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Absorption of obliquely incident sound in the intermediate state

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The absorption of shortwave sound $kR_0 \gg 1$ in the intermediate state in superconductors is investigated for oblique incidence of the sound on the interface of the two phases. It is shown that the magnitude and the character of the absorption depend very strongly on the angle of incidence α, so that at $a \ll R_0$ and angles α satisfying the condition $4.26\ k(aR_0)^{1/2}\sin\alpha \gg 1$, the sound absorption becomes of the order of the absorption in a normal metal in zero magnetic field (a is the thickness of the normal layer, R_0 is the Larmor radius). The contribution of surface electrons to the sound absorption is investigated; it is significant at $a > 2R_0$. It is shown that in this case magnetoacoustic oscillations are generated with a new period, strongly dependent on the angle α, on which frequent oscillations, due to electron-hole geometric resonance, are superimposed.

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I. INTRODUCTION

The absorption of sound in the intermediate state under various experimental conditions has been studied theoretically by A. F. Andreev. In particular, he investigated the sound absorption in pure samples, when $l \gg R_0$ (l is the free path length of the electrons, R_0 is their Larmor radius in the critical magnetic field), at low temperatures, at normal incidence of sound on the N-S interface.[1] It was shown by him that the sound absorption coefficient Γ is an oscillating function of the ratio R_0/a, where a is the thickness of the normal layer and the monotonic part of Γ remains of the order of the absorption coefficient in the normal metal in a critical magnetic field, multiplied by the specific volume of the normal phase a/d (d is the period of the structure). Ledenev *et al.*[2] made an attempt to find the predicted effect experimentally. However, the observed picture was extraordinarily complicated and actually did not admit of a unique interpretation.

In the present paper we shall show that the sound absorption coefficient depends strongly on the angle α between the sound-wave vector **k** and the normal to the N-S interface. Furthermore, even at very small angles, both the value of the monotonic part of the absorption coefficient and the character of the oscillations changes qualitatively. We shall assume everywhere that $\mathbf{k} \perp H$, $kR_0 \gg 1$, $ka \gg 1$.

We shall discuss qualitatively the observed phenomena. We first consider the case of large magnetic fields, when $a > 2R_0$. In this case, all the electrons in the normal layer can be divided into two groups (Fig. 1): 1) volume electrons, whose orbits do not touch the N-S interface (type c in Fig. 1), 2) surface electrons, which undergo Andreev reflection (types a and b). Correspondingly, the sound absorption coefficient consists of two parts: Γ_v and Γ_s, where

$$\Gamma_v = \Gamma(H_c)\frac{a}{d}\left\{\left(1-\frac{4R_0}{\pi a}\right)+\left(1-\frac{2R_0}{a}\right)\frac{\sin(2kR_0-\pi/4)}{(\pi kR_0)^{1/2}}\right\}. \quad (1)$$

Here $\Gamma(H_c)$ is the monotonic part of the sound absorption coefficient in the normal metal at $H = H_c$. The first component in the curly brackets is the fraction of the volume electrons, the second component describes the geometric resonance by volume electrons, and the coefficient in front of it is the fraction of resonance volume electrons. It is clear that Γ_v cannot depend on the angle of incidence of the wave α. The surface electrons, due to reflection from the N-S interfaces, drift along the wave. In one period, they are displaced by the vector **A**, $A_y = 4R\sin\varphi$ (Fig. 1). At normal incidence of the sound wave, this drift does not affect the sound absorption and the appearance of new drift trajectories does not lead to the appearance of oscillations since, upon averaging over the center of the Larmor orbit, they are strongly diminished and turn out to be of the order of $1/kR_0 \ll 1$ in comparison with the volume electrons. As a result,

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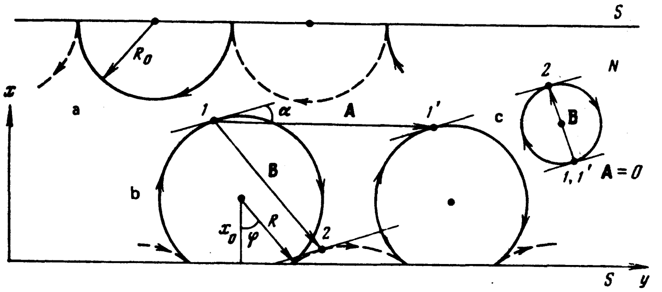

FIG. 1. Types of trajectories in the normal layer at $a>2R_0$: a) extremal surface trajectory; b) arbitrary surface trajectory; c) interior trajectory. The points 1, 2, 1′ are points of stationary phase. The vector A (from point 1 to point 1′) is the drift of the electron in the period T. The vector B—from point 1 to point 2. α is the angle of incidence of the wave on the N-S interface. The motion is periodic with period $T=2\pi/\Omega=2\pi\, mc/eH_c$, $x_0=R\cos\varphi$ is the distance between the center of the orbit and the N-S interface.

$$\Gamma_s=\Gamma(H_c)\frac{a}{d}\frac{4R_0}{\pi a}. \qquad (2)$$

Thus, in the case of normal incidence, all electrons of the normal phase make a contribution to the monotonic part of the sound absorption coefficient, but only the volume resonance electrons make such a contribution to the oscillating part:

$$\Gamma=\Gamma_v+\Gamma_s=\Gamma(H_c)\frac{a}{d}\left\{1+\left(1-\frac{2R_0}{a}\right)\frac{\sin(2kR_0-\pi/4)}{(\pi kR_0)^{1/2}}\right\}. \qquad (3)$$

In a time T, the electron moves from the point 1 to the point 1′. Here the phase change of the sound wave is $\mathbf{k}\cdot\mathbf{A}$. When the phase shift in the time τ between collisions is $\mathbf{k}\cdot\mathbf{A}\tau/T\ll1$, the drift of the particles is insignificant. Therefore, as long as the angle $\alpha\ll1/kl\ll1$, we have the case of normal incidence. In the opposite case ($\alpha\gg1/kl$), the electrons for which $\mathbf{k}\cdot\mathbf{A}\simeq2\pi n$ with accuracy to $T/\tau\ll1$ absorb the sound effectively. Therefore, the sound absorption coefficient is of the order of

$$\Gamma(H_c)\frac{T}{2\tau}\frac{4R_0}{\pi a}\frac{a}{d}=\Gamma(0)\frac{4R_0}{\pi d}.$$

This means that if we disregard the singular oscillations, then the surface electrons absorb the sound in the same way as in a normal metal without a magnetic field.

The exact expression for the sound absorption coefficient has the form

$$\Gamma_s=\Gamma(0)\frac{4R_0}{\pi d}f(4kR_0\sin\alpha,\alpha). \qquad (4)$$

We note that the quantity $4kR_0\sin\alpha$ is the maximum

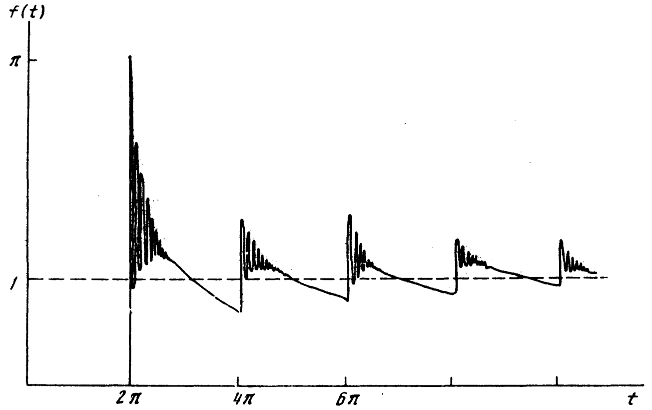

FIG. 2. Dependence of the normal sound absorption coefficient due to surface electrons on the quantity $t=4kR_0\sin\alpha$ at $\alpha\ll1$.

value of $\mathbf{k}\cdot\mathbf{A}$ which is achieved on extremal trajectories of the type a (Fig. 1).

The function $f(t)$ is shown in Fig. 2. It is seen from the drawing that when the phase lag $\mathbf{k}\cdot\mathbf{A}_{max}$ for the extremal trajectory becomes equal to $2\pi n$, the sound absorption coefficient undergoes a jump whose magnitude is $\Delta f_n=\pi/2n$. The situation here is similar to the magnetoacoustic oscillations in the normal metal. Moreover, near the jumps, there are rapid oscillations, which are connected with electron-hole resonances between the points 1 and 2 (Fig. 1). Similar phenomena in the normal metal in an oblique magnetic field were observed by Kaner and Fal'ko.[3,4]

A comparison of the amplitudes of the absorption-coefficient oscillations connected with the surface electrons and the volume electrons shows that at $2\pi^{-1}kR_0\sin\alpha\gtrsim1$,

$$\frac{\Gamma_s^{osc}}{\Gamma_v^{osc}}\sim\frac{\tau}{T(kR_0)^{1/2}}\left(\frac{a}{2R_0}-1\right).$$

Therefore, in the experiment, a basic contribution to the oscillations as a function of the parameters can be made by both the volume and the surface electrons, but the contribution of the surface electrons is small in the ratio T/τ. Upon decrease in the external magnetic field, the thickness of the normal layers decreases, and at some field the layer thickness becomes equal to the diameter of the Larmor orbit. At this point, the character of the motion of the electron changes: electrons with $p_z=0$, which make a small contribution t to the sound absorption, begin to be reflected from both N-S boundaries (examples of such trajectories are shown in Fig. 3).

We now discuss what happens when $a\ll2R_0$. Because of the Andreev reflection from the boundaries of the normal layer, the electrons drift parallel to the bound-

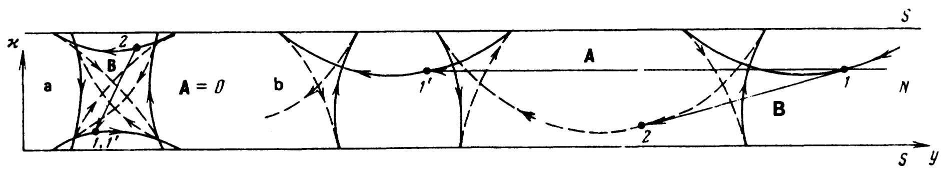

FIG. 3. Trajectory of the motion in the normal layer at $a<2R_0$: a) "standing" trajectory ($A=0$), b) "traveling" trajectory.

ary. In spite of the complicated character of the motion, it is periodic with a period T. However, if the wave is incident at the angle α, then the phase lag in one period will be $\mathbf{k}\cdot\mathbf{A}\sim k(aR_0)^{\frac{1}{2}}\sin\alpha$. Therefore, at angles $\alpha\ll(1/kl)(R_0/a)^{1/2}$, just as in the case of surface electrons, the sound absorption is the same as in normal incidence, and is described by the expressions obtained by Andreev.[1] At large angles, only the resonance trajectories for which $\mathbf{k}\cdot\mathbf{A}=2\pi n$ are again important.

We first consider the behavior of Γ for angles such that

$$T/\tau\ll kA_{max}<2\pi.$$

Here the basic contribution to the absorption is made by electrons which over a single period are not displaced perpendicular to the magnetic field (type a in Fig. 3). Here $\mathbf{k}\circ\mathbf{A}_{max}=4.26k(aR_0)^{1/2}\sin\alpha$. In this case,

$$\Gamma_t=\Gamma(0)\frac{a}{d}\frac{C}{k(aR_0)^{1/2}\sin\alpha}+\Delta\Gamma, \tag{5}$$

where C is a number of the order of unity, and $\Delta\Gamma$ is a small correction, which oscillates as a function of the ratio R_0/a with a period of unity. The ratio $\Delta\Gamma/\Gamma_t\sim(a/R_0)^{1/2}\ll1$.

At large sound-wave incidence angles such that $\mathbf{k}\cdot\mathbf{A}_{max}>2\pi$, Γ_t changes in a complicated way, remaining all the time of the order of $(a/d)\Gamma(0)$, and when the angles become so large that $\mathbf{k}\cdot\mathbf{A}_{max}\gg2\pi$, the absorption coefficient ceases to depend on the angle of incidence: $\Gamma_t=(a/d)\Gamma(0)$. This is a consequence of the infinite electron motion due to reflection from the boundaries of the normal layer.

Thus, even beginning with angles $\alpha\sim1/k(aR_0)^{1/2}\ll1$, the quantity Γ_t becomes of the order of the absorption coefficient in the normal metal without a magnetic field, while the oscillations turn out to be small of the order of $(a/R_0)^{1/2}\ll1$. However, even at $a\ll R_0$, there exist volume electrons with $p_z\neq0$, whose trajectories do not touch the boundaries. These electrons make a contribution

$$\Gamma_v=\Gamma(0)\frac{a}{d}\frac{a}{\pi R_0}\frac{\tau}{T} \tag{6}$$

to the absorption, and

$$\frac{\Gamma_v}{\Gamma_t}\sim\frac{\tau}{T}\frac{a}{R_0}, \tag{7}$$

while the total absorption coefficient is the sum of all the contributions:

$$\Gamma=\Gamma_t+\Gamma_v.$$

2. SOUND ABSORPTION COEFFICIENT

For the calculation of Γ we use the kinetic equation. In the case of longitudinal sound, the interaction of the electrons with the field of the sound wave is described by the deformation potential tensor $\lambda_{ik}(p)$. The equation has the form

$$\frac{\partial\psi}{\partial t}+\mathbf{v}\frac{\partial\psi}{\partial\mathbf{r}}+\frac{\partial\psi}{\partial t_1}+\frac{\psi}{\tau}=g, \tag{8}$$

where

$$g=\Lambda_{ik}(\mathbf{p})\dot{u}_{ik},\quad \Lambda_{ik}(\mathbf{p})=\lambda_{ik}(\mathbf{p})-\overline{\lambda_{ik}(\mathbf{p})},$$

$u_{nm}=u_{nm}^0e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}$ is the deformation tensor in the sound wave, the bar indicates averaging over the directions of the momentum, t_1 is the time on the trajectory, and ψ is a nonequilibrium contribution to the electron distribution function, determined by the condition

$$f=f_0-\frac{\partial f_0}{\partial\varepsilon}\psi.$$

As usual, the term $\partial\psi/\partial t$ can be neglected in comparison with $\mathbf{v}\partial\psi/\partial\mathbf{r}$ in terms of the smallness of s/v_F (s is the sound velocity).

The boundary condition corresponding to the Andreev reflection at the N-S interface[5] is

$$\psi(\mathbf{n},\xi)+\psi(-\mathbf{n},-\xi)\,|_{\text{on the boundary}}=0, \tag{9}$$

where $\mathbf{n}=\mathbf{p}/|\mathbf{p}|$, $\xi=\epsilon-\mu$, v_F is the Fermi velocity. This boundary condition guarantees the equality of the flux of electrons reaching a given point on the interface at a fixed angle to the flux of holes leaving it at the same angle.

The solution of Eq. (8) with the boundary condition (9) is of the form

$$\psi=\int_{-\infty}^{t_1}\tilde{g}(t_2)\exp\left(\frac{t_2-t_1}{\tau}\right)dt_2, \tag{10}$$
$$\tilde{g}(t_2)=\pm g(t+t_2-t_1,\ \mathbf{p}(t_2),\ \mathbf{r}+\mathbf{r}(t_2)-\mathbf{r}(t_1)).$$

where the plus sign is chosen if the particle is located on the electron part of the trajectory at the time t_2 (the solid lines in Figs. 1 and 3), and the minus sign if it is on the hole part (dashed lines).

Using the periodicity of the function $g(t_2)$ and the smallness of $\gamma=T/\tau\ll1$, we can transform (10) to the form

$$\psi=\frac{1}{e^{i\mathbf{k}\mathbf{A}+\gamma}-1}\int_{t_1}^{t_1+T}\tilde{g}(t_2)dt_2. \tag{11}$$

As usual,[6] the condition $kR\gg1$ allows us to use the method of stationary phase for the calculation of the integral in (11), which leads to the following final expression for ψ:

$$\psi=\frac{1}{e^{i\mathbf{k}\mathbf{A}+\gamma}-1}\sum_{t_1<t_\alpha<t_1+T}\tilde{g}(t_\alpha)\left(\frac{2\pi}{|\mathbf{k}\mathbf{v}_\alpha'|}\right)^{1/2}\exp\left[\frac{\pi i}{4}\operatorname{sign}(\mathbf{k}\mathbf{v}_\alpha')\right],$$

where t_α are the points of stationary phase,

$$|\psi|^2=\frac{2\pi}{|e^{i\mathbf{k}\mathbf{A}+\gamma}-1|^2}\sum_{\substack{t_1<t_\alpha\\ t_\beta<t_1+T}}\frac{\tilde{g}(t_\alpha)\tilde{g}^*(t_\beta)}{(|\mathbf{k}\mathbf{v}_\alpha'||\mathbf{k}\mathbf{v}_\beta'|)^{1/2}}\times\exp\left\{\frac{\pi i}{4}[\operatorname{sign}(\mathbf{k}\mathbf{v}_\alpha')-\operatorname{sign}(\mathbf{k}\mathbf{v}_\beta')]\right\}. \tag{12}$$

In the case of a spherical Fermi surface, which we shall consider, we have

$$\Lambda_{ik}=\Lambda\left(\delta_{ik}-\frac{3p_ip_k}{p^2}\right),\quad \dot u_{nm}=k\omega u_0\frac{k_mk_n}{k^2}e^{ikr-i\omega t}.$$

There exist two points of stationary phase, at which

$$\mathbf{kv}'=kR\Omega^2,\quad |g|=\Lambda k\omega u_0,$$

so that in this case,

$$|\psi|^2=\frac{(\Lambda k\omega u_0)^2}{|e^{i\mathbf{kA}+\gamma}-1|^2}\frac{4\pi}{kR\Omega^2}[1+\sin\Phi],\qquad(13)$$

where $\Phi=\mathbf{k}\cdot\mathbf{B}$ if the instants of time t_1 and t_2 pertain both to the electron or both to the hole parts of the trajectory, and $\Phi=\mathbf{k}\cdot\mathbf{B}+\pi/2$, if one of the instants pertains to the electron and the other to the hole part of the trajectory (the vector $\mathbf{B}$ is shown in Figs. 1 and 3).

As is well known,[6]

$$\Gamma=-\frac{1}{2I}\int\frac{|\psi|^2}{\tau}\frac{\partial f_0}{\partial\varepsilon}\frac{d^3\mathbf{p}}{(2\pi\hbar)^3},$$

where $I=\rho\omega^2u_0^2s/2$ is the energy flux in the sound wave (s is the sound velocity). The absorption coefficient is measured experimentally, averaged over the period of the structure; therefore, after integration over ϵ and transformation to a new variable $R=(p_F^2-p_z^2)^{1/2}/m\Omega$, we obtain

$$\Gamma=\frac{4\Gamma(0)}{\pi}\int_0^d\frac{dx}{d}\int_0^T\frac{dt_1}{\tau}\int_0^{R_0}\frac{dR}{(R_0^2-R^2)^{1/2}}\frac{1+\sin\Phi}{|e^{i\mathbf{kA}+\gamma}-1|^2}\qquad(14)$$

where $\Gamma(0)=m^2\Lambda^2\omega/4\pi\rho s^2\hbar^3$ is the absorption coefficient in the normal metal in the absence of a magnetic field. We can immediately separate the contribution of the volume electrons from Eq. (14). For them

$$\mathbf{kA}=0,\quad \Phi=\mathbf{kB}=2kR=2kR_0\sin\theta,$$

therefore,

$$\Gamma_v=\frac{4\Gamma(0)}{\pi}\frac{\tau}{T}\frac{a}{d}\int d\theta\left[1-\frac{2R_0}{a}\sin\theta\right][1+\sin(2kR_0\sin\theta)].\qquad(15)$$

The limits of integration are $0\leqslant\theta\leqslant\pi/2$ in the case $a>2R_0$, and $0\leqslant\theta\leqslant\arcsin(a/2R_0)$ in the case $a<2R_0$.

Carrying out the integration, we arrive at the expression (1) in the case $a>2R_0$ and in the case $a\ll 2R_0$ we have

$$\Gamma_v=\Gamma(0)\frac{\tau}{T}\frac{a}{d}\frac{a}{\pi R_0}\left(1+O\left(\frac{1}{ka}\right)\right).\qquad(16)$$

As has already been noted above, we assume everywhere that $ka\gg 1$. In the case of oblique incidence, when $\mathbf{k}\cdot\mathbf{A}_{\max}\gg\gamma$, the expression (14) can be transformed to the form

$$\Gamma=\frac{4\Gamma(0)}{\gamma}\sum_{n=-\infty}^{+\infty}\int_0^d\frac{dx}{d}\int_0^T\frac{dt_1}{\tau}\int_0^{R_0}\frac{dR}{(R_0^2-R^2)^{1/2}}[1+\sin\Phi]\delta(\mathbf{kA}-2\pi n).\qquad(17)$$

This latter expression will be considered by us below in several limiting cases. We note that if we are interested in the oscillating contribution to Γ, due to the component $\sin\Phi$ in the square brackets, then, for (17) to be valid we must have the stronger condition $\alpha\gg\gamma$.

A. Large magnetic fields ($a>2R_0$)

The surface electrons are those electrons whose orbit centers lie at a distance x_0 that is less than their radius R from the boundary. It is clear that all the quantities entering into (17) depend on x_0, and not explicitly on the coordinate x and the time on the trajectory. We therefore introduce new variables (Fig. 1)

$$x_0=R_0\sin\theta\cos\varphi,\quad R=R_0\sin\theta,\qquad(18)$$
$$x=x_0-R_0\sin\theta\cos\Omega t_1.$$

The quantity $\mathbf{k}\cdot\mathbf{A}$ in this case becomes simply

$$\mathbf{kA}=4kR_0\sin\alpha\sin\varphi\sin\theta,$$

and $\mathbf{k}\cdot\mathbf{B}=2kR_0\sin\theta(\varphi-\alpha)$. As a result, we get for Γ_s, after integration over t_1, with account of the two boundaries,

$$\Gamma_s=\frac{2R_0}{d}\Gamma(0)\sum_{n=-\infty}^{+\infty}\int_{-1}^{1}d\cos\theta\int_{-1}^{1}d\cos\varphi$$
$$\times\{1-\cos[2kR_0\sin\theta\cos(\varphi-\alpha)]\}\delta(4kR_0\sin\alpha\sin\varphi\sin\theta-2\pi n).\qquad(19)$$

Carrying out the integration over φ and transforming the resultant integral, we get the expression

$$\Gamma_s=\frac{4R_0}{\pi d}\Gamma(0)\sum_{n=0}^{[1/a_1]}a_1a_n\int_{a_n}^{1}\frac{du}{[(1-u^2)(u^2-a_n^2)]^{1/2}}$$
$$\times\left\{1-(-1)^n\cos\left(\pi n\operatorname{ctg}\alpha\frac{(1-u^2)^{1/2}}{u}\right)\right\},\qquad(20)$$

where $a_n=\pi n/2kR_0\sin\alpha$. $[x]$ is the integer part of x.

Integration of the first component in (20) gives

$$\int_{a_n}^{1}\frac{du}{[(1-u^2)(u^2-a_n^2)]^{1/2}}=K(\sqrt{1-a_n^2}),\qquad(21)$$

where $K(x)$ is a complete elliptic integral of the first kind. The second integral is easily transformed to the form

$$I=\frac{(-1)^n}{a_n}\int_0^1\frac{dt}{(1-t^2)^{1/2}}\left(1+\frac{1-a_n^2}{a_n^2}t^2\right)^{-1/2}\cos\left(\frac{\pi\operatorname{ctg}\alpha}{a_1}(1-a_n^2)^{1/2}t\right).\qquad(22)$$

At $\alpha\ll 1$, but such that all the inequalities given above are satisfied, the expression (22) is simplified to the form

$$I=(-1)^n\frac{\pi}{2}J_0\left(\frac{\pi\operatorname{ctg}\alpha}{a_1}(1-a_1^2n^2)^{1/2}\right);\qquad(23)$$

here $J_0(x)$ is the Bessel function.

We note that the oscillations described by this component are most important just in the region of small angles, since their amplitude falls off at large angles.

Substituting (21) and (23) in (20), we finally obtain

$$\Gamma_s=\frac{4R_0}{\pi d}\Gamma(0)\sum_{n=0}^{[1/a_1]}a_1^2n\left[K(\sqrt{1-a_1^2n^2})\right.$$
$$\left.-(-1)^n\frac{\pi}{2}J_0\left(\frac{\pi\operatorname{ctg}\alpha}{a_1}(1-a_1^2n^2)^{1/2}\right)\right]=\frac{4R_0}{\pi d}\Gamma(0)f(4kR_0\sin\alpha). \tag{24}$$

A plot of f as a function of $4kR_0\sin\alpha=2\pi/a$ is shown in Fig. 2. At large angles, when $2k\pi^{-1}R_0\sin\alpha\gg 1$, the contribution of the surface electrons turns out to be equal to $(4R_0/\pi d)\Gamma(0)$ and is small in comparison with the contribution of the volume electrons in terms of the parameter T/τ. At angles $\alpha>(\pi/2kR_0)^{1/2}$, ordinary geometric resonance sets in on the surface trajectories, i.e., when both resonance points are located on a single electron or hole trajectory. Just as for the monotonic part, at large α this oscillating contribution Γ_s is small in the parameter T/τ in comparison with the oscillating contribution of the volume electrons. However, in the narrow region of magnetic fields, when $(a-2R_0)/a\ll T/\tau$, the contribution Γ_s turns out to be the principal one, since the volume electrons disappear:

$$\Gamma_s=\frac{4R_0}{d}\Gamma(0)\frac{\sin(2kR_0-\pi/4)}{(\pi kR_0)^{1/2}}(1-\cos\alpha). \tag{24'}$$

B. Weak magnetic fields ($a\ll 2R_0$)

If $a\ll 2R_0$, then the electron is multiply reflected from both boundaries. For the calculation of the sound absorption coefficient in the expression (17) it is convenient to transform to the new variables r, t_1 such that

$$x=a[x_0-r\cos\Omega t_1],\quad R=ar,\quad r_0=R_0/a. \tag{25}$$

As a result, with account of the transformation of the regions of integration and the periodicity of $|\psi|^2$ with period a as a function of R, we obtain

$$\Gamma_t=\frac{a}{d}4\Gamma(0)\sum_{n=-\infty}^{+\infty}\int_0^{r_0}\frac{dr}{(r_0^2-r^2)^{1/2}}\int_0^1 dx_0[1+\sin\Phi]\delta(\mathbf{kA}-2\pi n]. \tag{26}$$

Calculation shows (see the Appendix) that $a\ll 2R_0$, i.e., at $r\gg 1$,

$$\mathbf{kA}=qr^{1/2}[F(r-x_0)+F(r+x_0)], \tag{27}$$

where

$$F(z)=\frac{4}{\pi^{1/2}}\int_0^\infty\frac{\sin\pi z\operatorname{ch}t-\cos\pi z\operatorname{sh}t}{\sin^2\pi z+\operatorname{sh}^2 t}\sqrt{t}\,dt. \tag{28}$$

[The function $F(z)$ is shown in Fig. 4], $q=ka\sin\alpha$. At $\alpha r^{1/2}\gg 1$,

$$\mathbf{kB}=qr^{1/2}F(r-x_0)\sim ka\alpha r^{1/2}\gg 1, \tag{29a}$$

and at $\alpha r^{1/2}\ll 1$,

$$\mathbf{kB}\sim ka\gg 1. \tag{29b}$$

Since $F(z+1)=-F(z)$, one of the roots of the equation $\mathbf{k}\cdot\mathbf{A}=0$ is $x_0=\frac{1}{2}$. Then, integrating over x_0, we obtain

$$\Gamma_t=\frac{4\Gamma(0)}{q}\frac{a}{d}\left\{\int_0^{r_0}\frac{dr}{[r(r_0^2-r^2)]^{1/2}}\frac{1+\sin\Phi(x_0=1/2,r)}{2|F'(r-1/2)|}\right.$$
$$\left.+\sum_{n=-\infty}^{+\infty}{}'\int_0^{r_0}\frac{dr}{[r(r_0^2-r^2)]^{1/2}}\frac{1+\sin\Phi(x_n(r),r)}{|F'(r+x_n(r))-F'(r-x_n(r))|}\right\}. \tag{30}$$

Here $x_n(r)$ is the root of the equation

$$r^{1/2}q[F(r-x_n(r))+F(r+x_n(r))]=2\pi n. \tag{31}$$

The prime on the sum indicates that the term with $x_0=\frac{1}{2}$ at $n=0$ is omitted. If $r^*<r-[r]<\frac{1}{2}$, where r^* is determined from the relation $F(r^*)=0$, then there is still another root in the equation (31) with $n=0$ in addition to $x_0=\frac{1}{2}$. Equation (31) at $n\neq 0$ at different r can have one, several, or no solutions. In particular, at

$$\mathbf{kA}_{max}=4.26kar_0^{1/2}\sin\alpha<2\pi \tag{32}$$

Eq. (31) has no roots. We first consider just this case.

Equation (32) along with the condition $ka\gg 1$ shows that $\alpha r^{1/2}\ll 1$. Here $\mathbf{k}\cdot\mathbf{B}\sim ka\gg 1$ and represents a piecewise-linear function of r with period 2. Therefore, the oscillations connected with the term proportional to $\sin\Phi$ will be small in comparison with the monotonic part in the parameter $1/\sqrt{ka}$. This component does not generally make a contribution to the monotonic part.

Thus,

$$\Gamma_t=\frac{4\Gamma(0)}{q}\frac{a}{d}\left\{\int_0^{r_0}\frac{dr}{[r(r_0^2-r^2)]^{1/2}}\frac{1}{2|F'(r-1/2)|}\right.$$
$$\left.+\int_{\substack{0\le r\le r_0\\ r^*\le r-[r]\le 1/2}}\frac{dr}{[r(r_0^2-r^2)]^{1/2}}\frac{1}{|F'(r+x_0(r))-F'(r-x_0(r))|}\right\} \tag{33}$$

One can easily calculate the monotonic part by using the fact that $|F'(r)|$ and $|F'[r+x_0(r)]-F'[r-x_0(r)]|$ are periodic functions of r with period 1. We have

$$\Gamma_{t\,mon}=\frac{4\Gamma(0)}{q}\frac{a}{d}\int_0^{r_0}\frac{dr}{[r(r_0^2-r^2)]^{1/2}}\left\{\int_0^1\frac{dt}{2|F'(t-1/2)|}\right.$$
$$\left.+\int_{r^*}^{1/2}\frac{dt}{|F'(t+x_0(t))-F'(t-x_0(t))|}\right\}. \tag{34}$$

Finally, we obtain

$$\Gamma_{t\,mon}=\Gamma(0)\frac{a}{d}\frac{C}{k(aR_0)^{1/2}\sin\alpha} \tag{35}$$

where

$$C=\frac{[\Gamma(1/4)]^2}{\sqrt{2\pi}}\left[\int_0^1\frac{dt}{F'(t)}+2\int_{r^*}^{1/2}\frac{dt}{|F'(t+x_0(t))-F'(t-x_0(t))|}\right].$$

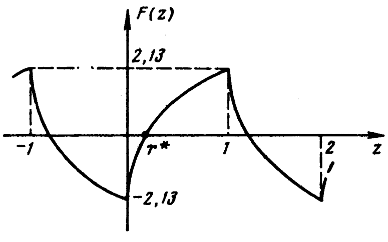

FIG. 4. Plot of the function $F(z)$.

We emphasize that this result is true at

$$T/\tau \ll k(aR_0)^{1/2} \sin\alpha < 2\pi.$$

So far as the oscillating part Γ_f is concerned, it is small in comparison with the monotonic in terms of the parameter $(a/R_0)^{1/2}$. This part is periodic in R_0 with period a and undergoes jumps at the points $R_n = a(r^* + n)$. However, it has not been possible to calculate the explicit form of the oscillating part.

In the opposite limiting case, when $\mathbf{k}\cdot\mathbf{A}_{max} \gg 2\pi$, we can substitute integration for summation in Eq. (26):

$$\Gamma_t = \frac{a}{d} 4\Gamma(0) \int_0^{r_0} \frac{dr}{(r_0^2 - r^2)^{1/2}} \int_0^1 \frac{dx_0}{2\pi} = \frac{a}{d}\Gamma(0). \qquad (36)$$

Thus, in this case the presence of the magnetic field is not reflected in the absorption coefficient, which is equal to the absorption coefficient in the normal metal in the absence of the magnetic field.

In the intermediate case $\mathbf{k}\cdot\mathbf{A}_{max} \geqslant 2\pi$,

$$\Gamma_t \sim (a/d)\Gamma(0)$$

and besides the monotonic part the sound absorption coefficient contains numerous oscillations with small amplitudes of the order of $(a/R_0)^{1/2}$ and jumps when the magnetic field satisfies the condition

$$k(aR_0)^{1/2} F\left(\frac{R_0}{a}\right) \sin\alpha = \pi n.$$

We thus see that at large angles of incidence of the sound wave, the periodicity of the motion becomes unimportant and the specifics of the Andreev reflection, with accuracy to small oscillations, do not appear.

APPENDIX

For the calculation of the quantity $\mathbf{k}\cdot\mathbf{A}$ we note that because of the peculiar character of the Andreev reflection the vector $\mathbf{A}$ can be represented as (Fig. 5)

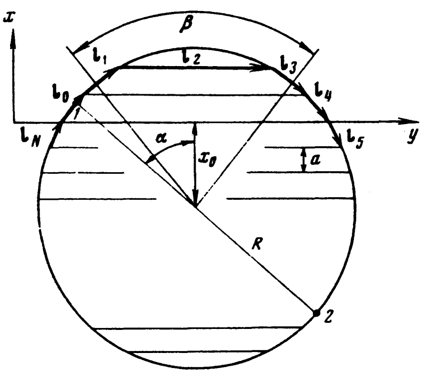

FIG. 5. Illustration for calculation of the vector A at $a \ll 2R_0$. The displacement between the nth and the $(n+1)$st reflection from the interface is $(-1)^n \mathbf{1}_n$; $\beta \sim (a/R)^{1/2} \ll 1$ is the angle in which lie the important $\mathbf{1}_n$; x_0 is the distance from the N-S interface to the center of curvature of the orbit near the point 1.

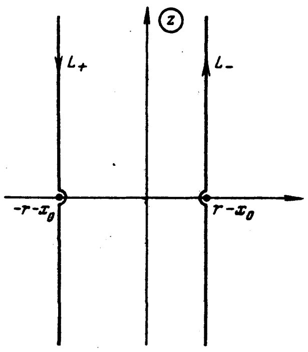

FIG. 6. Contour of integration C in the integral of (A.3), consisting of the two straight lines L_+ and L_-.

$$\mathbf{A} = \sum_{n=0}^{N} (-1)^n \mathbf{1}_n. \qquad (A.1)$$

It is evident that, because of the periodicity of the motion, $A_x = 0$ and only the y-component differs from zero. Then A_y can be represented as the sum of chords joining the ends of the vectors $\mathbf{1}_n$:

$$A_y = 4a \sum_{n=[-r-x_0]}^{[r-x_0]} (-1)^n [r^2 - (n+x_0)^2]^{1/2}; \qquad (A.2)$$

x_0 is shown in Fig. 5.

We can transform the sum on the definition (A2) into the contour integral, shown in Fig. 6:

$$A_y = 4a \int_C \frac{[r^2 - (z+x_0)^2]^{1/2}}{2i \sin \pi z} dz. \qquad (A.3)$$

When $r \gg 1$, Eq. (A.3) can be put in the form (27) by a change of variables. We note that only $\mathbf{1}_n$ lying in the narrow angular range $\beta \sim r^{-1/2} \ll 1$ make a contribution to A_y (Fig. 5). For the calculation of $\mathbf{B}$, we sum $\mathbf{1}_n$ from the point 1 to the point 2. When $\alpha \gg \beta \sim r^{-1/2}$, both these points lie outside the important region of summation, and therefore $\mathbf{B}$ can be represented as the sum of $\mathbf{1}_n$ over the upper semicircle, which corresponds to integration over the straight line L (on Fig. 6). This leads to the following expression for B_y:

$$B_y = ar^{1/2} F(r - x_0). \qquad (A.4)$$

Under these conditions $(\alpha \gg r^{-1/2}) B_x \sim a$ and therefore $\mathbf{k}\cdot\mathbf{B}$ is given by the expression (29).

The function $F(z)$ has the property (Fig. 4)

$$F(z+1) = -F(z). \qquad (A.5)$$

As $z \to 0$,

$$F(z) = -\frac{4}{\pi}\left(1 - \frac{1}{2\sqrt{2}}\right)\zeta(3/2) + 4\sqrt{2|z|}\,\theta(z) \approx -2.13 + 4(2|z|)^{1/2}\theta(z),$$

where $\zeta(x)$ is the Riemann zeta function.

On the cut $0 \leqslant z \leqslant 1$, $F(z)$ is a monotonically increasing function.

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Investigation of the nonequilibrium mixed state of superconducting niobium with pinning centers

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The interaction of magnetic flux vortices that enter the sample upon increase in the magnetic field with the frozen-in magnetic flux is studied by using the anisotropy of the impedance of type-II superconductors. If the pinning is effective, the magnetic flux of the sample in the intermediate state remains parallel to the frozen-in flux. The manner in which the total magnetic flux is confined by the pinned vortices when the magnetic field is rotated relative to an immobile sample is investigated. The experimental results can be interpreted in terms of plastic deformation of the vortex lattice.

As is well known, the surface impedance of type-II superconductors depends on the inclination of the vortex lattice relative to the surface of the sample.[1] We have used this property for the study of the behavior of the vortex lattice in superconducting niobium containing pinning centers in various concentrations. Vinikov *et al.*[2] have shown that the critical current changes strongly in superconducting niobium subjected to chemical polishing, different cooling regimes, and annealing in a high vacuum. This change was attributed to the formation of niobium hydride. Thus in niobium it is easy in practice to change the concentration of pinning centers in one and the same sample.[2]

In type-II superconductors in the mixed (intermediate) state containing pinning centers, a large or small part of the vortices are pinned, depending on the concentration of the centers. These pinned vortices remain as a frozen flux upon decrease in the magnetic field to zero. In the present work, we have studied the interaction between vortices of the magnetic flux entering into the sample upon increase in the magnetic field and frozen vortices whose orientation does not coincide with the direction of the field. We also studied how the pinned vortices contain the entire magnetic flux in the rotation of a magnetic field of constant magnitude.

PREPARATION OF THE SAMPLES

The niobium samples were cut from a single-crystal ingot with resistance ratio $\rho_{300K}/\rho_{4.2K}=250$ and were subjected to mechanical polishing. The case-hardened surface layer was then removed by chemical polishing in a mixture of nitric and fluoric acids. The chemical polishing in the present work is important since a significant amount (~1 at.%) of hydrogen is dissolved in the niobium because of it.[2,3] This dissolved hydrogen was used to produce the pinning centers in the niobium. At a temperature of $T=245$ K, bonding of the hydrogen into niobium hydride occurred. Below this temperature, inclusions of the hydride phase are observed in the niobium matrix; these reached sizes of 10^{-3} to 10^{-5} cm[2] depending on the cooling rate. The change in volume of the hydride phase in comparison with the matrix amounted to ~10% and led to plastic deformation of the niobium near the place of formation, a fact also reflected in the present results. Since the niobium hydride is not a superconductor in the investigated temperature range, its segregations in the matrix constitute the pinning centers. The concentration of the dissolved hydrogen and the corresponding pinning centers varied with the temperature of the chemical polishing. We have carried out chemical polishing at room temperature ($t=23$ °C) and at a temperature of 0 °C. After completion of the cycle of measurements, the samples were subjected to annealing in a vacuum of 10^{-8} Torr at a temperature of 800 °C. This annealing had practically no effect on the dislocation structure of the niobium, but led to complete removal of the hydrogen from the sample. The dislocation clusters remaining at the location of hydride formation are also pinning centers, but they are less effective in pinning the magnetic flux vortices. One of the samples was annealed immediately after chemical polishing, i.e., it did not contain "controlled" pinning centers (at $T=4.2$ K, neither niobium hydride nor clusters of dislocations produced by the previously existing formations were produced). For an explanation of the effect of the residual impurities on the studied effect, a cycle of measurements was performed on a sample of niobium with a resistance ratio $\rho_{300K}/\rho_{4.2K}=1500$. The results showed the insensitivity of the studied effects to the residual resistance of our samples. The data on the investigated samples are given in the table.

METHOD OF MEASUREMENT

We studied the active part of the surface impedance of superconducting niobium in the intermediate state.

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