kagan[1,3] have constructed, in their known cycle of papers, a consistent adiabatic theory of the phonon spectrum of metals with allowance for the Coulomb and multiparticle interactions. It was shown in these papers that many properties of metals, including polyvalent ones, can be calculated with high accuracy from first principles.

At the same time, the phonon spectrum of a metal can contain also nonadiabatic effects. Thus, if the Fermi surface has flat sections, then the quantity $\varepsilon_{p+q} - \varepsilon_p \approx \mathbf{q} \cdot \mathbf{v}$ is a constant on such a section and can be comparable with the phonon frequency ω_q. Since the phase volume determined from the condition $\omega_q \approx \mathbf{q} \cdot \mathbf{v}$ is finite in this case, the nonadiabatic contribution of such electrons to the phonon spectrum becomes substantial. The character of the singularities of the phonon spectrum produced in this situation was first analyzed by Afanas'ev and Kagan.[4]

Nonadiabatic effects become particularly strong in the presence of a magnetic field, when the electron motion acquires a quasi-one-dimensional character. Owing to the quantization of the energy of the transverse motion in the magnetic field, the longitudinal electron velocity on the Fermi surface can assume only discrete values $v_z = v_n$ (z is the direction of the vector $\mathbf{H}$). The fact that the discrete velocity with the smallest absolute value can vanish leads to certain singularities in the density of the electronic states on the Fermi surface. The state-density increase due to the electrons with low velocities attests to the important role of these electrons in the formation of the phonon spectrum. The applicability of the adiabatic approximation for such electrons is determined by the velocity ratio s/v_n, and at $v_n \lesssim s$ their interaction with the lattice becomes essentially nonadiabatic. Since the interaction of the electrons with the ions is not weak, it follows that the absence of a small adiabatic parameter in a magnetic field makes necessary a multiparticle formulation of the phonon-spectrum problem from the very outset, and calls for a consistent allowance for Fermi-liquid effects.

The quantization of the electronic states in a magnetic field is also the cause of the well known phenomenon of giant quantum oscillations of the collisionless absorption of acoustic[5] and electromagnetic[6] waves in metals. This phenomenon is due to the quasi-one-dimensional character of the electron motion in the magnetic field and to the quantization of its longitudinal velocity on the Fermi surface. The absence of absorption in those phase-space regions where the energy conservation law $\omega_q = q_z v_n$ is not satisfied leads to the existence of singular electronic excitations of the acoustic type, dubbed quantum waves, with a dispersion law $\omega = q_z u_n$, $u_n \approx (v_n + v_{n+1})/2$.[7-10] Since excitations with velocities comparable with the speed of sound appear in the electron system, the assumption of phonon adiabaticity is incorrect for this reason, too.

In addition, the premise that the deformation potential is constant in a magnetic field is incorrect. The point is that the deformation potential is determined to a considerable degree by effects of screening of the Coulomb interaction by conduction electrons. This circumstance is reflected in the appearance of the dielectric constant in the denominator of the expression for the deformation potential.[11] As the frequency of one of the quantum waves is approached, the dielectric constant tends to zero, and the value of the deformation potential increases strongly. Consequently, the deformation potential in a magnetic field cannot be regarded as constant even at acoustic frequencies. Yet it is just this assumption which is inherent in the Fröhlich model.

By virtue of the foregoing circumstances, the Fröhlich model is unsuitable for a consistent study of the phenomena connected with electron-phonon interaction in a magnetic field, particularly for the study of the singularities of the phonon spectrum of metals. Moreover, it must be emphasized that an electron-phonon interaction theory based on the concept of bare phonons and a constant deformation potential is unsuitable for a quantitative description of the strong electronic renormalization of the phonon spectrum, brought about by the singularities in the density of the electronic states on the Fermi surface.

It is necessary to review in this connection the results of Blank and one of us[12] as well as a number of other studies in which the Fröhlich model was applied literally to the analysis of the phonon spectrum of metals in magnetic fields. Those of the results which are connected with the calculation of the "simple loop" and with allowance for small corrections to the phonon spectrum are correct. As to the conclusion that the spectrum is strongly renormalized and that other strong effects due to electron-phonon interaction are possible, these cannot be regarded as correct. It would also be necessary to analyze anew the line shape of the giant quantum oscillations of strong absorption in those cases when the absorption is so large that the damping decrement becomes comparable with the frequency.

It can thus be stated that the question of a consistent investigation of the strong singularities of the phonon spectrum and of the giant quantum oscillations of sound absorption in metals in magnetic field has not yet been solved. The solution of this problem is closely connected with the problem of a correct analysis of the acoustic and quantum waves in metals.

The purpose of the present paper is a study of these phenomena. A consistent theory can be constructed on the basis of the electron-ion model of the metal. In these phonons are not introduced *a priori*, but are the result of a solution of the dispersion equation. Such a model, in the absence of a magnetic field, was investigated by a number of workers.[1,3,13-15]

In the next section we derive a dispersion equation for the longitudinal oscillations in the electron-ion system of the metal with account taken of the Fermi-liquid interaction between the electrons. Section 3 contains an analysis of the phonon spectrum in the absence of a magnetic field, a comparison with the results of Brovman and Kagan,[1,3] as well as a discussion of the transition to the Fröhlich model. The last section considers finally the spectrum of the longitudinal oscillations of a met-

al in a quantizing magnetic field, when strong adiabatic effects due to the interaction of the electron Fermi-liquid with the lattice become significant.

It must be emphasized that the description proposed below for the interaction between the electrons is not connected with perturbation theory, since it is not weak at metal densities. In other words, the parameter $e^2/\hbar v_F$ is not regarded as small, and the polarization operator of the conduction electrons was obtained without using the random-phase approximation. The results of the work are valid under the same assumptions as the Landau Fermi-liquid theory. Landau's theory, as is well known, uses not the smallness of the parameter $e^2/\hbar v_F$, but the absence of damping of the excitations on the Fermi surface. As to the actual calculation of the Landau function f, while difficult to carry out theoretically in consistent fashion for real metal, it can nonetheless be estimated by various approximate methods with reasonable accuracy.[3] In a magnetic field, under conditions of quasiclassical quantization, the interelectron interaction is taken into account essentially by the same method as used at $H=0$ by Brovman and Kagan.[1,3]

2. DISPERSION EQUATION FOR LONGITUDINAL OSCILLATIONS OF THE ELECTRON FERMI LIQUID OF A METAL

We start with an expression for the Hamiltonian of the lattice electrons and ions. We confine ourselves for simplicity to a monovalent metal with a cubic lattice:

$$H=\sum_{\alpha}\frac{\mathbf{p}_\alpha^2}{2m_0}+\sum_{\alpha,j}\Phi(\mathbf{r}_\alpha-\mathbf{R}_j)+H_{ii}+H_c. \qquad (2.1)$$

Here m_0 is the mass of the free electron, the subscripts α and j number the electrons and ions, respectively; $\Phi(\mathbf{r}-\mathbf{R}_j)$ is the electron-ion interaction energy; H_{ii} includes the kinetic energy of the ions, the energy of their Coulomb repulsion, and the non-Coulomb part of the interaction of the ion shells; H_c is the energy of the electron Coulomb repulsion. We assume that the energy of interaction of each electron with the homogeneous positive background has already been subtracted from the electron-ion energy, and that the energy of the corresponding charged background has been eliminated from the energies of the electron-electron interaction (H_c) and ion-ion interaction (H_{ii}).

We shall consider hereafter only small longitudinal oscillations. We expand the Hamiltonian (2.1) in powers of the small displacements $\delta\mathbf{R}_j=\mathbf{R}_j-\mathbf{R}_{j0}$ from the true ion equilibrium positions[1)] $\mathbf{R}_{j0}$. The equilibrium lattice configuration should be obtained from the condition that the total energy of the metal be a minimum (with account taken of the electron contribution to this energy) as a function of the ion coordinates. This problem was solved by Brovman and Kagan.[1,3] In the expansion of H_{ii} in powers of $\delta\mathbf{R}_j$ we confine ourselves to the quadratic terms (it can be shown[3] that the linear term of the expansion is zero because of the lattice symmetry). We obtain the potential $\Phi(\mathbf{r}-\mathbf{R}_j)$ accurate to terms linear in $\delta\mathbf{R}_j$ (the quadratic terms are small in the parameter $\delta R/a\sim(m/M)^{1/4}$ and correspond to third-order anharmonicities).

We proceed next to the second-quantization representation for the Bloch electrons and the lattice vibrations. In this representation, (2.1) takes the form

$$H=\sum_{\mathbf{p}}\varepsilon_{\mathbf{p}}a_{\mathbf{p}}^{+}a_{\mathbf{p}}+\sum_{\mathbf{q}}\omega_{\mathbf{q}}b_{\mathbf{q}}^{+}b_{\mathbf{q}}+\sum_{\mathbf{q}}V_{\mathbf{q}}\rho_{-\mathbf{q}}(b_{\mathbf{q}}+b_{-\mathbf{q}}^{+})+\frac{1}{2}\sum_{\mathbf{q}}v(\mathbf{q})\rho_{-\mathbf{q}}\rho_{\mathbf{q}}. \qquad (2.2)$$

Here $a_{\mathbf{p}}^{+}$ and $a_{\mathbf{p}}$ are the creation and annihilation operators of a Bloch electron with quasimomentum $\mathbf{p}$ and energy $\varepsilon_{\mathbf{p}}$, the latter being an eigenvalue of the Hamiltonian

$$\mathbf{p}^2/2m_0+\sum_j\Phi(\mathbf{r}-\mathbf{R}_{j0});$$

the electron-density fluctuation operator is

$$\rho_{\mathbf{q}}=\sum_{\mathbf{p}}a_{\mathbf{p}-\mathbf{q}}^{+}a_{\mathbf{p}};$$

$b_{\mathbf{q}}^{+}$ and $b_{\mathbf{q}}$ are the Bose creation and annihilation of a longitudinal lattice vibration with frequency $\omega_{\mathbf{q}}$ and wave vector $\mathbf{q}$. The frequency $\omega_{\mathbf{q}}$ is determined by solving the pure imaginary problem with the Hamiltonian H_{ii}. Finally, $V_{\mathbf{q}}$ and $v(\mathbf{q})$ are the respective matrix elements of the discrete electron-ion and electron-electron Coulomb interaction, and are calculated with the aid of Bloch wave functions. We assume that the dependences of $V_{\mathbf{q}}$ and $v(\mathbf{q})$ on the momentum $\mathbf{p}$ is a smooth one and can be disregarded.

We must emphasize the difference between our present approach and the method used by Brovman and Kagan.[1,3] In their theory the initial states were those of the free electrons, while the interaction with the ions was taken into account by expanding the total energy in terms of the small parameter $V_{\mathbf{K}}/\varepsilon_F$, where $\mathbf{K}\neq0$ is the reciprocal-lattice vector. In our case the initial states are those of the electrons in the periodic field of the crystal. In addition, the theory of Brovman and Kagan was adiabatic from the very outset. We cannot use their results directly, since we are interested in strong nonadiabatic effects.

In a quantizing magnetic field, the states of the electron are classified by an aggregate of quantum numbers $t=\{n,p_y,p_z\}$ (we chose a Landau gauge with a vector potential $\mathbf{A}=(0,Hx,0)$). The spin dependence of the energy is neglected. In the t-representation it is necessary to replace in the Hamiltonian (2.2) only the first term by $\sum_t\varepsilon_t a_t^{+}a$ and ρ_q by $\sum_{t,t'}M_{t't}a_{t'}^{+}a_t$, where a_t^{+} and a_t are the creation and annihilation of the operators in the state $|t\rangle$, $M_{t't}$ is the matrix of a plane wave between the states t' and t: $M_{t't}=\langle t'|\exp(-i\mathbf{qr})|t\rangle$.

Proceeding to the derivation of the dispersion equation, we must bear in mind that the interaction between the conduction electrons in a metal is not weak, and the electrons must be regarded as a charged Fermi liquid. The dispersion equation can be easily written down if the polarization operator of the electrons in the metal is known. The calculation of this equation for a charged Fermi liquid is an important and sufficiently difficult problem even in the absence of a magnetic field. We present below a derivation of the dispersion equation un-

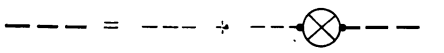

FIG. 1.

der conditions of quasiclassical quantization of the electronic states in a magnetic field. The interaction of the electrons with one another and with the ions is taken into account in the spirit of the Landau Fermi-liquid theory.

The spectrum of the longitudinal oscillations is determined by the poles of the exact phonon Green's function $D(\mathbf{q}, \omega, \mathbf{H})$. The Dyson equation for $D(\mathbf{q}, \omega, \mathbf{H})$ becomes much simpler if it is recognized that the phonon propagator $D(\mathbf{q}, \mathbf{q}', \omega, \mathbf{H})$ is diagonal in $\mathbf{q}$ and $\mathbf{q}'$ even in the presence of a magnetic field, $D(\mathbf{q}, \mathbf{q}', \omega, \mathbf{H}) = \delta_{\mathbf{q}\mathbf{q}'} D(\mathbf{q}, \omega, \mathbf{H})$. This property of the function D in the case of a quadratic dispersion law for electrons with a Frölich interaction with the phonons was proved in[12,16] The proof is actually based on the fact that the translation of the system by an arbitrary vector introduces in the Hamiltonian only a change that can be cancelled by an appropriate gauge transformation. In other words, in a homogeneous magnetic field the momentum transfer is conserved as before, even though the electron propagator depends not only on the coordinate difference. This proof is easily extended also to electrons in a periodic field with a Hamiltonian in the form (2.2).

Thus, the Dyson equation for the photon propagator has also in a magnetic field the usual form (Fig. 1):

$$D(\mathbf{q}, \omega, \mathbf{H}) = D_0(\mathbf{q}, \omega) + |V_{\mathbf{q}}|^2 D_0(\mathbf{q}, \omega)\, \Pi(\mathbf{q}, \omega, \mathbf{H}) D(\mathbf{q}, \omega, \mathbf{H}). \quad (2.3)$$

The thick dashed lines denote the exact phonon propagator, and the thin dashed lines correspond to the function $D_0 = 2\omega_{\mathbf{q}}/(\omega^2 - \omega_{\mathbf{q}}^2)$. The points represent simple $V_{\mathbf{q}}$ vertices which, by definition, are not included in the polarization operator Π represented by the crossed circle.

To take explicit account of the interelectron interaction, we separate from Π the compact part S, which is represented by the aggregate of all the compact diagrams, i.e., diagrams not cut into two parts along the thin phonon line and along the line $v(\mathbf{q})$ of the Coulomb interaction between the electrons. The latter will be represented by a wavy line, and the compact part by a shaded circle. The equation for Π then takes the form of Fig. 2. whence

$$\omega^2 - \omega^2 + \frac{2\omega_{\mathbf{q}} |V_{\mathbf{q}}|^2 S(\mathbf{q}, \omega, \mathbf{H})}{1 + v(\mathbf{q}) S(\mathbf{q}, \omega, \mathbf{H})} = 0 . \quad (2.4)$$

The equation for S is in turn of the form shown in Fig. 3. The thick lines denote the exact electron propagators, while the shaded square (Fig. 4) represent the exact electron four-point diagram $\Gamma(12, 34)$. The quantity Γ is graphically represented by the aggregate of all compact diagrams with four ends, containing all the possible combination of the phonon, electron, and Coulomb lines.

Thus, Γ includes all the types of short-range forces between the conduction electrons in the metal, i.e., Γ

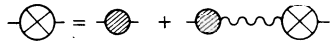

FIG. 2.

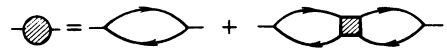

FIG. 3.

includes both the short-range part of the direct Coulomb interaction and the indirect interaction of the electrons via the bands. By definition, singular elements having a Coulomb singularities of the type q^{-2} are excluded from Γ as $q \to 0$. The short-range interaction between the electrons will be described in the language of the Fermi-liquid theory. The quantity S can then be regarded as the exact polarization operator of a system of fermions with short-range forces, in which the fermion propagator coincides with the exact Green's function of the conduction electron in the metal, and the complete four-point diagram coincides with Γ.

We confine ourselves henceforth to an analysis of the longitudinal-oscillation spectrum at small ω and q. The quantity S is calculated in the Appendix and is of the form

$$S(\mathbf{q}, \omega, \mathbf{H}) = \frac{S_0(\mathbf{q}, \omega, \mathbf{H})}{1 + \langle f^{(0)} \rangle S_0(\mathbf{q}, \omega, \mathbf{H})}. \quad (2.5)$$

We have introduced here the quantity

$$S_0(\mathbf{q}, \omega, \mathbf{H}) = \frac{2V_0}{(2\pi L)^2} \sum_{n,n'=0}^{\infty} M_{nn'}^2 \left(\frac{q_\perp^2 L^2}{2} \right) \times \int_{-\infty}^{\infty} dp_z \frac{f[E_n(p_z - q_z)] - f[E_{n'}(p_z)]}{E_{n'}(p_z) - E_n(p_z - q_z) + \omega}, \quad (2.6)$$

which will be called hereafter a simple loop, bearing it in mind that for non-interacting particles it is proportional to the longitudinal susceptibility. V_0 in (2.6) is the normalization volume, $q_\perp^2 = q_x^2 + q_y^2$, $L = (c/eH)^{1/2}$ is the magnetic length, and $f(E)$ is the equilibrium Fermi function. The dispersion law for true quasiparticles in a magnetic field is

$$E_n(p_z) = (n + 1/2)\Omega + p_z^2/2m, \quad \Omega = eH/cm_c, \quad (2.7)$$

where Ω is the cyclotron frequency, m_c is the cyclotron mass, and m is the effective mass in the z-axis direction (the contribution of the Fermi-liquid interaction is allowed-for in m and m_c). The modulus of the matrix element on the oscillator wave functions is given by

$$M_{nn'}(x) = |M_{n'}| = e^{-x/2} x^{|n-n'|/2} L_{\min(n,n')}^{|n-n'|}(x), \quad (2.8)$$

where $L_n^{\alpha}(x)$ is a generalized Laguerre polynomial normalized to unity.

The quantity $\langle f^{(0)} \rangle$ in (2.5) is the averaged correlation function of the Landau Fermi-liquid interaction between the electrons. The mean value $\langle f^{(0)} \rangle$ is defined by formula (A.12) and takes into account the change in the region of averaging on the Fermi surface as the state-

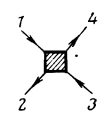

FIG. 4.

density singularities are approached. Thus, at $H=0$ the quantity $\langle f^{(0)}\rangle$ reduces in fact to the correlation function $f_{pp'}$ averaged over the Fermi surface. In the case of a quantizing magnetic field, however, in the immediate vicinity of the state-density singularities, the averaging is over only the extremal section $p_z=0$ of the Fermi surface.

Substituting (2.5) in the dispersion equation (2.4) we get

$$\omega^2-\omega_q^2+\frac{2\omega_q|V_q|^2S_0(\mathbf{q},\omega,\mathbf{H})}{1+[v(\mathbf{q})+\langle f^{(0)}\rangle]S_0(\mathbf{q},\omega,\mathbf{H})}=0. \tag{2.9}$$

It is seen that allowance for the Fermi-liquid interaction leads, first, to the appearance, in the simple energy loop, of true quasiparticles (with effective masses, etc.), second, to an additive increment $\langle f^{(0)}\rangle$ to the Coulomb interaction $v(q)$.

It must be emphasized that formula (2.5) holds only in the case when the damping of the quasiparticles on the Fermi surface is equal to zero. The proof of the important statement that under the conditions of quasiclassical quantization the damping actually vanishes on the Fermi surface was obtained by Luttinger.[17]

3. PHONONS IN A METAL IN THE ABSENCE OF A MAGNETIC FIELD. TRANSITION TO THE FRÖHLICH MODEL

In the absence of a magnetic field the spectrum of the longitudinal oscillations of a metal contains at least two branches, which differ substantially in frequency. The low-frequency branch corresponds to slow lattice vibrations; these are the longitudinal phonons. The other, high-frequency branch is connected with oscillations of the electron density and describes the Langmuir plasma oscillations of the electrons. In an uncharged Fermi liquid, the high-frequency branch of the spectrum is the Landau zero sound, while in a charged isotropic Fermi system there is no zero sound connected with the charge-density oscillations.[18] We consider here only the acoustic part of the phonon branch.

The spectrum and the damping of the phonons can be easily obtained from Eq. (2.9). We need for this purpose explicit expressions for ω_q, $v(\mathbf{q})$, V_q and $S_0(\mathbf{q},\omega)$ at small $\mathbf{q}$ and ω. The spectrum of the lattice vibrations ω_q is determined only by the ion-ion interactions and begins with the ion plasma frequency ω_i

$$\omega_q^2=\omega_i^2(1+q^2\alpha),\quad \omega_i^2=4\pi ne^2/M, \tag{3.1}$$

where n is the electron and ion density, while α has the dimension of length squared and its magnitude and sign are determined from the solution of the purely ionic problem (see, e.g.,[1,3]); in order of magnitude we have $|\alpha|\sim a^2$, where a is the lattice constant.

For small q we have furthermore

$$v(\mathbf{q})=\frac{4\pi e^2}{q^2}(1+q^2\beta). \tag{3.2}$$

The appearance of the term with β in (3.2) is due to the use of Bloch wave functions to calculate $v(\mathbf{q})$, and therefore β contains the small parameter $|V_{\mathbf{K}}|^2/\varepsilon_F^2$. The pseudopotential V_q for $q\to 0$ is equal to

$$V_q=-iq(2\omega_qV_0nM)^{-1/2}(4\pi ne^2/q^2-b),\quad b>0. \tag{3.3}$$

The parameter b describes the energy of electron repulsion by the ion shell.

Finally, in the case of an isotropic Fermi surface, small ω and q, and $\omega\ll qv_F$ the simple loop is given by

$$S_0(\mathbf{q},\omega)=V_0\nu_{cl}(\varepsilon_F)(1+i\pi\omega/2qv_F),\quad \nu_{cl}(\varepsilon_F)=3n/2\varepsilon_F, \tag{3.4}$$

where $\nu_{cl}(\varepsilon_F)$ is the density of the electronic states on the Fermi level at $H=0$.

A phonon spectrum with a linear dependence of ω on q is obtained from (2.9) because ω_i^2 is cancelled by the corresponding term of the last member of the equation. This cancellation is due to the electric quasineutrality of the metal. Substituting (3.1)–(3.4) in (2.9) we obtain the spectrum and the damping of the phonons:

$$\omega=qs,\quad \Gamma=-\operatorname{Im}\omega=\omega\frac{\pi}{12}\frac{mv_F}{Ms}. \tag{3.5}$$

The velocity of the longitudinal sound is given by the expression

$$s^2=\frac{1}{M}\left[4\pi ne^2(\alpha+\beta)+2b+n\langle f^{\mathrm{a}}\rangle+\frac{n}{\nu_{cl}(\varepsilon_F)}\right]\equiv s_*^2+\frac{n}{M\nu_{cl}(\varepsilon_F)}, \tag{3.6}$$

where s_*^2 denotes the sum of the first three terms of (3.6).

We note that when the coupling of the electrons with the lattice is excluded ($\alpha,\ \beta,\ b\to 0$) formula (3.6) goes over into the known Landau expression for the square of the sound velocity in an uncharged Fermi liquid.[19] The result represented by (3.6) coincides essentially with that of Brovman and Kagan.[1,3]

We consider now the manner of changing from the dispersion equation (2.9) to the corresponding equation of the Fröhlich model. It is known that the latter should be of the form

$$\omega^2-s_0^2q^2=-2\omega_q|\Lambda_q|^2S_0(\mathbf{q},\omega), \tag{3.7}$$

where s_0 is the unrenormalized speed of sound and

$$\Lambda_q=-iq\Lambda(2s_0qV_0nM)^{-1/2}, \tag{3.8}$$

Λ is the deformation potential (in the isotropic case $|\Lambda|=2\varepsilon_F/3$). Equation (2.9) can be recast in an equivalent form

$$\omega^2-q^2\frac{4\pi e^2}{M}\left\{n\alpha+\frac{4\pi ne^2\beta+2b+n\langle f^{\mathrm{a}}\rangle}{S_0^{-1}(\mathbf{q},\omega)+v(q)+\langle f^{\mathrm{a}}\rangle}\frac{1}{q^2}\right\}=\frac{4\pi ne^2}{MS_0(\mathbf{q},\omega)[1+(v(\mathbf{q})+\langle f^{\mathrm{a}}\rangle)S_0(\mathbf{q},\omega)]}S_0(\mathbf{q},\omega). \tag{3.9}$$

Equation (3.9) differs from (3.7) in the sign of the first part. To reduce (3.9) to the form (3.7) we subtract from both halves of (3.9) twice the value of the right-hand side. We recognize next that in the Fröhlich model the electron screening radius is assumed equal to zero and the adiabatic approximation ($\omega\to 0$) is used. The coefficient of q^2 in the left-hand side (the square of the un-

renormalized velocity s_0^2) and the multiplier of $S_0(\mathbf{q}, \omega)$ in the right-hand side (the effective electron-phonon coupling constant) are then taken at $\omega = 0$. We then obtain Eq. (3.7), in which the deformation potential is $|\Lambda| = 2\varepsilon_F/3$ (as it should), and the unrenormalized sound velocity s_0 is connected with the observed velocity s by the relation

$$s_0^2 = s^2 + \frac{\Lambda^2}{nM}\nu_{cl}(\varepsilon_F) = s_*^2 + \frac{2n}{M\nu_{cl}(\varepsilon_F)}. \tag{3.10}$$

It is seen from the foregoing analysis to what tricks we must resort to set Eq. (2.9) in correspondence with the Fröhlich model. The contribution of the electrons to the unrenormalized sound velocity s_0 is taken into account here twice: the last term of (3.10), due to the electrons, turns out to be twice as large as the corresponding term in (3.6) (this circumstance was pointed out already by Brovman and Kagan[11]).

In addition, there is a leeway in the transition to the Fröhlich model. It is due to the fact that in some of the factors of (3.9) the simple loop $S_0(\mathbf{q}, \omega)$ is replaced by the static quantity $S_0(\mathbf{q}, 0)$, while in others the dependence of $S_0(\mathbf{q}, \omega)$ on ω is preserved (if we take into account the ω-dependence in the terms in which it was neglected above, we obtain an incorrect expression for the damping and a different value of Λ). There are in fact no grounds for such substitutions. They are particularly inadmissible, and lead to incorrect results, in those cases when nonadiabatic effect play an important role. This is precisely the situation in a quantizing magnetic field.

4. LONGITUDINAL OSCILLATIONS IN A STRONG MAGNETIC FIELD

We proceed to consider longitudinal oscillations in a metal placed in a quantizing magnetic field. We consider ourselves to the quasiclassical case $\Omega \ll \varepsilon_F$, when we can disregard the change of the Fermi level in the magnetic field. It was noted in the preceding section that at $\mathbf{H} = 0$ there exist in a metal only two weakly-damped longitudinal modes—phonons and plasmons. This result is a direct reflection of the fact that the charged-particle system comprising the metal consists of two subsystems with two different velocities, ionic s and electronic v_F. In a magnetic field one should expect even from general considerations an increase in the number of weakly damped longitudinal oscillations.[7-10] In fact, quantization of the energy of the transverse motion of the electrons (relative to the magnetic field) leads to quantization of their longitudinal velocity on the Fermi surface: $v_z = \pm v_n$, $n = 0, 1, 2, \ldots, N$, where N is the number of Landau level under the Fermi surface:

$$N + \Delta = \varepsilon_F/\Omega - 1/2, \quad 0 \leq \Delta < 1, \tag{4.1}$$

and Δ is the fractional part of the quantity $\varepsilon_F/\Omega - \frac{1}{2}$. As a result, the electron subsystem consists in turn of $N+1$ subsystems, each characterized by a definite velocity along $\mathbf{H}$. This discrimination of the electrons leads to the existence of longitudinal quantum waves with acoustic-type spectra even in a free electron gas, when there are no ion oscillations ($M \to \infty$).[7-9] The spectrum and damping of these waves are obtained from the condition that the denominator of the second term in (2.4) vanish. The number of such oscillations is $N+1$, one of which corresponding to plasmons and the remaining N to "electron sounds." In anisotropic metals, transverse quantum waves should exist besides the longitudinal ones.[10] If the longitudinal waves propagate obliquely the picture becomes more complicated because of the interaction of the quantum waves with the transverse oscillations.[20]

We confine ourselves here to an investigation of the simplest case of long-wave longitudinal oscillations propagating along the vector $\mathbf{H}$. Then the simple loop is given by the formulas

$$\operatorname{Re} S_0(\mathbf{q}, \omega, \mathbf{H}) = \frac{2mV_0}{(2\pi L)^2 q}\sum_{n=0}^{N}\ln\left|\frac{(v_n + q/2m)^2 - u^2}{(v_n - q/2m)^2 - u^2}\right|, \tag{4.2}$$

$$\operatorname{Im} S_0(\mathbf{q}, \omega, \mathbf{H}) = \frac{\pi}{2}\frac{u}{v_F}\nu_{cl}(\varepsilon_F)\gamma(\mathbf{q}, \omega, \mathbf{H}). \tag{4.3}$$

Here $v_n = v_F[1 - (n + \frac{1}{2})\Omega/\varepsilon_F]^{1/2}$ and $u = \omega/q$ is the phase velocity of the wave. The quantity $\gamma(\mathbf{q}, \omega, \mathbf{H})$ is the ratio of the power absorbed by the electrons in the magnetic field to the power absorbed at $\mathbf{H} = 0$ [21]:

$$\gamma(\mathbf{q}, \omega, \mathbf{H}) = \frac{q\Omega}{m}\sum_{n=0}^{N}\int_0^\infty d\varepsilon\,\frac{f(\varepsilon) - f(\varepsilon+\omega)}{\omega}\int_{-\infty}^{\infty} dp_z\,\delta[\varepsilon - E_n(p_z)]\,\delta[\varepsilon + \omega - E_n(p_z + q)]. \tag{4.4}$$

Formulas (4.2) and (4.4) do not take the scattering of the electrons into account and pertain to absolute zero temperature. The scattering and for the thermal smearing of the Fermi distribution are accounted for in preceding papers.[10,21] At $T = 0$, the quantity $\gamma(\mathbf{q}, \omega, \mathbf{H})$ as a function of the square of the phase velocity u^2 takes the form of a system of narrow rectangular pulses of width $2v_n q/m$, whose centers are located at $u^2 = v_n^2$. Inside the pulses we have $\gamma(\mathbf{q}, \omega, \mathbf{H}) = \Omega/\omega$, and outside $\gamma(\mathbf{q}, \omega, \mathbf{H}) = 0$.

It is seen from (4.2) that $\operatorname{Re} S_0(\mathbf{q}, \omega, \mathbf{H})$ also changes strongly with the phase velocity. In Fig. 5, curves 1 show the dependence of $\operatorname{Re} S_0$ on u^2. At $u = 0$ the quantity

$$\operatorname{Re} S_0(\mathbf{q}, 0, \mathbf{H}) = \nu_{qu}(\varepsilon_F, \mathbf{H}) = \frac{4V_0}{(2\pi L)^2}\sum_{n=0}^{N}\frac{1}{v_n} \cong \frac{3n}{2\varepsilon_F}V_0\left(1 + \frac{1}{2(N\Delta)^{1/2}}\right) \tag{4.5}$$

is the quantum density of state of the electrons on the Fermi surface in a quantizing magnetic field. The vertical asymptotes, near which the function $\operatorname{Re} S_0$ becomes discontinuous, actually represent narrow strong-absorption intervals, in which $\gamma(\mathbf{q}, \omega, \mathbf{H}) = \Omega/\omega$ (the width of these intervals on the figure is equal to zero). Curves 2 represent the right-hand side of the dispersion equation

$$\frac{1}{V_0}S_0(\mathbf{q}, \omega, \mathbf{H}) = \frac{q^2}{4\pi e^2}\frac{u^2 - \omega_t^2/q^2}{s_*^2 - u^2}. \tag{4.6}$$

The intersection points $a_1, a_2, \ldots, a_N$ of curves 1 with the horizontal line $-q^2/4\pi e^2$ are the zeros of the dielectric constant, i.e., they yield the spectrum of the quantum

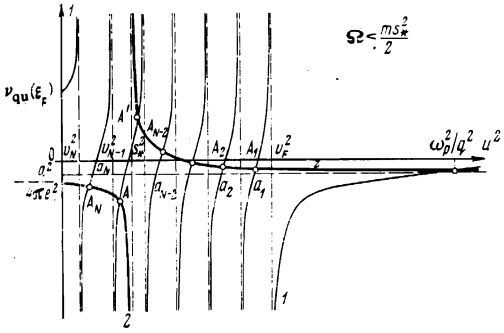

FIG. 5. Procedure for the solution of the dispersion equation in a magnetic field at $\Delta_0 = ms_*^2/2\Omega > 1$: curves 1—$V_0^{-1}\,\mathrm{Re}S_0(\mathbf{q}, \omega, \mathbf{H})$, 2—right-hand side of (4.6); $a_1, a_2, \ldots, a_N$—points of intersection of curves 1 with the line $q^2/4\pi e^2 = \text{const}$ and representing the spectrum of the quantum wave in the immobile lattice ($M \to \infty$); $A_1, A_2, \ldots, A_N$—quantum wave in vibrating lattice: A and A'—magnetic phonons.

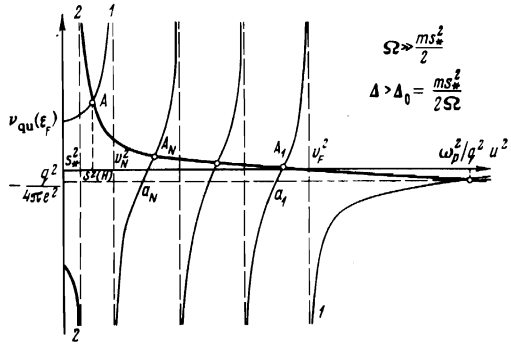

FIG. 7. The notation is the same as before, but there is one magnetic phonon A at $\Delta > \Delta_0$.

waves and of the plasma oscillations of the free electron Fermi liquid ($M \to \infty$).

The points of intersection of curves 1 and 2 are solutions of Eq. (4.6) and determine the spectrum of coupled oscillations in which phonons take part in addition to the already mentioned waves.[2)] That the waves are indeed coupled is seen from Fig. 5, which shows in the interval (v_n^2, v_{n-1}^2), in place of one solution corresponding to the point a_n, two solutions (the points A and A'). We shall call such two oscillations "magnetic phonons" to distinguish them from the remaining solutions, which we shall call as before quantum waves. The onset of magnetic phonons is due to the interaction of the lattice vibrations with the quantum wave whose velocity differs least from s_* (we recall that s_* is determined by the elasticity of the metal from which the pure electronic elasticity has been subtracted). The situation illustrated in Fig. 5 is realized in weak magnetic fields $\Omega < ms_*^2/2$. Of course, for magnetic phonons to exist at such Ω we need very low temperatures and pure samples, so as to satisfy the quantization conditions $\Omega \gg T$ and $\Omega \gg \tau^{-1}$ (τ is the free-path time). A detailed quantitative analysis of the properties of magnetic phonons and of the influence of the temperature and scattering will be published in a separate paper. We confine ourselves here mainly to a graphic investigation of the dispersion equation.

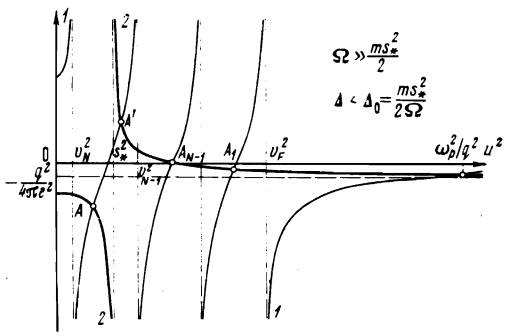

FIG. 6. Procedure for the solving Eq. (4.6) in a stronger field $\Delta_0 \ll 1$, $\Delta > \Delta_0$. The notation is the same as in Fig. 5.

Figures 6 and 7 show plots of the behavior of magnetic phonons in stronger magnetic fields $\Omega \gg ms_*^2/2$. In such fields, the magnetic-phonon velocities are smaller than the velocities of all quantum waves. In the case shown in Fig. 7 we have $v_N < s_*$ or $\Delta < \Delta_0 = ms_*^2/2\Omega$, and there are two magnetic phonons, whereas at $\Delta > \Delta_0$ there is only one (Fig. 6). In the latter case it is easy to find an analytic expression for the velocity and the damping of the magnetic phonon. If $s_* \ll v_N$ ($\Delta \gg \Delta_0$), then

$$s^2(\mathbf{H}) = s_*^2 + \frac{2\varepsilon_F}{3M}\frac{\nu_{cl}(\varepsilon_F)}{\nu_{qu}(\varepsilon_F, \mathbf{H})} = s^2(0) - \frac{2\varepsilon_F}{3M}(1+2(N\Delta)^{1/3})^{-1}, \qquad (4.7)$$

$$\Gamma(\mathbf{H}) = \Gamma(0)\left[\frac{\nu_{cl}(\varepsilon_F)}{\nu_{qu}(\varepsilon_F \mathbf{H})}\right]^2 \gamma(\mathbf{q}, \omega, \mathbf{H}). \qquad (4.8)$$

The amount by which the damping (4.8) differs from zero depends on the allowance for the scattering of the electrons by the impurities and on the extent to which the temperature is finite.

It must be stated that (4.7) and (4.8) differ substantially from the corresponding expressions in the Fröhlich model[12] in that the quantum density of states enters in the denominators of the expressions for the velocity and the damping. In particular, near the singularity of the density of states, where $N\Delta \to 0$ and $\nu_{qu}(\varepsilon_F, \mathbf{H}) \to \infty$, the square of the velocity of the magnetic phonon remains finite and positive, whereas in the Fröhlich model it becomes negative and its absolute value increases in proportion to $\nu_{qu}(\varepsilon_F, \mathbf{H})$. The decrease of the speed of sound in accord with formula (4.7) is explained by the fact that the effective number of the electrons that participate in the formation of the phonon spectrum increases on the Fermi surface. This strengthens the screening of the interaction between the electrons, a screening which this interaction weakens and by the same token decreases the electron contribution to the elasticity of the crystal. Since the square of the Debye radius is inversely proportional to the state density on the Fermi surface, it follows that with increasing $\nu_{qu}(\varepsilon_F, \mathbf{H})$ the velocity of the magnetic phonon decreases. Far from the state-density singularity we have $s(\mathbf{H}) = s$, i.e., the phonon velocity is the same as at $\mathbf{H} = 0$.

In contrast to the Fröhlich model, the phonons interact even during the pulses of the strong collisionless absorption at $\Delta = \Delta_0$, where $\gamma = \gamma_{max} = \Omega/\omega$ and

$$\frac{\operatorname{Im} S_0}{\operatorname{Re} S_0}=\frac{\pi}{2}\frac{\Omega}{qv_F}\gg 1. \tag{4.9}$$

The solution of the dispersion equation (4.6) is

$$u^2=s_*^2+nV_0/MS_0(\mathbf{q},\omega,\mathbf{H}), \tag{4.10}$$

from which we get

$$s^2(\mathbf{H})=s_*^2+\frac{4}{3\pi^2}\frac{mv_F^2}{M}\left(\frac{qv_F}{\Omega}\right)^2,\quad \Gamma(\mathbf{H})=\frac{mv_F^2}{3\pi Ms_*^2}\left(\frac{qv_F}{\Omega}\right)\omega. \tag{4.11}$$

It is seen that the phonon damping $\Gamma(\mathbf{H})$ is small in this region compared with the frequency ω on account of the parameter $qv_F/\Omega\ll 1$. The relative smallness of γ during the pulses is due to the fact that the simple loop $S_0(\mathbf{q},\omega,\mathbf{H})$ is contained in the denominator of the second term of (4.10), also that the appearance of a large imaginary part of the function S_0 leads to a damping that is smaller the larger $\operatorname{Im} S_0$.

We thus arrive at the conclusion that in a quantizing magnetic field, even in the quasiclassical situation, the interaction of the electrons with the lattice is essentially nonadiabatic, and the nonadiabatic effects play an important role in the region of small ω and q. Under these conditions the velocity of the long-wave phonons in the metal cannot be determined from the thermodynamics, since it is essential to take into account the strong frequency dependence of the elastic moduli of the metal. In other words, the dynamic (in the sense of the dependence on ω) and static longitudinal elastic moduli are substantially different if the speed of sound is close to one of the electronic velocities v_n. This constitutes the principal difference between the phonon spectrum of metals in a magnetic field and the adiabatic situation (at $\mathbf{H}=0$) that has been exhaustively analyzed by Brovman and Kagan.[1,3]

We note that in the magnetic-field region where $s_*\ll v_N$ and the nonadiabatic effects can be neglected, the speed of sound (4.7) is determined by the static (thermodynamic elastic modulus, which coincides with the dynamic modulus. Quantization of the electronic states manifests itself in this case in strong oscillations of the elastic moduli of the metal. This question was analyzed by Lazarev and the present authors.[23]

We have disregarded above the spin splitting of the electronic levels in the magnetic field. If this splitting

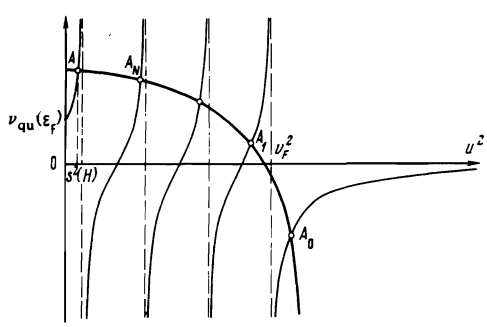

FIG. 8. Solution of the dispersion equation in the Fröhlich model. A—phonon, A_0—zero sound.

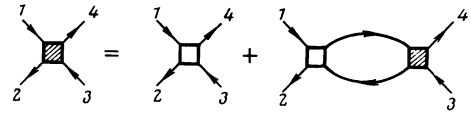

FIG. 9.

is taken into account, then the number of quantum waves is doubled. Nonetheless, the analysis described above is applicable also to this case and yields analogous results.

We discuss in conclusion the results of the quantization of the electronic states in a magnetic field in the case of the Fröhlich model. If we substitute in the dispersion equation (3.7) the expression for the simple model (4.2), then we arrive at the situation shown schematically in Fig. 8. Besides the fact that in this model there are no solutions of the plasma-oscillation type and a solution corresponding to zero sound appears (the point A_0), the existence of the phonon root A depends on the magnetic field intensity. In fact, the point A exists only if $\nu_{qu}(\varepsilon_F,\mathbf{H})<s_0/2q|\Lambda_q|^2$. Owing to the oscillations $\nu_{qu}(\varepsilon_F,\mathbf{H})$, this condition may be violated; the square of the sound velocity then becomes negative even before a change occurs in the number of Landau levels below the Fermi surface. It follows from the entire foregoing analysis that such conclusions stem from an unsubstantiated application of the Fröhlich model to the analysis of the electronic properties of crystals in a magnetic field.

The authors thank I. M. Lifshitz and Yu. Kagan for a discussion of the results.

APPENDIX: CALCULATION OF THE FUNCTION $S(\mathbf{q},\omega,H)$

Following the Landau Fermi-liquid theory,[24,25] we introduce the effective electron-electron interaction I (12, 34), which is an aggregate of diagrams made up of Γ which are not cut in two along two electron lines. Then Γ satisfies the integral equation of Fig. 9, where the light rectangle represents the effective interaction I. This equation can be symbolically written in the form

$$\Gamma=I+IR\Gamma, \tag{A.1}$$

where R corresponds to the product of two internal electron propagators.

We confine ourselves hereafter to the analysis of the spectrum of longitudinal oscillations at small ω and q. The limiting value of Γ as $\omega\to 0$ and $q\to 0$ is well known[24] to depend on the order of the limiting transition, in view of the ambiguity of the limit of R. At small ω ($\omega\ll\varepsilon_F$), the matrix elements of R take the form

$$R_{tt'}(E,E')=-\sigma_{tt'}(E,E')+R_{tt'}^{(0)}(E,E'), \tag{A.2}$$
$$\sigma_{tt'}(E,E+\omega)=z_tz_{t'}\frac{f(E_{t'})-f(E_t)}{E_t-E_{t'}+\omega}\delta(E-E_t),$$

where E_t is the energy of the true quasiparticles of the electron Fermi-liquid of a metal with a Fermi distribution function $f(E)$. Relation (A.2) is proved by a method similar to that used at $\mathbf{H}=0$ by Luttinger and Nozieres.[25]

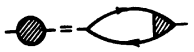

FIG. 10.

All the singularities that appear in the oscillation spectrum are due to the quantity σ, which is the singular part of R. The quantity $R^{(0)}$ remains single-valued as $\omega \to 0$ and $q \to 0$. Under the conditions of the quasiclassical quantization, it depends little on the magnetic field, so that $\mathbf{H}$ in $R^{(0)}$ can be made to tend to zero. We note that quantities of the type $R^{(0)}$ coincide as $\omega \to 0$ and $q \to 0$ with the usual ω-limit of the corresponding functions. z_t in (A.2) denotes the residue of the exact electronic propagator; z_t and z_t^{-1} have no singularities as functions of the magnetic field[17,26] and can also be replaced by their limits as $\mathbf{H} \to 0$. The existence of formula (A2) is due to the important circumstance that even in a magnetic field the imaginary part of the self-energy of an electron vanishes on the Fermi surface under the conditions of quasiclassical quantization. This statement was proved by Luttinger.[17]

To calculate the susceptibility $S(\mathbf{q}, \omega, \mathbf{H})$, we express it in terms of the vertex function T (12, 3) represented by the shaded triangle, using the relation (Fig. 10)

$$-S=MRT. \tag{A.3}$$

The vertex function satisfies the obvious equation (Fig. 11)

$$T=M+IRT. \tag{A.4}$$

At small ω, Eq. (A.4) can be rewritten in the equivalent form

$$T=T^{(0)}-\Gamma^{(0)}\sigma T, \tag{A.5}$$

where $\Gamma^{(0)}$ must be obtained from the equation $\Gamma^{(0)}=I+IR^{(0)}\Gamma^{(0)}$, and $T^{(0)}=M+\Gamma^{(0)}R^{(0)}M$. We replace the quantities $\Gamma^{(0)}$ and $T^{(0)}$ by their limits as $\mathbf{H} \to 0$. We then change over in (A.5) to the momentum representation, which is the most convenient in the quasicalassical situation. We then carry out a multiplicative renormalization of the quantities T, Γ, and σ by means of the formulas

$$T=z^{-1}\tau, \quad \Gamma=z^{-1}fz^{-1}, \quad \sigma=z^{-1}\zeta. \tag{A.6}$$

As a result of this renormalization, the residues of the electron propagators no longer enter explicitly in the definition of ζ and in the equations for the renormalized quantities τ, f, and others. The equation for τ becomes

$$\tau=\tau^{(0)}-f^{(0)}\zeta\tau, \tag{A.7}$$

and the relation (A.3) takes the form

$$S=\tau^{(0)}\zeta\tau, \tag{A.8}$$

where ζ differs from (A.2) in the absence of the residues z_t and $z_{t'}$. It is seen from (A.8) that to calculate S it is necessary to know $\tau^{(0)}$ and τ on the Fermi surface. It follows from Ward's identity that on the Fermi surface $\tau^{(0)} \to 1$ as $\omega \to 0$ and $q \to 0$.[25] Consequently $S=\zeta\tau$, and τ is determined from Eq. (A.7), in which $\tau^{(0)}$ is replaced by unity. In the momentum representation we

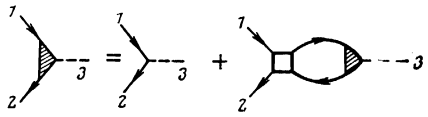

FIG. 11.

have

$$\tau_p=1-\sum_{p_1p_2} f^{(0)}_{pp_1}\zeta_{p_1p_2}\tau_{p_2}. \tag{A.9}$$

Here

$$\zeta_{p_1p_2}=\sum_{t_1t_2}\langle t_2|p_1-q\rangle\langle p_1|t_1\rangle\zeta_{t_1t_2}\langle t_1|p_2\rangle\langle p_2-q|t_2\rangle, \tag{A.10}$$

and $\langle \mathbf{p}|t\rangle$ is the wave function of the electron in the state t in the p-representation.

It is difficult to find the exact solution of the integral equation (A.9). The function τ, however, has an important property that allows us to construct a reasonable approximation of the solution of (A.9). This property is the smoothness of the function τ near the Fermi surface, whereas the function ζ is singular at small ω and q. Following the variational method of[27,28], we write down this approximate solution in the form

$$\tau=\frac{1}{1+\langle f^{(0)}\rangle S_0(\mathbf{q}, \omega, \mathbf{H})}, \tag{A.11}$$

$$\langle f^{(0)}\rangle=\sum_{p_1,p_2,p_3,p_4}\zeta_{p_1p_2}f^{(0)}_{p_2p_3}\zeta_{p_3p_4}\Big/\Big(\sum_{p_1,p_2}\zeta_{p_1p_2}\Big)^2. \tag{A.12}$$

We note that (A.11) coincides with the correct result in all cases when (A.9) admits of an exact solution. The relation (A.11) is therefore a reasonable approximation and gives the best approximation of the true solution from the point of view of the variational method of[27,28]. Substituting (A.11) in (A.8) we arrive at formula (2.5).

[1)] The change in the equilibrium positions of the ions can be neglected under conditions of quasiclassical quantization because this effect is connected with a metal-volume change proportional to the integral of the electron density states. We shall henceforth disregard oscillations of this kind.

[2)] The question of the coupling of quantum and sound waves was considered phenomenologically by Zyryanova *et al.*[22] on the basis of premises concerning the deformation interaction of the electrons with the lattice.

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Statistical thermodynamics of formation of a new phase. II. Theory of boiling of volatile liquids

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The fluctuating growth of a vapor-filled macroscopic bubble in a moderately superheated or decompressed volatile liquid is treated as two-dimensional diffusion of a germ of a new phase in the space of its variables, viz., the volume v and the pressure p of the vapor in it. The relief of the free energy of a "liquid + bubble with vapor" over the (v, p) plane is investigated in the vicinity of the labile equilibrium of the system, and the two-dimensional equilibrium distribution function of the germs with respect to their variables is determined. The nondiagonal diffusion tensor in (v, p) space near the saddle point is also calculated. A two dimensional stationary equation of the Kramers-Zel'dovich type of the kinetics of formation of a new phase is solved and an expression is obtained for the probability of homogeneous nucleation at arbitrary viscosity and volatility of the liquid far from its critical point. Various limiting cases are considered.

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1. INTRODUCTION

The analysis of the kinetics of formation of a new phase[1–8] has led to the development of a new method of describing the kinetics of a first-order phase transition in which the growth of a macroscopic germ of a new phase is treated as diffusion over the germ-size axis. The difference between the equation of the kinetics of new-phase formation (the Fokker-Planck equation) and the ordinary diffusion equation lies in the fact that the germ-size axis is not homogeneous: a certain force field is superimposed on it and is governed by the "supersaturation" of the investigated system. A natural macroscopic model of the kinetics of new-phase formation is therefore diffusion in the field of external forces.

The task of determining the rate of formation of the new phase in this approach breaks up into two stages: (a) determination of the coefficient of diffusion of the germ over the size axis as a function of the germ size; (b) determination and investigation of the "potential relief" on this axis, the relief being specified by the position of the system on the Van-der-Waals diagram in its metastable region. We make use here essentially of the fact, first pointed out by Gibbs, that the potential relief on the size axis is a potential barrier that separates the quasi-single-phase region of the size axis from the two-phase region. It is the diffusion flux from one region to the other which determines the kinetics of new-phase formation. Both stages of the solution of the nucleation problem become much more complicated if a single variable no longer suffices for a macroscopic description of the new-phase germ. This is the situation with

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