Effect of cooling type-I superconductors in the dynamic intermediate state

Yu. V. Sharvin and I. L. Broneva

Institute of Physics Problems, USSR Academy of Sciences
(Submitted June 27, 1977)
Zh. Eksp. Teor. Fiz. 73, 2274–2280 (December 1977)

The temperature difference T_1-T_2 produced between the ends of a cylindrical intermediate-state sample as a result of longitudinal motion of the superconducting and normal domains under the influence of a current J at $T \ll T_c$ have been calculated and measured. The problem of the temperature distribution in the sample is solved in general form under the assumption that the thermal conductivity has a cubic dependence on the temperature. For an indium sample of 4 mm diameter and an approximate electron mean free path in the normal phase 10 mm the results of measurements at $T = 0.34 \pm 0.01$ K are in approximately sufficient agreement with the calculations by the parabola $T_1-T_2 = -4\times10^{-5}J + 1.5\times10^{-5}J^2$, where J is in amperes. The distance between the thermometers on the sample was 8 cm.

PACS numbers: 74.55.+h, 74.30.Ek, 74.70.Gj

Motion of superconducting (s) and normal (n) domains the intermediate state under the influence of a current can produce in a sample a temperature gradient due to transport of the electron entropy concentrated in the n-domains.[1, 2] We have undertaken a more detailed investigation of this effect for samples of high purity in the temperature region $T \ll T_c$, where its observation is aided by the low thermal conductivity of the layered intermediate-state structures in a direction perpendicular to the layers, since the electronic component of the thermal conductivity in the s layers is already almost nonexistent, and the phonon component decreases rapidly (like T^3, provided that the phonon mean free path is determined by the scattering near the domain walls).[3–7]

Total Andreev reflection of the electrons[8] from the moving ns boundary causes the heat of the phase transition to be transferred, in motion towards the n phase, to the electrons, and to be given up by the electrons on the opposite n-layer boundary. Owing to the high thermal conductivity of the n phase, the temperature gradient produced in this phase is negligibly small in this case. The significant sources of the irreversible increase of the entropy in this process are the Joule heat and the heat transfer through the s layers. We have obtained experimental and theoretical estimates of the temperature difference $T_2 - T_1$ produced under these conditions between the ends of a cylindrical sample in which the s and n layers, which are perpendicular to its axis are displaced by a direct current having an average density j. In practice, the attained cooling effect is small ($(T_2 - T_1)_{max}/T_2 \sim 10^{-4}$) and could become appreciable only if the sample resistance were to be decreased by several orders of magnitude.

1. MACROSCOPIC EQUATION FOR THE TEMPERATURE IN THE SAMPLE

We make the following assumptions:

1. The temperature T averaged over the volume of several domains depends only on the coordinate x, which changes along the sample axis from x_1 to x_2. The sample is thermally insulated, with the exception of the end $x = x_2$, where $T(x_2) = T_2$ is specified.

2. The concentration C_n of the n phase does not depend on x.

3. The thermal conductivity averaged over the s and n domains is $\varkappa = \lambda T^3$, where λ is constant.

4. The specific heat and entropy per unit volume are equal to $C_n\gamma T$.

5. The layer velocity is $v = uj$, where u and j are constant.

6. The sample resistivity ρ is independent of T; $\rho = C_n\rho_n$, where ρ_n is the residual resistivity of the bulk n phase at $H = H_c$, as follows from the presence of total Andreev reflection of the electrons from the sn boundary at $T \ll T_c$.[9]

To write down the equation, we use the known general relations between the thermoelectric coefficients. If the current flows along the sample axis, then the absorption and release of heat at its ends as a result of entropy transport by the n layers can be regarded as a unique Peltier effect. The amount of Thomson heat that should be released from a unit volume of the sample is

$$W_T = -T\frac{d}{dT}\left(\frac{\Pi}{T}\right) j\nabla T,$$

where Π is the Peltier coefficient (see, e.g., [10], p. 103). In our case $\Pi = C_n\gamma T^2 u$ [1)] (since the heat of the sn transition per unit volume at $T \ll T_c$ is equal to γT^2) and

$$W_T = -C_n\gamma T u j dT/dx.$$

Recognizing that, besides the Thomson heat, Joule heat $C_n\rho_n j^2$ is also released in the sample we obtain the heat-conduction equation

$$\frac{d}{dx}\lambda T^3\frac{dT}{dx} = C_n\gamma T u j\frac{dT}{dx} - C_n\rho_n j^2. \qquad (1)$$

 0038-5646/77/4606-1191$02.40

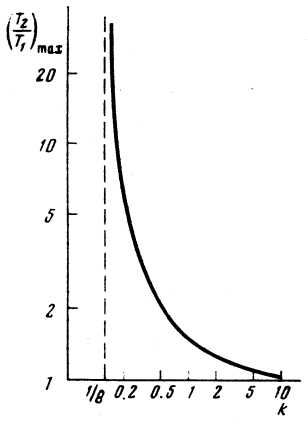

FIG. 1.

Its first integral can be expressed in terms of the variables $\vartheta = T^2$ and $\zeta = (C_n\gamma uj/\lambda)x$ in the form

$$d\vartheta/d\zeta = 1 - 2k\zeta/\vartheta, \tag{2}$$

where $k = \rho_n\lambda/C_n\gamma^2u^2$ is a dimensionless parameter. The arbitrary integration constant is contained in (2) implicitly, since the position of the point $\zeta = 0$ on the sample axis remains arbitrary.

The boundary condition on the thermally insulated end is that the heat of the transition be equal to the heat influx due to thermal conductivity:

$$C_n\gamma T^2uj = \lambda T^3 dT/dx$$

at $x = x_1$, i.e., at $\zeta = \zeta_1$ or $d\vartheta/d\zeta = 2$, or $\vartheta(\zeta_1) = 0$. At $x = x_2$ we have $\vartheta(\zeta_2) = T_2^2$.

The equation can be integrated if we make the change of variable $\vartheta = \eta\zeta$. At $k > \frac{1}{8}$ the ratio T_2/T_1 has a maximum at the optimal value of $\zeta_2 - \zeta_1$. The dependence of $(T_2/T_1)_{max}$ on k is shown in Fig. 1. In the practically realized case $k \gg 1$ we have

$$(T_2/T_1)_{max} = 1 + 1/2k,$$

and as $k \to \frac{1}{8}$

$$(T_2/T_1)_{max} = \pi/2\sqrt{8k-1}.$$

At $k \leqslant \frac{1}{8}$ the equation has solutions that satisfy the condition $\vartheta(\zeta_1) = 0$, for example the particular solutions

$$\vartheta = \zeta[1 \pm \sqrt{1-8k}]/2.$$

If the change of temperature over the sample length is $|T_2 - T_1| \ll T_1$, then ϑ in the right hand-side of (2) can be regarded as constant when the latter is integrated. We then obtain for the temperature difference between two points on the sample an expression that can be conveniently compared with the experimental data:

$$\Delta T = T(L_1) - T(L_2) = -\alpha J + {}^1/_2\beta J^2, \tag{3}$$

where

$$\alpha = \frac{C_n\gamma u}{\lambda TS}(L_2 - L_1),$$

$$\beta = \frac{C_n\rho_n}{\lambda T^3S^2}(L_2^2 - L_1^2),$$

L_1 and L_2 is the distance of the points from the thermally insulated end, S is the sample cross section, and J is the current through the sample.

2. ESTIMATE OF THE MAGNITUDE OF THE EFFECT

The layer displacement velocity is $v = cE_t/H_c$, where E_t is the n-phase electric field component tangential to the ns boundary. We can distinguish between two physically different mechanisms that cause the motion: 1) E_t can be determined by the resistivity, and then we have in the isotropic case, with j directed along the layers, $v = (c\rho_n/H_c)j$, i.e., $u = c\rho_n/H_c$.[12,13] The temperature gradient must obviously be attributed in this case to the Ettingshausen effect. For this effect to be realized in our case, the current contacts should be located on the lateral surface of the sample. 2) E_t can be also the result of the Hall effect, which is quite appreciable in uncompensated metals. In the isotropic case, with j directed perpendicular to the layers, we have $u = cR$, where R is the Hall constant.[13-15]

Equations (1) and (2) are valid for both cases at the corresponding values of u. We present estimates of k in both cases for an isotropic metal with a quadratic dispersion of the electrons. For the averaged thermal conductivity of the layered structure we use the expression $\varkappa = \frac{1}{4}c_sv_sd$, where d is the period of the structure, while c_s and v_s are the phonon specific heat and the averaged sound velocity in the s phase, which can be expressed in terms of the number of atoms per unit volume n_a and the Debye temperature Θ (see, e.g.,[16]). This relation, which yields $\varkappa \propto T^3$, is valid under the assumption that the phonons cross freely the s layers and are effectively scattered by the electrons in the n layers, as is the case when $T \gtrsim 10^{-2}$ K. The results obtained by Zavaritskiĭ for lead and tin[4] show that such an estimate yields the correct order of magnitude of $\varkappa$ for pure samples in the intermediate state at $T \ll T_c$. To obtain the value $k = k_H = \rho_n\lambda/C_n\gamma^2c^2R^2$ corresponding to the Hall mechanism of layer motion, we use the customary expressions for ρ and γ of an isotropic metal and put $R = 1/cn_ee$, assuming the electron mean free path l_e in the n phase to exceed the Larmor radius. Putting for simplicity $n_e = n_a$, we obtain, apart from a coefficient of order of unity,

$$k_H = (T_F/\Theta)^2 d/l_e,$$

where T_F is the Fermi temperature.

In the presence of total Andreev reflection, l_e can greatly exceed d. Usually $d \sim 10^{-2}$ cm,[4,13] and for the samples of the presently attained purity with $l_e \sim 1$ cm we get $k_H \sim 10^3$.

In the case of a resistive mechanism of the layer motion we can obtain for k the estimate

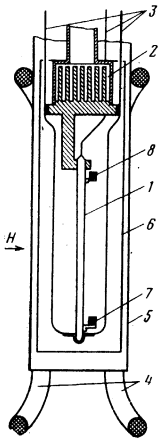

FIG. 2.

$$k=k_\Omega=\lambda H_c^2/C_n c^2\gamma^2\rho_n$$

for a metal with a small number of dissolved impurities which have a negligible effect on the thermal conductivity or dimensions of the layer and cause no pinning of the layers. The quantity H_c in the expression for k_Ω will be expressed for the sake of clarity in terms of the correlation parameter ξ_0, using the standard formulas of microscopic superconductivity theory for an isotropic metal (see, e.g., [17]). Assuming $n_e=n_a$, we obtain, apart from a coefficient of the order of unity,

$$k_\Omega=\frac{e^2}{\hbar c}\frac{v_F}{c}\left(\frac{T_F}{\Theta}\right)^2\frac{l_e d}{\xi_0^2}$$

at $l_e\lesssim\xi_0$. At $l_e\sim\xi_0\sim10^{-4}$ cm and at the usual values of the remaining parameters we obtain $k_\Omega\sim10^3$. The mean free path $l_e\sim10^{-4}$ cm corresponds to an impurity content of the order of 10^{-1}–10^{-2}%.

The entropy transport in the resistive case—the Ettingshausen effect—was observed by Solomon and Otter for the alloy Sn + 0.05% in the region of high temperatures $0.46\leqslant T/T_c\leqslant0.97$, although they noted an appreciable pinning of the domains in the investigated samples. The results agree in order of magnitude with the theory of Andreev and Dzhikaev, [18] which connects the magnitude of the effect with the measured values of the thermal conductivity in the n and s states under the condition $l_e\ll d$.

The Peltier effect was not investigated either theoretically (Dzhikaev's paper, [19] devoted to the case $l_e\gg d$, does not take the Hall effect into account) or experimentally. In the experiments described below, our aim was to observe the cooling effect using the Hall layer-motion mechanism.

3. EXPERIMENT

The samples were single crystals of high-purity indium in the form of cylinders of 4 mm diameter and approximate length 10 cm, grown in glass molds coated with thin layers of finely dispersed silicon dioxide to prevent sticking of the metal to the glass. The residual resistivity of the n phase of the samples in the intermediate state, determined from observations of the helicon damping (see below), was 1.4×10^{-10} Ω-cm for sample No. 1 and 5×10^{-11} Ω-cm for sample No. 2.

The instrument used to observe the thermoelectric effect is illustrated in Fig. 2. Sample 1 was soldered with pure indium to a copper cold finger constituting an integral part of the He^3 bath 2. The upper parts of the current leads, niobium-titanium wires of 0.3 mm diameter, were in good thermal contact with the He^3 bath. One of the wires was soldered with indium at the junction of the sample to the cold finger, while two others, symmetrically placed and carrying equal currents, were soldered with lead to a lead cap into which the lower end of the sample was brazed. This contact remained superconducting in the magnetic field H that caused the sample to change to the intermediate state. The field was produced by a pair of superconducting coils 4 mounted on the outer surface of the vacuum chamber 5. The field inhomogeneity over the length of the sample did not exceed 2%. The chamber was placed in a glass dewar with liquid helium. The lower end of the sample was secured with caprone strings to two stainless steel tubes soldered to the He^3 bath (not shown in the figure). The screen 6 shielded the sample from parasitic heat fluxes. Carbon thermometers 7 and 8 were soldered to the sample at distances $L_1=1$ cm and $L_2=9$ cm from its lower end. The thermometer resistance was measured with an R309 thermometer at a current 5×10^{-8} A with a nanovoltmeter F128/2 as the null indicator. The thermometers were calibrated against the vapor pressure of He^3 with a McLeod manometer. At $T\sim0.34$ K, their sensitivity was $\sim5\times10^6$ Ω/deg. To prevent mechanical vibrations from affecting the thermometer readings, the low-temperature part of the setup was fastened on a massive concrete support.

Small changes of the thermometer resistance during the time of the experiment were registered with an automatic recorder connected to the nanovoltmeter. By continuously recording the thermometer readings when the current through the sample was turned on and off made it possible to exclude the effect of the bath-temperature drift due to the change in the level of the liquid helium in the bath and in the dewar. The settling time of the temperature gradient in the sample after turning on the current was of the order of 30 sec.

The heater used to measure the thermal conductivity was mounted on the thermally insulated end of sample No. 1. In the first experiments, microcontacts were also mounted on the lateral surface of the sample[1,13] to confirm that the layers move through the sample under the influence of the current.

The temperature difference ΔT between the upper and lower thermometers at $T=0.34\pm0.01$ K is shown in Fig. 3 for different values of the current J through sample No. 2. Positive values of J correspond to motion of positive charges from the thermally insulated end of the sample. The transverse magnetic field was 0.75 H_c. The results can be approximated by the parabola (3) with $\alpha=4\cdot10^{-5}$ deg/A and $\beta=3\cdot10^{-5}$ deg/A^2, shown by the solid curve in the figure. Figure 3 shows also the value of $[\Delta T(J)-\Delta T(-J)]/2$ for samples No. 1 and

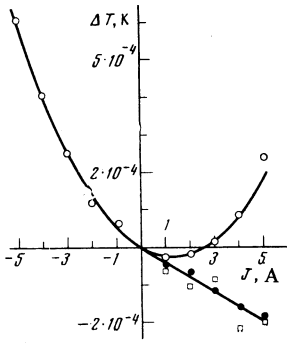

FIG. 3.

No. 2. It is seen that for both samples the entropy transport by the current leads to an approximately equal lowering of the temperature of the thermally insulated end, proportional to the current, while the ohmic resistance of the sample differs by a factor 2.8.

The values of α and β for sample No. 1 were respectively 4.5×10^{-5} deg/A and 1×10^{-4} deg/A^2. Owing to the large resistance of the sample, we observed no negative values of ΔT in this case.

4. DISCUSSION OF RESULTS

For comparison with experiment, we shall use formulas (3) to calculate the values of α and β/α. We assume $\gamma = 1.15 \cdot 10^{-4}$ J/cm^{3}deg^2 [20] and $u = 1/n_a e = 1.62 \cdot 10^{-4}$ cm^3/sec-A. This value of u is theoretical upper limit of this quantity. The experimentally observed values were $u = 1.1 \cdot 10^{-4}$ cm^3/sec-A [13] and $u = 1.45 \cdot 10^{-4}$ cm^3/sec-A. [15] The thermal conductivity of sample No. 2, unfortunately, was not measured directly, and we possess only approximate measurements of $\varkappa$ for sample No. 1 at $T = 0.35$ K, which yield the value $\lambda = 4 \cdot 10^{-2}$ W/deg^4 cm with an error on the order of ±25%. These data yield an estimate $\alpha = (3-5)\times10^{-5}$ deg/A, which agrees with the observed values of α.

The value of λ does not enter in the ratio β/α, but in this case it is necessary to know the resistance of the samples in the intermediate state. For our samples we measured the logarithmic damping decrement Δ of the helicons in the intermediate state (see [21]), and the resistivity values cited above were obtained from the relation

$$\rho_n = \frac{\Delta}{2\pi} \frac{H_c}{n_a e c}.$$

The resultant values of β/α greatly exceeded the experimental values, by 2.3 and 2.4 times for samples No. 1 and No. 2, respectively. It must apparently be assumed that the method of measuring helicon damping in cylindrical samples [21] leads in the case of indium to a more than twofold overestimate of the resistivity. The electron mean free path in our samples also seems to differ from the initial estimates and is about 10 mm for sample No. 2 as $T \to 0$.

The authors are sincerely grateful to L. M. Shpel'ter for preparing the high-grade samples, to R. K. Nikolaev and A. D. Bronnikov of the Solid State Physics Institute for supplying the high purity indium, and to A. F. Andreev and Yu. K. Dzhikaev for a discussion of the results.

[1)] It follows from this, in particular, that in the intermediate state there exists a differential thermoelectric power in a direction perpendicular to the layers:

$$Q = \Pi/T = C_n \gamma T u = C_n \gamma T c R,$$

where R is the Hall constant. The quantity Q depends linearly on C_n, but at $H = H_c$ it should decrease jumpwise to a value corresponding to the n phase (by an estimated factor 1.5 in the case of free electrons with constant mean free path). The dependence of Q on H for indium in the intermediate state, observed by Dee and Guenault [11] agrees quite well with this scheme.

[1] Yu. V. Sharvin, Pis'ma Zh. Eksp. Teor. Fiz. **2**, 287 (1965) [JETP Lett. 2, 183 (1965)].
[2] P. R. Solomon and F. A. Otter, Jr., Phys. Rev. **164**, 608 (1967).
[3] K. Mendelssohn and J. L. Olsen, Proc. Phys. Soc. A **63**, 2 (1950); Phys. Rev. **80**, 859 (1950).
[4] N. V. Zavaritskiĭ, Zh. Eksp. Teor. Fiz. 38, 1673 (1960) [Sov. Phys. JETP **11**, 1207 (1960)].
[5] A. A. Abrikosov and N. V. Zavaritskiĭ, Supplement to the Russian translation of D. Shoenberg's "Superconductivity," IIL, 1953.
[6] S. J. Laredo and A. B. Pippard, Proc. Cambr. Phil. Soc. **51**, 368 (1955).
[7] J. L. Olsen, A. Waldvogel, and P. Wyder, Helv. Phys. Acta **39**, 361 (1966).
[8] A. F. Andreev, Zh. Eksp. Teor. Fiz. **46**, 1823 (1964) [Sov. Phys. JETP **19**, 1228 (1964)].
[9] A. F. Andreev, Zh. Eksp. Teor. Fiz. **51**, 1510 (1966) [Sov. Phys. JETP **24**, 1019 (1967)].
[10] A. A. Abrikosov, Vvedenie v Teoriyu normal'nykh metallov (Introduction to the Theory of Normal Metals), Nauka, 1972.
[11] R. H. Dee and A. M. Guenault, Proc. Fourteenth Intern. Conf. on Low Temp. Phys. 2, Otaniemi, 1975, p. 129.
[12] C. J. Gorter, Physica (Utrecht) **23**, 45 (1957).
[13] Yu. V. Sharvin and I. L. Landau, Zh. Eksp. Teor. Fiz. **58**, 1943 (1970) [Sov. Phys. JETP **31**, 1047 (1970)].
[14] A. F. Andreev and Yu. V. Sharvin, Zh. Eksp. Teor. Fiz. **53**, 1499 (1967) [Sov. Phys. JETP **26**, 865 (1968)].
[15] I. P. Krylov, Zh. Eksp. Teor. Fiz. **69**, 1058 (1975) [Sov. Phys. JETP **42**, 538 (1975)].
[16] L. D. Landau and E. M. Lifshitz, Statisticheskaya fizika (Statistical Physics), Pt. 1, Nauka, 1976. [Pergamon].
[17] P. De Gennes, Superconductivity of Metals and Alloys, Benjamin, 1965.
[18] A. F. Andreev and Yu. K. Dzhikaev, Zh. Eksp. Teor. Fiz. **60**, 298 (1971) [Sov. Phys. JETP **33**, 163 (1971)].
[19] Yu. K. Dzhikaev, Zh. Eksp. Teor. Fiz. **68**, 295 (1975) [Sov. Phys. JETP 41, 144 (1975)].
[20] J. R. Clement and E. H. Quinnell, Phys. Rev. **92**, 258 (1953).
[21] I. P. Krylov, I. L. Bronevoĭ, and Yu. V. Sharvin, Pis'ma Zh. Eksp. Teor. Fiz. **19**, 588 (1974) [JETP Lett. **19**, 306 (1974)].

Translated by J. G. Adashko