dicated concentration. A $\Delta x=0.3\%$ supercooled solution was obtained by Landau *et al.*[14] at $P=0.5$ atm and $T=0.03$ K. In an experiment performed by Watson *et al.*[2] a 9.28% solution, after the disappearance of the interphase boundary at $P=7$ atm, again split up into two phases at $P=16$ atm on the stratification line. However, as was noted by the authors, the mixture did not hold out until full equilibrium was reached.[2] Unfortunately, as far as we know, no systematic experimental investigations of the considered phenomenon with whose results we could have compared the results of the present work have so far been performed.

The experimental investigation of the phenomenon involves the necessity to penetrate fairly far into the metastable region. This can, apparently, be done by forcing the solution to flow out slowly from the vessel through a "porous membrane" (a capillary or a system of capillaries). In this case He^4 will mainly flow out, and the solution in the vessel will grow richer in He^3. This corresponds to displacement parallel to the horizontal axis in the x-T plane.

In conclusion, we express our gratitude to A. F. Andreev and S. T. Boldarev for useful discussions.

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1) Strictly speaking, for $0<T<T_c$ there exists a temperature region in which, for high supersaturations, the mean free path, $\lambda\approx\hbar v_F\epsilon_F T^{-2}$, of the Fermi excitations turns out to be large compared to the nucleus dimensions (i.e., $\lambda\gg R$) and in which, consequently, the hydrodynamic description of the normal component is incorrect. It can be shown, however, that allowance for this circumstance changes the result insignificantly, and the hydrodynamic approximation yields virtually the correct answer in the dangerous region also.

2) The rate, $W(\Delta x,T)$, of nucleation contains the dimensional factor $N_0\omega_0$. In the equation $W(\Delta x,T)=1$ and the formulas (8) following from it, the value of $N_0\omega_0$ is taken in the cgs system, which corresponds to the characteristic scales of the quantities for the experiments in this region and gives an enormous value: $\ln N_0\omega_0\approx 80$. Therefore, a change of the time scale by several orders of magnitude virtually does not change the position of the "rapid-nucleation" curve.

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Quasitwo-dimensional electron-hole liquid in strong magnetic fields

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A study is made of the formation of an electron-hole liquid in quasitwo-dimensional systems in magnetic fields strong enough to ensure that the one-particle spectrum is completely discrete. Thermodynamic functions of this liquid are also calculated. In contrast to other systems, the exchange interaction is sufficient to form a liquid in the case considered and the correlation effects associated solely with virtual transitions to higher levels are negligible. The phase diagram is obtained and a "van der Waals" theory of liquid-gas phase transitions is developed for the investigated system. The possibility of experimental observation of the predicted effects is discussed.

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The behavior of electron and electron-hole systems in strong magnetic fields is of considerable interest. In particular, there have been many investigations of the kinetic properties (see, for example, [1-4]) and Wigner crystallization[5-8] of quasi two-dimensional electron systems in strong magnetic fields. We shall consider the formation of a liquid in a quasi two-dimensional electron-hole system in a transverse strong magnetic field. The system may be, for example, a quantized semimetal film with equilibrium electrons e and holes

h, a semiconductor film with nonequilibrium e and h, or a layer semiconductor. We shall show that such systems have properties basically different from those of three-dimensional e–h systems in strong magnetic fields H, investigated earlier.[9,10] Some of the results given below have already been published in a brief communication.[11]

We shall assume that

$$r_H \ll a_{e,h}, \tag{1}$$

i.e., that the characteristic energy $\omega_{e,h}$ of particles in a magnetic field is considerably higher than the characteristic energy of the Coulomb interaction whose order of magnitude, as shown below, is $\tilde{e}^2/r_H$; here, $\hbar=1$, $r_H=(c/eH)^{1/2}$ is the magnetic length, $a_{e,h}=1/m_{e,h}\tilde{e}^2$ are the effective Bohr radii of an electron and a hole, $m_{e,h}$ are the effective masses, $\omega_{e,h}=eH/m_{e,h}c$ are the cyclotron frequencies, $\tilde{e}^2=e^2/\varkappa$, and $\varkappa$ is the effective permittivity (for example, in the case of a quantized film, this is the permittivity of the ambient medium).

Quantization of the Landau spectrum of noninteracting particles in a quasi two-dimensional system makes the spectrum completely discrete and, for $r_s > r_H \cdot 2^{1/2}$, all the particles are in the lowest level since the multiplicity of degeneracy of (each) level is $L_xL_y/2\pi r_H^2$;[12] here, $r_s^2=L_xL_y/\pi N$, where N is the number of electrons equal to the number of holes, and L_x and L_y are the dimensions of the system. Thus, the motion of the particles in such a system is "quasi zero-dimensional."

We shall show that the system under discussion has the following properties.

1. Since the particles do not have kinetic energy, even the exchange attraction is sufficient to form an e–h liquid. The exchange attraction splits the system into drops, each of which is compressed to a density such that all the vacancies in the lowest level are occupied (i.e., $r_s=r_H\cdot 2^{1/2}$) but further compression is not favored because of a strong rise in the energy associated with the filling of the next level. This "zero-dimensional" aspect makes the properties of the liquid independent of m_e and m_h.

2. The correlation energy is associated only with virtual transitions to the next levels and in strong fields H the energy of a two-dimensional system falls as $H^{-1/2}$, whereas the exchange energy rises as $H^{1/2}$, so that the Hartree–Fock approximation is asymptotically exact in the limit $H\to\infty$. (It should be noted that the converse is true of a three-dimensional system in a strong field H: the formation of a liquid is due to the correlation effects and the exchange energy is negligible near the equilibrium density.[10])

3. In view of the negligible influence of the correlation, there is no charge screening and collective excitations are practically indistinguishable from one-particle excitations.

4. If the initial density (in $H=0$) is so high that the n levels are filled in sufficiently strong fields, a self-bound liquid forms at the highest of the levels.

5. In contrast to conventional Fermi systems, whose specific heat obeys $c_v \propto T$, the system under consideration is characterized by $c_v \propto \exp(-\omega_{e,h}/T)$ for $T<\omega_{e,h}$ if the correlations are ignored; this is due to the absence of a Fermi surface for such a degenerate system.

We shall consider (see §§1–3) a purely two-dimensional e–h system in a strong magnetic field at finite temperatures. In §1, we shall deal with the properties of the system in the Hartree–Fock approximation. In §2, we shall calculate the correlation corrections which are negligible if $r_H \ll 10a_{e,h}$, i.e., for a condition weaker than that given by Eq. (1). In §3, we shall study the thermodynamic properties of the system and develop a "van der Waals" theory of liquid–gas phase transitions in the system. In §4, we shall show that the results obtained are also applicable to quasi two-dimensional systems if the distances between the transverse quantization levels are large enough. In the Conclusions, we shall consider the conditions under which the predicted effects may be observed experimentally.

§1. HARTREE-FOCK APPROXIMATION

Following Eq. (1), we shall take the zeroth approximation to be a noninteracting electron–hole gas in a magnetic field. We shall consider the Coulomb interaction by means of the temperature diagram technique.[13] We are interested in the energy of the system in the limit $T\to 0$ and its thermodynamic characteristics at $T\neq 0$. Since the degree of degeneracy of the system is infinite, we cannot apply the conventional diagram methods at $T=0$ because, for example, it is not clear how to define consistently the zeroth-approximation Green functions. In the thermodynamic technique, there is no such difficulty because averaging is carried out over a Gibbs ensemble. All the results obtained are valid at a temperature as low as we please, with the exception of the nonphysical isolated point $T=0$.

We shall confine our attention to the temperatures at which the Landau levels are not mixed:

$$T \ll \omega_{e,h}. \tag{2}$$

We shall ignore the impurity-induced broadening of the Landau levels.

In the Landau gauge ($A_x=-Hy, A_y=A_z=0$), the Hamiltonian is

$$\mathscr{H}=\mathscr{H}_0+\mathscr{H}_{int},$$

$$\mathscr{H}_0=\frac{1}{2m_e}\int\psi_1^+(\mathbf{r})\left[\hat{\mathbf{p}}+\frac{e}{c}\mathbf{A}\right]^2\psi_1(\mathbf{r})\,d^2\mathbf{r}+\frac{1}{2m_h}\int\psi_2^+(\mathbf{r})\left[\hat{\mathbf{p}}-\frac{e}{c}\mathbf{A}\right]^2\psi_2(\mathbf{r})\,d^2\mathbf{r},$$

$$\mathscr{H}_{int}=\frac{1}{2}\int\frac{e^2}{|\mathbf{r}_1-\mathbf{r}_2|}\{\psi_1^+(\mathbf{r}_1)\psi_1^+(\mathbf{r}_2)\psi_1(\mathbf{r}_2)\psi_1(\mathbf{r}_1)+\psi_2^+(\mathbf{r}_1)\psi_2^+(\mathbf{r}_2)\times\psi_2(\mathbf{r}_2)\psi_2(\mathbf{r}_1)-2\psi_1^+(\mathbf{r}_1)\psi_2^+(\mathbf{r}_2)\psi_2(\mathbf{r}_2)\psi_1(\mathbf{r}_1)\}\,d^2\mathbf{r}_1\,d^2\mathbf{r}_2.$$

Here, ψ_1^+, ψ_2^+ and ψ_1, ψ_2 are the creation and annihilation operators of electrons and holes, respectively. The Hamiltonian does not include the interaction of spins with the magnetic field and it is simplified by omitting the spin indices: since $\mathscr{H}_0$ and $\mathscr{H}_{int}$ commute with the spin operator, the motion of particles with different spins can be considered independently.

We shall first allow for the interaction of particles in the Hartree-Fock approximation and, in §2, we shall show that the corrections to this approximation are small. The integrated Hartree-Fock equation for the temperature Green functions is shown graphically in Fig. 1. In this equation, the straight (Hartree) terms cancel out because of the electrical neutrality conditions, so that the equations for the electron and hole Green functions are independent. Since the field H suppresses the translational invariance, it is convenient to use the mixed $y-p_x$ representation and carry out the Fourier approximation only in respect of $x-x'$. In this representation, the equation corresponding to Fig. 1 can be written in the form

$$G(y,y';p_x,\omega)=G_0(y,y';p_x,\omega)-\frac{T}{2\pi}\lim_{\tau\to+0}\sum_{\omega'}\int dy_1\,dy_2\,dp_x'$$
$$\cdot\{G_0(y,y_1;p_x,\omega)G(y_1,y_2;p_x',\omega')G(y_2,y';p_x,\omega)V(y_1-y_2,p_x-p_x')e^{i\omega'\tau}\}. \tag{3}$$

In this equation (K_0 is the MacDonald function),

$$V(y,p_x)=\int_{-\infty}^{\infty}\frac{2\pi\tilde{e}^2}{(p_x^2+q_y^2)^{1/2}}e^{iq_yy}\frac{dq_y}{2\pi}=2\tilde{e}^2K_0(yp_x) \tag{4}$$

represents the Coulomb interaction in the mixed representation.

The Green function G_0 of noninteracting electrons (holes) will be expanded in terms of the Landau wave functions $\varphi_{np_x}(x,y)$:

$$\varphi_{np_x}(x,y)=\frac{e^{\mp ip_xx}}{L_x^{1/2}}\chi_{np_x}(y),$$
$$\chi_{np_x}(y)=\frac{\exp[-{}^1/_2(y/r_H-p_xr_H)^2]}{(r_H\pi^{1/2}2^n n!)^{1/2}}H_n\left(\frac{y}{r_H}-p_xr_H\right)$$

(H_n is the Hermitian polynomial; the "$-$" sign applies to electrons and "+" to holes). In the mixed $y-p_x$ representation,

$$G_0(y,y';p_x,\omega)=\sum_{n=0}^{\infty}\frac{\chi_{np_x}(y)\chi_{np_x}(y')}{i\omega-n\omega_{e,h}+\mu},$$

where $\omega=\pi T(2k+1)$ and μ is the chemical potential of the particles (measured, for a nonequilibrium system, from the edge of the relevant band).

As shown in the Appendix, Eq. (3) has the exact solution:

$$G(y,y';p_x;\omega)=\sum_{n=0}^{\infty}\frac{\chi_{np_x}(y)\chi_{np_x}(y')}{i\omega-n\omega_{e,h}-\varepsilon_n+\mu}, \tag{5}$$

where the corrections ϵ_n to one-particle energy levels, associated with the Coulomb interaction, satisfy the system of equations

$$\varepsilon_n=-\frac{\tilde{e}^2(2\pi)^{1/2}}{2r_H}\sum_{m=0}^{\infty}I_{nm}f_m(T). \tag{6}$$

Here,

$$f_m(T)=\left[\exp\left(\frac{m\omega_{e,h}+\varepsilon_m-\mu}{T}\right)+1\right]^{-1}$$

are the Fermi factors, representing the populations of the modified levels,

$$I_{00}=1,\quad I_{01}=\frac{1}{2},\quad I_{11}=\frac{3}{4};\quad I_{n0}=\frac{(2n-1)!!}{(2n)!!},\quad n\geqslant1;$$
$$I_{n1}=\frac{(2n-{}^1/_2)(2n-3)!!}{(2n)!!},\quad n\geqslant2.$$

We shall consider the range of densities corresponding to the level populations $m=0$ or 1; in this range, subject to the condition (2), the functions $f_m(T)$ are exponentially small for $m\geqslant2$ and the corresponding terms make no contribution to Eq. (6). The system (6) should be supplemented by the normalization equation for the determination of the chemical potential:

$$N=\frac{TL_x}{2\pi}\lim_{\tau\to+0}\sum_{\omega}\int dy\,dp_x\,G(y,y;p_x,\omega)e^{i\omega\tau}. \tag{7}$$

Substituting Eq. (5) in Eq. (7), we obtain

$$\frac{2r_H^2}{r_s^2}=\sum_{m=0}^{\infty}f_m(T). \tag{7'}$$

The range of densities in which only the levels $n=0$ or 1 are filled (in the limit $T\to0$) is defined by the conditions $N\leqslant N_0$ and $N_0<N\leqslant2N_0$, respectively ($N_0=L_xL_y/2\pi r_H^2$ is the degree of degeneracy), i.e., $r_s\geqslant r_H\cdot2^{1/2}$ (region 1) and $r_H\cdot2^{1/2}\geqslant t_s\geqslant r_H$ (region 2).

We can easily see from Eqs. (6) and (7) that the population of the modified levels depends on r_s exactly in the same way as the population of the Landau levels in a noninteracting gas. (This is the "zero-dimensional" analog of the Landau theorem on the identical dependences of the Fermi momenta of interacting and noninteracting quasiparticles on the total number of particles.) Equations (6) and (7) define the chemical potential μ as a function of T and r_s^2 ($v=\pi r_s^2$ is the specific volume) and the corrections ϵ_n to the Landau levels, i.e., the modified spectrum of one-particle excitations. The solution of these equations gives, in region 1,

$$\varepsilon_0=-\frac{\tilde{e}^2(2\pi)^{1/2}}{r_s^2}r_H,\quad \varepsilon_n=-\frac{\tilde{e}^2(2\pi)^{1/2}}{r_s^2}r_HI_{n0},\quad n\geqslant1, \tag{8a}$$

and, in region 2,

$$\varepsilon_0=-\frac{\tilde{e}^2(2\pi)^{1/2}}{2r_H}\left[1+\frac{1}{2}\left(\frac{2r_H^2}{r_s^2}-1\right)\right],$$

$$\varepsilon_1=-\frac{\tilde{e}^2(2\pi)^{1/2}}{2r_H}\left[\frac{1}{2}+\frac{3}{4}\left(\frac{2r_H^2}{r_s^2}-1\right)\right],$$

$$\varepsilon_n=-\frac{\tilde{e}^2(2\pi)^{1/2}}{2r_H}\left[I_{n0}+I_{n1}\left(\frac{2r_H^2}{r_s^2}-1\right)\right],\quad n\geqslant2. \tag{8b}$$

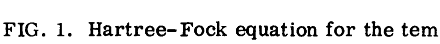

FIG. 1. Hartree-Fock equation for the temperature Green functions.

The chemical potential μ in regions 1 and 2 and in the intermediate matching region is

$$\mu_{e,h}=\varepsilon_0-T\ln\left(\frac{r_s^2}{2r_H^2}-1\right),\quad \frac{r_s^2}{2r_H^2}-1>2\exp\left(-\frac{\omega_{e,h}}{2T}\right),$$

$$\mu_{e,h}=\frac{\omega_{e,h}+\varepsilon_0+\varepsilon_1}{2}-\frac{T(r_s^2/2r_H^2-1)}{2}\exp\left(\frac{\omega_{e,h}}{2T}\right),$$

$$\left|\frac{r_s^2}{2r_H^2}-1\right|<2\exp\left(-\frac{\omega_{e,h}}{2T}\right), \tag{9}$$

$$\mu_{e,h}=\omega_{e,h}+\varepsilon_1-T\ln\frac{2r_s^2-2r_H^2}{2r_H^2-r_s^2},\quad 1-\frac{r_s^2}{2r_H^2}>2\exp\left(-\frac{\omega_{e,h}}{2T}\right).$$

These expressions are exact for all the values r_s under consideration, with the exception of very narrow ranges at the boundaries of the regions, where the smoothing of the solutions is permissible. It should be noted that, when the conditions (1) and (2) are obeyed, the expression for μ as well as the formulas for the derivatives with respect to μ can be matched to within small parameters $r_H/a_{e,h}$ or $T/\omega_{e,h}$. It should be noted that, at $T=0$, there is no matching region and the chemical potential (as well as several other thermodynamic quantities such as pressure) have a discontinuity at $r_s=r_H\cdot 2^{1/2}$, which is further evidence of the inadequacy of an analysis of the system only at absolute zero.

The thermodynamic properties of the system can be found completely if the chemical potential μ is known as a function of density and temperature. (We shall consider thermodymanics in §3.) Here, we shall only give the expression for the free energy in the Hartree-Fock approximation (per one electron-hole pair), calculated from

$$F=v\int_v^\infty\frac{\mu(v,T)}{v^2}dv.$$

In region 1, we have

$$F=-\frac{\tilde{e}^2(2\pi)^{1/2}}{r_s^2}r_H+2T\left[\left(\frac{r_s^2}{2r_H^2}-1\right)\ln\left(\frac{r_s^2}{2r_H^2}-1\right)-\frac{r_s^2}{2r_H^2}\ln\frac{r_s^2}{2r_H^2}\right], \tag{10a}$$

and, in region 2,

$$F=-\frac{\tilde{e}^2(2\pi)^{1/2}}{2r_H}+\left(\omega_e+\omega_h-\frac{\tilde{e}^2(2\pi)^{1/2}}{2r_H}\right)\left(1-\frac{r_s^2}{2r_H^2}\right)$$
$$-\frac{3\tilde{e}^2(2\pi)^{1/2}}{16r_H}\left(\frac{r_s}{r_H}-\frac{2r_H}{r_s}\right)^2+T\frac{r_s^2}{r_H^2}\left[\left(2-\frac{2r_H^2}{r_s^2}\right)\right.$$
$$\left.\times\ln\left(2-\frac{2r_H^2}{r_s^2}\right)+\left(\frac{2r_H^2}{r_s^2}-1\right)\ln\left(\frac{2r_H^2}{r_s^2}-1\right)\right]. \tag{10b}$$

It is clear from Eq. (10) that, in the limit $T\to 0$, the quantity F has an abrupt minimum at the boundary between regions 1 and 2 (i.e., at $r_s=r_H\cdot 2^{1/2})^{1)}$ which corresponds to the formation of a liquid; its binding energy (per one $e-h$ pair) is $E_0=\tilde{e}^2(2\pi)^{1/2}/2r_H$.

§2. CORRELATION EFFECTS

Before analyzing the properties of an $e-h$ liquid, we shall show that the correlation corrections are small. A consistent procedure involves calculations of the corrections to the Hartree-Fock-Green function and determination, from the normalization condition, of the corresponding corrections to the chemical potential. However, we shall use the diagram technique for a fixed number of particles and calculate the correction to the free energy as the sum of vacuum diagrams based on the Hartree-Fock-Green functions. We shall then show that the corresponding correction to the chemical potential is

$$\Delta\mu=\left(\frac{\partial\Delta F}{\partial N}\right)_{T,v}\ll\mu_{HF},$$

which justifies the use of this technique. The expression for the correlation corrections will only be given for region 1, where the population of the Landau levels with $n\neq 0$ is exponentially small; the corrections for the other regions are of the same order of magnitude.

We shall first calculate the analytic continuation $\Pi^R(\epsilon)$ of a single-loop polarization operator $\mathcal{T}(i\epsilon_k)$ from the upper half-plane ϵ to the real axis ($\epsilon_k=2\pi kT$). Knowing $\Pi^R(\epsilon)$ allows us to determine not only the correlation energy but also the permittivity $\kappa(\epsilon,p)$, i.e., the screening of the interaction, collective excitation (plasmon) spectrum, etc.

The single-loop polarization operator is

$$\mathcal{T}(i\varepsilon_k,p_x;y,y')=-\frac{T}{2\pi}\sum_\omega\int dp_x'G(y,y':p_x,\omega)G(y',y;p_x+p_x',\omega+\varepsilon_k).$$

We apply the standard technique[14] to transform the sum into an integral and, calculating the analytic continuation in ϵ, we obtain—after elementary integration over the frequencies and application of Eq. (5)—

$$\Pi^R(\varepsilon,p_x;y,y')=\sum_{m,n=0}^{\infty}\frac{f_m(T)-f_n(T)}{\varepsilon+(m-n)\omega_{e,h}+\varepsilon_m-\varepsilon_n+i\delta}\pi_{mn}(p_x;y,y'), \tag{11}$$

where

$$\pi_{mn}(p_x;y,y')=\frac{1}{2\pi}\int dp_x'\,\chi_{np_x'}(y)\chi_{np_x'}(y')\chi_{mp_x+p_x'}(y)\chi_{mp_x+p_x'}(y'). \tag{12}$$

Since the functions $f_m(T)$ are exponentially small for $m\neq 0$, it is sufficient to calculate only the contribution of the terms π_{0n} and π_{m0}. It is found that the integrals (12) depend on only $y-y'$. Calculation of the integrals (12) and application of the Fourier transformation give $\Pi^R(\epsilon,\mathbf{p})$ in the pure momentum representation (we are omitting the exponentially small terms):

$$\Pi^R(\varepsilon,\mathbf{p})=\sum_{n=1}^{\infty}\frac{2f_0(T)(n\omega_{e,h}+\varepsilon_n-\varepsilon_0)}{\varepsilon^2-(n\omega_{e,h}+\varepsilon_n-\varepsilon_0)^2+i\delta\,\mathrm{sign}\,\varepsilon}$$
$$\times\left(\frac{p^2r_H^2}{2}\right)^n\frac{\exp(-p^2r_H^2/2)}{2\pi r_H^2n!}. \tag{11'}$$

It is important to note that, in contrast to a three-dimensional system in a strong magnetic field when Π is dominated by the contribution of the correlations associated with virtual transitions beyond a one-dimensional Fermi surface within the limits of the lowest Landau level,[10] the "quasi zero-dimensional" system under consideration here is characterized by the fact that only virtual transitions from the lowest Landau level to all the other levels contribute to Π, and there are no correlations within the same level. This is the physical reason for the smallness of the correlations.

The contribution to the correlation energy is made by diagrams of the type shown in Fig. 2. Since the polarization operator is proportional to p^2, the ring diagrams (Figs. 2a and 2c) do not exhibit the usual Coulomb divergence in the limit $p \to 0$. Therefore, it is sufficient to consider only the second-order ring diagram (Fig. 2a); any diagrams of higher orders are characterized by an additional small factor $r_H/a_{e,h}$. For simplicity, we shall give the expression for $m_e = m_h$, $a_0 \equiv a_e = a_h$, $\omega_0 \equiv \omega_e = \omega_h$. The contribution of the diagram in Fig. 2a to the energy per one e–h pair is as follows:

$$\Delta F^{2a} = -\frac{TL_xL_y}{4N}\sum_{\varepsilon}\int\frac{d^2p}{(2\pi)^2}\left[\mathcal{T}(\varepsilon,\mathbf{p})\frac{2\pi\tilde{e}^2}{p}\right]^2$$

$$= -\frac{L_xL_y}{4\pi N}\int d\varepsilon\int d^2p\,\operatorname{cth}\frac{\varepsilon}{2T}\operatorname{Im}[\Pi^R(\varepsilon,\mathbf{p})]^2\frac{\tilde{e}^4}{p^2}.$$

We shall ignore the residue at zero which makes a contribution with an additional small factor T/ω_0; we shall also omit the terms with an additional small factor r_H/a_0, so that simple steps give

$$\Delta F^{2a} = -\frac{\tilde{e}^2(2\pi)^{1/2}}{2r_s^2}r_H\frac{r_H}{a_0(2\pi)^{1/2}}\sum_{m,n=1}^{\infty}\frac{(n+m-1)!}{(n+m)(2n)!!(2m)!!}$$

$$= -\frac{\tilde{e}^2(2\pi)^{1/2}}{2r_s^2}r_H\frac{r_H}{a_0}\frac{\ln^2 2}{(2\pi)^{1/2}}. \qquad (13)$$

It is interesting to note that Eq. (13) includes an important contribution to transitions from the lowest Landau level to a number of higher levels (allowance for the 0–1 transition alone gives a factor of 1/8 instead of $\ln^2 2 \approx 1/2$).

We must also include another second-order diagram which is the "envelope" shown in Fig. 2b. It corresponds (in the Matsubara technique) to the expression

$$\Delta F^{2b} = \frac{T^3L_x}{4N}\sum_{klmn}\sum_{\omega\omega'\varepsilon}\Big\{G_k(y,y_1;\omega,p')G_l(y_1,y';\omega+\varepsilon,p+p')$$

$$\times G_m(y',y_1';p+p'',\omega'+\varepsilon)G_n(y_1',y;\omega',p'')$$

$$\times\frac{\exp[iq(y-y')]}{((p'-p'')^2+q^2)^{1/2}}\frac{\exp[iq_1(y_1-y_1')]}{(p^2+q_1^2)^{1/2}}\Big\}.$$

Here, $\{\ldots\}$ represents integration over all (five) momentum and (four) coordinate variables. Summation over the frequencies and fairly cumbersome integration gives (as before, we are omitting exponentially small terms)

$$\Delta F^{2b} = \frac{\tilde{e}^2(2\pi)^{1/2}}{2r_s^2}r_H\frac{r_H}{a_0}\gamma, \qquad (14)$$

where

$$\gamma = \pi^{1/2}\sum_{k=2}^{\infty}\frac{1-2^{1-k}}{k}\left[\frac{(2k-1)!!}{(2k)!!}\right]^2 F\left(\frac{1}{2},k+\frac{1}{2};k+1;\frac{1}{2}\right)\approx 0.32$$

(F is the hypergeometric function).

As in Eq. (13), the expression (14) includes contributions due to virtual transitions from the lowest Landau level to various other levels. We shall bear in mind that there are three diagrams of the type shown in Fig. 2a, which consist-respectively-of two electron loops, two hole loops, and an electron-hole loop (the last diagram makes a double contribution), and two diagrams (electron and hole) of the type shown in Fig. 2b. Thus, the correlation corrections are negative and

$$\left(\frac{4\ln^2 2}{(2\pi)^{1/2}}-2\gamma\right)\frac{r_H}{a_0} = 0.13\frac{r_H}{a_0} \qquad (15)$$

times smaller than the Hartree-Fock energy. The corresponding corrections to the chemical potential are characterized by the same small factor. It should be noted that the corrections are characterized not only by the parametric small factor r_H/a_0 but also by an additional numerical small factor (this also applies to the corrections of higher orders) associated with a weak overlap of the wave functions of the particles located in various levels. This demonstrates the validity of the adopted theory when magnetic fields weaker than those defined by Eq. (1) are considered:

$$r_H \ll 10a_0. \qquad (16)$$

When we know the polarization operator $\Pi^R(\epsilon,\mathbf{p})$, we can immediately determine the longitudinal permittivity $\kappa(\epsilon,p)$ which-like Π^R- is analytic in the upper half-plane ϵ:

$$\varkappa(\varepsilon,\mathbf{p}) = [1-\Pi^R(\varepsilon,\mathbf{p})2\pi\tilde{e}^2/p]\varkappa.$$

In the case of Π^R, we can clearly use the single-loop approximation since the higher-order corrections are small. We can easily see that, at low frequencies ($\epsilon \ll \omega_0$) for any momentum, we have $\Pi^R\tilde{e}^2/p \ll \tilde{e}^2/r_H\omega_0 = r_H/a_0 \ll 1$ since, at these frequencies, we have $\kappa(\epsilon,\mathbf{p}) = \kappa$, i.e., the Coulomb interaction is not screened. The poles of $\kappa(\epsilon,\mathbf{p})$ coincide with the poles of the Green function G and define the spectrum of one-particle excitations. The zeros $\kappa(\epsilon,\mathbf{p})$ generally define the spectrum of collective excitations ("plasmons"). We can easily obtain the relevant dispersion law:

$$\varepsilon_{(n)}^2 = (n\omega_0+\varepsilon_n-\varepsilon_0)^2 + 2\frac{(pr_H)^{2n-1}}{(2n)!!}\exp\left(-\frac{p^2r_H^2}{2}\right)(n\omega_0+\varepsilon_n-\varepsilon_0)\frac{\tilde{e}^2}{\pi r_s^2}r_H. \qquad (17)$$

Thus, only interlevel collective excitations can exist in the system and the frequencies of these excitations depend weakly on the momentum, coinciding (for $p = 0$) with the frequencies of one-particle excitations. (In

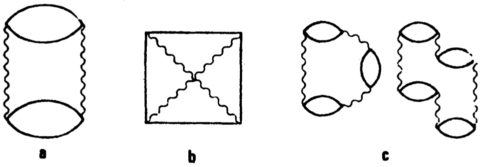

FIG. 2. Correlation corrections to the free energy: a—direct second-order diagram; b—exchange second-order diagram; c—higher-order ring diagrams.

fact, the collective excitations in this system are indistinguishable from one-particle excitations and are of the same nature: they involve the excitation of particles from the filled ground Landau level to higher levels.) There are naturally no plasma oscillations in the same Landau level since the particles do not have kinetic energy.

§3. THERMODYNAMIC PROPERTIES OF THE INVESTIGATED SYSTEM

The chemical potential $\mu(r_s^2, T)$ enables us to determine all the thermodynamic properties. Following the treatment in §2, we shall use the Hartree-Fock approximation. The free energy (10) is given in §1. A simple combination of Eqs. (9) and (10), in accordance with the formula[2)] $P=(\mu-F)/v$ gives, for $r_s>r_H$ (when the Landau levels with $n=0$ and 1 are filled), the expression for the pressure as a function of temperature and volume.

This equation of state is exact in the limit $H\rightarrow\infty$ (the corrections are of the order of r_H/a_0). Like the isotherms of the van der Waals equation, the isotherms of this equation of state include regions in which the thermodymanic inequality $(\partial P/\partial v)_T<0$ is not satisfied (here, $v=\pi r_s^2$ is the two-dimensional specific volume). These regions appear formally because we are calculating the equation of state assuming that the substance consists of a single phase, whereas, normally, at such values of the pressure and temperature, the system in question becomes stratified, forming gaseous and liquid phases, which are in thermodynamic equilibrium. The correct dependence $P(v)$ is found[15] from the Maxwell rule. Figure 3 shows the dependences $P(\rho)$, where $\rho/\pi=1/\pi r_s^2$ is the density. Curve I applies at a temperature $T_1\ll E_0 = \tilde{e}^2(2\pi)^{1/2}/2r_H$.

We can see that it corresponds to two phase-coexistence regions [in which we formally have $(\partial P/\partial\rho)_T<0$]. Region 1 corresponds to the liquid–gas equilibrium in the level $n=0$, and region 2 to the corresponding equilibrium in the $n=1$ level (i.e., when the zeroth level is filled and liquid is formed in the first level). Curve II corresponds to $T_2=(3/16)E_0$, at which the inflection disappears in region 2, i.e., this temperature is critical for the liquid in the $n=1$ level. Curve III corresponds to $T_3=(1/4)E_0$, which is critical for the liquid in the $n=0$ level.

We can see that, when the density is sufficiently high so that the lowest level is completely filled, a self-bound liquid forms in the next level. This is physically clear: the high positive energy ω_0 which the particles have in the $n=1$ level is the same for all the particles and is simply a new reference point for measuring the energy; the liquid forms under the action of the exchange attraction between the particles (exactly as in the lowest level). An increase in density gives rise to phase-coexistence regions in the higher levels (at sufficiently low temperatures) so that a liquid always forms in the highest of the levels being filled. Naturally, if the density is sufficiently high so that the particles do not all occupy the lowest Landau level, they may be transferred to the lowest Landau level corresponding to a different spin configuration or to the next transverse quantization level (see §4), depending on which of these possibilities is preferable from the thermodynamic point of view. Nevertheless, the qualitative conclusions are still the same: a self-bound e–h liquid forms at the highest of the levels being filled provided only conditions (1) and (2) are satisfied, i.e., provided there is no thermal or Coulomb mixing of the levels.

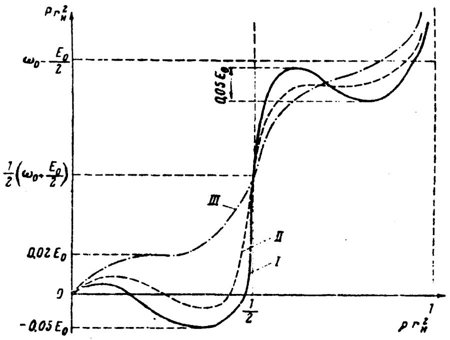

FIG. 3. Isotherms of the equation of state of an e–h liquid: I–$T_1\ll E_0$; II–$T_2=(3/16)E_0$, which is critical for a liquid in the $n=1$ level; III–$T_3=1/4(E_0)$, which is critical for a liquid in the $n=0$ level. For clarity, the isotherms are not to scale. The phase equilibrium regions corresponding to different levels cannot coexist because equilibrium of the "e-h liquid in the $n=1$ level–gas in the $n=0$ level" type is impossible.

It is interesting to note some thermodynamic features of the system. The free energy (10) has the form $F=a+bT$, apart from exponentially small terms, so that it is clear from $E=F+TS$ that the energy $E=a$ and the entropy $S=-b$ are independent of temperature. The entropy S does not vanish for any (nonequilibrium) value of r_s and this is a consequence of the ground-state degeneracy. For equilibrium values of $r_s=r_H\cdot 2^{1/2}$, it follows from Eq. (10) that $S=0$ (with exponential precision). We can easily show that S also vanishes for $r_s=r_H[2(n+1)]^{1/2}$ $(n=0,1,2,\ldots)$, when the n levels are filled completely. Physically, this follows from the fact that there is no degeneracy in the case of completely filled levels.

It should also be noted that, since E is independent of T, the specific heat is $c_v=(\partial E/\partial T)_v=0$, whereas, in the case of nondegenerate many-electron (or electron-hole) systems, we have $c_v\propto T$. The unusual behavior of the specific heat of our system is due to the fact that particles do not have kinetic energy, i.e., they do not have "degrees of freedom" (it is understood that, because of the interlevel transitions, the specific heat does not vanish completely but the contribution of these transitions to the specific heat and to all other thermodynamic quantities is exponentially small).

§4. ALLOWANCE FOR TRANSVERSE MOTION

We have considered so far a two-dimensional system. We shall show that allowance for transverse motion in quasi two-dimensional systems simply results in small corrections to two-dimensional motion. To be specific, we shall consider a quantized film. We shall model the transverse motion in an infinitely deep potential well of width d (in this case, the properties of quantized motion are only slightly sensitive to the nature of the model potential). We shall first assume that all the particles are at the lowest transverse-motion level. The two-dimensional potential (4) is then replaced in all the diagrams with the effective potential

$$V_d(p_x, y-y')=\int_0^d dz\int_0^d dz'\int\frac{dp_y\,dp_z}{4\pi^2}\frac{4\pi\tilde{e}^2}{p_x^2+p_y^2+p_z^2}$$

$$\times\Phi_0^2(z)\Phi_0^2(z')\exp[ip_y(y-y')+ip_z(z-z')],$$

where $\Phi_0(z)=(2/d)^{1/2}\sin(\pi z/d)$ is the wave function of the ground state of transverse motion. Integration gives

$$V_d(p_x, y-y')=\int\frac{dp_y}{2\pi}\exp[ip_y(y-y')]\frac{4\pi\tilde{e}^2}{d^2}$$

$$\times\left\{\frac{d}{p^2}-\frac{1-e^{-dp}}{p^3[1+(pd/2\pi)^2]}\right\}. \tag{18}$$

In the limit $d\to 0$, Eq. (18) reduces, as expected, to Eq. (4).

The solution of the Hartree-Fock equation (3) with the potential (18) replacing (4) has still the same form as before [Eqs. (5) and (6)] but the integrals I_{nm} now depend on the parameter d/r_H. Expanding these integrals in terms of this parameter and restricting the treatment to the first approximation, we obtain-as in §1-the energy E in the limit $T\to 0$:

$$E(d)=E_0\left[1+\left(\frac{2}{\pi}\right)^{1/2}\left(\frac{1}{6}-\frac{1}{(2\pi)^3}\right)\frac{d}{r_H}\right]=E_0\left(1+0.11\frac{d}{r_H}\right). \tag{19}$$

We can see that allowance for the transverse motion in the $0.11d/r_H\ll 1$ case has practically no influence on the thermodynamic characteristics of the system obtained in the Hartree-Fock approximation.

One should also bear in mind that the correlation corrections include not only contributions of transitions to higher Landau levels but also to higher transverse quantization levels. An analysis similar to that given in §2 easily shows that this contribution is again negligible if $d\lesssim r_H\lesssim a_0$. This condition means that the transverse quantization energy $1/md^2\gtrsim\tilde{e}^2/r_H$ is the characteristic energy of interaction in a magnetic field. It is interesting to note that, in the limit of infinitely strong fields, the transverse quantization effects should disappear since $\tilde{e}^2/r_H\to\infty$ and a thin film should behave as a three-dimensional object.

CONCLUSIONS

The effects described above would be easiest to observe in quantized films.[16,17] Some experiments on such films have already been carried out[18-21] (in particular, films have been studied in magnetic fields[20,21]).

We shall now estimate the range of parameters in which the predicted effects may be observed. For a_0 *amounting* to, for example, 100 Å, the fields to which the theory developed above applies are, on the basis of Eq. (16), about 0.7×10^5G. The thickness of quantized films should also be ~100 Å. The critical temperature of the liquid-gas transition in such fields is ~10°K. If (in $H=0$) the initial density is $\rho>\rho_0(H)=eH/\pi c$ in the fields in which the above theory is applicable, a liquid forms in a Landau level with $n\neq 0$ and an increase in the field should produce effects associated with the dropping of the liquid to lower levels (in nonequilibrium systems, such a change may be manifested in a luminescence spectrum). In $H\sim 10^5G$, the equilibrium density is $\rho_0(H)\sim 10^{11}-10^{12}\text{cm}^{-3}$, which corresponds to the bulk density of $10^{17}-10^{18}\text{cm}^{-3}$. If the initial density is less, a liquid forms in a lower level, splitting into drops when the field is increased. The formation of a liquid may be deduced from the appearance of a corresponding line in the infrared spectrum. Moreover, the electron specific heat may vanish in strong fields.

An increase in the magnetic field causes the cyclotron resonance frequency (the frequency of transitions of particles from the ground to the first excited state) to increase in accordance with the law $eH/mc+\tilde{e}^2(2\pi)^{1/2}\times r_H/2r_s^2$, i.e., to deviate considerably [on condition that Eq. (16) applies] from the usual law eH/mc, which is only valid in weak fields.[22] When two phases exist in the system, they are characterized by different values of r_s, so that the cyclotron resonance spectrum should have two lines corresponding to these phases.

APPENDIX

We shall substitute function (5) in Eq. (3). After integration over the frequencies, we apply elementary transformations to obtain

$$\sum_{k=0}^{\infty}\frac{\chi_{kp}(y)\chi_{kp}(y')}{i\omega-k\omega_{e,h}-\varepsilon_k+\mu}=\sum_{k=0}^{\infty}\frac{\chi_{kp}(y)\chi_{kp}(y')}{i\omega-k\omega_{e,h}+\mu}$$

$$-\frac{\tilde{e}^2(2\pi)^{1/2}}{2r_H}\sum_{k,n,m=0}^{\infty}\frac{f_n(T)I_{km}{}^n\chi_{kp}(y)\chi_{mp}(y')}{(i\omega-k\omega_{e,h}-\varepsilon_k+\mu)(i\omega-m\omega_{e,h}+\mu)}. \tag{A.1}$$

We shall demonstrate the diagonality of the integral (where $p\to pr_H$ is a dimensionless quantity)

$$I_{km}{}^n=\frac{2}{(2\pi)^{1/2}}\int dy_1\,dy_2\,dp'\,dq\,\chi_{kp}(y_1)\chi_{np'}(y_1)$$

$$\times\chi_{np'}(y_2)\chi_{mp}(y_2)\frac{\exp[iq(y_1-y_2)]}{[q^2+(p-p')^2]^{1/2}} \tag{A.2}$$

in respect of the indices k and m. We shall consider the specific case when $k, m\geqslant n$ and then [subject to Eq. (4)]

$$\int_{-\infty}^{\infty}dy_1\,\chi_{kp}(y_1)\chi_{np'}(y_1)\exp(iqy_1)=\text{const}\cdot\exp\left[-\frac{(p-p')+q^2}{4}+\frac{iq(p+p')}{2}\right]$$

$$\times\int_{-\infty}^{\infty}dy\exp(-y^2)H_k\left(y+\frac{iq-p'+p}{2}\right)H_n\left(y+\frac{iq+p'-p}{2}\right)$$

$$=\text{const}\cdot\exp\left[-\frac{(p-p')^2+q^2}{4}+\frac{iq(p+p')}{2}\right]$$

$$\times\left(\frac{p'-p}{2}+\frac{iq}{2}\right)^{k-n}L_n^{k-n}\left[\frac{(p-p')^2+q^2}{4}\right],$$

where L_n^{k-n} is the associated Laguerre polynomial. Substituting this and the conjugate expression in Eq. (A.2), we find that simple transformations yield

$$I_{km}{}^{n}=\mathrm{const}\int\frac{dp\,dq}{(p^2+q^2)^{1/2}}\left(\frac{p+iq}{2}\right)^{k-n}\left(\frac{p-iq}{2}\right)^{m-n}\exp\left(-\frac{p^2+q^2}{2}\right)$$

$$\times L_n^{k-n}\left(\frac{p^2+q^2}{4}\right)L_n^{m-n}\left(\frac{p^2+q^2}{4}\right)=\mathrm{const}\int_0^{2\pi}\exp[i\varphi(k-m)]\,d\varphi$$

$$\times\int_0^{\infty}\exp\left(-\frac{u^2}{2}\right)\left(\frac{u}{2}\right)^{m+k-2n}L_n^{k-n}\left(\frac{u^2}{4}\right)L_n^{m-n}\left(\frac{u^2}{4}\right)du.$$

We can see that, for $k\neq m$, we have

$$\int_0^{2\pi}\exp[i\varphi(k-m)]\,d\varphi=0,$$

which demonstrates the diagonality of I^n_{km}. Then, after simple transformations, we obtain Eq. (6) from Eq. (A.1), which proves that Eq. (5) is the solution of Eq. (3). (We have introduced here $I^n_{km}\equiv\delta_{mk}I_{mn}$.) The calculation of the integrals I_{mn} for $n=0$ or 1 is relatively simple but time-consuming. We then obtain

$$I_{00}=1,\quad I_{01}=\frac{1}{2},\quad I_{11}=\frac{3}{4};\quad I_{m0}=\frac{(2m-1)!!}{(2m)!!},\quad m\geqslant 1;$$

$$I_{m1}=\frac{(2m-{}^1/_2)\,(2m-3)!!}{(2m)!!},\quad m\geqslant 2.$$

---

[1] As pointed out, at $T=0$, the chemical potential μ has a discontinuity for $r_s=r_H.2^{1/2}$ but, for any value of $T\neq 0$, no matter how low, we find that the value of μ, like any other thermodynamic quantity, is continuous for any r_s.

[2] It should be noted that Eq. (10) for F is approximate at the boundary between regions 1 and 2 because $(\partial F/\partial v)_T$, deduced from Eq. (10), is continuous at the equilibrium point. Therefore, it is convenient to use the above formula in which smoothing out is carried out to the same accuracy as in Eq. (9).

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