the breakdown value.[3]

3)The substitution $\epsilon_0 \to \epsilon_0 \cos^2\psi$ easily generalizes these results to the case when the wave vector x lies in the x, z plane and makes an angle ψ with the x axis.

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Electron paramagnetic resonance on localized magnetic moments in gapless superconductors

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The question of the coupled motion of the localized magnetic moments and the magnetic moments of the conduction electrons in type-II superconductors in the vortex state is studied by the temperature–Green-function method. The relaxation processes that occur under conditions of an "electron bottleneck" in a system of magnetic impurities and conduction electrons coupled by exchange interaction are considered. It is shown that the dynamic nature of the interaction of the localized magnetic moments with the magnetic moments of the conduction electrons leads to the narrowing of the magnetic-resonance line of the paramagnetic impurities on going from the normal to the superconducting state, whereas the existing theory, which does not take the dynamic interaction into consideration, predicts just the opposite line behavior—broadening.

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1. INTRODUCTION

The dynamic properties of the localized magnetic moments and the magnetic moments of the conduction electrons in superconducting alloys have recently been studied intensively.[1-5] In the first experiments on electron paramagnetic resonance on the magnetic moments of impurities in superconductors a broadening of the line was observed on going from the normal to the superconducting phase. The line broadening is in qualitative accord with the theory of nuclear magnetic resonance in the superconducting state. According to the Bardeen-Cooper-Schrieffer (BCS) model, the increase in the rate of relaxation of the nuclear spins in the superconducting phase occurs owing to the coherence effects and the high density of states of the quasiparticles at the energy-gap boundaries. The application of the theory of nuclear relaxation to electron relaxation is justified by the profound analogy between the phenomena of nuclear-magnetic and electron-paramagnetic resonances, with the only difference that, in the case of electron relaxation, the role of the hyperfine interactions with the conduction electrons is played by the exchange interactions.

On the other hand, a narrowing of the line was observed in the study of electron paramagnetic resonance on the magnetic moments of Er and La on going from the normal to the superconducting phase.[4] This effect is quite unexpected, since it sharply contradicts the earlier-performed experiments and the existing theory. It has been suggested[5] that the line narrowing is partially due to the dynamic nature of the interaction between the localized magnetic moments and the magnetic moments of the conduction electrons (the "electron bottleneck" effect). The magnetic resonance of paramagnetic impurities in the normal phase, including the case when the conditions for an electron bottleneck are fulfilled, has been well studied.[6-11]. As regards the theoretical study of electron paramagnetic resonance in the superconducting state, there is Maki's paper,[12] in which a computation is carried out of the dynamic response of the conduction electrons in dirty gapless superconductors. The dynamic properties of the magnetic moments of the impurities were neglected in this work.

In the present paper we study on a microscopic basis the problem of the coupled motion of the magnetic moments of the impurities and conduction electrons in type-II superconductors in the vortex state and consider the relaxation processes that occur in the system under the conditions of an electron bottleneck. These results are obtained by solving a system of equations for the dynamic susceptibilities of the magnetic moments of the impurities and conduction electrons in the superconducting state. The equations for the susceptibilities are presented in closed form in the gapless region of the superconducting phase. The Feynman-diagram technique and the pseudo-Fermi representation for the localized-spin operators[13] are used in the computation.

2. SYSTEM OF EQUATIONS FOR THE DYNAMIC SUSCEPTIBILITIES OF THE MAGNETIC MOMENTS OF THE IMPURITIES AND CONDUCTION ELECTRONS IN A SUPERCONDUCTOR

The properties of the localized magnetic moments and the conduction electrons in a superconductor in the presence of a magnetic field are described by the Hamiltonian

$$\mathscr{H}=\mathscr{H}_0+\mathscr{H}_1+\mathscr{H}_2,$$

$$\mathscr{H}_0=\sum_\alpha\int d\mathbf{r}\Psi_\alpha^+(\mathbf{r})\left[-\frac{1}{2m}(\nabla-ie\mathbf{A})^2+\frac{1}{2}\omega_e\sigma_z\right]\Psi_\alpha(\mathbf{r})+\omega_s\sum_i S_i^z,$$

$$\mathscr{H}_1=-|g|\int\Psi_+^+(\mathbf{r})\Psi_-^+(\mathbf{r})\Psi_-(\mathbf{r})\Psi_+(\mathbf{r})d\mathbf{r},$$

$$\mathscr{H}_2=J\sum_{i\alpha\beta}\Psi_\alpha^+(\mathbf{r}_i)\mathbf{S}_i\sigma_{\alpha\beta}\Psi_\beta(\mathbf{r}_i)+V\sum_{j\alpha}\Psi_\alpha^+(\mathbf{r}_j)\Psi_\alpha(\mathbf{r}_j)$$

$$+ip_0^{-2}V_{so}\sum_{\substack{k\alpha\beta\\ uvw}}(\nabla_u\Psi_\alpha^+(\mathbf{r}_k))\sigma_{\alpha\beta}^w(\nabla_v\Psi_\beta(\mathbf{r}_k))\varepsilon_{uvw},$$

where $\mathscr{H}_0$ is the Hamiltonian of the free motion of the conduction electrons and the localized spins in the magnetic field H, $\mathscr{H}_1$ is the BCS interaction, $\mathscr{H}_2$ determines the interaction of the conduction electrons with the magnetic and nonmagnetic impurities, $\Psi_\alpha^+(\mathbf{r})$ and $\Psi_\alpha(\mathbf{r})$ are the electron field operators, ω_s and ω_e are the Zeeman frequencies of the magnetic impurities and the conduction electrons, S_i is the spin operator of the i-th ion (here we consider only the $S=\frac{1}{2}$ case), J is the exchange integral, V and V_{s0} are the constants of the potential and spin-orbit scattering of the conduction electrons on the impurities, $|g|$ is the BCS coupling constant, and ϵ_{uvw} is an antisymmetric tensor.[12] The indices i, j, k respectively run through the locations of the magnetic impurities, the impurities with the Coulomb potential, and the impurities with the spin-orbit potential.

The transverse dynamic susceptibility is expressible, within the framework of linear-response theory, in terms of the retarded Green function for the magnetic-moment operators:

$$\chi^{+-}(\omega)=\langle[M^+M^-]\rangle,$$
$$M^\pm=M_s^\pm+M_e^\pm=g_s\mu_B\sum_i S_i^\pm+{}^1/_2 g_e\mu_B\sigma_{q=0}, \qquad (1)$$

g_s and g_e are the g factors of the magnetic moments of an impurity and a conduction electron respectively.

The function (1) can be represented in the form

$$\chi^{+-}(\omega)=\chi_s^{+-}(\omega)+\chi_e^{+-}(\omega),$$
$$\chi_s^{+-}(\omega)=\langle[M_s^+M^-]\rangle,\quad \chi_e^{+-}(\omega)=\langle[M_e^+M^-]\rangle. \qquad (2)$$

We shall calculate the correlation functions $\langle[M_s^+M^-]\rangle$ and $\langle[M_e^+M^-]\rangle$ on the basis of the Feynman-diagram technique, for which purpose let us, following Abrikosov,[13] introduce the pseudo-Fermi representation for the localized-spin operators

$$\mathbf{S}_i=\sum_{\sigma\sigma'}c_\sigma^+(i)\mathbf{S}_{\sigma\sigma'}c_{\sigma'}(i),$$

where c_σ^+ and c_σ are the field operators of the spin-Fermions and the $\mathbf{S}_{\sigma\sigma'}$ are the Pauli matrices.

Let us introduce the following averages

$$C_{\sigma\gamma}(i,j)=-\langle Tc_\sigma(i)c_\gamma^+(j)\rangle,\quad G_{\alpha\beta}(x,y)=-\langle T\Psi_\alpha(x)\Psi_\beta^+(y)\rangle,$$
$$F_{\alpha\beta}(x,y)=\langle T\Psi_\alpha(x)\Psi_\beta(y)\rangle,\quad F_{\alpha\beta}^+(x,y)=\langle T\Psi_\alpha^+(x)\Psi_\beta^+(y)\rangle.$$

Here the $C_{\sigma\gamma}(i,j)$ are single-particle Green's functions for the impurities. Diagrammatically, we shall denote the $C_{\sigma\gamma}(i,j)$ by dashed lines. The $G_{\alpha\beta}(x,y)$ are single-particle Green's functions for the conduction electrons. In diagrams we represent these functions by a continuous line with two arrows in the same direction. The functions $F_{\alpha\beta}(x,y)$ and $F_{\alpha\beta}^+(x,y)$ are specific—for the superconducting state—conduction electron Green's functions. Diagrammatically, we shall represent these functions by a continuous line with two arrows pointing inwards or outwards.

The derivations of the diagrammatic equations satisfied by the functions $\langle[M_s^+M^-]\rangle$ and $\langle[M_e^+M^-]\rangle$ are presented in Fig. 1a and b, respectively. The diagrammatic equations have been constructed up to second order in the coupling constants J, V, and V_{s0}. As can be seen from Fig. 1, to find the quantities $\langle[M_s^+M_-]\rangle$ and $\langle[M_e^+M^-]\rangle$, it is, generally speaking, necessary to know three new quantities that differ in the diagrams in the directions of the arrows. The derivation of the equations for these quantities is carried out in the same way as the derivation of the equation for the function $\langle[M_e^+M^-]\rangle$. The meanings of the terms in the diagrammatic equations can be given as follows. The first term on the right-hand side of the equation in Fig. 1a is the susceptibility of the magnetic impurities without allowance for the interaction with the conduction electrons. The second and third terms, which are corrections of second order in the exchange interaction J, are responsible for the relaxation pro-

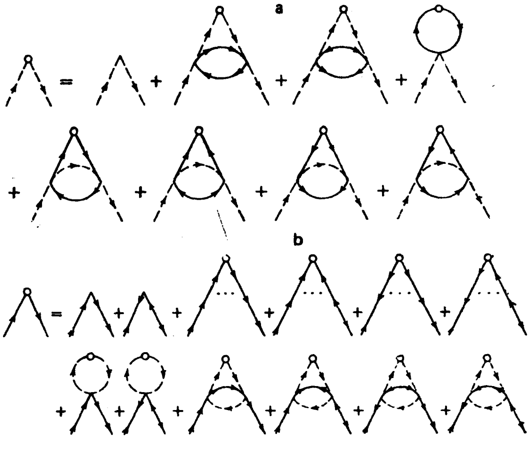

FIG. 1. Diagrammatic equations for the coupled dynamic spin susceptibilities of the magnetic impurities, (a), and the conduction electrons, (b), in superconductors.

cess in which no transfer of spin excitations occurs between the magnetic impurities and the spin system of the conduction electrons. Such a process will be dominant in the case when the magnetic moments of the impurities and the conduction electrons differ considerably in magnitude. The fourth term represents an interaction that is of first order in J. The fifth to the eighth diagrams, which represent off-diagonal interactions of second order in J, describe the transfer of spin excitations between the magnetic-impurity and conduction-electron systems. The contribution of such diagrams is important in the case when the magnetic moments of the impurities and the conduction electrons are close in magnitude. In the study of the relaxation processes under conditions of an electron bottleneck, these diagrams have the dominant value. The diagrams shown in Fig. 1b can be interpreted in similar fashion. Notice only that the third to the sixth terms are corrections that are of second order in both the exchange interaction J and the interactions V and V_{s0}. In Fig. 1b these terms have, for brevity, been combined.

All the Green functions entering into the diagrammatic equations are dressed in their self-energy parts, which have been computed in the Born approximation. In computing the self-energy parts (and, subsequently, in solving the equations for the susceptibilities), we assumed that the impurities were arranged quite randomly and that there was no correlation between the magnetic impurities.

3. COMPUTATION OF THE DYNAMIC SUSCEPTIBILITIES

The equations given in Fig. 1 determine the dynamic susceptibilities of the magnetic moments of the impurities and conduction electrons in the superconducting state in an arbitrary temperature interval. It is impossible to find the solution to the equations in the general form because of complications that arise in the evaluation of the frequency sums. However, in the temperature range $T_c - T \ll T_c$, which is characterized by gapless superconductivity,[14] a formal expansion of the equation in power series in the superconducting order parameter Δ is possible, and this allows the summation over the frequencies to be carried out quite easily (see Ref. 12). Integrating the analytic expressions over the conduction-electron momenta, and carrying out the summation over the frequencies, we obtain for the susceptibilities in bulk gapless superconductivities the following equations:

$$\begin{gathered}
[\omega_s(1+\lambda\chi_e^0)-\omega]\chi_s^{+-}(\omega)=g_sM_s^z(1+\lambda\chi_e^{+-}(\omega))\\
+\frac{i}{T_{se}}[\chi_s^{+-}(\omega)-\chi_s^0(1+\lambda\chi_e^{+-}(\omega))]-\frac{g_s}{g_e}\frac{i}{T_{es}}[\chi_e^{+-}(\omega)-\chi_e^0(1+\lambda\chi_s^{+-}(\omega))],\\
[\omega_e(1+\lambda\chi_s^0)-\omega]\chi_e^{+-}(\omega)=g_eM_e^z(1+\lambda\chi_s^{+-}(\omega))+i(1/T_{es}+1/T_{so})\\
\times[\chi_e^{+-}(\omega)-\chi_e^0(1+\lambda\chi_s^{+-}(\omega))]-\frac{g_e}{g_s}\frac{i}{T_{se}}[\chi_s^{+-}(\omega)-\chi_s^0(1+\lambda\chi_e^{+-}(\omega))],
\end{gathered} \tag{3}$$

where $M_s^z=\omega_s\chi_s^0$, $M_e^z=\omega_e\chi_e^0$, χ_s^0 and χ_e^0 are the static susceptibilities of the impurities and the conduction electrons:

$$\chi_s^0=c_Mg_s^2S(S+1)/3T,$$

$$\chi_e^0=\frac{1}{2}g_e^2N_0\left[1+\frac{e(1-T/T_c(H))S_1(t)}{8\pi^2T_c(H)\sigma\{[2k_2^2(t)-1]\beta_A+n\}}\frac{\Psi^{(2)}(1/2+\rho)}{\Psi^{(1)}(1/2+\rho)}\right],$$

$$\frac{1}{T_{se}}=4\pi TJ^2N_0^2\left[1+\frac{1-T/T_c(H)}{4\pi\sigma D\{[2k_2^2(t)-1]\beta_A+n\}}S_3(t)\right],$$

$$\frac{1}{T_{es}}=\frac{8}{3}c_M\pi J^2\langle S_z^2\rangle N_0\left[1+\frac{1-T/T_c(H)}{4\pi\sigma D\{[2k_2^2(t)-1]\beta_A+n\}}S_2(t)\right],$$

$$\frac{1}{T_{so}}=\frac{4}{3\tau_{so}}\left[1-\frac{1-T/T_c(H)}{4\pi\sigma D\{[2k_2^2(t)-1]\beta_A+n\}}S_1(t)\right],$$

$\lambda=2J/g_eg_s$ is the molecular-field constant,[10,11] $\Psi^{(1)}(z)$ and $\Psi^{(2)}(z)$ are trigamma and tetragamma functions, $\rho=DeH_{c2}/2\pi T$, $D=v_F^2\tau/3$ is the diffusion coefficient, H_{c2} is the upper critical magnetic field, N_0 is the density of states of the conduction electrons at the Fermi surface,

$$\frac{1}{2\tau}=c_1\pi N_0|V|^2,\quad \frac{1}{2\tau_{so}}=c_2\pi N_0|V_{so}|^2\int\frac{\sin^2\theta\,d\Omega}{4\pi}.$$

Further, c_M, c_1, and c_2 are respectively the concentrations of the magnetic impurities, the impurities with a Coulomb potential, and the impurities with a spin-orbit potential, $\sigma=Ne^2\tau_{tr}/m$ is the conductivity in the normal state, $k_2(t)$ is the second Ginzburg-Landau parameter,[15] n is the demagnetizing factor,[16] $\beta_A=1.16$, $T_c(H)$ is the transition temperature in the magnetic field, $t=T_c(H)/T_{c0}$, and T_{c0} is the transition temperature in the absence of a magnetic field. Finally,

$$\begin{gathered}
S_1(t)=\{[\rho\Psi^{(1)}(1/2+\rho)]^{-1}-1\},\\
S_2(t)=S_1(t)\{1+2\rho\Psi^{(2)}(1/2+\rho)/\Psi^{(1)}(1/2+\rho)\},\\
S_3(t)=S_1(t)\{1+3\rho\Psi^{(2)}(1/2+\rho)/\Psi^{(1)}(1/2+\rho)\}.
\end{gathered}$$

The functions $S_1(t)$, $S_2(t)$, and $S_3(t)$ are shown in Fig. 2.

From Eqs. (2) and (3) it is easy to derive an expression for the dynamic susceptibility, $\chi^{+-}(\omega)$, which describes the total response of the system of magnetic impurities and exchange-interaction-coupled conduction electrons in type-II superconductors that are in the vortex state in a magnetic field of strength slightly lower than the upper critical field H_{c2},

$$\chi^{+-}(\omega)=\frac{(\varepsilon_s-\omega)\eta_e+\xi_s\eta_e+(\varepsilon_e-\omega)\eta_s+\xi_e\eta_s}{(\varepsilon_s-\omega)(\varepsilon_e-\omega)-\xi_e\xi_s}. \tag{4}$$

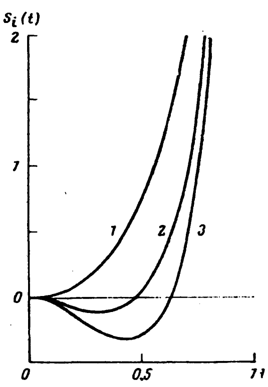

FIG. 2. The functions $S_{i=1,2,3}(t)$, which arise in the computation of the magnetic-resonance linewidth of paramagnetic impurities in gapless superconductors: 1—$S_1(t)$, 2—$S_2(t)$, 3—$S_3(t)$.

Here

$$\varepsilon_s=\omega_s(1+\lambda\chi_e^0)-i\frac{1}{T_{se}}-i\lambda\frac{g_s}{g_e}\frac{1}{T_{es}}\chi_e^0,$$

$$\varepsilon_e=\omega_e(1+\lambda\chi_s^0)-i\left(\frac{1}{T_{es}}+\frac{1}{T_{so}}\right)-i\lambda\frac{g_e}{g_s}\frac{1}{T_{se}}\chi_s^0,$$

$$\xi_s=\lambda g_s M_s^z-i\frac{g_s}{g_e}\frac{1}{T_{es}}-i\lambda\frac{1}{T_{se}}\chi_s^0,$$

$$\xi_e=\lambda g_e M_e^z-i\frac{g_e}{g_s}\frac{1}{T_{se}}-i\lambda\ \frac{1}{T_{es}}+\frac{1}{T_{so}}\Big)\chi_e^0,$$

$$\eta_s=g_s M_s^z-i\frac{1}{T_{se}}\chi_s^0+i\frac{g_s}{g_e}\frac{1}{T_{es}}\chi_e^0,$$

$$\eta_e=g_e M_e^z-i\left(\frac{1}{T_{es}}+\frac{1}{T_{so}}\right)\chi_e^0+i\frac{g_e}{g_s}\frac{1}{T_{se}}\chi_s^0.$$

4. DISCUSSION OF THE RESULTS

With the equations for the dynamic susceptibilities, (3), may be set in correspondence the following equations for the magnetization:

$$\frac{d\mathbf{M}_s}{dt}=g_s[\mathbf{M}_s\times(\mathbf{H}_{int}+\lambda\mathbf{M}_e)]-\frac{1}{T_{se}}(\mathbf{M}_s-\chi_s^0(\mathbf{H}_{int}+\lambda\mathbf{M}_e))$$
$$+\frac{g_s}{g_e}\frac{1}{T_{es}}(\mathbf{M}_e-\chi_e^0(\mathbf{H}_{int}+\lambda\mathbf{M}_s)),\tag{5}$$
$$\frac{d\mathbf{M}_e}{dt}=g_e[\mathbf{M}_e\times(\mathbf{H}_{int}+\lambda\mathbf{M}_s)]-\left(\frac{1}{T_{es}}+\frac{1}{T_{s0}}\right)(\mathbf{M}_e-\chi_e^0(\mathbf{H}_{int}+\lambda\mathbf{M}_s))$$
$$+\frac{g_e}{g_s}\frac{1}{T_{se}}(\mathbf{M}_s-\chi_s^0(\mathbf{H}_{int}+\lambda\mathbf{M}_e)),$$

where $\mathbf{M}_s$ and $\mathbf{M}_e$ are the magnetizations of the impurities and conduction electrons and $\mathbf{H}_{int}$ is the magnetic field inside the sample.

The equations of motion of the magnetization in the superconducting phase, (5), have the same form as the equations in the normal phase.[9,10,11] Notice that the Bloch equations (5) have been written in a form that determines the relaxation of the magnetization to its equilibrium magnitude in the internal instantaneous field. In the normal phase, such a form of the equations has been considered in detail by Cottet *et al.*[10] It follows from the Eqs. (5) that the detailed-balance relation is, as was expected, also valid in the super-conducting phase:

$$T_{se}/T_{es}=g_e^2\chi_e^0/g_s^2\chi_s^0.$$

The line width of the electron paramagnetic resonance on magnetic impurities in the superconducting phase can, in the case of low paramagnetic-impurity concentrations, be represented in the form of two main contributions: a homogeneous broadening due to $(s-d)$ or $(s-f)$ exchange and a broadening due to the inhomogeneous distribution of the magnetic field in the sample when $H_{c1}<H<H_{c2}$.

The broadening caused by the inhomogeneous vortex field can easily be separated from the total width (see Ref. 17). The homogeneous width can be found from the relation (4). Since it is difficult to determine the general expression for the line width, let us consider the two limiting cases:

case (a)

$$1/T_{se}+1/T_{es}\ll 1/T_{so},\ |\omega_e-\omega_s|,$$

case (b)

$$1/T_{se}+1/T_{es}\gg 1/T_{so},\ |\omega_e-\omega_s|.$$

The case (a) has been well studied in a number of papers. The smallness of the exchange-interaction energy as compared to the energy of the spin-orbit scattering of the conduction electrons on the impurities leads to a situation in which a coupled motion of the spins of the impurities and conduction electrons does not arise. Owing to the smallness of the terms responsible for the transfer of magnetization between the systems, the equations for the susceptibilities $\chi_s^{+-}(\omega)$ and $\chi_e^{+-}(\omega)$ turn out to be independent of each other. This means that the response of the magnetic impurities and that of the conduction electrons can be studied separately. The susceptibility $\chi_e^{+-}(\omega)$, which determines the response of the conduction electrons in dirty gapless superconductors, has been studied by Maki.[12] In computing the relaxation rates, $1/T_{es}$ and $1/T_{s0}$, we refined the results of this work, which contains minor algebraic errors. The linewidth of the electron paramagnetic resonance on the magnetic moments of the impurities, which is determinable from the susceptibility $\chi_s^{+-}(\omega)$, can, as in the case of nuclear magnetic resonance, be expressed in terms of the Korringa relaxation rate. On going from the normal to the superconducting state, the Korringa relaxation rate increases, owing to the coherence effects and the high density of states of the quasiparticles at the energy-gap boundaries. The line broadening that was observed to occur in the first experiments on the magnetic resonance of paramagnetic impurities on going into the superconducting phase[1-3] is in good agreement with this fact.

The case (b) corresponds to the existence in the system of electron-bottleneck conditions. Owing to the fact that the energy of the exchange interaction of the d-(f)-electrons of the magnetic shell of an ion with the conduction s electrons is significantly higher than the conduction electron spin-lattice interaction energy, which is due to the scattering of the conduction electrons on the impurities with a spin-orbit potential, and owing to the closeness of the resonance frequencies, there arises a coupled motion of the localized magnetic moments and the magnetic moments of the conduction electrons. The system of magnetic impurities and conduction electrons is characterized in this case by a combined response, since the conduction-electron resonance is also excited whenever there is localized-spin resonance. The effective linewidth of the magnetic resonance under electron-bottleneck conditions is easily determined from Eq. (4) with the aid of the inequalities (b):

$$\left(\frac{1}{T_1}\right)_{\text{eff}} = \frac{4}{3\tau_{s0}}\left[1 - \frac{(1-T/T_c(H))S_1(t)}{4\pi\sigma D\{[2k_2^2(t)-1]\beta_A+n\}}\right] \times \left(1+\frac{g_e^2\chi_s^0}{g_s^2\chi_e^0}\right)^{-1}. \quad (6)$$

Since $S_1(t)$ is a positive-definite function for all t, the conclusion can be drawn from Eq. (6) that, under electron-bottleneck conditions, the linewidth of the magnetic resonance of the paramagnetic impurities decreases on going from the normal to the superconducting phase. Such a behavior of the magnetic-resonance linewidth is exactly contrary to the behavior of the linewidth in the case (a). If in case (a) broadening of the line occurs on going to the superconducting phase, then in case (b) the existence in the system of electron-bottleneck conditions leads to the narrowing of it.

The experimentally observed narrowing of the electron paramagnetic resonance line for the magnetic moments of Er and La on going from the normal to the superconducting phase is partially explained apparently by the dynamical character of the interaction between the magnetic impurities and the conduction electrons.

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Certain effects related to the appearance of a temperature superlattice in a semiconductor with hot electrons

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Certain transport phenomena arising in a semiconductor with a temperature superlattice as a result of the heating of the electron gas by radiation are studied. Three effects are predicted which are due to free convection in the electron gas in the presence of a weak electric field perpendicular to the luminous flux: the variation of the effective electrical conductivity in the direction of the field, the appearance of a potential difference between the illuminated and shady sides of the sample, and the appearance of a current in the direction perpendicular to the luminous flux and the field.

PACS numbers: 72.20.—i

§1. INTRODUCTION

It has been shown earlier[1,2] that the heating of the electron gas during the illumination of a sample can lead to the electronic analog of the well-known Bénard effect in hydrodynamics: under certain conditions there should arise steady convection of the carriers at a rate, u, that is a periodic function of the coordinates x and y (the direction of the luminous flux is chosen as the z axis). In this case the electron-temperature and (with a smaller amplitude) the electron-density distributions also become periodic. The constant of the resulting superlattice depends on the intensity of the heating light. In other words, there should arise in the sample a distinctive diffraction grating with a controllable spacing (diffraction can be experienced by other electro-magnetic waves, as well as by the heating light itself when it has the appropriate wavelength and the nonlinear effects are taken into consideration). It would be interesting, however, to ascertain what other consequences admitting of experimental verification the appearance of convection in the electron liquid leads to. Some of these consequences are studied in the present paper.

As before,[1,2] we shall consider a material with unipolar conductivity, all the formulas being written out for positively charged particles (which, however, does not prevent us from calling them electrons). This ap-

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