Here d is the dimensionality of space. It is logical to assume that the property of conformal invariance is possessed not only by the model studied but also by other systems in the critical region. Knowledge of the phenomenological Hamiltonian (31) can turn out to be useful if the system being investigated is in weakly nonuniform external conditions.

The Ward identities (14)–(15), (17)–(18), (17′)–(18′) and (22)–(23) are useful for establishing whether any particular system possesses the conformal symmetry (2′), (3′). In general, the description of strongly fluctuating systems with the aid of a locally defined correlation length may be of interest in the case of weakly nonuniform or slowly relaxing systems.

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[1)]In the first order in $1/N$ we have $\langle\varphi\rangle = \varphi_s = r_c^{-2}$. This implies that the system can be imagined to be a set of noninteracting regions of volume r_c^2, with energy equal to $-\frac{1}{2}$ (in our notation the factor $-T$ multiplying F has been omitted), which corresponds to the usual ideas about critical fluctuations.

[2)]This follows from the fact that

$$\delta F_c = \sum_{n=1}^{\infty} \int \frac{1}{n!} \mathcal{F}^n(x_1 \ldots x_n, r_c) \prod_{i=1}^{n} \delta \ln r_c(x_i)\, d^2x_i.$$

[3)]We recall that our definition of the free energy $F[\beta]$ differs from that of the usual $F_0[T]$: $F[\beta] = -\beta F_0[1/\beta]$. Therefore, $\tilde{F}$ does not coincide with the energy.

[4)]Because of the symmetry, $\partial P^{n+1}/\partial q_\nu|_{q=0} = \delta_\nu \tilde{P}^{n+1}$.

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The phase transition in a weakly disordered uniaxial ferromagnet

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By the renormalization-group method the exact temperature dependences of the susceptibility and specific heat (above T_c in zero external field) are found for the four-dimensional Ising model with short-range exchange forces and randomly distributed, rigidly fixed impurities. The stability of the impurity fixed point in $(d = 4-\epsilon)$-dimensional space is demonstrated and the critical exponents are calculated to second order in $\epsilon^{1/2}$.

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One of the few exactly soluble realistic problems is that of the phase transition in a three-dimensional easy-axis ferromagnet (or ferroelectric of the displacement type) with dipolar interaction and randomly distributed fixed impurities, the impurity concentration being considerably below the percolation threshold. Near the transition point the indirect interaction of the critical fluctuations of the order parameter via the impurities becomes important, and as a result the behavior of all the thermodynamic quantities is greatly changed from that in the impurity-free case. Although the interaction via the impurities is attractive in sign it does not violate the stability, and, therefore, a second-order phase transition occurs in the system. The temperature dependences of the uniform susceptibility r^{-1} and specific heat C of an impure easy-axis dipolar ferromagnet (in $d=3$ dimensions) and of the four-dimensional impure Ising model with short-range exchange forces have been obtained by the renormalization-group (RG) method in a paper[1] by Aharoni ($T > T_c$, $h = 0$):

$$r \sim \tau \exp\{-(D|\ln\tau|)^{1/2}\}, \tag{1}$$

$$C \sim \exp\{-2(D|\ln\tau|)^{1/2}\}\,|\ln\tau|^{1/2}, \tag{2}$$

where $\tau = (T - T_c)/T_c$; $D = 9/(81\ln(\frac{4}{3}) + 53)$ for the impure dipolar ferromagnet ($d=3$) and $D = \frac{6}{53}$ for the impure Ising model ($d=4$). (The equation of state and the dynamics of these systems have been considered in Refs. 2 and 3.)

However, the results (1) and (2) are in need of refinement. It is shown in this paper that the true singularities of the susceptibility and specific heat are described by the formulas (41) and (43), which differ from (1) and (2) by slowly varying logarithmic factors. This refinement is of interest because it can, apparently, be

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checked experimentally. In experiments with the pure uniaxial ferromagnets $LiTbF_4$[4-6] and $GdCl_3$[7,8] logarithmic corrections to the molecular-field approximation, in brilliant agreement with the theory of Larkin and Khmel'nitskiĭ,[9] have been discovered. To calculate the exponents in the powers of logarithms in (41) and (43) it is necessary to obtain the Gell-Mann–Low (GML) function to fourth order of perturbation theory in the invariant charges (the three-loop approximation), and this is not difficult to do for the impure Ising model if we make use of the results of Refs. 10, 11 on the $(\Phi_i\Phi_i)^2$ scalar field theory (see below). For the impure uniaxial dipolar ferromagnet the calculation of the fourth-order graph presents great difficulties and has not yet been carried out; the powers of the logarithms are, therefore, unknown, but it may be hoped that the results in the two cases will turn out to be numerically close.

Knowledge of the GML function in the three-loop approximation also enables us to prove the stability of the impurity fixed point (FP) in a $(4-\varepsilon)$-dimensional space and to obtain at the same time the values of the critical exponents to second order in $\varepsilon^{1/2}$.

The order of the exposition is as follows. First, the Gell-Mann–Low equations for the invariant charges in the impure Ising model are obtained to fourth order in the amplitudes ($d=4-\varepsilon$), the impurity FP is determined to within terms of order ε, its stability is demonstrated, and the critical exponents are calculated. Then, with the aid of the RG equations, the temperature dependences of the susceptibility and specific heat ($T>T_c$, $h=0$) and the behavior of the pair correlation function in the region of small momenta at the transition point are found in the four-dimensional problem.

The Hamiltonian of the $(4-\varepsilon)$-dimensional impure Ising model has the form

$$H\{\Phi(x),\Psi(x)\}=\frac{1}{2}\int d^dx\left[r_0\Phi^2+(\nabla\Phi)^2+\frac{8\pi^2}{3}\Lambda^\varepsilon v_0\Phi^4+2\Phi^2\Psi\right], \quad (3)$$

where $\Psi(x)\sim n(x)-\langle n(x)\rangle$ is a random variable describing the local fluctuations of the temperature in the mean-field approximation; $n(x)$ is the density of impurities, r_0 is a linear function of the temperature and Λ is the momentum cutoff. In constructing a diagram technique for impurity systems it is convenient to apply the effective-Hamiltonian method.[12] The free energy of a system with the Hamiltonian (3) is a functional of the quantity $\Psi(x)$:

$$F(\Psi)=-\ln\int d\Phi\exp[-H(\Phi,\Psi)]. \quad (4)$$

The experimentally observed free energy is obtained by averaging over the possible configurations of the impurities:

$$F=-\int d\Psi\,P(\Psi)\ln\int d\Phi\exp[-H(\Phi,\Psi)], \quad (5)$$

where $P(\Psi)$ is the distribution function of the impurities. In order to integrate over the variables Ψ in (5) we represent the logarithm in the form

$$\ln\int d\Phi\exp[-H(\Phi,\Psi)]=\frac{\partial}{\partial n}\left\{\int d\Phi\exp[-H(\Phi,\Psi)]\right\}^n\Big|_{n=0}$$
$$=\frac{\partial}{\partial n}\left\{\int\prod_{i=1}^n d\Phi_i\exp\Big[-\sum_{i=1}^n H(\Phi_i,\Psi)\Big]\right\}\Big|_{n=0}. \quad (6)$$

Substituting (6) into (5), we obtain

$$F=-\frac{\partial}{\partial n}\left\{\int d\sigma\exp[-H_{eff}(\sigma)]\right\}\Big|_{n=0}, \quad (7)$$

where

$$\sigma=\{\Phi_1,\ \Phi_2,\ \dots,\ \Phi_n\},$$
$$H_{eff}=\int d^dx\left[\frac{1}{2}(\nabla\sigma)^2+\frac{1}{2}r_0\sigma^2+\frac{4\pi^2}{3}\Lambda^\varepsilon v_0\sum_{i=1}^n\sigma_i^4\right]+G(\sigma), \quad (8)$$

$$G(\sigma)=-\ln\int d\Psi\,P(\Psi)\exp\left[-\int d^dx\,\sigma^2\Psi\right]. \quad (9)$$

Near the transition point the impurity-density fluctuations can be assumed to be δ-correlated and Gaussian; consequently,

$$G(\sigma)=\frac{4\pi^2}{3}\Lambda^\varepsilon u_0\int d^dx\,(\sigma^2)^2. \quad (10)$$

Nongaussian impurity-density correlations lead to the appearance of interactions of the type $(\sigma^2)^3$, $(\sigma^2)^4$ and higher orders, which are irrelevant in $(4-\varepsilon)$-dimensional space.[13,14] As is easily seen from (8) and (10), the impure Ising model is equivalent to an n-component Heisenberg ferromagnet with cubic anisotropy in the limit when the number of components goes to zero ($n\to 0$). The Ising vertex v plays the role of the cubic vertex, the impurity vertex u plays that of the isotropic vertex, and $u_0<0$, $v_0>0$.

The RG equations for the invariant charges, describing a system with the Hamiltonian (8) at the phase-transition point $T=T_c$, have the following appearance[14,15]:

$$\frac{du}{dt}=f(u,v),\quad \frac{dv}{dt}=g(u,v), \quad (11)$$

where

$$t=\ln\frac{\Lambda^2}{p^2},\quad u=p^{-\varepsilon}Z^2\Gamma_u^{(4)},\quad v=p^{-\varepsilon}Z^2\Gamma_v^{(4)},$$
$$u|_{t=0}=u_0,\quad v|_{t=0}=v_0, \quad (12)$$

$$f(u_0,v_0)=\frac{du(u_0,v_0,t)}{dt}\Big|_{t=0},\quad g(u_0,v_0)=\frac{dv(u_0,v_0,t)}{dt}\Big|_{t=0}. \quad (13)$$

By definition, the quantity Z is expressed in terms of the pair correlator:

$$\delta_{\alpha\beta}Z(p^2)=p^2G_{\alpha\beta}(p^2),\quad G_{\alpha\beta}(p^2)=\langle\sigma_\alpha(p)\sigma_\beta(-p)\rangle, \quad (14)$$

while the quantities Γ_u and Γ_v are related to the symmetric vertex function with no external lines:

$$\Gamma^{(4)}_{\alpha\beta\mu\nu}(p^2)=\Gamma_u^{(4)}(p^2)I_{\alpha\beta\mu\nu}+\Gamma_v^{(4)}(p^2)R_{\alpha\beta\mu\nu}$$
$$\Gamma^{(4)}_{\alpha\beta\mu\nu}(p^2)=\langle\sigma_\alpha(p_1)\sigma_\beta(p_2)\sigma_\mu(p_3)\sigma_\nu(p_4)\rangle \quad (15)$$

for $p_ip_j=(\delta_{ij}-\frac{1}{4})p^2$ and $p_1+p_2+p_3+p_4=0$; here,

$$I_{\alpha\beta\mu\nu}=\delta_{\alpha\beta}\delta_{\mu\nu}+\delta_{\alpha\mu}\delta_{\beta\nu}+\delta_{\alpha\nu}\delta_{\beta\mu},\quad R_{\alpha\beta\mu\nu}=\delta_{\alpha\beta}\delta_{\alpha\mu}\delta_{\alpha\nu}.$$

We give the scheme for calculating the critical exponents. In the region of small momenta, according to scaling theory,

$$Z\sim p^{2d_\sigma+2-d}=p^\eta, \tag{16}$$

where d_σ is the scaling dimension of the field σ and η is the Fisher exponent. On the other hand, for Z we have the Lie differential equation of the renormalization group:

$$\frac{d\ln Z}{dt}=-\eta(u,v). \tag{17}$$

At infinitely large values of t the invariant charges u, v tend to a stable fixed point u^*, v^* of the system of equations (11); therefore,

$$\eta=\eta(u^*,v^*). \tag{18}$$

To calculate the critical exponent γ of the susceptibility we introduce a vertex function (with no external lines) containing the composite operator $\sigma^2(x)$:

$$\Gamma^{(1,2)}_{\alpha\beta}(p^2)=\Gamma^{(1,2)}(p^2)\delta_{\alpha\beta}=\int d^dx_1\,d^dx_2\exp(-ip_1x_1-ip_2x_2)\times\langle\sigma^2(x_1)\sigma_\alpha(x_2)\sigma_\beta(0)\rangle|_{p_ip_j=(2\delta_{ij}-1)p^2}. \tag{19}$$

In analogy with (16) and (17), we have[15]

$$\frac{d\ln\Gamma^{(1,2)}}{dt}=-\rho(u,v), \tag{20}$$

$$\Gamma^{(1,2)}(p^2)\underset{t\to\infty}{\sim}p^{d_{\sigma^2}-2d_\sigma}, \tag{21}$$

where d_{σ^2} is the scaling dimension of the composite operator $\sigma^2(x)$, connected with the exponent γ by the relation[15]

$$\gamma^{-1}=\frac{d-d_{\sigma^2}}{2-\eta}. \tag{22}$$

From (20) and (21) we obtain

$$d_{\sigma^2}=2d_\sigma+\rho(u^*,v^*). \tag{23}$$

The results of Refs. 10, 11, in which the fourth-order graphs and the GML functions to fourth order of perturbation theory were calculated in a massless theory (i.e., $T=T_c$) with the Hamiltonian (8) with $v_0=0$, can be generalized without difficulty to the case of a $(4-\varepsilon)$-dimensional system ($\varepsilon\ll1$) with cubic anisotropy $v_0\neq0$. The Gell-Mann–Low functions (11) have the form (in the impurity system: $n=0$)

$$f(u,v)=\frac{\varepsilon}{2}u-(n+8)u^2-2uv+6(3n+14)u^3+44u^2v+\frac{10}{3}uv^2-\left[\left(44-20J+2\ln\frac{3}{4}\right)n^2+\left(240\zeta(3)-248J+20\ln\frac{3}{4}+808\right)n+1056\zeta(3)-704J+32\ln\frac{3}{4}+2496\right]u^4-\left[\left(158-56J+8\ln\frac{3}{4}\right)n+768\zeta(3)-592J+40\ln\frac{3}{4}+2136\right]u^3v-\left[\left(4J+\frac{2}{3}\ln\frac{3}{4}-\frac{14}{3}\right)n+96\zeta(3)-112J+\frac{40}{3}\ln\frac{3}{4}+460.867\right]u^2v^2-\left[24.667+\frac{4}{3}\ln\frac{3}{4}\right]uv^3 \tag{24}$$

$$g(u,v)=\frac{\varepsilon}{2}v-3v^2-12uv+(10n+164)u^2v+92uv^2+\frac{34}{3}v^3-\left[(18-24J)n^2+\left(192\zeta(3)-288J+24\ln\frac{3}{4}+780\right)n+2688\zeta(3)-1632J+48\ln\frac{3}{4}+5712\right]u^3v-\left[\left(180-108J+6\ln\frac{3}{4}\right)n+2304\zeta(3)-1512J+60\ln\frac{3}{4}+5148\right]u^2v^2-\left[576\zeta(3)-432J+20\ln\frac{3}{4}+1414\right]uv^3-\left[48\zeta(3)-36J+2\ln\frac{3}{4}+124\right]v^4, \tag{25}$$

where $\zeta(3)=1.202$ is the Riemann Zeta function and $J\simeq0.75$.[10] Putting $v=0$ in (24) we obtain the GML function of the single-charge $(\Phi_i\Phi_i)^2$ model; for $\varepsilon=0$ this function coincides with that found in[10]. If $n=2$, the functions (24) and (25) satisfy the exact symmetry relations

$$f\left(u+\frac{v}{2},-v\right)\Big|_{n=2}=f(u,v)|_{n=2}+\frac{1}{2}g(u,v)|_{n=2},$$
$$g\left(u+\frac{v}{2},-v\right)\Big|_{n=2}=g(u,v)|_{n=2},$$

which are valid to all orders of perturbation theory and for any ε. In the lowest, quadratic approximation in the invariant charges the system of equations (11) for $n=0$ is degenerate and does not have stable solutions corresponding to physical initial conditions. This degeneracy is accidental and does not obtain for the exact GML functions; to remove the degeneracy it is sufficient to calculate the functions $f(u,v)$ and $g(u,v)$ to third order in the amplitudes.

We seek the impurity FP in the form

$$u^*=-A\varepsilon^{1/2}+B\varepsilon+F\varepsilon^{3/2}, \tag{26}$$

$$v^*=4A\varepsilon^{1/2}+(D-4B)\varepsilon+(E-4F)\varepsilon^{3/2}. \tag{27}$$

Substituting (26) and (27) into (24) and (25) and equating the coefficients of $\varepsilon^{3/2}$ and ε^2 to zero, we obtain two systems of equations, for A, D and B, E, respectively; solving these we find the renormalized charges u^* and v^* to terms of order ε inclusive (we draw attention to the fact that in (24) and (25) the terms quadratic in u and v are of order $\varepsilon^{3/2}$, and not ε as might appear at first glance):

$$u^*=-\frac{1}{4}\left(\frac{6\varepsilon}{53}\right)^{1/2}+\frac{165+94.5\zeta(3)}{(53)^2}\varepsilon, \tag{28}$$

$$v^*=\left(\frac{6\varepsilon}{53}\right)^{1/2}-\frac{342+378\zeta(3)}{(53)^2}\varepsilon. \tag{29}$$

To calculate the coefficient F it is necessary to take into account the terms of fifth order in the invariant charges in (24) and (25) and also the dependence on ε of the coefficients in the GML functions.

The question of the stability of the FP (26), (27) is solved by linearizing the system (11) about u^*, v^* and finding the signs of the eigenvalues λ_1 and λ_2 of the matrix

$$\begin{pmatrix}\frac{\partial f}{\partial u}&\frac{\partial f}{\partial v}\\ \frac{\partial g}{\partial u}&\frac{\partial g}{\partial v}\end{pmatrix}_{|u=u^*,\,v=v^*} \tag{30}$$

In the two-loop approximation we have

$$\begin{pmatrix} 2\left(\frac{6\varepsilon}{53}\right)^{1/2} & \frac{1}{2}\left(\frac{6\varepsilon}{53}\right)^{1/2} \\ -12\left(\frac{6\varepsilon}{53}\right)^{1/2} & -3\left(\frac{6\varepsilon}{53}\right)^{1/2} \end{pmatrix} \quad \begin{matrix} \lambda_1 = 0 + O(\varepsilon), \\ \lambda_2 = -\left(\frac{6\varepsilon}{53}\right)^{1/2} + O(\varepsilon). \end{matrix}$$

The fact that λ_1 is equal to zero is a consequence of the above-mentioned degeneracy of the RG equations in the approximation quadratic in the invariant charges; it is, of course, not rigorous, but simply indicates the fact that to determine the sign of λ_1 the two-loop approximation is not sufficient and one must calculate the next order in the expansion in $\varepsilon^{1/2}$.

From (24), (25) and (28), (29) it is not difficult to find the matrix elements in (30) to terms of order ε and to show that, in this approximation, $\lambda_1 < 0$. Thus, since both eigenvalues are negative, the impurity fixed point is a stable zero.

Expanding the functions $\rho(u, v)$ and $\eta(u, v)$ in powers of u and v and confining ourselves to second order of perturbation theory, we find

$$\gamma^{-1} = (1 - 2u - v + 12u^2 + 12uv + 2v^2)\big|_{u=u^*,\, v=v^*}, \tag{31}$$

$$\eta = \left(4u^2 + 4uv + \frac{2}{3}v^2 - 96u^3 - 144u^2v - 54uv^2 - 6v^3\right)\Big|_{u=u^*,\, v=v^*}. \tag{32}$$

We substitute u^*, v^* from (28), (29) into (31), (32) and obtain the values of the critical exponents:

$$\gamma = 1 + \frac{1}{2}\left(\frac{6\varepsilon}{53}\right)^{1/2} + \frac{147 - 189\zeta(3)}{(53)^2}\varepsilon, \tag{33}$$

$$\eta = -\frac{\varepsilon}{106} + \left(\frac{6}{53}\right)^{1/2}\frac{216 + 63\zeta(3)}{(53)^2}\varepsilon^{3/2}. \tag{34}$$

The quantities γ and η were calculated to first order in $\varepsilon^{1/2}$ in Refs. 1, 18. All the other critical exponents can be expressed in terms of γ and η using the well-known relations of scaling theory. It is interesting that three constants ($\zeta(3)$, J and $\ln(\frac{3}{4})$) appear in the Gell-Mann–Low functions while only one ($\zeta(3)$) appears in the expressions for the fixed point and critical exponents.

The RG equations of the four-dimensional impure Ising model are obtained by taking the limit $\varepsilon \to 0$ in (11), (24) and (25). The solution of these equations that corresponds to the impurity critical behavior has the form

$$u = -\frac{1}{4}\left(\frac{6}{53t}\right)^{1/2} + \frac{489 + 189\zeta(3)}{(53)^2 t}, \tag{35}$$

$$v = \left(\frac{6}{53t}\right)^{1/2} - \frac{1638 + 756\zeta(3)}{(53)^2 t}, \tag{36}$$

where

$$t = \ln(\Lambda^2/x^2), \quad x^2 = \max\{r, p^2\}.$$

Using the Lie equation (17), and also (32), (35) and (36), we obtain the behavior of the pair correlation function at small momenta at the phase-transition point:

$$G(p^2) \sim p^{-2}|\ln p^2|^{1/106}. \tag{37}$$

We note that in "pure" systems ($d = 4$) the Ornstein–Zernike approximation[9] is usually valid for $G(p^2)$:

$$G(p^2) \sim p^{-2}.$$

The inverse susceptibility r obeys the equation

$$\frac{dr}{d\tau} = T(r), \tag{38}$$

where $T(r)$ is the vertex with two external lines and one angle, for which the following representation holds:

$$T = \exp\left\{\int_0^t dt'[-2u - v + 12u^2 + 12uv + 2v^2]\right\}. \tag{39}$$

The important point is that in the exponent in (39) we cannot confine ourselves to first order in the invariant charges, as was done in Ref. 1, but must take into account the second-order terms leading to additional, logarithmic factors in the temperature dependences of the specific heat and susceptibility. Substituting (35) and (36) into (39) and integrating, we obtain

$$T \sim \exp\left\{-\left|\frac{6}{53}\ln r\right|^{1/2}\right\}|\ln r|^{\omega}, \tag{40}$$

$$r \sim \tau \exp\left\{-\left|\frac{6}{53}\ln \tau\right|^{1/2}\right\}|\ln \tau|^{\omega}, \tag{41}$$

$$\omega = \frac{580.5 + 378\zeta(3)}{(53)^2}. \tag{42}$$

The singular part of the specific heat is determined in terms of the polarization operator:

$$\frac{d\Pi(0)}{dt} \sim T^2(r), \quad C_{sing} \sim \Pi(0) \sim \exp\left\{-2\left|\frac{6}{53}\ln \tau\right|^{1/2}\right\}|\ln \tau|^{1/2+2\omega}. \tag{43}$$

The expressions obtained for the thermodynamic quantities of a disordered uniaxial ferromagnet are valid only for "weakly nonuniform" systems (the impurity concentration is considerably below the percolation threshold) and in a comparatively narrow temperature region near T_c, the size of this region being determined by the impurity concentration. In principle, an experimental study of disordered systems could be carried out on, e.g., a substance such as $LiTbF_4$ with added nonmagnetic atoms by measuring the specific heat[4] or magnetization[6] with the use of high-sensitivity optical methods.

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Temperature and magnetic-field dependences of the resistivity of Ga-doped $CdCr_2Se_4$ single crystals

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The electrical resistivity ρ of $Cd_{1-x}Ga_xCr_2Se_4$ single crystals ($x = 0.017$, 0.019, 0.037, 0.048, and 0.091) was investigated in the temperature range 4.2–300°K in magnetic fields 0–50 kOe. The dependence $\rho(T)$ was found to be nonmonotonic with a minimum below the Curie point T_c and a maximum in the region of T_c. The position of the maximum of the $\rho(T)$ curve shifted toward higher temperatures on increase of the magnetic field. In the region of the minimum, and particularly near the maximum, the behavior of $\rho(T)$ indicated a giant negative magnetoresistance: ρ in $H = 50$ kOe near T_c was one or two orders of magnitude less than ρ in $H = 0$. The observed anomalies were attributed to the presence of ferrons in this compound.

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An investigation was made of the resistivity of $Cd_{1-x}Ga_xCr_2Se_4$ single crystals ($x = 0.017$, 0.019, 0.037, 0.048, and 0.091) in a wide temperature range from 4.2 to 220°K, applying magnetic fields up to 50 kOe. The samples, application of ohmic contacts, and the method used to measure ρ were all described earlier.[1,2]

The magnetic field was produced in a superconducting solenoid. During measurements a sample was placed in an enclosure with double walls where vacuum down to 10^{-3} Torr was maintained. The necessary temperature was produced by an electric heater wound bifilarly on a single-crystal quartz rod and the sample was bonded to the end of this rod. The temperature was measured with a copper–copper-iron thermocouple.

Figure 1 shows the temperature and magnetic-field dependences of ρ obtained for a sample of $Cd_{0.983}Ga_{0.017}Cr_2Se_4$. It is clear from Fig. 1a that the temperature dependence of ρ is complex; when the temperature is increased from 4.2°K, the value of ρ first falls, passes through a minimum, and then rises steeply by about four orders of magnitude reaching a maximum in the region of the Curie point, and then falls again.

The resistivity depends strongly on the magnetic field H at temperatures beginning from the minimum and ending in the region of the maximum; the dependence

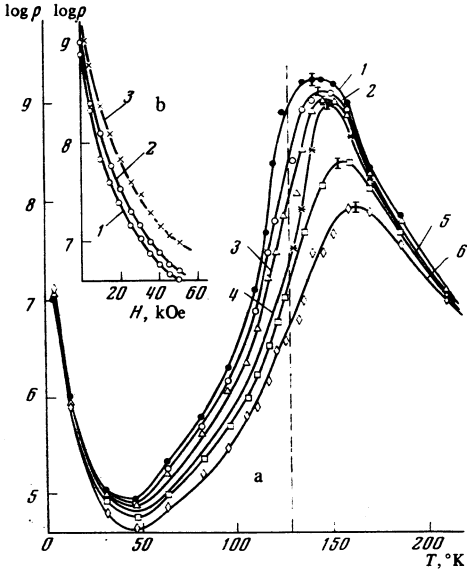

FIG. 1. Properties of $Cd_{0.983}Ga_{0.017}Cr_2Se_4$. a) Temperature dependence of the logarithm of the resistivity in various magnetic fields: 1) $H = 0$; 2) 5 kOe; 3) 10 kOe; 4) 20 kOe; 5) 30 kOe; 6) 50 kOe. The dashed line represents the temperature at which α has its maximum value. b) Dependence of log ρ on the magnetic field at various temperatures: 1) $T = 124.5$°K; 2) 128°K; 3) 132°K.

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