and Breach, New York, 1974, p. 59.
[3]G. Martinez, in: Physics of IV–VI Compounds and Alloys (ed. by S. Rabii), Gordon and Breach, New York, 1974, p. 215.
[4]G. Martinez, Phys. Rev. B 8, 4678 (1973).
[5]N. B. Brandt, O. N. Belousova, V. P. Zlomanov, Ya. G. Ponomarev, and E. P. Skipetrov, Fiz. Tverd. Tela (Leningrad) 19, 437 (1977) [Sov. Phys. Solid State 19, 250 (1977)].
[6]J. O. Dimmock, in: Proc. Intern. Conf. on Physics of Semimetals and Narrow-Gap Semiconductors, Dallas, 1970 (ed. by D. L. Carter and R. T. Bate), in: J. Phys. Chem. Solids 32, Suppl. 1, 319 (1971).
[7]E. O. Kane, J. Phys. Chem. Solids 1, 249 (1957).
[8]J. Melngailis, T. C. Harman, and W. C. Kernan, Phys. Rev. B 5, 2250 (1972).
[9]I. V. Kucherenko, V. N. Moiseenko, and A. P. Shotov, Fiz. Tekh. Poluprovodn. 11, 162 (1977) [Sov. Phys. Semicond. 11, 90 (1977)].
[10]V. N. Ivanov, I. V. Kucherenko, V. N. Moiseenko, M. S. Taktakishvili, and A. P. Shotov, Fiz. Tekh. Poluprovodn. 9, 690 (1975) [Sov. Phys. Semicond. 9, 453 (1975)].
[11]O. Beckman and Y. Shapira, Phys. Lett. 21, 282 (1966).
[12]P. Kästner, Phys. Status Solidi B 53, 753 (1972).
[13]Ya. G. Ponomarev, Prib. Tekh. Eksp. No. 6, 218 (1966).
[14]O. N. Belousova, Kand. diss. (Thesis for Candidate's Degree), Moscow State University, 1975.
[15]N. B. Brandt and Ya. G. Ponomarev, Zh. Eksp. Teor. Fiz. 55, 1215 (1968) [Sov. Phys. JETP 28, 635 (1969)].
[16]L. D. Jennings and C. A. Swenson, Phys. Rev. 112, 31 (1958).
[17]N. B. Brandt, S. V. Kuvshinnikov, N. Ya. Minina, and E. P. Skipetrov, Prib. Tekh. Eksp. No. 6, 160 (1973).
[18]T. C. Harman, in: Proc. Intern. Conf. on Physics of Semimetals and Narrow-Gap Semiconductors, Dallas, 1970 (ed. by D. L. Carter and R. T. Bate), in: J. Phys. Chem. Solids 32, Suppl. 1, 363 (1971).
[19]A. P. Shotov, Proc. Fourth Conf. on Solid State Devices, Tokyo, 1972, publ. by Japanese Society of Applied Physics, Tokyo (1973), in: Oyo Butsuri 42, Suppl., 282 (1973).
[20]K. F. Cuff, M. R. Ellett, C. D. Kuglin, and L. R. Williams, Proc. Seventh Intern. Conf. on Physics of Semiconductors, Paris, 1964, Vol. 1, Physics of Semiconductors, publ. by Dunod, Paris; Academic Press, New York (1964), p. 677.
[21]S. Bermon, Phys. Rev. 158, 723 (1967).
[22]A. A. Abrikosov, J. Low Temp. Phys. 8, 315 (1972).
[23]O. N. Belousova and E. P. Skipetrov, Tezisy dokladov 1-ĭ Vsesoyuznoĭ konf. "Poluchenie i svoĭstva poluprovodnikovykh soedineniĭ $A^{II}B^{VI}$ i $A^{IV}B^{VI}$ i tverdykh rastvorov na ikh osnove" (Abstracts of Papers presented at First All-Union Conf. on Preparation and Properties of II–VI and IV–VI Semiconductor Compounds and Solid Solutions Based on Them), M., 1977, p. 10.
[24]G. M. T. Foley and D. N. Langenberg, Preprint, University of Pennsylvania, Philadelphia, 1976.
[25]J. C. Ramage, R. A. Stradling, A. Aziza, and M. Balkanski, J. Phys. C 8, 1731 (1975).

Translated by A. Tybulewicz

Oscillations of the impedance of thin tungsten plates in a magnetic field

O. A. Panchenko, V. V. Vladimirov, P. P. Lutsishin, and M. A. Mukhtarov

Institute of Physics, Ukrainian Academy of Sciences
(Submitted 16 July 1977)
Zh. Eksp. Teor. Fiz. **74**, 658–664 (February 1978)

The derivative with respect to the magnetic field of the surface impedance of thin tungsten plates (the skin layer depth being comparable with the thickness) in circularly polarized radio-frequency fields ($f = 5$ MHz) is investigated experimentally and theoretically. The magnetic field is perpendicular to the (001) face of the sample. It is shown that in both polarizations the impedance oscillations are due to a group of holes lying at the inflection of the Fermi surface octahedron. The "–" polarization oscillation series can be attributed entirely to the Gantmakher-Kaner radio-frequency effect [Sov. Phys. JETP **21**, 1053 (1965)]. The impedance oscillations in the "+" polarization are due to both the Gantmakher-Kaner effect and to doppleron excitation [McGraddy *et al.*, Phys. Rev. **141**, 437(1966); Fisher *et al.*, Sov. Phys. JETP **33**, 410 (1971)].

PACS numbers: 72.15.Gd

1. It is known that weakly attenuating electromagnetic waves, called dopplerons, can exist in metals near the threshold of Doppler-shifted cyclotron resonance.[1,2] The dopplerons are most clearly observed in compensated metals, where the helicons cannot propagate. Tungsten is one such metal. The excitation of dopplerons can lead to an oscillatory dependence of the impedance of the plate on the magnetic field.

In the present work, we have investigated the surface resistance of thin single-crystal samples of tungsten ($D \sim \delta$, D is the thickness of the plate, δ is the skin depth) in the radiofrequency region of the spectrum. A circularly polarized exciting field was used in the experiments. A sharply delineated series of oscillations of the derivative of the impedance with respect to the magnetic field was observed in strong magnetic fields in both polarizations. The period of the oscillations in the "+" polarization (direction of rotation of the radiofrequency field corresponds to the cyclotron rotation of the holes) changes by about 5% with the magnetic field, and in the "–" polarization it is practically independent of the field. In very strong fields, the two periods coincide and correspond to the cyclotron displacements of the holes located at the inflection points of the octahedron of the Fermi surface. The conclusion has been drawn, on the basis of these results, that all the observed phenomena are connected with a single group of carriers. It will be shown that the series of oscillations with constant period ("–" polarization) is due to the Gantmakher-Kaner ef-

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fect.[3] Oscillations in the "+" polarization are connected both with the excitation of the doppleron due to Doppler-shifted cyclotron resonance of the holes of the octahedron, and to the associated Gantmakher-Kaner effect whose amplitude depends on the direction of rotation of the radio-frequency field.[3] There is satisfactory agreement between the theoretical and experimental results.

2. Measurements of the high-frequency impedance of thin plates ($\rho_{300\ \mathrm{K}}/\rho_{4.2\ \mathrm{K}} \approx 10^5$) of tungsten ($D = 450\ \mu$) have been carried out at the temperature of liquid helium. The method of preparation of samples and carrying out the experiment have been described previously.[4,5]

The results of recording the derivatives of the real part of the surface impedance with respect to the magnetic field, dR/dH, as a function of the applied field H are shown in Fig. 1. The plots in both polarizations were obtained at equal amplification coefficients of the measuring apparatus. Figure 2 shows the experimental dependence of the period of oscillation on the magnetic field. It is clearly seen that the period in the "−" polarization does not depend on the magnetic field and corresponds to the cyclotron displacement of the holes at the inflection of the octahedron of the Fermi surface.[6] ($\Delta HD \approx 21$ Oe· cm, $(2\pi)^{-1}(\partial S/\partial p_z)_{\mathrm{ext}} = -0.491\,\mathrm{Å}^{-1}$,[7] where S is the cross section area of the Fermi surface as intercepted by the plane $p_z = \mathrm{const}$). The period of oscillation in the "+" polarization is significantly increased with increase in the magnetic field (~5%) and approaches the value in the "−" polarization.

Such a character of the dependence of the period on the magnetic field is typical for the oscillations produced by the excitation of the doppleron connected with the Doppler-shifted cyclotron resonance of the carriers having a maximum shift after one cyclotron period. Just such a group (section A of the hole octahedron[7]) makes the greatest contribution to the non-local conductivity. The results (Figs. 1 and 2) indicate that both series of oscillations are due to the single group of carriers.[1)] The absence of dispersion of the period in the

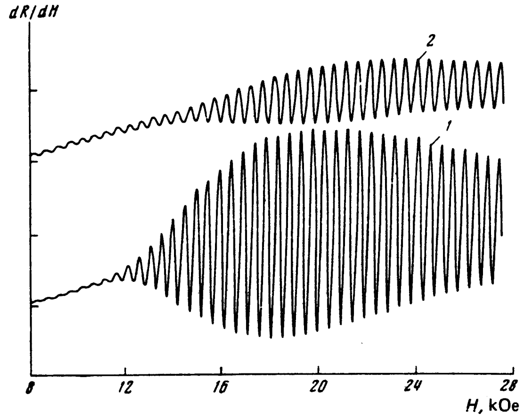

FIG. 1. Dependence of the derivative of the surface resistance of a thin single crystal plate of tungsten on the value of the constant magnetic field H. The normal to the surface of the plate and the vector **H** are parallel to the ⟨001⟩ axis, the thickness of the plate D = 450 μ, the frequency of the alternating field f = 5 MHz; 1—"+" polarization, 2—"−" polarization.

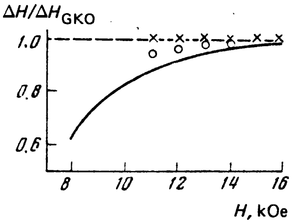

FIG. 2. Dependence of the period of oscillations of the derivative of the surface resistance on the magnetic field. The open circles correspond to the "+" polarization, the crosses to the "−" polarization, the continuous curve denotes the calculated dependence of the period in the "+" polarization.

"−" polarization has made possible the conclusion that this series of oscillations is produced by the single-particle Gantmakher-Kaner effect. This effect should arise in both polarizations.

3. We proceed to the theoretical analysis of the results. The real Fermi surface of tungsten consists of an electron jack, six hole ellipsoids, and a hold octahedron.[6] In calculations of the nonlocal conductivity, we neglect the contribution of the hole ellipsoids (their total volume is ~10% of the volume of the octahedron[6]). The electron contribution to the conductivity is taken into account in the local limit, while the dispersion law for the hole octahedron in the vicinity of the Fermi surface is chosen in the form

$$\varepsilon(\mathbf{p}) = \frac{\hbar^2}{2m}\left[r - a\cos 4\varphi - b\cos\frac{\pi p_z}{d}\right]^2, \tag{1}$$

where **p** is the wave vector, $r = (p_x^2 + p_y^2)^{1/2}$, $\varphi = \arctan(p_y/p_x)$, $-d \leq p_z \leq d$, the quantities m, a, b, d are constant parameters, and $2d$ is the diameter of the Fermi surface along the z axis. The z axis is oriented along the magnetic field and coincides with the axis of fourth-order symmetry ⟨001⟩. The numerical values of the parameters $(2m\epsilon_F)^{1/2}, a, b, d$ are chosen from the requirements of equality of the phase volume, of the derivative $(\partial S/\partial p_z)_{\mathrm{ext}}$ and of the two linear dimensions ($r - a - b, r + a - b$) to the real values which are given in the literature.[6,8] These parameters are respectively equal to

$$\hbar^{-1}(2m\varepsilon_F)^{1/2} = 0.56\ \mathrm{Å}^{-1},\quad a = 0.075\ \mathrm{Å}^{-1},\quad b = 0.115\ \mathrm{Å}^{-1},\quad d = 0.43\ \mathrm{Å}^{-1}.$$

It should be noted that on the jack there are groups of electrons which undergo larger displacements over the cyclotron period than the considered group of holes of section A.[7] Corresponding estimates given in the Appendix show that we can neglect the collisionless absorption of the wave by these electrons.

The nonsphericity of the Fermi surface, which is taken into account in our model by the parameter a, leads to the the possibility of the existence of multiple dopplerons.[9] In our experiments, no multiple dopplerons were observed. We shall therefore assume below that $a = 0$ and in the Appendix we estimate the amplitude of the multiple dopplerons.

The dispersion equation for waves propagating along the magnetic field is of the form

$$c^2k^2-4\pi i\omega\sigma_\pm(k)=0, \quad (2)$$

c is the velocity of light in vacuum, k and ω are the wave vector and the wave frequency, respectively, and $\sigma_\pm(k)$ is the nonlocal conductivity tensor for circular polarizations of the field.

The conductivity tensor, obtained from the general expression,[10] is equal in our model of the Fermi surface (without account of multiple Doppler-shifted cyclotron resonances) to

$$\sigma_\pm(k)=\sigma_{xx}\pm i\sigma_{xy}=\frac{Nec}{H}\left\{\mp i\left[\frac{1}{(1-\xi^2)^{1/2}}-1\right]+\gamma\left[\frac{1}{(1-\xi^2)^{1/2}}+1\right]\right\}, \quad (3)$$

where

$$N=\frac{2}{(2\pi)^3}\int_{-d}^{d}S\,dp_z$$

is the hole concentration, e is the absolute value of the electron charge, $\xi=k(v_z/\Omega)_{\text{ext}}$, $\Omega=eH/mc$ is the cyclotron frequency, v_z is the projection of the hole velocity on the z axis, $\gamma=\nu/\Omega$, and ν is the frequency of collisions of carriers with scatterers. The expression (3) was obtained in the limit $|1-\xi^2|>\gamma$, $\omega\ll\nu\ll\Omega$.

In the "+" polarization, there is an almost true solution ($|\xi'|\gg\xi''$) of the dispersion equation, describing the doppleron:

$$\xi'=\operatorname{Re}\xi=-\left[\frac{1}{2}-\frac{1}{h}+\frac{1}{2}\left(1+\frac{4}{h}\right)^{1/2}\right]^{1/2}, \quad (4)$$

$$\xi''=\operatorname{Im}\xi=-\gamma\frac{\xi'[1+(1-(\xi')^2)^{3/2}]}{3(\xi')^2-2[1-(1-(\xi')^2)^{3/2}]}. \quad (5)$$

Here

$$h=\left(\frac{H}{H_0}\right)^3,\quad H_0^3=\frac{4\pi\omega Nc}{e}(mv_z)^2_{\text{ext}}.$$

The quantity measured in the experiments is the derivative of the surface resistance with respect to the magnetic field. We calculate it, making use of the expression for the impedance in the case of specular reflection of the carriers from the boundary of the sample. It should be noted that the real character of the reflection of carriers is close to diffuse in our experiments; however, in the case of bilateral antisymmetry excitation, when the thickness of the skin layer is comparable with the thickness of the sample, the reflection conditions have a weak effect on the amplitude of the doppleron. Thus, for example, it has been shown[11] that the amplitude of the doppleron increases under these conditions, by about 30% upon change in the specularity coefficient from 0 to 0.5. Qualitatively, this can be explained in the following fashion. Of the entire skin layer, only that spatial harmonic whose length is close to the cyclotron shift of the resonance group of carriers is "useful" for the doppleron and the Gantmakher-Kaner wave (for this harmonic, decompensation of the Hall conductivities takes place). The amplitude of this harmonic depends on the distribution of the field in the skin layer. In the case of bilateral antisymmetric excitation, when $\delta\sim D$, the field distribution is such that the depth of the effective skin layer $\delta_{\text{eff}}=D/2$ (the field is equal to zero at the center of the plate) and depends weakly on the reflection conditions at the boundary. Under conditions in which the thickness of the skin layer is considerably less than the thickness of the sample, the character of the scattering can depend strongly on the amplitude of the doppleron, as is shown in the theoretical researches of Refs. 12 and 13. In the calculations, we have taken into account the character of the scattering by the introduction of the effective path length: $1/l^*=1/l+1/D$, where l is the free path length.

According to Fisher *et al.*,[2] the expression for the impedance of the plate in the case of bilateral antisymmetric excitation is of the form

$$Z_\pm=-8i\omega\int_{-\infty}^{+\infty}\frac{1-e^{ikD}}{c^2k^2-4\pi i\omega\sigma_\pm(k)}dk. \quad (6)$$

The doppleron contribution to the impedance is

$$\Delta Z_+=\frac{4H^2}{Nc^2(mv_z)_{\text{ext}}}\exp\left[i\left(\frac{\Omega}{v_z}\right)_{\text{ext}}\xi D\right]\xi^{-2}\left(\frac{\partial F}{\partial\xi}\right)^{-1}, \quad (7)$$

here

$$F=\frac{1}{\xi^2}\left\{\frac{1}{(1-\xi^2)^{1/2}}-1+i\gamma\left[\frac{1}{(1-\xi^2)^{1/2}}+1\right]\right\},$$

$\xi=\xi'+i\xi''$, where ξ' and ξ'' are determined by the expressions (4) and (5). The period of the impedance oscillations is

$$\Delta H=\Delta H_{\text{GKO}}\Big/\left(-\xi'-3h\frac{\partial\xi'}{\partial h}\right), \quad (8)$$

where

$$\Delta H_{\text{GKO}}=\frac{2\pi c(mv_z)_{\text{ext}}}{eD}=\frac{\hbar c}{eD}\left(\frac{\partial S}{\partial p_z}\right)_{\text{ext}}$$

is the period of the Gantmakher-Kaner oscillations.

The oscillations, connected with the excitation of the doppleron, of the derivative of the surface impedance dR/dH [$R=\operatorname{Re}(\Delta Z_+)$] for a plate with $D=450\mu$, excitation frequency $f=\omega/2\pi=5$ MHz and effective path length $l^*=200\mu$, are shown in Fig. 3 (curve 1). For comparison, the result of the calculation of the experimental records of the derivative of the impedance in the polarizations "+" and "−", is shown in Fig. 3 (curve 2). The qualitative form of curves 1 and 2 (Fig. 3) is approximately the same. However, the theoretical excitation threshold of the doppleron in the magnetic field is lower than the one observed experimentally. The reason for this discrepancy is the following. The lower threshold field is determined by the value at which the Hall current, which is connected with the nonlocal decompensation of the electron and hole conductivities, becomes equal to the dissipative conductivity. Since the electron conductivity is taken into account in the calculations in the local limit, the theoretical decompensation of the Hall conductivities at fixed magnetic field exceeds the real. Figure 2 shows the calculated dependence of the period of the oscillations (the solid

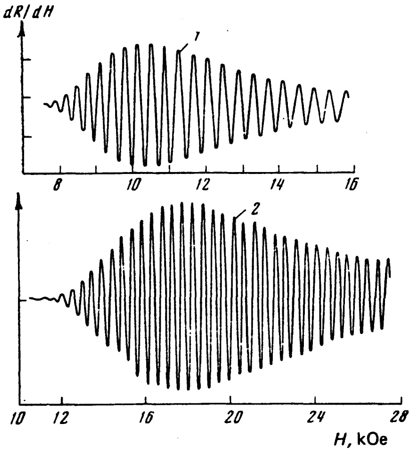

FIG. 3. Dependence of the derivative of the surface resistance of a thin single crystal plate of tungsten on the constant magnetic field. The vector **H** is directed along the normal to the surface of the plate and coincides with the ⟨001⟩ axis, the thickness of the plate $D = 450\,\mu$, the frequency of the alternating field $f = 5$ MHz. 1—calculated curve for doppleron oscillations ("+" polarization), 2—result of substraction of the experimental recordings of the derivatives of the surface resistance in the "+" and "−" polarizations.

curve) in the "+" polarization on the magnetic field, which agrees qualitatively with the observed.

The contribution to the impedance from the Gantmakher-Kaner effect in the region of fields $H \sim 10$ kOe is described by the expression

$$\Delta Z_{\mathrm{GK}} \approx \frac{4H^2}{Nc^2(mv_z)_{ext}} \pi^{1/2}(-1+i)\exp\left(-\frac{D}{l^*}\right)\left(\frac{u}{2\pi D}\right)^{3/2}\exp\left(i\frac{2\pi D}{u}\right), \quad (9)$$

where

$$\frac{u}{2\pi} = \left(\frac{v_z}{\Omega}\right)_{ext}.$$

Comparison of the formulas (7) and (9) shows that in the region of strong magnetic fields, the amplitudes of the doppleron oscillations and those connected with the Gantmakher-Kaner effect are close in value. A comparison of the experimental data (Fig. 1, curve 2 and Fig. 3, curve 2) confirms this conclusion. In the range of fields where the amplitude of the doppleron is a maximum, the measured ratio of the amplitudes of the doppleron and the oscillations in the "+" polarization is approximately equal to 5. The calculated value is about 12 ($\xi' = -0.96$). These estimates allow us to draw the conclusion that the oscillations of the impedance in the "−" polarization are due to the Gantmakher-Kaner effect.

We express our deep gratitude to E. A. Kaner for valued critical remarks.

APPENDIX

1. The dissipative conductivity connected with collisionless absorption is determined by the expression

$$\sigma_{xx}(k) \approx \frac{1}{8\pi^2}\frac{ec}{H}\left\{S\left|\frac{\partial S}{\partial p_z}\bigg/\frac{\partial^2 S}{\partial p_z^2}\right|\right\}_{p_z=p_z(k)}, \quad (\mathrm{A.1})$$

where $p_z(k)$ is determined by the condition for Doppler shifted cyclotron resonance of carriers with the wave:

$$1 - \frac{\hbar ck}{2\pi eH}\frac{\partial S}{\partial p_z} = 0. \quad (\mathrm{A.2})$$

The estimates made by means of Eq. (1), with the use of the data for $\partial S/\partial p_z$ in Ref. 7, show that the collisionless damping is comparable with the collision damping at $\gamma \sim 10^{-3}$. Actually, in tungsten in the range of fields $H \sim 10$ kOe, $\gamma \gtrsim 10^{-2}$; therefore, we shall neglect this damping in the calculations.

2. The transverse corrugation of the Fermi surface leads to the existence of multiple Doppler shifted transverse resonances.[14] In our model, the conductivity tensor in the case $a \neq 0$ is of the form

$$\sigma_\pm(k) = \frac{Nec}{H}\left\{\mp i\left[\frac{1}{(1-\xi^2)^{1/2}} - 1 + 4\mu^2\left(-\frac{1}{(9-\xi^2)^{1/2}} + \frac{1}{(25-\xi^2)^{1/2}} + \frac{2}{15}\right)\right]\right.$$
$$\left. + \gamma\left[\frac{1}{(1-\xi^2)^{3/2}} + 1 + 4\mu^2\left(\frac{3}{(9-\xi^2)^{3/2}} + \frac{5}{(25-\xi^2)^{3/2}}\right)\right] + O(\mu^4)\right\},$$
$$\mu = \hbar a/(2m\varepsilon_F)^{1/2}. \quad (\mathrm{A.3})$$

It follows from this expression that the multiple Doppler shifted cyclotron resonances appear in the region $|n^2 - \xi^2| \lesssim \mu^4 \sim 10^{-3} - 10^{-4}$ $(n = 3, 5)$. Since $\gamma \gtrsim 10^{-2}$, in our experiments the multiple dopplerons were not observed.

In the collisionless limit $\gamma \ll \mu^4$ weakly damped waves can exist, near multiple Doppler shifted cyclotron resonances. Analytical estimates for the ratio of the amplitudes of the single and triple dopplerons that are maximal in the magnetic field yield $A_3/A_1 \lesssim 10^{-2}$, while the triple doppleron is estimated in the limit $\gamma = 0$, i.e., this is its maximum value at the given corrugation μ. The amplitude of the five-fold doppleron is much less than the triple because of the presence of the resonant group of electrons (section H of the jack) with displacements close to the wavelength $(-(\partial S/\partial p_z)_A/(\partial S/\partial p_z)_H \simeq 4.8$[7]).

The dependence of the maximum aplitude of the multiple doppleron on the transverse corrugation μ in the limit $\gamma = 0$ is weak:

$$A_3(\mu) = A_3(1+\varkappa\mu^{4/3})^{2/3}(1-\delta_{\mu,0}),$$
$$\varkappa \approx \left(\frac{2\pi D}{u_{max}}\right)^{1/2}, \quad \frac{u_{max}}{2\pi} = \frac{c}{eH_{max}}(mv_z)_{ext}, \quad \delta_{\mu,0} = \begin{cases}1, & \mu=0\\ 0, & \mu\neq 0\end{cases}, \quad (\mathrm{A.4})$$

$H_{\max}$ corresponds to the field at which the amplitude of the doppleron is maximum, and the dependence of $H_{\max}$ on μ can be neglected. For the thicknesses used in the experiment, $\varkappa \approx 3$. Thus the amplitude of the transverse corrugation strongly affects the possibility of existence of weakly damped waves near multiple Doppler shifted cyclotron resonances, but has little effect on their maximum amplitude.

Note. The authors offer sincere apology to Professor Carolan, whose earlier investigations [Low Temp. Phys. **13**, No. 4, 75 (1974)] were unknown to them and were therefore not cited in the article. In our article, unlike in Prof. Carolan's, we analyzed theoretically the experimental data and obtained more unequivocal conclusions concerning the character of doppleron excitation and the Gantmakher-Kaner effect when the RF emission is circularly polarized. Our experimental results differ also somewhat from those of Professor Carolan.

(signed) O. A. Panchenko, V. V. Vladimirov, P. P. Lutsishin, and M. A. Mukhtarov. 4 September 1978

[1)]In doppleron excitation, the period of the oscillations in strong magnetic fields is determined by the displacement of that group of carriers which makes the greatest contribution to the nonlocal conductivity. There is a group of electrons in tungsten with an extremal shift (the section I [7]); however, for these electrons, the quantity $\partial S/\partial p_z$ differs by 30% from the corresponding value for the holes of the section A (at the limiting points of the electron Fermi surface, this difference is even greater). Therefore, the oscillations in the "−" polarization cannot be connected with the excitation of the electron doppleron.

[1] J. C. McGroddy, J. L. Stanford and E. A. Stern, Phys. Rev. **141**, 437 (1966).

[2] L. M. Fisher, V. V. Lavrova, V. A. Yudin, O. V. Konstantinov and V. G. Skobov, Zh. Eksp. Teor. Fiz. **60**, 759 (1971) [Sov. Phys. JETP **33**, 410 (1971)].

[3] V. F. Gantmakher and É. A. Kaner, Zh. Eksp. Teor. Fiz. **48**, 1572 (1965) [Sov. Phys. JETP **21**, 1053 (1965)].

[4] V. V. Lavrova, S. V. Medvedev, V. G. Skobov, L. M. H. Fisher and V. A. Uydin, Zh. Eksp. Teor. Fiz. **64**, 1839 (1973) [Sov. Phys. JETP **37**, 929 (1973)].

[5] I. M. Vitebskiĭ, V. T. Vitchinkin, A. A. Galkin, Yu. S. Ostroukhov, O. A. Panchenko, D. T. Tsymbal and A. N. Cherkasov, Fiz. Nizkikh Temp. 1, 400 (1975) [Sov. J. Low Temp. Phys. **1**, 200 (1975)].

[6] R. F. Girvan, A. V. Gold, R. A. Phillips, J. Phys. Chem. Sol. **29**, 1485 (1968).

[7] D. E. Soule, and J. C. Abele, Phys. Rev. Lett. **23**, 1287 (1969).

[8] V. V. Boĭko and V. A. Gasparov, Zh. Eksp. Teor. Fiz. **61**, 2362 (1971) [Sov. Phys. JETP, **34**, 1266 (1972)].

[9] V. G. Skobov, L. M. Fisher, A. S. Chernov and V. A. Yudin, Zh. Eksp. Teor. Fiz. **67**, 1218 (1974) [Sov. Phys. JETP **40**, 605 (1975)].

[10] É. A. Kaner and V. G. Skobov, Adv. in Phys. **17**, 605 (1968).

[11] Yu. S. Ostroukhov, O. A. Panchenko and A. A. Kharlamov, Zh. Eksp. Teor. Fiz. **70**, 1838 (1976) [Sov. Phys. JETP **43**, 957 (1976)].

[12] I. F. Voloshin, S. V. Medvedev, V. G., Skobov, L. M. Fisher and A. S. Chernov, Pis'ma Zh. Eksp. Teor. Fiz. **23**, 553 (1976) [JETP Lett. **23**, 507 (1976)].

[13] I. F. Voloshin, S. V. Medvedev, V. G. Skobov, L. M. Fisher and A. S. Chernov, Zh. Eksp. Teor. Fiz. **71**, 1555 (1976) [Sov. Phys. JETP **44**, 814 (1976)].

[14] D. S. Falk, Phys. Rev. B **3**, 1973 (1971).

Translated by R. T. Beyer

Penetration of a magnetic field into a Josephson junction

G. F. Zharkov and S. A. Vasenko

P. N. Lebedev Institute of Physics, USSR Academy of Sciences

(Submitted 18 July 1977)
Zh. Eksp. Teor. Fiz. **74**, 665–680 (February 1978)

All the possible types of solutions to the nonlinear equation for the distribution of a static magnetic field in a Josephon barrier of finite width are described. For each type of solution corresponding to the boundary-value problem, the Cauchy problem, which allows a unique association of a definite set of "initial" data with each solution, is formulated and solved by numerical methods. The magnetization curves of the Josephson junction are found for several barrier-width values ($L = 1, 4, 10$). The question of the stability of the static solutions, including those that are anomalous in comparison with the usual Meissner-type solutions, is investigated. Examples of the numerical solution of the nonstationary, nonlinear equation that illustrate the dynamics of the establishment of the static solutions are given.

PACS numbers: 74.30.Ci, 74.50.+r

The question of the penetration of a static magnetic field into a Josephson junction has been repeatedly discussed in the literature.[1-7] The one-dimensional linear vortex structure that arises in the barrier in the case of an infinitely wide barrier has been analytically described by Kulik,[2,3] while for the case of a barrier of finite width some examples of the field distribution in the junction have been obtained with the aid of numerical methods by Owen and Skalapino.[4] A number of distinctive features of the penetration of a magnetic field into a junction of finite width have been noted in a paper by one of the present authors[7] (in particular, the unevenness of the entry of the individual vortices into the junction, as well as the presence of "superheating" and "supercooling" fields that limit the existence domain of a given number of vortices in a weak superconductor).

The present paper is devoted to a more detailed—in comparison with Refs. 4 and 7—study of the character of the penetration of a magnetic field into a Josephson barrier of finite width. The boundary-value problem for the nonlinear equation governing the steady-state magnetic-field and current distributions inside the barrier turns out to be nonunique: for specified values of the field at the junction edges the equation has several solutions describing different field distributions inside the barrier. In the present paper the boundary-value problem is reduced to an equivalent Cauchy problem; this procedure allows us to uniquely associate a definite set of "initial" data with each solution and find all the solutions of the problem. Below we describe all the possible types of solutions (Sec. 1) and find the integral relations that allow the determination of the initial data (i.e., the values of the function and its derivative at one of the barrier edges) in terms of the parameters of the boundary-value problem (Sec. 2). Dependences found with the aid of a computer (and, in a number of cases, analytically) are illustrated with graphs. Besides the usual Meissner-type solutions,[1-7] in which the field falls off into the barrier in comparison with its value at the edges, we describe anamalous solutions in which the field increases into the barrier, or else there obtains an asymmetric distribution of the field, as well as solutions corresponding to more complex field configurations, in particular, to an array of vortices with alternating signs.

We find the free-energy functions of the Josephson barrier in an external field for the various types of solutions (Sec. 3), and present graphs illustrating the shape

0038-5646/78/4702-0350$02.40