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[1)] A. F. Ioffe, Physicotechnical Institute, USSR Academy of Sciences, Leningrad.
[2)] P. N. Lebedev, Physics Institute, USSR Academy of Sciences, Moscow.

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Generation of high-frequency magnons by nonequilibrium electrons polarized opposite to the direction of magnetization

I. Ya. Korenblit and B. G. Tankhilevich

Leningrad Institute of Nuclear Physics, USSR Academy of Sciences
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A theoretical investigation is made of the generation of high-frequency magnons by nonequilibrium electrons with spins directed opposite to the magnetization in wide-band ferromagnets. It is shown that the isotropic case is characterized by narrowing of the magnon generation range at high electron pumping rates: the range of momenta of the generated magnons decreases either exponentially with increase in the electron pumping rate or is inversely proportional to this rate, and the number of magnons in this range rises exponentially with the pumping rate in the first case and quadratically in the second. Typical momentum of the generated magnons is of the order of the momentum of an electron whose kinetic energy is equal to the *s–d* exchange interaction energy. If the magnon spectrum depends strongly on the angle between the magnon momentum and magnetic moment of a crystal, an increase in the pumping rate gradually produces an almost monochromatic beam of magnons whose momenta are directed along the magnetization. At a certain critical pumping rate the generation of magnons becomes avalanchelike and the magnon system becomes unstable.

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1. INTRODUCTION

Electromagnetic methods for the excitation of the spin system of a ferromagnet, capable of generating low-frequency spin waves with wave vectors $q<10^6\ \mathrm{cm}^{-1}$, are widely known.[1] We shall consider the process of generation of magnons by nonequilibrium electrons with $\downarrow$ spin ("against the field"), which—under certain conditions—can produce high-intensity beams of high-frequency almost monochromatic magnons.

We shall consider the process of relaxation of non-

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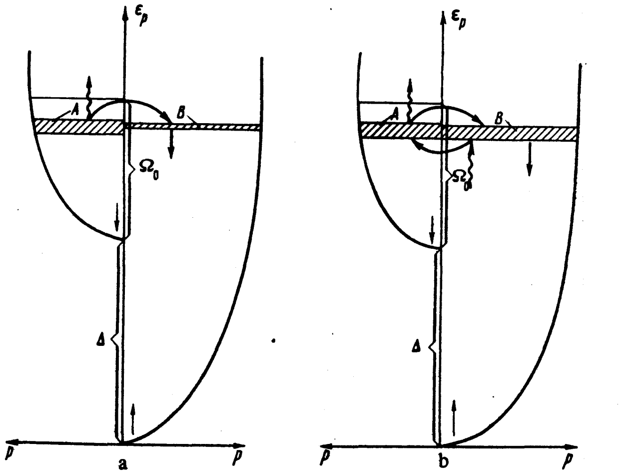

FIG. 1. Transitions of an electron transferred to the conduction subband with spin against the field. The kinetic energy ε of the electron is less than the optical phonon energy Ω_0. The wavy line represents a magnon and the continuous curve—an electron. The interaction with magnons may be weaker (a) or stronger (b) than their interaction with phonons and electrons.

equilibrium electrons transferred to a conduction subband with spin against the field in a wide-band ferromagnetic semiconductor (Fig. 1). We shall assume that the kinetic energy ε of these electrons is less than the energy of a longitudinal optical phonon Ω_0. Then, the most likely result is that an electron of this kind becomes transferred to a subband with ↑ spin ("parallel to the field") emitting a magnon in the process, and its new kinetic energy is approximately equal to the exchange gap Δ in the conduction band. Since $\Delta \gg \Omega_0 > \varepsilon$, the momentum q of the emitted magnon is of the order of $p_0 = (2m\Delta)^{1/2}$, where m is the effective electron mass. In the subband with spin parallel to the field the electron may: a) drop directly to the bottom of the band giving up the excess energy to phonons or electrons already present there (Fig. 1a) or b) before dropping to the bottom, it may absorb a magnon and return back to the subband with spin against the field (Fig. 1b).

The situation b) occurs when the probability of absorption of a magnon is higher than the probability of the electron–phonon or electron–electron interactions. Clearly, in the case a) the density of nonequilibrium electrons whose kinetic energy is of the order of Δ in the subband with spin parallel to the field (denoted by B in Fig. 1b) is much less than the density of nonequilibrium electrons with spin against the field (band A in Fig. 1a), whereas in the case b) these densities are approximately equal. If the electron population is inverted (i.e., if the electron density in the band A is higher than in the band B), stimulated emission of magnons predominates over their absorption by electrons.

The ratio of the magnon generation frequency $\Gamma_e(\mathbf{q})$ to the frequency of their relaxation (as a result of interaction with equilibrium magnons) $\Gamma_m(\mathbf{q})$ is a function of the wave vector $\mathbf{q}$. We shall first assume that $\Gamma_e(\mathbf{q})/\Gamma_m(\mathbf{q})$ is independent of the direction of $\mathbf{q}$. Then, the distribution function of the nonequilibrium magnons $N(\mathbf{q})$ is isotropic and has a maximum at the value of $q = q^*$ corresponding to the maximum value of $\Gamma_e(\mathbf{q})/\Gamma_m(\mathbf{q})$. Stimulated emission of magnons provides conditions favorable for preferential increase in the number of magnons with $q \approx q^*$ when the electron pumping rate is increased. Therefore, at a sufficiently high pumping rate, when the number of the nonequilibrium magnons exceeds the thermal background, the magnons are generated mainly in a narrow range of wave vectors $\Delta\mathbf{q}$ near q^*. Then, depending on the nature of the function $\Gamma_e(\mathbf{q})/\Gamma_m(\mathbf{q})$, the number of magnons in this interval rises on increase of the pumping rate either exponentially or quadratically, and the interval itself decreases either exponentially or inversely proportional to the pumping rate. The exponential narrowing of the range of frequencies in which this magnon generation takes place is considered in our earlier paper.[2]

The magnon generation rate is a function of the number of electrons in the subband with spin against the field, and this number is governed—in its turn—by the pumping rate and the total number of the emitted magnons. This feedback stabilizes the steady state of the magnon system in the isotropic case for any finite electron pumping rate.

If the ratio $\Gamma_e(\mathbf{q})/\Gamma_m(\mathbf{q})$ depends on the direction of the wave vector $\mathbf{q}$ and is maximal for a certain fixed value $q = q^*$, the magnon distribution function $N(\mathbf{q})$ becomes strongly anisotropic at high pumping rates and above a certain critical value of this rate the value of $N(\mathbf{q})$ becomes infinite, i.e., the generation of magnons with $q = q^*$ becomes avalanche-like and the magnon system becomes unstable.

An increase in the electron pumping rate increases the number of electrons in the state B in the subband with spin parallel to the field; then the processes involving magnon absorption and electron transfer to the subband with spin against the field become increasingly important and they suppress the narrowing of the magnon generation region. Therefore, the narrowing effect may be expected only if the frequency of electron loss from the state B to the bottom of the band is sufficiently high. This frequency should be much greater than the difference between the frequency of electron transitions from A to B (accompanied by magnon emission) or the frequency of return transitions (accompanied by magnon absorption).

We shall assume that the electron density in the states B of the subband with spin parallel to the field is low and we shall ignore the magnon absorption processes. Moreover, with the exception of Sec. 7, we shall assume that the interaction of electrons with longitudinal optical phonons is stronger than their interaction with magnons or electrons. This situation is typical of ferromagnetic semiconductors in which the lattice binding is largely ionic.

2. SYSTEM OF KINETIC EQUATIONS

We shall describe quantitatively the magnon generation process by deriving a system of kinetic equations for the electron $f_\sigma(\mathbf{p})$ ($\sigma = \uparrow, \downarrow$) and magnon $N(\mathbf{q})$ distribution functions. We shall allow for the interaction of electrons with magnons and optical phonons and for the interaction of magnons with electrons and other magnons. We shall derive the kinetic equations using the Keldysh diagram technique.[3] In the Keldysh technique the electron kinetic equation is obtained for the Green functions $G_\sigma^+(\mathbf{p},\varepsilon)$ and $G_\sigma^-(\mathbf{p},\varepsilon)$. It is convenient to introduce generalized electron distribution functions $f_\sigma(\mathbf{p},\varepsilon)$ using[4,5]

$$\left.\begin{aligned} G_\sigma^+(\mathbf{p},\varepsilon)&=f_\sigma(\mathbf{p},\varepsilon)\,(G_\sigma^a(\mathbf{p},\varepsilon)-G_\sigma^r(\mathbf{p},\varepsilon)),\\ G_\sigma^-(\mathbf{p},\varepsilon)&=(1-f_\sigma(\mathbf{p},\varepsilon))\,(G_\sigma^r(\mathbf{p},\varepsilon)-G_\sigma^a(\mathbf{p},\varepsilon)),\end{aligned}\right\} \tag{1}$$

where $G_\sigma^r(\mathbf{p},\varepsilon)$ and $G_\sigma^a(\mathbf{p},\varepsilon)$ are the retarded and advanced Green functions.

Near the mass surface the difference between these functions is

$$G_\sigma^a(\mathbf{p},\varepsilon)-G_\sigma^r(\mathbf{p},\varepsilon)=2\pi i\delta(\gamma_\sigma(\mathbf{p},\varepsilon)\,|\,\varepsilon-\tilde\varepsilon_\sigma(\mathbf{p},\varepsilon)), \tag{2}$$

where

$$\delta(\gamma_\sigma(\mathbf{p},\varepsilon)\,|\,\varepsilon-\tilde\varepsilon_\sigma(\mathbf{p},\varepsilon)) =\pi^{-1}\gamma_\sigma(\mathbf{p},\varepsilon)\,[(\varepsilon-\tilde\varepsilon_\sigma(\mathbf{p},\varepsilon))^2+\gamma_\sigma^2(\mathbf{p},\varepsilon)]^{-1}, \tag{3}$$

$$\gamma_\sigma(\mathbf{p},\varepsilon)=\operatorname{Im}\Sigma_\sigma^r(\mathbf{p},\varepsilon),\quad \tilde\varepsilon_\sigma(\mathbf{p},\varepsilon)=\varepsilon_\sigma(\mathbf{p},\varepsilon)+\operatorname{Re}\Sigma_\sigma^r(\mathbf{p},\varepsilon), \tag{4}$$

$\Sigma_\sigma^r(\mathbf{p},\varepsilon)$ is the self-energy part of the function G_σ^r. We shall assume that the interaction of electrons with optical phonons is stronger than the electron–magnon interaction. Therefore, we shall allow for the electron–magnon interaction in the lowest order of the perturbation theory; consequently, γ and $\tilde\varepsilon$ in Eqs. (2) and (3) are governed only by the interaction with optical phonons. The electron energy $\tilde\varepsilon_\sigma(\mathbf{p})$, renormalized by the electron–phonon interaction, will now be denoted simply by $\varepsilon_\sigma(\mathbf{p})$.

Since the kinetic energy of an electron with spin against the field is less than the energy of an optical phonon, the damping of such electrons by optical phonons is $\gamma\downarrow(\mathbf{p},\varepsilon_\mathbf{p})$ and

$$\delta(\gamma_\downarrow(\mathbf{p},\varepsilon_\mathbf{p})\,|\,\varepsilon-\varepsilon_\downarrow(\mathbf{p}))=\delta(\varepsilon-\varepsilon_\downarrow(\mathbf{p})). \tag{5}$$

The magnon–electron and magnon–magnon interactions will also be allowed for in the lowest order of the perturbation theory, i.e., the magnon Green functions $D^+(\mathbf{q},\omega)$ and $D^-(\mathbf{q},\omega)$ are

$$\left.\begin{aligned} D^+(\mathbf{q},\omega)&=2\pi i\delta(\omega-\omega_\mathbf{q})N(\mathbf{q},\omega),\\ D^-(\mathbf{q},\omega)&=2\pi i\delta(\omega-\omega_\mathbf{q})\,(1+N(\mathbf{q},\omega)).\end{aligned}\right\} \tag{6}$$

The collision integral R^{em} of electrons with spin against the field interacting with magnons is obtained by applying the diagram technique rules[3]:

$$R^{em}(p)=-i\cdot 2SI^2\int d^4q\,[G_\uparrow^+(p-q)D^+(q)G_\downarrow^-(p)-G_\uparrow^-(p-q)D^-(q)G_\downarrow^+(p)], \tag{7}$$

where $q=(\mathbf{q},\omega)$, $p=(\mathbf{p},\varepsilon)$. Here, I is the s–d exchange interaction constant and S is the localized spin. Similarly, the integral R^{me} representing collisions of magnons with electrons is

$$R^{me}(q)=-i\cdot 2SI^2\int d^4p\,[G_\downarrow^+(p)G_\uparrow^-(p-q)D^-(q)-G_\downarrow^-(p)G_\uparrow^+(p-q)D^+(q)]. \tag{8}$$

In the derivation of Eqs. (7) and (8) the complete vertex of the electron–magnon interaction is replaced by a simple vertex. Estimates indicate that renormalization of the electron–magnon vertex due to the interaction of electrons with optical phonons is of the order of $\alpha\Omega_0\Delta^{-1}$, where α is the usual dimensionless constant of the coupling of electrons with optical phonons and the exchange gap is $\Delta=2IS\gg\Omega_0$. Thus, the kinetic equations obtained below are valid if $\alpha\ll\Delta\Omega_0^{-1}$.

We shall be interested in magnons with sufficiently large momenta whose relaxation is governed mainly by four-magnon exchange processes. In deriving the corresponding collision integral R^{mm} by the Keldysh technique we must bear in mind that the vertex matrix of the four-body interaction is diagonal and that $\gamma_{1111}=1$, $\gamma_{2222}=-1$:

$$\begin{aligned} R^{mm}(q)=&\iint d^4q'\,d^4q''\,|V(\mathbf{q},\mathbf{q}';\mathbf{q}'',\mathbf{q}+\mathbf{q}'-\mathbf{q}'')|^2\\ &\times[D^+(q+q'-q'')D^+(q'')D^-(q')D^-(q)-D^-(q+q'-q'')D^-(q'')D^+(q')D^+(q)],\end{aligned} \tag{9}$$

where $V(\mathbf{q},\mathbf{q}';\mathbf{q}'',\mathbf{q}+\mathbf{q}'-\mathbf{q}'')$ is the matrix element of the magnon–magnon interaction.[6]

We shall now introduce the electron and magnon distribution functions:

$$f_\sigma(\mathbf{p})=\frac{1}{2\pi i}\int_{-\infty}^{+\infty}G_\sigma^+(\mathbf{p},\varepsilon)\,d\varepsilon, \tag{10}$$

$$N(\mathbf{q})=\frac{1}{2\pi i}\int_{-\infty}^{+\infty}D^+(\mathbf{q},\omega)\,d\omega. \tag{11}$$

Substituting the Green functions (1) and (6) in the collision integrals R^{em}, R^{me}, and R^{mm} we obtain the following system of kinetic equations:

$$(1+N(\mathbf{q}))\,\Gamma_e(\mathbf{q})+\hat\Gamma_m(N(\mathbf{q}))=0, \tag{12}$$

$$f_\downarrow(\mathbf{p})\gamma_m(\mathbf{p})=g(\varepsilon_\mathbf{p}), \tag{13}$$

where the frequency of the magnon–electron relaxation is

$$\Gamma_e(\mathbf{q})=\frac{2SI^2v}{(2\pi)^3}\int d^3p\,\delta(\gamma(\mathbf{p}-\mathbf{q})\,|\,\varepsilon_\mathbf{p}^\downarrow-\omega_\mathbf{q}-\varepsilon_{\mathbf{p}-\mathbf{q}}^\uparrow)f_\downarrow(\mathbf{p}), \tag{14}$$

the frequency of the electron–magnon relaxation is

$$\gamma_m(\mathbf{p})=\frac{2SI^2v}{(2\pi)^3}\int d^3q\,\delta(\gamma(\mathbf{p}-\mathbf{q})\,|\,\varepsilon_\mathbf{p}^\downarrow-\omega_\mathbf{q}-\varepsilon_{\mathbf{p}-\mathbf{q}}^\uparrow)\,(1+N(\mathbf{q})), \tag{15}$$

and the magnon–magnon collision operator is

$$\begin{aligned} \hat\Gamma_m(N(\mathbf{q}))=&\frac{v^2}{(2\pi)^6}\iint d^3q'\,d^3q''\,|V(\mathbf{q},\mathbf{q}';\mathbf{q}'',\mathbf{q}+\mathbf{q}'-\mathbf{q}'')|^2\\ &\times\{N(\mathbf{q}'')N(\mathbf{q}+\mathbf{q}'-\mathbf{q}'')\,(1+N(\mathbf{q}'))\,(1+N(\mathbf{q}))-(1+N(\mathbf{q}+\mathbf{q}'-\mathbf{q}''))\\ &\times(1+N(\mathbf{q}''))N(\mathbf{q}')N(\mathbf{q})\}\,\delta(\omega_\mathbf{q}+\omega_{\mathbf{q}'}-\omega_{\mathbf{q}''}-\omega_{\mathbf{q}+\mathbf{q}'-\mathbf{q}''}).\end{aligned} \tag{16}$$

Here, v is the volume of a unit cell and $g(\varepsilon_\mathbf{p})$ is the generation function of electrons with spin against the field. We shall assume the latter to be the delta function:

$$g(\varepsilon_\mathbf{p})=g\delta(\varepsilon-\varepsilon_\mathbf{p}), \tag{17}$$

where g is the generation function amplitude.

In deriving Eqs. (12) and (13) we have ignored, following the comments in the Introduction, the terms proportional to $f_\downarrow$. Moreover, we have assumed that electrons with spin against the field are not degenerate. The damping of electrons with spin parallel to the field, due to the emission of optical phonons, is—according to Eq. (4)—given by the following expression in the lowest order of the perturbation theory:

$$\gamma_\uparrow(\mathbf{p},\varepsilon_p)=-{}^1/_2\frac{v}{(2\pi)^4}\Omega_0\,\mathrm{Im}\left\{i\int d^4k[G_{0\uparrow}^{s}(p-k)\mathscr{D}^{r}(k)+G_{0\uparrow}^{r}(p-k)\mathscr{D}^{s}(k)]\right\}, \tag{18}$$

where $G_{0\uparrow}^{s}$, $G_{0\sigma}^{\pm}$ are the free Green functions of electrons and $\mathscr{D}^{s}$, $\mathscr{D}^{+}$, $\mathscr{D}^{-}$ are the Green functions of phonons.[3] Equation (18) yields the well-known expression (here $\varkappa_0$ and $\varkappa_\infty$ are the static and high-frequency permittivities):

$$\gamma_\uparrow(\varepsilon_p)=\frac{\pi}{2}\alpha\Omega_0\frac{(2m\Omega_0)^{1/2}}{p}\ln\left|\frac{p+(p^2-2m\Omega_0)^{1/2}}{p-(p^2-2m\Omega_0)^{1/2}}\right|, \tag{19}$$

$$\alpha=\frac{e^2}{2\Omega_0}(2m\Omega_0)^{1/2}(\varkappa_\infty^{-1}-\varkappa_0^{-1}).$$

For electrons of energy $\varepsilon_p=\Delta$, the damping is

$$\gamma_\uparrow=\frac{\pi}{2}\alpha\Omega_0\left(\frac{\Omega_0}{\Delta}\right)^{1/2}\ln\frac{4\Delta}{\Omega_0}\ll\Delta. \tag{20}$$

The system of kinetic equations (12) and (13), together with Eq. (19), describes completely the dependences of the electron and magnon distribution functions on the pumping rate.

3. INTEGRAL EQUATION FOR $N(\mathbf{q})$

If we ignore the interaction of the nonequilibrium magnons with one another, we find that the operator $\hat{\Gamma}_m(N(\mathbf{q}))$ of Eq. (16) describes the relaxation of the nonequilibrium magnons because of their interaction with the equilibrium magnons ("test particle" relaxation) and it can be rewritten in the form

$$\hat{\Gamma}_m(N(\mathbf{q}))=-(N(\mathbf{q})-N^{(0)}(\mathbf{q}))\Gamma_m(\mathbf{q}), \tag{21}$$

where the equilibrium distribution function is $N^{(0)}(\mathbf{q})=(\exp(\omega_q/T)-1)^{-1}$ and the magnon–magnon relaxation frequency is given by [7]

$$\Gamma_m(\mathbf{q})\approx(T/T_c)^2\omega_q(aq)^2, \tag{22}$$

if the generated magnons are subthermal, and

$$\Gamma_m(\mathbf{q})\approx(T/T_c)^{5/2}T_c(aq)^3, \tag{23}$$

if the generated magnons are suprathermal. Here, ω_q is the magnon spectrum, a is the lattice constant, and the temperature is assumed to be $T\ll T_c$. It should be noted that in $\Gamma_m(\mathbf{q})$ we may include not only the four-magnon exchange processes but also other types of magnon relaxation.

In the case of low values of q, when the four-magnon processes become unimportant, $\Gamma_m(\mathbf{q})$ ceases to depend on q[1]. The following integral equation for $N(\mathbf{q})$ is obtained from Eqs. (12) and (13) if use is made of Eq. (21):

$$N(\mathbf{q})=(N^{(0)}(\mathbf{q})+\Gamma_e(\mathbf{q})\Gamma_m^{-1}(\mathbf{q}))(1-\Gamma_e(\mathbf{q})\Gamma_m^{-1}(\mathbf{q}))^{-1}; \tag{24}$$

$$\Gamma_e(\mathbf{q})=g\int d^3p\,\delta(\gamma(\mathbf{p}-\mathbf{q})|\varepsilon_p^{\downarrow}-\omega_q-\varepsilon_{p-q}^{\uparrow})\delta(\varepsilon-\varepsilon_p)Z^{-1}(\mathbf{p}), \tag{25}$$

$$Z(\mathbf{p})=vT\int d^3q\,\delta(\gamma(\mathbf{p}-\mathbf{q})|\varepsilon_p^{\downarrow}-\omega_q-\varepsilon_{p-q}^{\uparrow})(1+N(\mathbf{q})). \tag{26}$$

Equation (24) resembles formally the expression for the magnon distribution function under parametric pumping conditions. However, in our case the frequency Γ_e is itself a functional of $N(\mathbf{q})$ since the number of emitted magnons depends on $f_\downarrow(\mathbf{p})$, which is the distribution function of electrons with spin against the field and which—according to Eqs. (13) and (15)—is itself governed not only by the pumping rate $g(\varepsilon_p)$ but also by some average $Z(\mathbf{p})$ of the distribution function of the emitted magnons. Therefore, the behavior of $N(\mathbf{q})$ is different from that in the case of parametric pumping.

4. ASYMPTOTIC BEHAVIOR OF THE DISTRIBUTION FUNCTION $N(\mathbf{q})$ AT HIGH PUMPING RATES

We shall first consider the isotropic situation when $\Gamma_m(\mathbf{q})$ and $\Gamma_e(\mathbf{q})$ are independent of the direction of $\mathbf{q}$. Then,

$$\Gamma_e(q)=\frac{2\pi^2Tv}{Z(p_\varepsilon)}\int d\Omega_p\delta(\gamma(\mathbf{p}-\mathbf{q})|\varepsilon_q^{\downarrow}-\omega_q-\varepsilon_{p-q}^{\uparrow}); \tag{27}$$

$p_\varepsilon=(2m\varepsilon)^{1/2}$. Here, $\nu=2^{-1/2}\pi^{-2}gvm^{3/2}\varepsilon^{1/2}$ is the number of electrons transferred to the subband with spin against the field (per unit time and per unit cell). Substituting Eq. (27) into Eq. (24), and then Eq. (24) into Eq. (26), we obtain the following transcendental equation describing the dependence of Z on the pumping rate ν:

$$Z=\int_0^\infty\frac{dq\Phi_1(q)}{1-\nu\Phi(q)/Z}, \tag{28}$$

where

$$\Phi_1(q)=vTq^2(1+N^{(0)}(q))\varphi(q), \tag{29}$$

$$\Phi(q)=2\pi^2T\Gamma_m^{-1}(q)\varphi(q), \tag{30}$$

$$\varphi(q)=\int d\Omega_q\delta(\gamma(\mathbf{p}-\mathbf{q})|\varepsilon_p^{\downarrow}-\omega_q-\varepsilon_{p-q}^{\uparrow})=\int d\Omega_p\delta(\gamma(\mathbf{p}-\mathbf{q})|\varepsilon_p^{\downarrow}-\omega_q-\varepsilon_{p-q}^{\uparrow}). \tag{31}$$

We can easily see that the functions $\Phi(q)$ and $\Phi_1(q)$ are bounded.

We shall introduce $\zeta(\nu)=Z(\nu)/\nu$, where ζ satisfies the equation

$$\nu=\int_0^\infty\frac{dq\Phi_1(q)}{\zeta(\nu)-\Phi(q)}. \tag{32}$$

Figure 2 shows the graphical solution of Eq. (32). The curve represents the right-hand side of Eq. (32) considered as a function of ζ and the horizontal line cuts off (on the ordinate) an intercept equal to ν. The point of intersection of the line and curve gives the solution of the equation. For low values of ν, we have

$$\zeta(\nu)\gg\Phi(q),\quad \lim_{\nu\to0}Z(\nu)=\int_0^\infty dq\,\Phi_1(q). \tag{33}$$

An increase in ν reduces the function ζ which tends to a certain constant value ζ_∞, which coincides with the

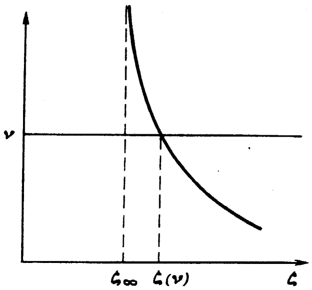

FIG. 2. Graphical solution of Eq. (32) for $\zeta(\nu)$ in the isotropic case.

upper limit of $\Phi(q)$. Consequently, asymptotically, we find that

$$Z(\nu)=\zeta_\infty\nu+Z_1(\nu),\quad \lim_{\nu\to\infty} Z_1(\nu)=0. \tag{34}$$

The asymptotic behavior of the distribution function $N(q)$ requires also the determination of the dependence of Z_1 on ν. There are two possible situations.

1. The upper limit of the function $\Phi(q)$ may be its maximum. We shall use q^* to denote the point at which this maximum occurs and we shall expand $\Phi(q)$ in the vicinity of q^*:

$$\Phi(q)=\Phi(q^*)-{}^1/_2|\Phi''(q^*)|(q-q^*)^2+\ldots \tag{35}$$

We shall substitute this expansion into Eq. (28) for Z and assume that $\Phi_1(q)$ is a smooth function of q near $q=q^*$; after integration we obtain:

$$Z(\nu)=\Phi(q^*)\nu+\frac{\pi^2\Phi_1^2(q^*)}{|\Phi''(q^*)|}\frac{1}{\nu}. \tag{36}$$

This formula is valid if the higher terms of the expansion are ignored in Eq. (35) and if the second term in Eq. (36) is less than the first, i.e., if $\nu>\nu_c^{\rm I}=\max(\nu_1,\nu_2)$,

$$\nu_1=\Phi_1(q^*)\Phi'''(q^*)/(\Phi''(q^*))^2, \tag{37}$$

$$\nu_2=\pi\Phi_1(q^*)/(|\Phi''(q^*)|\Phi(q^*))^{1/2}. \tag{38}$$

Thus, at high pumping rates the distribution function $N(q)$ is

$$N(q)=\frac{\Phi(q)+N^{(0)}(q)\Phi(q^*)}{\Phi(q^*)-\Phi(q)+\pi^2\Phi_1^2(q^*)/|\Phi''(q^*)|\nu^2}. \tag{39}$$

Hence,

$$N(q)=(1+N^{(0)}(q))\left(\frac{\nu}{\nu_2}\right)^2,\quad |q-q^*|\ll\Delta q=\frac{\nu_2}{\nu}\left(\frac{\Phi(q^*)}{|\Phi''(q^*)|}\right)^{1/2}, \tag{40}$$

$$N(q)=\frac{\Phi(q)+N^{(0)}(q)\Phi(q^*)}{\Phi(q^*)-\Phi(q)},\quad |q-q^*|\gg\Delta q. \tag{41}$$

Consequently, at pumping rates higher than the critical value only those magnons are generated whose momenta lie within a narrow range near q^*. The width of this interval decreases as ν^{-1} and the number of magnons in this interval rises as ν^2.

2. Let us now assume that the upper limit of the function $\Phi(q)$ is reached at a point q^* such that $\Phi'(q^*)\neq 0$. For simplicity, we shall assume that $\varphi(q)=0$ if $q<q^*$. Then, expanding $\Phi(q)$ in terms of the powers of $q-q^*$, we shall rewrite Eq. (32) in the form

$$\nu=\int_{q^*}^{\infty}\frac{\Phi_1(q)\,dq}{\zeta-\zeta_\infty+|\Phi'(q^*)|(q-q^*)+O(q-q^*)}. \tag{42}$$

At high pumping rates, i.e., when $\zeta\to\zeta_\infty$, the contribution made to the above integral in the interval of q close to q^* is proportional to $(\zeta-\zeta_\infty)^{-1}$. Therefore, introducing the cutoff parameter q_1+q^* in Eq. (42), we obtain the following equation for ζ which is valid with logarithmic precision at high pumping rates:

$$\nu=\frac{\Phi_1(q^*)}{|\Phi'(q^*)|}\ln\left|\frac{\Phi'(q^*)q_1}{\zeta-\zeta_\infty}\right|. \tag{43}$$

Hence, it follows that if

$$\nu>\nu_c^{\rm II}=\Phi_1(q^*)/|\Phi'(q^*)|, \tag{44}$$

then

$$Z=\nu\Phi(q^*)+\nu q_1|\Phi'(q^*)|\exp(-\nu/\nu_c^{\rm II}). \tag{45}$$

Consequently,

$$N(q)=\frac{\Phi(q)+N^{(0)}(q)\Phi(q^*)}{\Phi(q^*)-\Phi(q)+q_1\Phi'(q^*)\exp(-\nu/\nu_c^{\rm II})}, \tag{46}$$

and hence

$$\left.\begin{aligned}&N(q)=(1+N^{(0)}(q))\frac{\Phi(q^*)}{q_1|\Phi'(q^*)|}\exp\left(\frac{\nu}{\nu_c^{\rm II}}\right),\\ &q-q^*\ll q_1\exp\left(-\frac{\nu}{\nu_c^{\rm II}}\right).\end{aligned}\right\} \tag{47}$$

For $q-q^*\gg q_1\exp(-\nu/\nu_c^{\rm II})$, the distribution function is independent of the pumping rate and is described by Eq. (41).

Thus, if at a point where the function $\Phi(q)$ reaches its upper limit it has either a kink or changes very abruptly, the magnon generation interval decreases exponentially with increasing pumping rate and the number of magnons whose momenta are $q\approx q^*$ rises equally rapidly with the pumping rate. The critical pumping rate $\nu_c^{\rm II}$ is governed completely by the functions Φ_1 and Φ known for each specific case. In general, only the preexponential factors in Eqs. (46) and (47), depending on the cutoff parameter q_1, remain indeterminate. It follows from the foregoing analysis that no matter how fast the pumping rate, the frequency of magnon generation Γ_e is less than the frequency of magnon relaxation Γ_m, so that the magnon system is always stable. This is associated with the above-mentioned dependence of Γ_e on the level populations in the subband with spins against the field. As the pumping rate is increased, not only does the numerator in Eq. (25) become larger, but the function Z in the denominator increases and $N(q)$ rises with ν in such a way that the function Z remains finite.

5. DETERMINATION OF THE CRITICAL PUMPING RATE

Realization of the first or second regime and the critical pumping rate depend on the relationship between the electron energy ε in the subband with spin against the field and the damping of electrons by optical phonons in the subband with spin parallel to the field. We shall consider first the case of weak damping and go to the limit $\gamma \to 0$ in Eq. (31) for $\varphi(q)$. Then,

$$\varphi(q)=\begin{cases} 2\pi m/p_\varepsilon q, & p_0-p_\varepsilon\leq q\leq p_0+p_\varepsilon \\ 0, & q<p_0-p_\varepsilon, \quad q>p_0+p_\varepsilon \end{cases}; \tag{48}$$

$$\Phi(q)=\begin{cases} 4\pi^3 Tm/p_\varepsilon q\Gamma_m(q), & p_0-p_\varepsilon\leq q\leq p_0+p_\varepsilon \\ 0, & q<p_0-p_\varepsilon, \quad q>p_0+p_\varepsilon \end{cases}. \tag{49}$$

In the majority of cases $\Gamma_m(q)$ can be regarded as a power function of q: $\Gamma_m(q)=\Gamma_m(p_0)(q/p_0)^\beta$. In particular, if $\Gamma_m(q)$ is governed by the magnon-magnon exchange scattering and a gap in the magnon spectrum can be ignored, it follows that $\beta=4$ for the subthermal magnons [see Eqs. (22) and (23)]. It is clear from Eq. (49) that the upper limit of $\Phi(q)$ is reached at the point $q=q^*=p_0-p_\varepsilon$, where $\Phi'(q^*)\neq 0$. This means that at pumping rates higher than the critical value, only the magnons with momenta exponentially little different from q^* are generated and the number of magnons in this exponentially narrow interval rises exponentially with the pumping rate. The critical pumping rate can be found from Eqs. (29), (44), (48), and (49):

$$\nu_c^{II}=\frac{2}{\beta+1}\nu_0=\frac{1}{4\pi^2}\Gamma_m(p_0)\,(1+N^{(0)}(p_0))\,vp_0^3\frac{2}{\beta+1}. \tag{50}$$

Since in this case the function $\Phi_1(q)$ vanishes for $q>p_0+p$, and also we have $p_0+p_\varepsilon-q^*=2p_\varepsilon\ll p_0$, there is no need to introduce an indeterminate cutoff parameter for the integral (42), and q_1 in Eqs. (46) and (47) for $N(q)$ and Δq should be replaced with $2p_\varepsilon$, so that[2]

$$N(q)=(1+N^{(0)}(p_0))\frac{p_0}{2p_\varepsilon(\beta+1)}\exp\left(\frac{\beta+1}{2}\frac{\nu}{\nu_0}\right),\quad q^*<q<q^*+\Delta q, \tag{51}$$

$$\Delta q=2p_\varepsilon\exp\left(-\frac{\beta+1}{2}\frac{\nu}{\nu_0}\right). \tag{52}$$

The physical meaning of the characteristic pumping rate ν_0 can be understood as follows: The ratio of the rate of magnon generation to the rate of magnon relaxation Γ_e/Γ_m reaches its upper limit at $q=p_0-p_\varepsilon$ and its lower limit (where it vanishes) at $q=p_0+p_\varepsilon$, i.e., there is an excess generation at the left edge of the interval compared with the right edge. As a result of stimulated emission, this asymmetry increases with the pumping rate. Therefore, nonlinear generation begins when the difference between the number of the nonequilibrium magnons at the ends of the interval becomes comparable with the number of the equilibrium magnons, i.e., when

$$\Gamma_e N(q=p_0-p_\varepsilon)-\Gamma_e N(q=p_0+p_\varepsilon)\sim\Gamma_m N(p_0)$$

or $\Gamma_e p_\varepsilon/p_0\sim\Gamma_m$. It follows from Eqs. (26) and (27) that if $N(q)\sim N^{(0)}(q)$, then the magnon generation (emission) frequency is $\Gamma_e\sim Tvp_0m/p_\varepsilon p_0p_\varepsilon$, i.e., nonlinear generation begins at pumping rates

$$\nu\sim\Gamma_m p_0^3 v(1+N^{(0)}(p_0))\sim\nu_0.$$

We shall now allow for the influence of the damping of electrons whose spin is parallel to the field. If $\gamma\neq 0$, then $\Phi_1(q)$ and $\Phi(q)$ are finite for all values of q satisfying the inequality $\varepsilon_{p-q}>\Omega_0$. The function $\varphi(q)$ has a maximum at $q=p_0$ and the lower the value of γ, the sharper is this maximum. On the other hand, if $\beta>1$, the quantity $\Gamma_m^{-1}(q)$ rises on reduction of q. Therefore, the function $\Phi(q)$ may have a maximum at $q=p_0$ and also another maximum at low values of q. Estimates indicate that for $\alpha<(\Delta/\Omega_0)^{(5-\beta)/4}$, the upper limit of $\Phi(q)$ is reached at $q\approx p_0$, i.e., effectively only the magnons with the momenta $q\approx p_0$ are generated. Assuming that this inequality is satisfied, we shall replace $\gamma(p_\varepsilon-q)$ in Eq. (31) for $\varphi(q)$ with $\gamma(p_0)$, which makes it easy to carry out the integration:

$$\varphi(q)=\frac{2m}{p_\varepsilon q}\left[\operatorname{arctg}\left(\frac{\Delta-q^2/2m+p_\varepsilon q/m}{\gamma}\right)-\operatorname{arctg}\left(\frac{\Delta-q^2/2m-p_\varepsilon q/m}{\gamma}\right)\right]. \tag{53}$$

If $\gamma\ll 2(\varepsilon\Delta)^{1/2}$, the arguments of arc tangents in Eq. (53) are large so that

$$\varphi(q)=\frac{2m}{p_\varepsilon q}\left[\pi-\frac{\gamma}{\Delta-q^2/2m+p_\varepsilon q/m}-\frac{\gamma}{q^2/2m+p_\varepsilon q/m-\Delta}\right]. \tag{54}$$

The position of the maximum of the function $\varphi(q)$ depends on the ratio of γ and ε. If $\gamma\ll\pi(\beta+1)\varepsilon_1$, the maximum occurs at a point

$$q^*=p_0-p_\varepsilon(1-(\gamma/\pi(\beta+1)\varepsilon)^{1/2}),$$

and throughout the range of q—with the exception of a very narrow interval $p_0-p_\varepsilon\leq q\leq q^*$—the function $\varphi(q)$ is almost independent of γ. This means that at pumping rates exceeding ν_0 the generation interval Δq and the number of magnons $N(q)$ in this interval vary exponentially in accordance with Eqs. (51) and (52) until Δq contracts to a width of the order of

$$q^*-p_0+p_\varepsilon=(\gamma/\pi(\beta+1)\varepsilon)^{1/2}p_\varepsilon.$$

A further increase in the pumping rate results in a change from the exponential to power-law magnon generation, which is described by Eqs. (40) and (41) and now ν_c^I differs logarithmically from ν_0.

If $\pi(\beta+1)\varepsilon<\gamma<2(\varepsilon\Delta)^{1/2}$, the maximum of the function $\varphi(q)$ is displaced from the point $q^*\approx p_0-p_\varepsilon$ to the point $q^*\approx p_0$ and throughout the nonlinear range we have the power-law rise of $N(q)$. The critical pumping is then

$$\nu_c^I=2\pi\nu_0\left(\frac{\varepsilon}{\gamma}\right)^{1/2}\left(\frac{\varepsilon}{\Delta}\right)^{1/4},\quad \gamma>\pi(\beta+1)\varepsilon\left(\frac{\Delta}{\varepsilon}\right)^{1/4}. \tag{55}$$

Finally, if $\gamma>2(\varepsilon\Delta)^{1/2}$, the denominator $\delta(\gamma(\mathbf{p}_\varepsilon-\mathbf{q})\times|\varepsilon_p^\downarrow-\varepsilon_{p-q}^\uparrow-\omega_q)$ can be simplified by dropping the term $p_\varepsilon q/m\sim p_\varepsilon p_0/m$, which is small compared with γ; then, the integrand ceases to depend on the angles and $\varphi(q)$ is given by

$$\varphi(q)=4\frac{\gamma}{(\Delta-q^2/2m)^2+\gamma^2}. \tag{56}$$

The function $\Phi(q)$ still has a maximum at $q\approx p_0$ and the critical pumping rate is

$$\nu_c^I=\pi\nu_0\gamma/\Delta. \tag{57}$$

6. INFLUENCE OF THE ANISOTROPY. INSTABILITY OF THE MAGNON SYSTEM

Narrowing of the interval of effective magnon generation considered above is due to the fact that the magnon generation conditions are optimal for certain values $q=q^*$. Clearly, if the ratio of the magnon generation rate $\Gamma_c(\mathbf{q})$ to the relaxation rate $\Gamma_m(\mathbf{q})$ is an anisotropic function of the wave vector $\mathbf{q}$, the distribution function $N(\mathbf{q})$ becomes strongly anisotropic in the nonlinear region because of stimulated magnon emission. The anisotropy of $\Gamma_e(\mathbf{q})/\Gamma_m(\mathbf{q})$ may partly be due to the anisotropy of the magnon spectrum associated with the dipole–dipole interaction. As a rule, we have $Dp_0^2 \gg 8\pi\mu M$, where D is spin-wave "stiffness," μ is the Bohr magneton, and M is the saturation value of the moment. Therefore, the spectrum of magnons with wave vectors $q=p_0$ can be represented in the form

$$\omega_q = Dq^2(1+\lambda\sin^2\vartheta_q), \tag{58}$$

where $\lambda = 4\pi\mu M/\omega_{p_0} \ll 1$ and ϑ_q is the angle between the directions of the vectors $\mathbf{q}$ and $\mathbf{M}$. We shall assume that the generated magnons are subthermal; then, according to Eq. (22), the dependence of Γ_m on the angles is given by the same factor $(1+\lambda\sin^2\vartheta_q)$.

The quantity $Z(\mathbf{p})$ given by Eq. (26) now depends generally on the angle between the vectors $\mathbf{p}$ and $\mathbf{M}$ and it satisfies the equation

$$Z(\mathbf{p}) = \int TvN^{(0)}(\mathbf{q})\,\delta(\gamma(\mathbf{p}-\mathbf{q})|\varepsilon_p^{\downarrow}-\varepsilon_{p-q}^{\uparrow}-\omega_q) \times\left\{1-\frac{Tv}{\Gamma_m(\mathbf{q})}\int\frac{\delta(\gamma(\mathbf{p}-\mathbf{q})|\varepsilon_p^{\downarrow}-\varepsilon_{p-q}^{\uparrow}-\omega_q)\,g\,\delta(\varepsilon_p^{\downarrow}-\varepsilon)\,d^3p}{Z(\mathbf{p})}\right\}^{-1} d^3q, \tag{59}$$

which reduces to Eq. (28) for $\lambda=0$.

The general form of Eq. (59) cannot be analyzed. We can obtain its solution in two limiting cases, when the function Z is independent of the direction of $\mathbf{p}$.

1. We shall first consider the case when the damping γ is strong so that $\gamma \gg 2(\varepsilon\Delta)^{1/2}$. Then,

$$\delta(\gamma(\mathbf{p}-\mathbf{q})|\varepsilon_p^{\downarrow}-\varepsilon_{p-q}^{\uparrow}-\omega_q) = \frac{1}{\pi}\frac{\gamma}{(\Delta-q^2/2m)^2+\gamma^2}. \tag{60}$$

Equation (59) for $Z(\mathbf{p})$ becomes

$$Z(p_\varepsilon) = \frac{2T^2v}{D\gamma}\int_{-1}^{1} dt\int_0^\infty dq\left[\left(\left(\frac{p_0^2-q^2}{p_\gamma^2}\right)^2+1\right)(1+\lambda(1-t^2)) - \frac{8\pi^2Tv}{\Gamma_m(p_0)\gamma}\left(\frac{p_0}{q}\right)^4\frac{1}{Z(p_\varepsilon)}\right]^{-1}. \tag{61}$$

To within terms of higher order in respect of the parameter p_γ/p_0, we find from Eq. (61) the following transcendental equation for $\zeta(\nu) = Z(\nu)\nu^{-1}$.

$$\frac{\nu\lambda^{1/2}Dp_0\zeta(\nu)}{4\pi T^2vm} = \arcsin\left[\lambda^{1/2}\left(1+\lambda-\frac{8\pi^2T}{\Gamma_m(p_0)\gamma\zeta(\nu)}\right)^{-1/2}\right]. \tag{62}$$

The results of the preceding section for the isotropic case can be deduced from Eq. (62) if the argument of the arc sine is small, which corresponds to pumping rates $\nu \ll \pi\nu_0\gamma\Delta^{-1}\lambda^{-1/2}$. This inequality shows that even

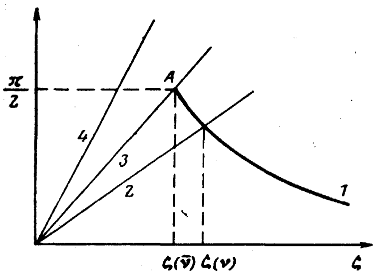

FIG. 3. Graphical solution of Eq. (62) for $\zeta(\nu)$ in the anisotropic case. Curve 1 represents the right-hand side of the equation. Straight lines 2, 3, and 4 represent the left-hand side of the equation; ν rises with the number of the curve.

a slight anisotropy becomes important if the pumping rate is sufficiently high. Figure 3 shows qualitatively the graphical solution of Eq. (62). This equation has a real solution if the pumping rate ν is less than a certain critical value $\bar\nu$, for which the straight line in Fig. 3 passes through the point A. It follows from Eq. (62) that

$$\bar\nu = \frac{\gamma\Gamma_m(p_0)vp_0^3N^{(0)}(p_0)}{8\Delta\lambda^{1/2}} = \frac{\pi^2}{4}\nu_0\frac{\gamma}{\Delta}\frac{1}{\lambda^{1/2}}, \tag{63}$$

$$Z(\bar\nu) = 8\pi^2T\bar\nu/\gamma\Gamma_m(p_0). \tag{64}$$

Substituting Eqs. (63) and (64) into Eqs. (24) and (25), we obtain the magnon distribution function at the critical pumping rate:

$$N(q) = N^{(0)}(q)\left[(1+\lambda\sin^2\vartheta_q) - \frac{\gamma^2}{(\Delta-\varepsilon_q)^2+\gamma^2}\left(\frac{p_0}{q}\right)^4\right]^{-1}. \tag{65}$$

If $q=p_0$ and $\vartheta_q=0$, the denominator of the above expression vanishes.

Thus, the steady-state solution of the system of kinetic equations (12) and (13) exists in this situation only for pumping rates lower than $\bar\nu$. When the pumping rate reaches $\bar\nu$, the number of those magnons whose momenta are equal to p_0 and which are directed along the magnetization rises exponentially with time to some value which is clearly limited by nonlinear processes in the nonequilibrium magnon system itself.

We recall that in the isotropic case the magnon system is stable at any pumping rate because of feedback relationship between the magnon emission frequency and the population of the band A in the subband with spin against the field. In the anisotropic situation considered here this feedback is ineffective since the divergence of $N(q)$ for a fixed direction of the wave vector q does not result in divergence of the function Z and, therefore, the rate of loss of electrons from the subband with spin against the field—proportional, according to Eqs. (15) and (26), to the function Z—remains finite. Thus, this example shows that an instability of the magnon system may appear because magnons are not generated directly by photons but by an intermediate system, which are the electrons with spin against the field.

2. In the second case the damping goes to the limit $\gamma\to 0$ and then the dependence of Z on the direction of p can be ignored only if $p_\varepsilon \ll p_0\lambda$ and $\nu \ll \nu_0\ln(p_0\lambda/p_\varepsilon)$. Then, in the delta functions in the numerator of Eq. (59)

we can substitute $p=0$; consequently, we obtain $q=p_0$. In other words, this approximation implies that all the generated magnons have the same momentum p_0. In this approximation the ratio Γ_e/Γ_m is the function of just one variable, the angle ϑ_q, and it reaches its upper limit at the points where $\vartheta_q=0$ or π, where the derivative is

$$\frac{d}{d\vartheta}\frac{\Gamma_e}{\Gamma_m}\neq 0.$$

Therefore, the divergence of the generated magnon beam decreases exponentially with increasing pumping rate. Calculations similar to those in Sec. 4 show that for $\nu>2\nu_0 p_\varepsilon/p_0\lambda^{1/2}$,

$$N(\mathbf{q})=\frac{N^{(0)}(p_0)}{\lambda}\left[\sin^2\vartheta_q+2\exp\left(-\frac{\lambda^{1/2}\nu p_0}{2\nu_0 p_\varepsilon}\right)\right]^{-1}, \tag{66}$$

i.e., the number of magnons with momenta in the angular range

$$\Delta\vartheta<\exp\left(-\frac{\nu}{4\nu_0}\frac{p_0}{p_\varepsilon}\lambda^{1/2}\right)$$

rises exponentially with increasing ν. However, we recall that this solution is valid only if $\nu\ll\nu_0\ln(p_0\lambda/p_\varepsilon)$, because the nonmonochromaticity of the magnon beam increases with rising ν.

7. STRONG ELECTRON-ELECTRON INTERACTION

If the density of nonequilibrium electrons at the bottom of the conduction band is sufficiently high, the electron–electron collision frequency may be higher than the electron–magnon frequency. This has two important consequences. First, the population of the levels in the band B of the subband with spin parallel to the field is low irrespective of the degree of interaction of electrons with optical phonons. Second, the electron distribution function in the subband with spin against the field is Maxwellian (or Fermi-type) with an effective temperature T^* identical with the electron temperature at the bottom of the subband with spin parallel to the field but generally different from the lattice temperature:

$$f_\downarrow(\varepsilon_p)=C\exp(-\varepsilon_p/T^*). \tag{67}$$

The normalization constant C is found from the condition of balance of the number of electrons pumped to the subband with spin against the field and transferred to the subband with spin parallel to the field (the latter process is accompanied by magnon emission). Consequently, we can find C by substituting in Eq. (13) the expression (67) for $f_\downarrow(\varepsilon_p)$ and summing over all the states. If we confine ourselves to the case of small damping, $\gamma\ll T^*$, we find that in the first order in γ/T^* the normalization constant is given by

$$C^{-1}=\frac{1}{2\pi^3}\frac{v^2S\Gamma m}{\nu}\int_0^\infty dp\,p\exp\left(-\frac{\varepsilon_p}{T^*}\right)\int_{p_0-p_T}^{p_0+p_T}dq\,q(1+N(q)), \tag{68}$$

where $p_T=(2mT^*)^{1/2}$.

Equations (14), (67), and (68) give us the frequency of relaxation of magnons by interaction with electrons:

$$\Gamma_e(q)=\frac{2\pi^2T^*\nu}{qv}\exp\left[-\frac{1}{8mT^*}\left(q-\frac{p_0^2}{q}\right)^2\right]\times\left[\int_0^\infty d\varepsilon_p\exp\left(-\frac{\varepsilon_p}{T^*}\right)\int_{p_0-p_T}^{p_0+p_T}dq\,q(1+N(q))\right]^{-1}. \tag{69}$$

We shall introduce

$$Z=\frac{v^{2/3}}{T^*}\int_0^\infty d\varepsilon_p\exp\left(-\frac{\varepsilon_p}{T^*}\right)\int_{p_0-p_T}^{p_0+p_T}dq\,q(1+N(q)). \tag{70}$$

According to Eqs. (24) and (70), Z satisfies the equation

$$Z=\frac{v^{2/3}}{T^*}\int_0^\infty d\varepsilon_p\exp\left(-\frac{\varepsilon_p}{T^*}\right)\int_{p_0-p_T}^{p_0+p_T}\frac{dq\,q(1+N^{(0)}(q))}{(1-\nu\Phi_2(q)/Z)} \tag{71}$$

$$\Phi_2(q)=\frac{2\pi^2}{\Gamma_m(q)aq}\exp\left[-\frac{1}{8mT^*}\left(q-\frac{p_0^2}{q}\right)^2\right]. \tag{72}$$

The function $\Phi_2(q)$ has a maximum at $q=p_0$ (to within terms of the order of p_T/p_0).

The structure of the integrand in Eq. (71) is the same as in Eq. (28) and, therefore, at high pumping rates the main contribution to the integral is made by q near p_0, so that integration with respect to dq can be carried out from zero to infinity. Consequently, Eq. (71) reduces to the form completely analogous to Eq. (28). Thus, we reach the conclusion that in the case when the electron–electron interaction predominates over the electron–magnon interaction and the damping γ is weak, the nature of the dependence of $N(q)$ on ν at high pumping rates ($\nu\gg 2\pi\nu_0p_T/p_0$) is the same as in the case when the electron–magnon interaction predominates and the damping is $\gamma>\varepsilon$:

$$N(q)=\begin{cases}\left(\dfrac{\nu}{2\pi\nu_0}\dfrac{p_0}{p_T}\right)^2(1+N^{(0)}(p_0)), & |q-p_0|<\Delta q\\[2mm] \dfrac{1+N^{(0)}(p_0)}{1-\Phi(q)/\Phi(q^*)}, & |q-p_0|\gg\Delta q\end{cases}; \tag{73}$$

$$\Delta q=2\pi\frac{\nu_0}{\nu}\frac{p_T}{p_0}p_T. \tag{74}$$

8. CONCLUSIONS

The results obtained in the preceding sections are based on the assumption that the population of the states B in the subband with spin parallel to the field is small compared with the population of the state A in the subband with spin against the field; in other words, it is assumed that the frequency of dropping of electrons to the bottom of the band with spin parallel to the field due to the electron–phonon or electron–electron interactions is high compared with the frequency of the electron–magnon collisions. However, we can show that the narrowing of the magnon generation interval occurs also when a "softer" condition is satisfied. In fact, it is sufficient that the frequency of dropping of electrons to the bottom of the conduction band be greater than the difference between the frequencies of arrival of electrons from A in B (accompanied by magnon emission) and loss from B to A (accompanied by magnon absorption). If the damping γ_t is much less than ε and the dropping of electrons is due to interaction with optical phonons, this condition reduces to the inequality

$$\alpha \gg (IS/\Omega_0)^{1/2} Impv. \quad (75)$$

We shall now give some numerical estimates. The ferromagnetic semiconductor EuO is characterized[8] by the following parameters: the permittivities are $\varkappa_0 = 25$ and $\varkappa_\infty = 5$; the optical phonon energy is $\Omega_0 = 0.08$ eV; the exchange gap in the conduction band is $\Delta = 0.6$ eV[9]; the magnon mass is $330m_0$[10]; the effective electron mass is $m = 0.35m_0$.[11] Then, $\alpha = 1.4$, the damping constant is $\gamma \approx 0.06$ eV, and the criterion $\gamma \gg 2(\varepsilon\Delta)^{1/2}$ is satisfied by the electrons in the subband with spin against the field where their energy is $\varepsilon \ll 0.01$ eV. The wave vector of the generated magnons is $q \approx p_0 = 2.5 \times 10^7$ cm^{-1}, $\omega_{p_0} = 7.4$°K, and at $T > 10$°K the magnons are subthermal. If we assume that $\Gamma_m(p_0) \approx 10^8$ sec^{-1}, we find that $\nu_0 = 10^{29}$ sec$^{-1}\cdot$cm^{-3} and the narrowing of the generation interval begins from pumping rates $\nu_c^{I} = \pi\gamma\nu_0/\Delta\nu_0/3$. We also have $\lambda \approx 0.1$ so that the instability of the magnon system begins at $\bar{\nu} = \nu_0$. If $\varepsilon \gg 0.01$ eV (but $< \Omega_0$), the inequality $\gamma \ll \pi(\beta+1)\varepsilon$ applies and we then have $p/p_0 \lesssim \lambda$. Therefore, at pumping rates of the order of ν_0 the angular divergence of the generated magnon beam and its radius decrease exponentially.

For $CdCr_2Se_4$ we can use the parameters taken from Ref. 12 and we find that $\alpha \approx 0.1$. The inequality (75) is satisfied if the energy is $\varepsilon \ll 0.01$ eV. Then, the damping is given by $\gamma \ll \pi(\beta+1)\varepsilon$, i.e., the generation interval at pumping rates $\nu > \nu_0$ should decrease exponentially in width. The critical pumping ν_0 is still of the order of of $10^{28}-10^{29}$ sec$^{-1}\cdot$cm^{-3}.

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