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Ionization instabilities of an electromagnetic wave

V. B. Gil'denburg and A. V. Kim

Institute of Applied Physics, Academy of Sciences of the USSR
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An analysis is made of instabilities of a homogeneous high-frequency discharge maintained by the field of a traveling plane electromagnetic wave in a cold gas of electronegative molecules; a low electron collision frequency is assumed. A general dispersion equation is obtained for arbitrary wave perturbations of the field and plasma density. The maximum increments are found, as well as the characteristic scales of the main types of instability, which are large-scale transverse and longitudinal modulation, backscattering, and small-scale stratification in the direction of the electric field vector.

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1. One of the important tasks in the theory of high-frequency discharges is the determination of the main types of instability of possible steady-state postbreakdown states.[1-3] We shall discuss instabilities of a homogeneous discharge maintained by the field of a traveling plane electromagnetic wave in a cold gas of electronegative molecules. The wave frequency ω is assumed to be high compared with the electron collision frequency $\nu(\omega \gg \nu)$ and the plasma is regarded as a nonabsorbing medium with a real permittivity $\epsilon = 1 - N/N_c > 0$ (N is the electron density and $N_c = m\omega^2/4\pi e^2$ is the critical value of this density). The adopted homogeneous absorption-free model makes it possible to reveal instabilities of real discharges due to perturbations of characteristic scale smaller than the wave attenuation length. A similar problem has been considered by Gurevich and Shvartsburg[2] for the opposite limiting case of $\nu \gg \omega$, $\mathrm{Re}\,\epsilon = 1$ assuming a specific type of perturbation (long-wavelength modulation in the longitudinal direction).

We shall begin from the vector wave equation for the slowly varying (with time) complex amplitude of the electric field $\mathbf{E}(\mathbf{r}, t)$

$$\Delta\mathbf{E}+\nabla\left(\frac{1}{\varepsilon}\mathbf{E}\,\nabla\varepsilon\right)+\frac{\omega^2}{c^2}\left(\varepsilon\mathbf{E}-\frac{2i}{\omega}\frac{\partial\mathbf{E}}{\partial t}\right)=0 \qquad (1)$$

and from the rate equation for the electron density

$$\frac{\partial N}{\partial t}=D\,\Delta N+(\nu_i-\nu_a)N, \qquad (2)$$

in which the frequency of ionization by electron impact ν_i is regarded as a given function of the field amplitude $[\nu_i = \nu_i(|E|)]$,[1)] whereas the diffusion coefficient D and the frequency of capture by neutral molecules ν_a, both depending much less strongly on $|E|$, are assumed to be constant.

We shall introduce dimensionless variables by the substitution:

$$\nu_a t\to t,\quad \frac{\omega}{c}\mathbf{r}\to\mathbf{r},\quad \frac{N}{N_c}\to N,\quad \frac{E}{E_c}\to E. \qquad (3)$$

Here, E_c is the amplitude (known as the breakdown value) corresponding to $\nu_i(|E|) = \nu_a$, i.e., the amplitude at which a homogeneous discharge is in equilibrium. In terms of the new variables, Eqs. (1) and (2) become

$$-2i\delta\frac{\partial\mathbf{E}}{\partial t}+\Delta\mathbf{E}+\nabla\left(\frac{1}{\varepsilon}\mathbf{E}\,\nabla\varepsilon\right)+\varepsilon\mathbf{E}=0, \qquad (4)$$

$$-\frac{\partial N}{\partial t}+L^2\,\Delta N+f(|E|)N=0. \qquad (5)$$

Here,

$$\delta=\frac{\nu_a}{\omega}\ll 1,\quad \varepsilon=1-N,\quad L=\frac{\omega}{c}\left(\frac{D}{\nu_a}\right)^{1/2}$$

is the dimensionless diffusion capture length; $f(|E|) = \nu_i/\nu_a - 1$; for $|E| = 1$, we have $f = 0$ and $df/d|E| > 0$.

Let us assume that, under steady-state conditions the field is a plane wave of unit (breakdown) amplitude $E = y_0\exp(-i\epsilon_0^{1/2}x)$, traveling along the x axis in a homogeneous plasma with an arbitrary value of $N = N_0 < 1$ ($\epsilon = \epsilon_0 = 1 - N_0 > 0$).[2)] We shall investigate the stability of this state in the presence of small perturbations. Assuming that

$$N=N_0+N_1(\mathbf{r},t),\quad E_y=(1+E_1(\mathbf{r},t))\exp(-i\varepsilon_0^{1/2}x)$$

and linearizing Eqs. (4) and (5), we obtain the following equations for the perturbations E_1 and N_1:

$$-2i\delta\frac{\partial E_1}{\partial t}+\Delta E_1-2i\varepsilon_0^{1/2}\frac{\partial E_1}{\partial x}-\frac{1}{\varepsilon_0}\frac{\partial^2 N_1}{\partial y^2}-N_1=0, \qquad (6)$$

$$-\frac{\partial N_1}{\partial t}+L^2\,\Delta N_1+\frac{1}{2}\alpha N_0(E_1+E_1^*)=0 \qquad (7)$$

(here $\alpha = df/d|E|$ for $|E| = 1$), or, representing E_1 in the form $E_1 = u_1 + iv_1$, where u_1 and v_1 are real functions:

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$$2\delta \frac{\partial v_1}{\partial t} + \Delta u_1 + 2\varepsilon_0^{1/2}\frac{\partial v_1}{\partial x} - \frac{1}{\varepsilon_0}\frac{\partial^2 N_1}{\partial y^2} - N_1 = 0, \tag{8}$$

$$-2\delta \frac{\partial u_1}{\partial t} + \Delta v_1 - 2\varepsilon_0^{1/2}\frac{\partial u_1}{\partial x} = 0, \tag{9}$$

$$-\frac{\partial N_1}{\partial t} + L^2\Delta N_1 + \alpha N_0 u_1 = 0. \tag{10}$$

For perturbations of the $\exp(\gamma t - i\boldsymbol{\varkappa}\cdot\mathbf{r})$ type, we find that the system (8)–(10) yields a cubic dispersion equation which describes the time constant γ as a function of an arbitrary real wave vector $\varkappa$:

$$(\gamma+\varkappa^2L^2)\left[\varkappa^2+\frac{4}{\varkappa^2}(\delta\gamma - i\varkappa_x\varepsilon_0^{1/2})^2\right] = \alpha N_0\left(\frac{\varkappa_y^2}{\varepsilon_0}-1\right). \tag{11}$$

The coefficient αN_0 on the right-hand side of Eq. (11) can be regarded as a parameter of the nonlinear coupling between normal solutions of the field equation (6) and of the diffusion equation for the electron density (7). If $\alpha N_0 = 0$, the roots of Eq. (11) are negative or purely imaginary ($\mathrm{Re}\gamma \leq 0$, no instabilities). If $\alpha N_0 > 0$, some roots have positive parts, i.e., the discharge becomes unstable.

2. We shall consider solutions of Eq. (11) for various directions and values of the wave vector $\boldsymbol{\varkappa}$. We shall give the dispersion equations and the approximate expressions for the roots in various ranges of the values of $\varkappa_x$, $\varkappa_y$, and $\varkappa_z$, and of the parameter αN_0 (we recall that γ and $\varkappa$ are dimensionless; the dimensional quantities are obtained by multiplying them by ν_a and ω/c, respectively).

(1) If $\varkappa = \varkappa_z$, $\varkappa_x = \varkappa_y = 0$ (perturbations along the magnetic field of the wave):

$$(\gamma+\varkappa^2L^2)(\varkappa^2+4\delta^2\gamma^2/\varkappa^2) = -\alpha N_0, \tag{12}$$

$$\gamma_1 = -\varkappa^2L^2 - \alpha N_0/\varkappa^2, \quad 2\delta|\gamma_1| \ll \varkappa^2, \tag{13}$$

$$\gamma_{2,3} = \pm i\frac{\varkappa^2}{2\delta} + \frac{\alpha N_0}{2\varkappa^2(1+4\delta^2L^4)}, \quad \delta\alpha N_0 \ll \varkappa^4\left(\frac{1}{2}+\delta L^2\right). \tag{14}$$

For small values of $\varkappa$, when $\delta\alpha N_0 \gg \varkappa^4(\frac{1}{2}+\delta L^2)^3$, we have

$$\gamma_1 = -\left(\frac{\alpha N_0\varkappa^2}{4\delta^2}\right)^{1/3}, \quad \gamma_{2,3} = \left(\frac{\alpha N_0\varkappa^2}{4\delta^2}\right)^{1/3}\left(\frac{1}{2}\pm i\frac{\sqrt{3}}{2}\right). \tag{15}$$

An instability is due to roots $\gamma_{2,3}$ whose real parts $\mathrm{Re}\gamma_{2,3}$ in the $\delta L^2 \ll 1$ case have a maximum in the region of $\varkappa \sim (\delta\alpha N_0)^{1/4}$:

$$(\mathrm{Re}\,\gamma_{2,3})_{max} \sim \mathrm{Im}\,\gamma_{2,3} \sim (\alpha N_0/\delta)^{1/2}. \tag{16}$$

(2) If $\varkappa = \varkappa_y$, $\varkappa_x = \varkappa_z = 0$ (perturbations along the electric field of the wave):

$$(\gamma+\varkappa^2L^2)\left(\varkappa^2+\frac{4\delta^2\gamma^2}{\varkappa^2}\right) = -\alpha N_0\left(1-\frac{\varkappa^2}{\varepsilon_0}\right), \tag{17}$$

$$\gamma_1 = -\varkappa^2L^2 - \frac{\alpha N_0}{\varkappa^2}\left(1-\frac{\varkappa^2}{\varepsilon_0}\right), \quad \delta|\gamma_1| \ll \frac{1}{2}\varkappa^2, \tag{18}$$

$$\gamma_{2,3} = \pm i\frac{\varkappa^2}{2\delta} + \frac{\alpha N_0(1-\varkappa^2/\varepsilon_0)}{2\varkappa^2(1+4\delta^2L^4)}, \quad \delta\alpha N_0 \ll \frac{\varkappa^4(1/2+\delta L^2)}{1-\varkappa^2/\varepsilon_0}. \tag{19}$$

In the range of small values of $\varkappa$ we have Eqs. (15) and (16). If $\varkappa^2 < \epsilon_0$, we find that the instability is associated (as in the preceding case) with the roots $\gamma_{2,3}$. However, if $\varkappa^2 > \epsilon_0$, an aperiodic instability can only arise from the root γ_1, which reaches its maximum value

$$\gamma_{1\,max} = \alpha N_0/\varepsilon_0 - 2L(\alpha N_0)^{1/2} \tag{20}$$

at the point

$$\varkappa = \varkappa_m = (\alpha N_0)^{1/4}/L^{1/2}. \tag{21}$$

This instability ($\gamma_{1\,max} > 0$) begins from the threshold value of αN_0:

$$\alpha N_0 > (\alpha N_0)_{th} = 4\varepsilon_0^2L^2, \tag{22}$$

which corresponds to $\varkappa_m = (2\epsilon_0)^{1/2}$ and if $\alpha N_0 \gg (\alpha N_0)_{th}$, it exists in a band $\epsilon_0 < \varkappa^2 < \alpha N_0/\epsilon_0 L^2$.

(3) If $\varkappa = \varkappa_x$, $\varkappa_y = \varkappa_z = 0$ (perturbation along the direction of propagation of the wave):[3]

$$(\gamma+\varkappa^2L^2)\left[\varkappa^2+\frac{4}{\varkappa^2}(\delta\gamma - i\varkappa\varepsilon_0^{1/2})^2\right] = -\alpha N_0, \tag{23}$$

$$\gamma_1 = -\varkappa^2L^2 - \frac{\alpha N_0}{\varkappa^2-4\varepsilon_0}, \quad |\varkappa^2-4\varepsilon_0| \gg \frac{8\varepsilon_0^{1/2}}{\varkappa}\delta|\gamma_1|, \tag{24}$$

$$\gamma_{2,3} = \frac{i\varkappa}{\delta}\left(\varepsilon_0^{1/2}\pm\frac{\varkappa}{2}\right) \pm \frac{\alpha N_0(\varepsilon_0^{1/2}\pm\varkappa/2)}{4\varkappa[(\varepsilon_0^{1/2}\pm\varkappa/2)^2+\delta^2\varkappa^2L^4]}, \quad \delta\alpha N_0 \ll \varkappa^4 A, \tag{25}$$

where A is the smaller of the two quantities

$$\delta L^2 + |1/2 \pm \varepsilon_0^{1/2}/\varkappa|, \quad 4\delta^2L^4 + 4(1/2 \pm \varepsilon_0^{1/2}/\varkappa)^2.$$

We note that $\gamma_1(\varkappa) = \gamma_1^*(-\varkappa)$ and $\gamma_3(\varkappa) = \gamma_2^*(-\varkappa)$, so that it is sufficient to consider the functions $\gamma_{1,2,3}(\varkappa)$ in the range $\varkappa > 0$.

When $\varkappa$ is small and

$$\delta\alpha N_0 \gg 4\varkappa(\varepsilon_0^{1/2} + \varkappa\delta L^2 + {}^1/_2\varkappa)^3,$$

the functions $\gamma_{1,2,3}$ are described by Eq. (15). For somewhat larger values of $\varkappa$, we have

$$\gamma_1 = \alpha N_0/4\varepsilon_0, \quad \delta\alpha N_0 \ll 4\varkappa\varepsilon_0^{3/2}, \quad 4\varepsilon_0\varkappa^2L^2 \ll \alpha N_0, \quad \varkappa^2 \ll 4\varepsilon_0, \tag{26}$$

$$\gamma_{2,3} = i\frac{\varkappa}{\delta}\varepsilon_0^{1/2} \pm \left(\frac{\alpha N_0\varkappa}{8\delta\varepsilon_0^{1/2}}\right)^{1/2}, \quad \varepsilon_0^{1/2}\varkappa^3 \ll \delta\alpha N_0 \ll 4\varkappa\varepsilon_0^{3/2}, \; \varkappa\delta L^2 \ll \varepsilon_0^{1/2}. \tag{27}$$

The maximum of $\mathrm{Re}\gamma_2$ in the $\epsilon_0 \sim 1$, $\delta L^2(\alpha N_0\delta)^{1/3} \ll 1$ case corresponds to $\varkappa \sim (\delta\alpha N_0)^{1/3}$ and its order of magnitude is

$$(\mathrm{Re}\,\gamma_2)_{max} \sim (\alpha^2N_0^2/\delta)^{1/3}. \tag{28}$$

The corresponding imaginary part is $\mathrm{Im}\gamma_2 \sim (\alpha N_0/\delta^2)^{1/3}$.

It is clear from Eqs. (24) and (25) that the maxima of the increments occur also in the vicinity of the point $\varkappa = \varkappa_0 = 2\epsilon_0^{1/2}$. For small values of αN_0, in the range of validity of Eq. (25), the maximum of $\mathrm{Re}\gamma_3$ is reached at the point $\varkappa = 2\epsilon_0^{1/2}(1+2\delta L^2)$, where on condition that

$\alpha N_0 \ll 4\varkappa_0^4\delta L^4$, $\delta L^2 \ll 1$, we have

$$\gamma_3 = -2i\varkappa\varepsilon_0^{1/2} L^2 + \alpha N_0/32\varepsilon_0\delta L^2. \qquad (29)$$

The width of this maximum is $|\varkappa - \varkappa_0| = 2\varkappa_0\delta L^2$. The root γ_1 corresponding to small values of αN_0 and to $\varkappa \approx \varkappa_0$ is negative [$\gamma_1 = -\varkappa^2 L^2$ for $\alpha N_0 \ll 4\varkappa_0^4\delta L^4(1+\delta L^2)$]. For values of αN_0 lying within the range $4\varkappa_0^4\delta L^4 \ll \alpha N_0 \ll 4\varkappa_0^4\delta^{-1}$, near the point $\varkappa = \varkappa_0[|\varkappa^2 - 4\epsilon_0| \ll 2(\delta\alpha N_0)^{1/2}]$, we have

$$\gamma_1 = 2i\varkappa\varepsilon_0^{1/2}/\delta + \alpha N_0/4\varkappa^2, \qquad (30)$$

$$\gamma_3 = (\alpha N_0/8\delta)^{1/2}(1-i). \qquad (31)$$

We must bear in mind that this analysis, based on the use of the functional dependence $\nu_i(|E|)$, is valid only for perturbation scales $\varkappa^{-1}$ exceeding the characteristic heat conduction length and for times $|\gamma|^{-1}$ long compared with the electron temperature relaxation time. In the case of dimensionless $\varkappa$ and γ, the corresponding conditions are

$$\varkappa\omega/c \ll \delta_T^{1/2}/l_f, \quad |\gamma|\delta \ll \delta_T\nu/\omega \ll 1, \qquad (32)$$

where δ_T is the fraction of the energy lost by an electron in a collision with a molecule and l_f is the mean free path of electrons. The inequalities (32) impose certain restrictions on the permissible range of wave numbers $\varkappa$ in the above expressions for the roots of the dispersion equation. For example, Eqs. (14) and (19) for $\gamma_{2,3}$, containing a small parameter δ in the denominator of the imaginary part, are valid only in the range $\varkappa^2 \ll \delta_T\nu/\omega \ll 1$, so that the instabilities defined by these equations should be regarded in our calculations as of the large-scale type ($\varkappa \ll 1$). Small-scale instabilities ($\varkappa > 1$) are given only by the roots γ_1 for $\varkappa = \varkappa_y$ (stratification in the direction of the vector **E**) and by the roots $\gamma_{1,2,3}$ for $\varkappa = \varkappa_x \approx 2\epsilon_0^{1/2}$ (opposite wave instability).

3. The above results allow us to identify three main types of electrodynamic instabilities in discharges.

1. The first is a large-scale instability with maximum values of the increments $\mathrm{Re}\gamma$ attained in the range of small values of $\varkappa$: in the case of transverse perturbations ($\varkappa_x = 0$) at $\varkappa \sim (\delta\alpha N_0)^{1/4}$, whereas in the case of longitudinal perturbations ($\varkappa = \varkappa_x$) at $\varkappa \sim (\delta\alpha N_0)^{1/3}$ [see Eqs. (16) and (28)]. In fact, this is an instability of electromagnetic perturbations which are almost exactly matched directly to the pump wave $\exp(-i\epsilon_0^{1/2}x)$.

2. The next is a backscattering instability for which the increment maximum [Eqs. (29) and (31)] is reached at $\varkappa \approx 2\epsilon_0^{1/2}$, i.e., near the point of matching of the perturbation wave $E_1\exp(-i\epsilon_0^{1/2}x)$ with the opposite (reflected) magnetic wave $\exp(i\epsilon_0^{1/2}x)$. Perturbations of the electron density in this instability are striations traveling parallel to the front of the electromagnetic pump wave at a velocity $v = \varkappa^{-1}\,\mathrm{Im}\gamma$ in the same direction as this wave [roots (28) and (30)] or opposite to it [roots (29) and (31)].

3. The third is a threshold aperiodic small-scale instability with increments (18) and (20), producing stationary plane striations perpendicular to the electric field vector **E**. This instability is not related to any wave matching: it is purely quasistatic and is in fact due to a plasma resonance effect, i.e., to the increase in the component of **E** normal to the plasma layer on reduction in ϵ. It is fully analogous to the well-known modulation (stratification) instability in a plasma with a "striction" nonlinearity (when $\partial\epsilon/\partial|E|^2 > 0$).[4,5] Such a small-scale instability in nonlinear media with $\partial\epsilon/\partial|E|^2 < 0$ was considered by Gil'denburg and Litvak.[6]

A comparison of the values of $\mathrm{Re}\gamma_{max}$ for various types of perturbations shows that in the range of the parameters

$$\delta L^2 \ll 1, \quad \delta\alpha N_0 \ll 1, \quad \alpha N_0/\delta \ll 4(2\varepsilon_0^{1/2}L)^4$$

the greatest increment is exhibited by the large-scale transverse instability of type 1 [$\varkappa_{y,z} \sim (\delta\alpha N_0)^{1/4}$, $\mathrm{Re}\gamma_{max} \sim (\alpha N_0/\delta)^{1/2}$]. If the last of the above three inequalities is disobeyed in the range $\alpha N_0/\delta \gtrsim 4(2\epsilon_0^{1/2}L)^4$, the increment $\mathrm{Re}\gamma_{max}$ for the type 2 instability (backscattering) becomes of the same order of magnitude.

However, we must bear in mind that in a real spatially confined discharge the type 1 large-scale instability may appear only if its longitudinal ($l_\parallel$) and transverse ($l_\perp$) dimensions are very large ($\omega\varkappa l_\parallel/c > 1$, $\omega\varkappa l_\perp/c > 1$). The type 2 opposite-wave instability is of the small-scale type (for $\epsilon_0 \approx 1$ and $\varkappa \approx 2$) but it may be important for sufficiently large values of $l_\parallel$ ($\omega c^{-1}\mathrm{Re}\gamma l_\parallel\delta > 1$), when perturbations manage to grow significantly in the time that a signal takes to pass through the discharge region. It follows that in the case of the discharges of moderate dimensions the main instability is the type 3 aperiodic small-scale one, the condition of whose appearance is [when the threshold condition (22) is satisfied] is simply that the transverse dimension of the discharge be greater than the wavelength.

We shall conclude by noting that, in addition to electrodynamic instabilities in a cold discharge for which the main processes are those involving changes in the field intensity with the electron density, in the case of discharges operating for some time the important processes may also include various "kinetic" instabilities, which have been investigated thoroughly for discharges in static fields (see, for example, the papers of Allis[7] and of Golubev *et al.*[8] as well as the literature cited there) and which are associated with changes in the rates of some elementary processes in a gas when energy is transferred from hot electrons to various degrees of freedom of molecules and ions.

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[1]We are in fact ignoring the spatial and temporal nonlocality of the dependence of the electron temperature (or other kinetic characteristics governing ν_i) on the amplitude, which imposes certain restrictions (see below) on the frequencies and wave numbers of the perturbations considered later.

[2]The electron density N and its distribution in a steady-state spatially confined discharge are established so that for given external sources the field in the discharge region is equal to

the breakdown value.[3]

[3)]The substitution $\epsilon_0 \to \epsilon_0 \cos^2\psi$ easily generalizes these results to the case when the wave vector x lies in the x, z plane and makes an angle ψ with the x axis.

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Electron paramagnetic resonance on localized magnetic moments in gapless superconductors

A. A. Kosov and B. I. Kochelaev

Kazan' State University

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The question of the coupled motion of the localized magnetic moments and the magnetic moments of the conduction electrons in type-II superconductors in the vortex state is studied by the temperature–Green-function method. The relaxation processes that occur under conditions of an "electron bottleneck" in a system of magnetic impurities and conduction electrons coupled by exchange interaction are considered. It is shown that the dynamic nature of the interaction of the localized magnetic moments with the magnetic moments of the conduction electrons leads to the narrowing of the magnetic-resonance line of the paramagnetic impurities on going from the normal to the superconducting state, whereas the existing theory, which does not take the dynamic interaction into consideration, predicts just the opposite line behavior—broadening.

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1. INTRODUCTION

The dynamic properties of the localized magnetic moments and the magnetic moments of the conduction electrons in superconducting alloys have recently been studied intensively.[1-5] In the first experiments on electron paramagnetic resonance on the magnetic moments of impurities in superconductors a broadening of the line was observed on going from the normal to the superconducting phase. The line broadening is in qualitative accord with the theory of nuclear magnetic resonance in the superconducting state. According to the Bardeen-Cooper-Schrieffer (BCS) model, the increase in the rate of relaxation of the nuclear spins in the superconducting phase occurs owing to the coherence effects and the high density of states of the quasiparticles at the energy-gap boundaries. The application of the theory of nuclear relaxation to electron relaxation is justified by the profound analogy between the phenomena of nuclear-magnetic and electron-paramagnetic resonances, with the only difference that, in the case of electron relaxation, the role of the hyperfine interactions with the conduction electrons is played by the exchange interactions.

On the other hand, a narrowing of the line was observed in the study of electron paramagnetic resonance on the magnetic moments of Er and La on going from the normal to the superconducting phase.[4] This effect is quite unexpected, since it sharply contradicts the earlier-performed experiments and the existing theory. It has been suggested[5] that the line narrowing is partially due to the dynamic nature of the interaction between the localized magnetic moments and the magnetic moments of the conduction electrons (the "electron bottleneck" effect). The magnetic resonance of paramagnetic impurities in the normal phase, including the case when the conditions for an electron bottleneck are fulfilled, has been well studied.[6-11]. As regards the theoretical study of electron paramagnetic resonance in the superconducting state, there is Maki's paper,[12] in which a computation is carried out of the dynamic response of the conduction electrons in dirty gapless superconductors. The dynamic properties of the magnetic moments of the impurities were neglected in this work.

In the present paper we study on a microscopic basis the problem of the coupled motion of the magnetic moments of the impurities and conduction electrons in type-II superconductors in the vortex state and consider the relaxation processes that occur in the system under the conditions of an electron bottleneck. These results are obtained by solving a system of equations for the dynamic susceptibilities of the magnetic moments of the impurities and conduction electrons in the superconducting state. The equations for the susceptibilities are presented in closed form in the gapless region of the

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