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Trap charge exchange waves in compensated germanium

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We have observed experimentally, at 90 K, impedance oscillations due to excitation of trap charge-exchange waves in n-Ge compensated with gold. It is shown that the observed impedance singularities (shift of the oscillations to lower frequencies with increasing dc voltage and with decreasing sample length, decrease of the oscillation period with decreasing frequency, change of frequency with changing conductivity) agree with the "inverse" dispersion law $\omega^{-1} = kv\tau\tau_M$ that is characteristic of these waves.

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1. It was shown earlier[1] that in a compensated monopolar semiconductor it is possible to excite weakly damped trap charge-exchange waves. The main feature of these waves is their "inverse" dispersion law. The frequency ω and the wave vector k are connected by the relation[1]

$$\omega^{-1}=kv\tau\tau_M, \tag{1}$$

where $v=\mu E$ is the electron drift velocity, is their lifetime, $\tau_M=\epsilon/4\pi\sigma$, and $\sigma=en\mu$ is the conductivity. In this paper we present experimental proof of the existence of these waves.

The onset of charge-exchange waves should lead to singularities in the behavior of the impedance of a crystal.[2,3] If a traveling wave-charge wave is present in the sample, a phase shift appears between the current and the voltage, and the admittance has accordingly a reactive component. The phase shift due to the wave vanishes when the sample spans an integral number of waves. The sample impedance will therefore oscillate with changing frequency of the alternating field. This reasoning is apparently valid for all waves propagating in a homogeneous medium. However, the impedance oscillations will have a different character as a function of the nature of the wave. For charge-exchange waves, the impedance singularities are due to the dispersion relation (1). It was shown[2] that the susceptance of the sample corresponds to a capacitance greatly exceeding the geometrical value. Under conditions when the conductivity is controlled by trapping on one compensated level of the impurity, the expression for the low-frequency capacitance can be approximated by

$$C=C_0\frac{\tau v}{d}\left(1-\cos\frac{d}{\omega\tau v\tau_M}\right)+C_0, \tag{2}$$

where $C_0=\epsilon S/4\pi d$ is the geometric capacitance of the sample, S is the cross-section area, d is the sample length, and ϵ is the permittivity. Expression (2) is valid for short samples ($d\ll v\tau, v\tau_M$) and if diffusion is neglected. When $d/\omega\tau v\tau_M=2\pi m$ (m are natural numbers), i.e., precisely when the sample length is equal to an integer number of charge-exchange wavelengths, the capacitance of the sample is minimal.

Some other singularities of the impedance are also obvious consequences of the dispersion law (1). The oscillations of the frequency dependence of the capacitance should shift towards lower frequencies with increasing constant field E or with decreasing density n of the free electrons, while the maxima and the minima should come closer together with decreasing frequency. Since the charge-exchange wavelength depends on n and E, the capacitance of the sample should oscillate also if n and E vary and the frequency ω is fixed. At low frequencies the conductivity also acquires an oscillating increment

$$\frac{\Delta(\mathrm{Re}\,Y)}{X_0}=\frac{\omega\tau v\tau_M}{d}\sin\frac{d}{\omega\tau v\tau_M}. \tag{3}$$

It is seen from (2) and (3) that the frequencies of the ReY oscillations should become higher than those of

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ImY.

2. We investigated expermentally the impedance of germanium samples with partially compensated gold level (E_c-0.2) eV. The measurements were made at a temperature 90 K, at which the equilibrium carrier density is small. The illumination through an indium-arsenide filter ensured n-type crystal conductivity controlled by the trapping on the (E_c-0.2) eV level only, since the influence of the upper level of gold (E_c-0.04) eV could be neglected because of the intense thermal generation. Non-rectifying contacts were made of an In+2% Sb alloy.

A direct and small alternating voltage were applied to the sample. The real part ReY of the admittance and the equivalent capacitance of the sample $C = \mathrm{Im}Y/\omega$ were determined from the phase shift between the current and the voltage and from their amplitudes.

Figure 1 shows the frequency dependences of the active and reactive parts of the admittance (curves 1 and 2). Both admittance components oscillate with changing frequency f and, as follows from the theory, the relative amplitudes of the ReY oscillations is much smaller. For the sake of clarity, the same figure shows the oscillations of the difference Re$(Y - Y_0)$, where Y_0 is the low-frequency admittance. The amplitude of the ImY and ReY oscillations decreases with decreasing frequency, and the minima of the Re$Y(f)$ plot are shifted towards higher frequencies in comparison with Im$Y(f)$ in accord with formulas (2) and (3).

Figure 2 shows the frequency dependence of the capacitance C of the sample for different values of the dc voltage. At a sufficiently large bias, the character of the $C(f)$ dependence corresponds to expression (2). At low frequencies one can see the capacitance oscillations; then the capacitance reaches a certain constant value, and finally, at high frequencies it decreases like $1/f^2$. When the dc voltage is increased, the oscillations shift towards lower frequencies.

Figure 3a shows the frequency dependence of the capacitance for one and the same sample at various illuminations. It is seen that when the admittance is tripled the oscillations shift proportionally towards higher frequencies. With decreasing sample length d, the capacitance oscillations shift to lower frequencies. It is

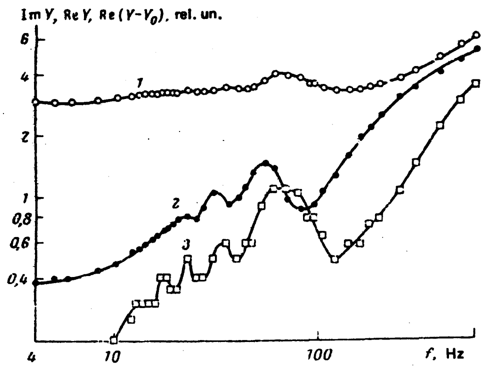

FIG. 1. Frequency dependences of Re Y, Im Y, and Re$(Y - Y_0)$: 1—Re Y, 2—Im Y, 3—Re$(Y - Y_0)$; Y_0 is the low-frequency admittance.

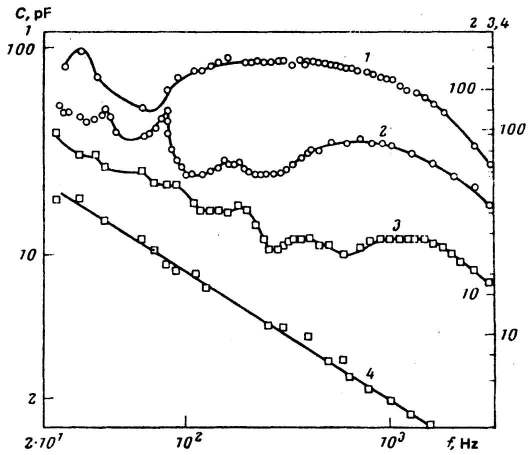

FIG. 2. Frequency dependence of capacitance for different values of the applied voltage: 1—E=800 V/cm; 2—400 V/cm; 3—200 V/cm; 4—40 V/cm. The numbers on the curves correspond to the numbers of the scales.

seen from the presented curves (Fig. 1–3) that the period of the oscillations decreases with decreasing frequency in accordance with the inverse dispersion law.

The dependence of the capacitance on the bias voltage at fixed frequencies is also oscillatory (Fig. 4, curves 1 and 2). Curve 3 of Fig. 4 corresponds to a high frequency at which the oscillations vanish. With increasing dc voltage the capacitance of the sample increases on the average.

The appearance of oscillations in the frequency dependence of the impedance is itself an indication of space-charge wave excitation in the investigated samples. The singularities of the behavior of the impedance indicate that these waves satisfy the inverse dispersion law. The shift of the oscillations on the frequency scale with changing dc voltage, illumination intensity, or sample length confirms expressions (1) and (2). Thus, our data demonstrate that it is precisely trap space-charge waves which were excited in the compensated germanium.

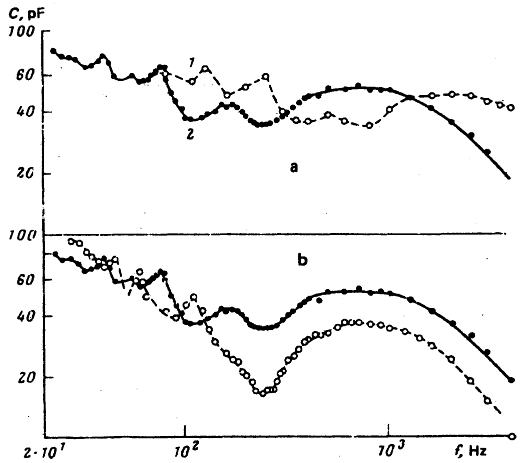

FIG. 3. Dependence of the capacitance on the frequency at different values of the conductivity (Fig. a): 1—$\sigma_0 = 1.2 \times 10^{-5}$ $\Omega^{-1}\mathrm{cm}^{-1}$; 2—$4 \times 10^{-6}$ $\Omega^{-1}\mathrm{cm}^{-1}$; σ_0 is the conductivity in a weak constant field, E=400 V/cm; b—at different directions of the direct current; E=400 V/cm.

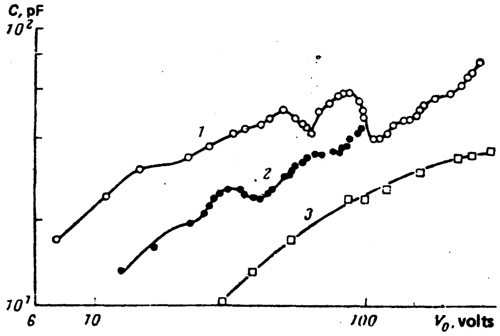

FIG. 4. Capacitance vs. voltage at various frequencies: 1—f=80 Hz, 2—300 Hz, 3—3000 Hz.

For a quantitative comparison of the experimental data with the theory it is necessary to take into consideration the fact that the electron lifetimes τ and mobilities μ of the samples investigated by us depend on the electric field because of the sample heating.

Figure 5 (curve 1) shows the current-voltage characteristic of the sample. It is sublinear because of the increase of the cross section for electron trapping by the repelling gold ions in the electric field[4] and because of the $\mu(E)$ dependence. The electron lifetime τ (curve 2) decreases with increasing field. The values of τ were determined from the frequency dependence of the generation-recombination noise. Curve 3 shows the field dependence of the mobility.

In the earlier calculation of the impedance[2] no account was taken of the field dependences of μ and τ. We shall show below that these dependences do not influence the main singularities of the impedance of a sample in which a trap space-charge wave is excited, but the frequencies of the extrema and the values of the capacitance are altered somewhat.

3. The calculation procedure is similar to that used in[2]. The initial equations are those of Poisson, of the conservation of the total current, and of the capture kinetics for a monopolar semiconductor. We linearize these equations, neglecting diffusion. In the case of compensated crystals this assumption corresponds to the condition[2] $D \ll vd$ and $D \ll v^2\tau\tau_M/\tau_1$, where D is the diffusion coefficient, $\tau_1 = \tau N\nu(1-\nu)/n$ is the characteristic time of the impurity charge exchange, N is the concentration and ν is the impurity filling factor. As a result we have[1)]

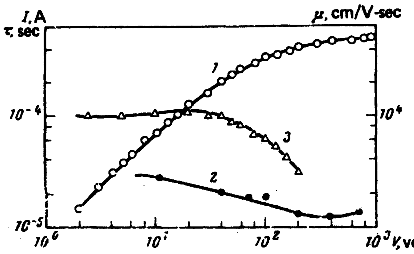

FIG. 5. Current-voltage characteristic (curve 1) and dependence of the lifetime τ (curve 2) and of the electron mobility μ (curve 3; in units of cm^2/V-sec) on the voltage for a Ge:Au sample.

$$\frac{\partial \delta E}{\partial x} = \frac{4\pi e}{\varepsilon}(N\delta\nu + \delta n),$$

$$\delta j = \sigma \frac{d\ln v}{d\ln E}\delta E + ev\delta n + \frac{\varepsilon}{4\pi}\frac{\partial \delta E}{\partial t}, \tag{4}$$

$$\left(\frac{1}{\tau_1} - i\omega\right) N\delta\nu = \frac{\delta n}{\tau} - \frac{n}{\tau}\frac{d\ln\tau}{d\ln E}\frac{\delta E}{E}.$$

As shown in[2], when diffusion is neglected it suffices to use as the boundary condition the following conditions for the injecting contact:

$$\delta E_{\text{rp}}/E_j = -\delta n_{\text{rp}}/n, \tag{5}$$

where $E_j = kT/el_j$ is the characteristic contact field and l_j is the length of the space-charge contact region. Solving the system (4) for δj and δE and using the condition (5), we obtain an expression for the impedance:

$$Z = \frac{1}{S\delta j}\int_0^d \delta E\, dx = \frac{d}{S\sigma}\left[\frac{d\ln v}{d\ln E} + \frac{d\ln\tau}{d\ln E}\frac{1}{1-i\omega\tau} - i\omega\tau_M\right]^{-1}$$
$$\times\left[1 - \frac{1-e^{-ikd}}{ikd}\left(\frac{E}{E_j} - \frac{d\ln\tau}{d\ln E}\frac{1}{1-i\omega\tau}\right) \Big/ \left(\frac{d\ln v}{d\ln E} + \frac{E}{E_j} - i\omega\tau_M\right)\right], \tag{6}$$

where

$$k = -\frac{i}{v\tau_M}\left(\frac{d\ln v}{d\ln E} + \frac{d\ln\tau}{d\ln E}\frac{1}{1-i\omega\tau_M}\right)\left(1 + \frac{\tau_1}{\tau}\frac{1}{1-i\omega\tau_1}\right). \tag{7}$$

The real part of k coincides with the wave number of a trap charge-exchange wave of frequency ω excited by the n^+-n junction that injects the majority carriers into the semiconductor volume.

Let us transform expression (6) to conform to our experiments. We use for this purpose the following conditions, which are always satisfied in the experiment:

$$\frac{\tau}{\tau_1} \ll 1, \quad \frac{\tau_M}{\tau} \ll 1, \quad \frac{d}{v\tau} \ll 1, \quad \frac{d}{v\tau_M} \ll 1, \quad \omega\tau_M \ll 1, \quad \omega\tau_1 \gg 1.$$

In addition, we must know the value of the junction field E_j. An estimate similar to that used in [2] yields for E_j a value 100 V/cm. This field can also be determined in experiment by measuring the active conductivity at high frequencies (see [2]). The value of E_j turned out to be 60 ± 10 V/cm and was practically independent of the voltage. Thus, it can be assumed in our case that $E/E_j \gg 1$. Calculating now from formula (6) the conductivity at low frequencies ($\text{Re}kd \gg 1$), we get

$$C = C_0\left\{-\frac{\tau}{\tau_M}\frac{d\ln\tau}{d\ln E} + \frac{\tau v}{d}\left[1 - \exp(-k_i d)\cos\left(\frac{\sigma_d}{\sigma}\frac{d}{\omega\tau v\tau_M}\right)\right]\right\}, \tag{8}$$

$$\text{Re}\,Y = \frac{S\sigma_d}{d}\left[1 - \frac{\sigma}{\sigma_d}\frac{\omega\tau v\tau_M}{d}\exp(-k_i d)\sin\left(\frac{\sigma_d}{\sigma}\frac{d}{\omega\tau v\tau_M}\right)\right], \tag{9}$$

where $\sigma_d = \sigma d\ln(v\tau)/d\ln E$ is the static differential conductivity of the sample, and

$$k_i = \text{Im}\,k \cong \frac{1}{v\tau_M}\left(\frac{d\ln v}{d\ln E} + \frac{1}{\omega^2\tau\tau_1}\frac{\sigma_d}{\sigma}\right).$$

In the expression for the capacitance there appear, first, the factor σ_d/σ and second, an additional term (the first in the curly bracket of (8)) due to the field dependence of the lifetime τ and determined by the delay in the establishment of the density of the hot electron. The capacitance connected with the charge-exchange wave will exceed this "recombination" capaci-

TABLE I.

V_0, V	$C_{theor.}$ pF	$C_{exp.}$ pF	$f_{1theor.}$ Hz	$f_{1 exp.}$ Hz
50	30	30	600	600
100	45	52	220	240
200	60	82	60	70

tance if $d \ll v\tau_M$, i.e., when the flight time of the electrons through the sample is short compared with the Maxwellian relaxation time. The same condition ensures smallness of the damping $k_i d$.

4. It is seen from (8) that the minimal values of the capacitance should occur at the frequencies

$$f_m = \sigma_d d / m\pi e v\tau, \quad m = 1, 2, \ldots, \tag{10}$$

at which the sample spans an integral number of charge-exchange wavelengths. Owing to the inverse dispersion law (1), the position of the first minimum ($m = 1$) corresponds to the highest frequency f_1. The table lists the experimental values of f_1 and the theoretical ones obtained by formula (10), and are all in good agreement. The table lists also the maximal values of the capacitance, corresponding to the high-frequency plateau (see Fig. 2). These values are also in satisfactory agreement. With increasing dc voltage, the capacitance increases approximately like τv (see also curve 3 of Fig. 4.)

For all the experimental curves presented above, the sample capacitance amounts to dozens of picofarads and greatly exceeds the geometric capacitance. When the sample dimensions are changed, its capacitance (on the plateau) changes in accord with expression (8). Figure 6 shows the dependence of the capacitance on the ratio S/d^2. It is seen that the capacitance increases in proportion to S/d^2, as it should, and it can reach several thousand picofarads. The geometrical capacitance, whose values are shown for comparison on the same figure, depends on the sample dimensions like the ratio S/d. Thus, the observed behavior of the capacitance of compensated germanium agrees with the theory not only qualitatively but also quantitatively. There are, however, also some discrepancies. According to (10) the frequencies f_m corresponding to neighboring capacitance minima should be related like successive integers. Experiment shows that for the first two minima this holds true and $f_1/f_2 = 2$. But the higher-order minima shift towards the lower frequencies more strongly than called for by the calculation. Figure 3b show the frequency dependences of the capacitance at various polarities of the bias applied to the sample. Both curves show oscillations whose first minima have practically the same frequencies, but the frequencies of the higher-order minima differ significantly.

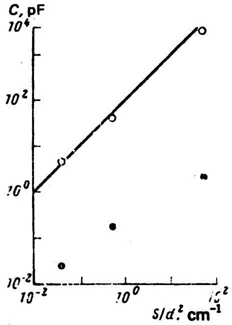

FIG. 6. Dependence of the capacitance of the sample (○) and of its geometric capacitance (●) on the sample dimensions; $E = 300$ V/cm.

Next, the swing of the observed capacitance oscillations turns out to be much less than called for by the theory. In addition, when the frequency is lowered the capacitance increases approximately like $f^{-\alpha}$, where α ranges from 0.5 to 1 for different samples (curve 4 of Fig. 2). These discrepancies between the experimental data and theory can be due to inhomogeneity of the impurity distribution over the sample, and also to the difference between the injection properties of various junctions, particularly to the possible injection of holes.

The results show that to excite trap charge-exchange waves there are no special and difficult-to-satisfy conditions. It is in fact sufficient for the free-carrier density to be much lower than the densities of the impurities that control their capture. In a compensated semiconductor, owing to the excitation of the charge-exchange waves even at relatively low frequencies (and at distances on the order of the lengths of these waves at large distances), the local neutrality inside the crystal is violated. The absence of neutrality and the excitation of the charge-exchange waves should manifest themselves not only in the behavior of the impedance, but also in other nonstationary processes in compensated semiconductors.

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[1)] All the equations are written for a positive charge. For an n-type semiconductor it is necessary to reverse the signs of e and μ.

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