Resonant formation of μ-mesic molecules of hydrogen

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In a state with orbital angular momentum $J = 1$ and with vibrational quantum number $v = 1$, highly excited levels with energy $\epsilon_{Jv}(dd\mu) = -2.2$ eV and $\epsilon_{Jv}(dt\mu) = -1.1$ eV have been found in the mesic molecules $dd\mu$ and $dt\mu$. The presence of these states leads to resonant formation of mesic molecules. The rate of resonant formation of the mesic molecules is calculated and found to be $\lambda_{dd\mu} = 0.8 \times 10^6$ sec^{-1} and $\lambda_{dt\mu} \approx 10^8$ sec^{-1}. The obtained value of $\lambda_{dd\mu}$ agrees well with the measured value. The value obtained for $\lambda_{dt\mu}$ means that a single μ^- meson can catalyze $\sim 10^2$ deuterium and tritium fusion reactions.

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1. INTRODUCTION

The usual mechanism of production of μ-mesic molecules is the electric dipole $E1$ transition with conversion of the atomic electron from the state $J = 0$ of the continuous spectrum of the μ-mesic atom + nucleus system into a bound state of the μ molecule with orbital angular momentum $J = 1$,[1,2] for example

$$d\mu + H_2 \rightarrow [(pd\mu)pe]^+ + e. \tag{1}$$

This results in the production of a unique molecular ion, one of the nuclei of which becomes the produced mesic molecule $pd\mu$. The energy released when the $pd\mu$ molecule is formed is carried away by the Auger electron. The recently calculated[3] rates of formation of mesic molecules, $\lambda_{pp\mu} = 2.2 \times 10^6$ sec^{-1} and $\lambda_{pd\mu} = 5.9 \times 10^6$ sec^{-1}, corresponding to this mechanism, agree well with the measured values.[4-7]

At the same time, the calculated value[3,8-10] $\lambda_{dd\mu} = 0.4 \times 10^5$ sec^{-1} is close to the experimental[11,12] $\lambda_{dd\mu} = 0.8 \times 10^5$ sec^{-1} at liquid-hydrogen temperature and differs by more than one order of magnitude from the value $\lambda_{dd\mu} = 0.8 \times 10^6$ sec^{-1} measured at room temperature.[13-15] Such large differences are attributed to the fact that the $dd\mu$ molecule is produced with the aid of the special resonance mechanism, which was considered by Vesman.[16]

The gist of the resonance mechanism is that the energy released in the formation of the $dd\mu$ molecule is transferred to the excited vibrations of the "molecule" $[(dd\mu)d2e]$ produced when the mesic atom $d\mu$ collides with the molecule D_2, in accordance with the reaction

$$d\mu + D_2 \rightarrow [(dd\mu)d2e]^{\cdot}. \tag{2a}$$

This mechanism is possible only if at least one of the levels of the $dd\mu$ molecule has an energy lower than the electron ionization energy (~15 eV) and the D_2-molecule dissociation energy (~4.5 eV). For a long time, the theoretical calculations did not confirm the existence of such a level, although suspicions that it exists were expressed long ago.[1,9,15] Recently, however, an effective scheme of calculating the binding energy of the three-body system with Coulomb interaction has been developed in the adiabatic representation of the three-body problem[17] and has made it possible to determine the energy of this level with high accuracy. The calculated binding energy ϵ_{Jv} of the $dd\mu$ molecule in a state with orbital angular momentum $J = 1$ and vibrational quantum number $v = 1$ is

$$\epsilon_{Jv} = -1.96 \text{ eV.} \tag{3}$$

So far, there has been no indication whatever of the existence of an analogous level ($J = 1, v = 1$) of the $dt\mu$ molecule. The calculation scheme developed by us makes it possible to establish definitely the existence of this level and to obtain its binding energy:

$$\epsilon_{Jv} = -0.85 \text{ eV.} \tag{4}$$

It follows therefore that the dth molecule can be produced by a resonance mechanism analogous to (2a):

$$t\mu + D_2 \rightarrow [(dt\mu)d2e]^{\cdot} \tag{2b}$$

2. RESONANCE MECHANISM OF μ-MESIC MOLECULE PRODUCTION

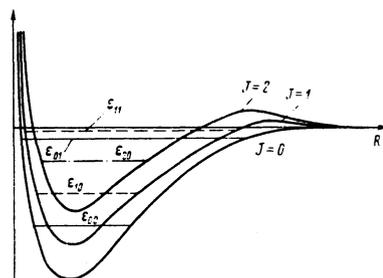Figure 1 shows the level scheme of the $dd\mu$ molecule. There are altogether five levels, two levels each with

FIG. 1. Energy levels ϵ_{Jv} of the mesic molecule $dd\mu$ in states with different orbital angular momenta J and vibrational numbers v.

TABLE I. Binding energy ϵ_{Jv} of the states (J, v) of the mesic molecules * $dd\mu$ and $dt\mu$.

J	v	$-\epsilon_{Jv},$ eV	
		$dd\mu$	$dt\mu$
0	0	324.97	319.07
0	1	35.62	34.67
1	0	226.25	232.23
1	1	1.96	0,85
2	0	85.62	102.26

*The table is based on the data of [18]. In the calculations we used the particle-mass values[19] $m_\mu = 206.769$, $M_d = 3670.481$, $M_t = 5496.918$ and the value Ry = 13.60535 eV. The mesic-atom energy unit is $\epsilon_\mu = 5626.33$ eV. The particles are assumed to have zero spin and no structure. The binding energy ϵ_{Jv} is reckoned respectively from the energies $E_{d\mu}$ and $E_{t\mu}$ of the isolated atoms $d\mu$ and $t\mu$.

vibrational quantum numbers $v = 0$ and $v = 1$ in states with orbital angular momentum $J = 0$ and $J = 1$, and one level with $J = 2$ and $v = 0$. The binding energies of these states of the molecules $dd\mu$ and $dt\mu$, calculated in accordance with the algorithm given in[7], are listed in Table I.

Particular interest attaches to the weakly bound states $(J = 1, v = 1)$ with anomalously low binding energy ~10^{-4} mesic-atom energy units. In Fig. 2 this level of the molecule $dd\mu$ is shown in magnified scale; ϵ_0 is the collision energy in the reaction $d\mu + d \to dd\mu$.

The reaction (2) proceeds from the initial states of the continuous spectrum of the system $d\mu + d$ (or $t\mu + d$) with $J = 0$ and from the ground state $(K = 0, v = 0)$ of the D_2 molecule into a bound state $(J = 1, v = 1)$ of the mesic molecule $dd\mu$ (or $dt\mu$) and an excited vibrational state v with orbital angular momentum $K = 1$ of the molecule $[(dd\mu)d2e]$ or $[(dt\mu)d2e]$ (see Fig. 2).

The rate of the resonant formation of the $dd\mu$ molecules, referred to the density N_0 of liquid hydrogen, is

$$\lambda = \sigma v N_0 [\text{sec}^{-1}], \tag{5}$$

where v is the relative velocity of the colliding particles and σ is the cross section for resonant production of mesic molecules in the reactions (2a) and (2b), and is given by the relation[16]

$$v d\sigma = 2\pi\hbar^{-1} |T_{fi}|^2 \delta(E_f - E_i) \gamma(\epsilon, \epsilon_T) d\epsilon. \tag{6}$$

Here E_f and E_i are the energies of the final and initial states of the system, $\gamma(\epsilon, \epsilon_T)$ is the Maxwellian distribution of the incident mesic atoms in the energies ϵ at a given temperature T corresponding to the average energy of the thermal motion $\epsilon_T = (3/2)kT$, where k = 8.167×10^{-5} eV/deg is the Boltzmann constant:
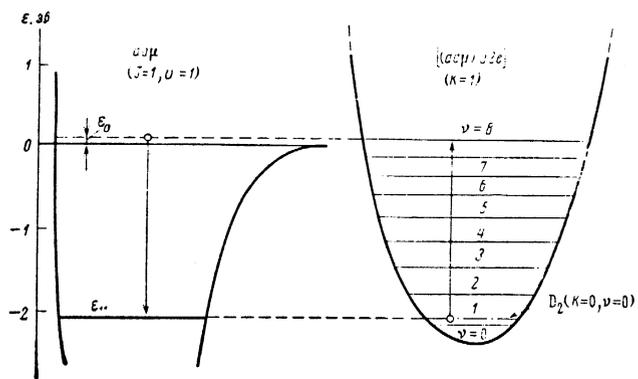

FIG. 2. Pattern of resonant production of $dd\mu$ molecule with transfer of the binding energy to excitation of vibrational levels v of the molecule $[(dd\mu)d2e]$.

$$\gamma(\epsilon, \epsilon_T) = \left(\frac{27}{2\pi} \frac{\epsilon}{\epsilon_T}\right)^{1/2} \frac{1}{\epsilon_T} \exp\left(-\frac{3\epsilon}{2\epsilon_T}\right). \tag{7}$$

The total energies of the system in the initial and final state are

$$E_i = \epsilon + E_0, \quad E_f = \epsilon_{11} + E_v, \tag{8}$$

where ϵ_{11} is the binding energy of the molecule $dd\mu$ (or $dt\mu$), E_v is the energy of the vibrational level v of the molecule $[(dd\mu)d2e]$ (or $[(dt\mu)d2e]$), and E_0 is the energy of the ground state of the molecule D_2. It follows from (6) that the reactions (2a) and (2b) are possible only for the condition $E_i = E_f$, i.e., at an incident mesic-atom energy $\epsilon = \epsilon_0$ (see Fig. 2) equal to

$$\epsilon_0 = \epsilon_{11} + E_v - E_0. \tag{8a}$$

The matrix element of the transition is

$$|T_{fi}|^2 = \sum_{m_K} \sum_{m_J} \left| \int d\mathbf{R}\, d\mathbf{r}\, d\rho\, \Psi^{(f)*}(\mathbf{r}, \mathbf{R})\, \psi^{(f)*}(\rho)\, \hat{H}_{int}\psi^{(i)}(\rho)\, \Psi^{(i)}(\mathbf{r}, \mathbf{R}) \right|^2, \tag{9}$$

where $\Psi^{(i,f)}(\mathbf{r}, \mathbf{R})$ and $\psi^{(i,f)}(\rho)$ are the wave functions of the initial (i) and final (f) states of the mesic molecule and the ordinary molecule, respectively. The summation is over all the projections of the orbital angular momenta J and K in the final state of the mesic molecule $dd\mu$ and $dt\mu$ and the molecules $[(dd\mu)d2e]$ and $[(dt\mu)d2e]$. For the definition of the vectors $\mathbf{r}$, $\mathbf{R}$, and ρ see Fig. 3.

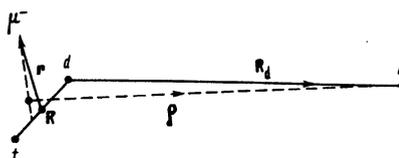FIG. 3. Coordinate system for the calculation of T_{fi} (9): $\mathbf{R}$ is the radius vector joining the nuclei t and d; $\mathbf{r}$ is the radius vector of the μ^- meson with origin at the geometric center of the segment $\mathbf{R}$, ρ is the radius vector joining the mass center of the $dt\mu$ system and the other nucleus of the molecule D_2, and $\mathbf{R}_d$ is the vector joining the nuclei in the molecule D_2. The following relations hold: $\rho \gg R$, r; $\rho \approx R_d$.

The interaction operator is (at $r \ll \rho$ and $R \ll \rho$)

$$H_{int} = d\rho / \rho^3,$$ (10)

where the dipole moment $d = d(r, R)$ of the system comprising the mesic atom and the nucleus of the D_2 molecule, relative to the mass center of the $dd\mu$ or $dt\mu$ molecule, is

$$d = -\frac{1}{2} \varkappa (1 - m_\mu / M_{tot}) R - (1 + m_\mu / M_{tot}) r.$$ (11)

Here m_μ is the muon mass, M_{tot} is the total mass of the μ-mesic molecule $dd\mu$ or $dt\mu$; $\varkappa = 0$ and $\varkappa = (M_d - M_t)/(M_d + M_t)$ respectively for the mesic molecules $dd\mu$ and $dt\mu$.

The mesic atoms $d\mu$ and $t\mu$, which move at normal temperature with velocities $\sim 10^5$ cm/sec, negotiate a region of interaction with the D_2-molecule nucleus of the order of $10a_\mu \approx 2 \times 10^{-10}$ cm within a time $\sim 10^{-15}$ sec. This time is much shorter than the time of the adiabatic restructuring of the wave function of the D_2 molecule, for which the frequency of the zero-point oscillations is $\sim 10^{14}$ sec^{-1}. It is therefore necessary to choose as the initial wave function $\psi^{(i)}(\rho)$ the function $\bar{y}_0(\rho)$ of the ground state of the D_2 molecule, and for the final state the wave functions $y_\nu(\rho)$ of the systems $[(dd\mu)d2e]$ and $[(dt\mu)d2e]$:

$$\Psi^{(i)}(\rho) = \rho^{-1} \bar{y}_0(\rho) / \sqrt{4\pi}.$$ (12a)

In the final state we have for the molecules $[(dd\mu)d2e]$ and $[(dt\mu)d2e]$ in the vibrational state ν with orbital angular momentum K respectively

$$\Psi^{(f)}(\rho) = \rho^{-1} y_\nu(\rho) Y_{Km_K}(\theta, \varphi).$$ (12b)

For the reaction (2a), the wave functions of the "mesic" part of the system in the initial and final states are respectively

$$\Psi^{(i)}(r, R) = \frac{1}{kR} \sum_{J=0} i^J (2J+1) \varphi_{1u}(r; R) \chi_{1u}^{(i)}(R) P_J(\cos\theta),$$ (13)

$$\Psi^{(f)}(r, R) = \frac{1}{R} \sum_j \varphi_{jg}(r; R) \chi_{jg}^{(f)}(R) Y_{Jm_J}(\theta, \Phi).$$

Here $\varphi_{jg}(r; R)$ and $\varphi_{ju}(r; R)$ are the even and odd solutions of the two-center problem and correspond to a state with a set of quantum numbers $j = [n_1 n_2 m]$ in accordance with the classification of the separated atoms.[20] The functions $\chi_{jg}^{(f)}(R)$ are the components of the vector function $\chi_g^{(f)} = \{\chi_{jg}^{(f)}\}$ corresponding to the level $(J=1, v=1)$ of the $dd\mu$ molecule when expanded in the basis $\varphi_{jg}(r; R)$. The solution $\chi_{1u}^{(i)}(R)$ is the first component of the wave function of the continuous spectrum of the $d\mu + d$ system in a state with orbital angular momentum J, and corresponds to the index $j = 1 = [000]$ of the principal term of the two-center problem.

For the reaction (2b), the relations (13) are valid subject to the substitutions

$$\varphi_{1u}(r; R) \chi_{1u}^{(i)}(R) \rightarrow \varphi_{1g}(r; R) \chi_{1g}^{(i)}(R) + \varphi_{1u}(r; R) \chi_{1u}^{(i)}(R),$$

$$\sum_j \varphi_{jg}(r; R) \chi_{jg}^{(f)}(R) \rightarrow \sum_j [\varphi_{jg}(r; R) \chi_{jg}^{(f)}(R) + \varphi_{ju}(r; R) \chi_{ju}^{(f)}(R)],$$ (14)

where the wave functions $\chi_{jg}(R)$ and $\chi_{ju}(R)$ represent the relative motion of the nuclei in the mesic molecule $dt\mu$.

The calculation of T_{fi} is analogous in many respects to the calculation in[3]. Ultimately, after separating the dimensional factors[1]), we obtain for the rate of resonant production of the mesic molecules $dd\mu$ and $dt\mu$ the expression

$$\lambda[\text{sec}^{-1}] = \beta \frac{8\pi^2}{3} (N_0 a_0^3) \left(\frac{m_e}{m_\mu}\right)^3 \left(\frac{m_\mu}{m_a}\right)^3 I_\nu^2 |d_{fi}|^2 \gamma(\epsilon_0, \epsilon_T) \frac{m_e e^4}{\hbar^3};$$ (15)

here $\gamma(\epsilon_0, \epsilon_T)$ is the Maxwellian distribution (7); $m_a^{-1} = m_\mu^{-1} + M_a^{-1}$, where M_a is the deuteron (or triton) mass; $N_0 = 4.25 \times 10^{22}$ cm^{-3} is the density of the liquid hydrogen; β is a statistical factor equal to $\frac{1}{3}$ for $dd\mu$ and 1 for $dt\mu$;[1-3] the quantities I_ν and d_{fi} were calculated respectively in atomic and mesic-atom units. The matrix elements I_ν of the dipole interaction are defined by the expression

$$I_\nu = \int_0^\infty \bar{y}_0(\rho) y_\nu(\rho) \frac{d\rho}{\rho^2}.$$

The matrix element d_{fi} is defined as

$$|d_{fi}|^2 = \frac{1}{4\pi} \sum_{m_J} \left| \int dR \, dr \, \Psi^{(f)*}(r, R) d(r, R) \Psi^{(i)}(r, R) \right|^2.$$ (16)

The value of d_{fi} in the case of the mesic molecule $dt\mu$ is (in units of $a_{m_a} = \hbar^2 / m_a e^2$)

$$d_{fi} = \frac{\varkappa}{2} \left(1 - \frac{m_\mu}{M_{tot}}\right) \frac{1}{k} \int dR \, R (\chi_{1g}^{(i)} \chi_g^{(f)} + \chi_{1u}^{(i)} \chi_u^{(f)})$$
$$+ \left(1 + \frac{m_\mu}{M_{tot}}\right) \frac{1}{k} \int dR (\chi_{1g}^{(i)} \chi_u^{(f)} + \chi_{1u}^{(i)} \chi_g^{(f)});$$ (17)

we have introduced here the notation

$$\chi_g^{(f)} = \sum \{\chi_{j\sigma_g}^{(f)} \langle j\sigma_g | r_z | 1u \rangle + \chi_{j\pi_g}^{(f)} \langle j\pi_g | r_x + ir_y | 1u \rangle\},$$

$$\chi_u^{(f)} = \sum \{\chi_{j\sigma_u}^{(f)} \langle j\sigma_u | r_z | 1g \rangle + \chi_{j\pi_u}^{(f)} \langle j\pi_u | r_x + ir_y | 1g \rangle\},$$ (18)

$$\langle i | r_k | j \rangle = \int dr \, \varphi_i^*(r; R) r_k \varphi_j(r; R),$$ (19)

$$j\sigma_{g,u} = [n_1 n_2 0]_{g,u}, \quad j\pi_{g,u} = [n_1 n_2 1]_{g,u}.$$

For the mesic molecule $dd\mu$ the value of d_{fi} is

$$d_{fi} = \left(1 + \frac{m_\mu}{M_{tot}}\right) \frac{1}{k} \int dR \, \chi_{1u}^{(i)} \chi_g^{(f)}.$$ (20)

3. CALCULATION OF WAVE FUNCTIONS

The wave functions $\bar{y}_0(\rho)$ and $y_\nu(\rho)$ of the D_2 molecule and of the systems $[(dd\mu)d2e]$ and $[(dt\mu)d2e]$ were calculated by us as solutions of the Schrödinger equation

$$y_\nu''(\rho) + 2M \left[E_\nu - \frac{K(K+1)}{2M\rho^2} - U(\rho)\right] y_\nu = 0$$

with reduced masses $M = M_d / 2m_e$ for $\bar{y}_0(\rho)$ and $M = M_{(d,t)}/$

TABLE II. Energy levels E_ν and integrals I_ν for the molecules $[(dd\mu)\,d2e]$ and $[(dt\mu)\,d2e]$ in a state with orbital angular momentum $K=1$.*

ν	[(ddμ) d2e] $-E_\nu$, eV	$I_\nu \cdot 10$	[(dtμ) d2e] $-E_\nu$, eV	$I_\nu \cdot 10$	ν	[(ddμ) d2e] $-E_\nu$, eV	$I_\nu \cdot 10$	[(dtμ) d2e] $-E_\nu$, eV	$I_\nu \cdot 10$
0	4.577	5.00	4.582	5.00	4	3.357	0.119	3.395	0.133
1	4.255	0.865	4.270	0.845	5	3.079	0.0663	3.123	0.0757
2	3.945	0.483	3.968	0.521	6	2.811	0.0390	2.862	0.0453
3	3.645	0.224	3.676	0.244	7	2.554	0.0239	2.610	0.0282
					8	2.307	0.0152	2.368	0.0182

*The binding energy $\bar{E}_0 = -4.556$ eV calculated by us for the ground state ($K=0$, $v=0$) of the D_2 molecule agrees well with the experimentally measured value of the dissociation energy.

m_e for $y_\nu(\rho)$, where

$$M_{(d)} = M_d^{-1} + (2M_d + m_\mu)^{-1}, \quad M_{(t)}^{-1} = M_d^{-1} + (M_d + M_t + m_\mu)^{-1}.$$

The effective potential $U(\rho)$ was taken from[21], and the SLIP program[22] was used in the calculations. The results of the calculation of the energies $\bar{E}_0$ and E_ν and of the integrals I_ν, are listed in Table II.[2]

The wave functions $\varphi_{j_g}(\mathbf{r}; R)$ and $\varphi_{j_u}(\mathbf{r}; R)$ of the two-center problem were calculated in accordance with the algorithms of [23,24]. They are numbered by a set of parabolic quantum numbers $j = [n_1 n_2 m]_{g,u}$ and by the parity values $P_{g,u} = (-)^l$, where the orbital angular momentum is $l = 2n_2 + m$ or $l = 2n_2 + m + 1$, depending on the symmetry of the function relative to the inversion $\eta \rightarrow -\eta$.[20] The matrix elements (19) were calculated in accordance with algorithms from [25-28] with accuracy $\sim 10^{-7}$ or better.

The wave functions $\chi_{i_g}^{(f)}(R)$ and $\chi_{j_u}^{(f)}(R)$ are obtained as the solutions of the system of equations

$$\sum_{j=1}^{N} \left\{ \left(\frac{d^2}{dR^2} - 2M\varepsilon_{j_v} \right) \delta_{ij} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} V_{i_g j_g}^J & V_{i_g j_u}^J \\ V_{i_u j_g}^J & V_{i_u j_u}^J \end{pmatrix} \right\} \begin{pmatrix} \chi_{j_g} \\ \chi_{j_u} \end{pmatrix} = 0,$$
$$M = M_e / m_{a_1}, \quad M_0^{-1} = M_a^{-1} + M_b^{-1} \tag{21}$$

with boundary conditions

$$\chi_{j_g}(0) = \chi_{j_u}(0) = \chi_{j_g}(R_m) = \chi_{j_u}(R_m) = 0$$

at $R_m \gg 1$.

The form of the effective potentials $V_{ij}^J \equiv V_{ij}^J(R)$ is given in [17,29]. To solve the system (21) we used an algorithm based on a continuous analog of Newton's method.[30]

The results of the calculation of the binding energies ε_{11} of the levels ($J=1$, $v=1$) of the mesic molecules $dd\mu$ and $dt\mu$ are given in Table III. We used in the calculations the algorithm developed in [17], which makes it possible to calculate the binding energy of molecules accurate to 10^{-5} (i.e., ~ 0.1 eV).

The wave functions $\chi_{1_g}^{(i)}(R)$ and $\chi_{1_u}^{(i)}(R)$, which represent the systems $d\mu + d$ and $t\mu + d$ in the initial scattering state, are obtained from the system (21) at $N=1$ with the boundary conditions at $R \rightarrow \infty$. For the reaction $d\mu + d$ have

$$\chi_{1_u}^{(i)}(R) \underset{R \to \infty}{\sim} \sin(kR - \pi J/2 + \delta_J), \quad k = (2M\varepsilon)^{1/2} \tag{22}$$

TABLE III. Components* of the binding energy ϵ_{J_v} [eV] of the mesic molecules $dd\mu$ and $dt\mu$ in the state $(J=1, v=1)$.

Contribution	$dd\mu$	$dt\mu$	Note
$\epsilon_{11}^{(0)}$	-0.67	$+0.47$	Two-level excitation ($N=1$)
$\epsilon_{11}^{\mathrm{discr}}$	-1.62	-0.46	With allowance for the discrete spectrum of the two-center problem ($N=11$)
$\epsilon_{11}^{\mathrm{cont}}$	-0.34	-0.39	Contribution of the continuous spectrum of the two-center problem.
ϵ_{11}	-1.96	-0.85	Calculated binding energy
$\bar{\epsilon}_{11}$	-2.196	-1.09	Correct binding energy

*The value of $\epsilon_{11} = \epsilon_{11}^{\mathrm{discr}} + \epsilon_{11}^{\mathrm{cont}}$. The quantity $\epsilon_{11}^{\mathrm{discr}}$ was calculated from the system (21) at $N=11$ in steps $h=0.1$ on a base $R_m = 60$. The values of $\epsilon_{11}^{\mathrm{cont}}$ were determined by perturbation theory in accordance with the algorithm from [17]. The method of obtaining the correct values of the binding energy $\bar{\epsilon}_{11}$ is described in the text.

and for the reaction $t\mu + d$

$$\chi_{1_g}^{(i)}(R) = \frac{\chi_{1_g}^{(i)}(R) - \chi_{1_u}^{(i)}(R)}{\sqrt{2}} \underset{R \to \infty}{\sim} \sin\left(kR - \frac{\pi J}{2} + \delta_J\right),$$

$$\chi_{1_b}^{(i)}(R) = \frac{\chi_{1_g}^{(i)}(R) + \chi_{1_u}^{(i)}(R)}{\sqrt{2}} \underset{R \to \infty}{\rightarrow} 0. \tag{23}$$

In the calculations we used the phase-function method in an algorithm developed on this basis in [31].

4. CALCULATION OF THE RATES OF MESIC-MOLECULE PRODUCTION

The production rates of the μ-mesic molecules $dd\mu$ and $dt\mu$, are made up of resonant and nonresonant parts:

$$\lambda = \lambda^{\mathrm{nres}} + \lambda^{\mathrm{res}}.$$

The procedure of calculating the nonresonant part is described in [3]. In Table IV are given the results of calculations of the rate of production of mesic molecules $dd\mu$ and $dt\mu$ in quantum states with different J and v. We note the presence of relations that are linear in the energy in the production of mesic molecules from the state $J=1$ of the continuous spectrum of the mesic atom + nucleus system. As a result, λ^{nres} can be represented in the form

$$\lambda^{\mathrm{nres}}[\mathrm{sec}^{-1}] = \lambda_1 + \lambda_2 \varepsilon, \tag{24}$$

where ε is in electron volts, and the values of λ_1 and λ_2

TABLE IV. Rates of nonresonant production of the mesic molecules $dd\mu$ and $dt\mu$ in different states (J, v).

Type of Transition	Initial state J	Final state J	Final state v	$\lambda_{dd\mu}^{\mathrm{nres}}$, 10^6 sec^{-1} $\varepsilon=0.004$ eV	$\lambda_{dd\mu}^{\mathrm{nres}}$, 10^6 sec^{-1} $\varepsilon=0.04$ eV	$\lambda_{dt\mu}^{\mathrm{nres}}$, 10^6 sec^{-1} $\varepsilon=0.004$ eV	$\lambda_{dt\mu}^{\mathrm{nres}}$, 10^6 sec^{-1} $\varepsilon=0.04$ eV
E1	0	1	0	0.013	0.013	$5\cdot10^{-4}$	$5\cdot10^{-4}$
E1	1	0	1	0.009	0.091	0.060	0.503
E1	1	2	0	0.000	0.003	0.006	0.047
E0	0	0	1	0.017	0.017	0.033	0.033

TABLE V. Quantities that determine the rate of resonant production of mesic molecules $dd\mu$ and $dt\mu$.

Quantity	$dd\mu$	$dt\mu$	Quantity	$dd\mu$	$dt\mu$
β	1/3	1	T, K	400	500
ν	8	4	d_{ft}, mesic-atomunits	400	380
I_ν, at. un.	$1.52 \cdot 10^{-3}$	$1.33 \cdot 10^{-2}$	m_a	195.742	199.273
E_0, eV	-4.556	-4.556	λ_2, 10^6 sec^{-1}·eV	0.15	29.4
E_ν, eV	-2.307	-3.395	λ^{res}, 10^6 sec^{-1}	0.63	94
ε_{11}, eV	-2.196	-1.09	λ^{rres}, 10^6 sec^{-1}	0.17	1.0
ε_0, eV	0.053	0.07	λ^{max}, 10^6 sec^{-1}	0.80	95

FIG. 4. The wave functions $\chi_{1a}^{(0)} = 2^{-1/2}(\chi_{1\ell}^{(f)} - \chi_{1u}^{(f)})$ and $\chi_{1b}^{(0)} = 2^{-1/2}(\chi_{1\ell}^{(f)} + \chi_{1u}^{(f)})$ of the state $(J = 1, v = 1)$ of the mesic molecules $dd\mu$ and $dt\mu$, calculated from the system (21).

are as follows:

Molecule:	$dd\mu$	$dt\mu$
λ_1, sec^{-1}:	0.40	0.033
λ_2, sec^{-1} eV^{-1}:	2.3	13.8

Table V lists the quantities that determine the rates λ^{res} of resonant production of mesic molecules $dd\mu$ and $dt\mu$, calculated from formula (15). At $\epsilon_0 = \epsilon_T$, the value of λ^{res} reaches a maximum

$$\lambda^{res}(\epsilon_0 = \epsilon_T) = \beta \left(\frac{m_\nu}{m_a} \right)^3 I_\nu^2 |d_{ft}|^2 \frac{8.61}{\epsilon_T[\text{eV}]} \cdot 10^6 [\text{sec}^{-1}]. \quad (25)$$

We note some features of our calculations. As already mentioned, the accuracy of the calculations of the binding energies ϵ_{11} of the mesic molecules is of the order of 0.1 eV. At the same time, the resonant collision energy ϵ_0 is equal to several hundredths of an electron volt, and consequently cannot be calculated from relation (8) with the required accuracy. Therefore, in the calculation of $\lambda_{dd\mu}^{res}(\epsilon_0 = \epsilon_T)$ the resonant value of the energy $\epsilon_0 = 0.053$ eV was taken by us from experiment [15]. The quantity

$$\lambda^{max} = (\lambda^{rres} + \lambda^{res})_{\epsilon = \epsilon_T} = 0.80 \cdot 10^6 \text{ sec}^{-1}, \quad (26)$$

which was calculated by us from formulas (24)–(25) at this value of ϵ_0, agrees very well with the experimental value obtained in [15] only in the case when a vibrational level $\nu = 8$ is excited in the molecule $[(dd\mu)d2e]$ in the rotational state $K = 1$ in the course of the resonant transition.[3] Using the experimental value of the resonance energy ϵ_0, we can easily calculate from relation (8) at $\nu = 8$ the correct energy $\tilde\epsilon_{11}$ of the level $(J = 1, v = 1)$ of the $dd\mu$ molecule, which turns out to be

$$\varepsilon_{11} = -2.196 \text{ eV}. \quad (27)$$

The difference $\delta\epsilon = \tilde\epsilon_{11} - \epsilon_{11} = 0.24$ eV between the correct and the calculated values include both the computation errors and the contribution of the numerous corrections which have so far not been determined: relativistic corrections, corrections for polarization of the vacuum in the mesic molecules, corrections for the finite dimensions and the polarization of the nuclei, for the electron screening, and others. Although their relative contribution to $\delta\epsilon$ is still unknown, it is obvious that they are the same with high degree of accuracy for the two mesic molecules $dd\mu$ and $dt\mu$.

The close analogy between the $dd\mu$ and $dt\mu$ systems manifests itself in all stages of the calculation, for
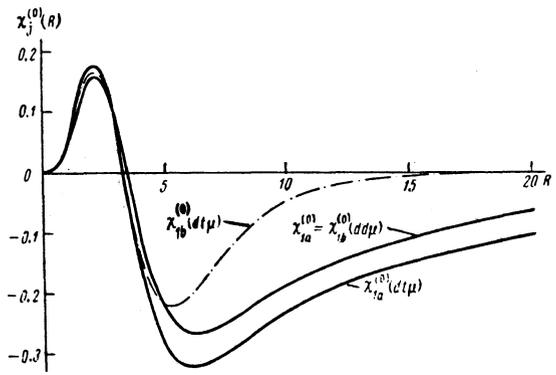
example in the calculation of the binding energy ϵ_{11}. As seen from Table IV, in the two-level approximation $(N = 1)$ of the system (21), we have $\epsilon_{11}^{(0)}(dd\mu) = -0.67$ eV, whereas in the $dt\mu$ system in place of the bound state there is a quasistationary level with energy $\epsilon_{11}^{(0)}(dt\mu) = 0.47$ eV and width $\Gamma \approx 0.15$ eV. When account is taken of the discrete spectrum of the two-center problem, i.e., when the system (21) is extended to $N = 11$, and also when account is taken of the continuous spectrum of the two-center problem, the levels $\epsilon_{11}^{(0)}$ drop respectively to the values (3) and (4) (see also Table III). It is easy to verify that the difference $\epsilon_{11} - \epsilon_{11}^{(0)}$ are equal to each other with good accuracy:

$$\varepsilon_{11}(dd\mu) - \varepsilon_{11}^{(0)}(dd\mu) = -1.29 \text{ eV}, \quad \varepsilon_{11}(dt\mu) - \varepsilon_{11}^{(0)}(dt\mu) = -1.32 \text{ eV}. \quad (28)$$

Using the above-mentioned analogy of the systems $dd\mu$ and $dt\mu$, (see, e.g., Fig. 4, which shows the wave functions of these systems) and the value of $\delta\epsilon$ obtained for the molecule $dd\mu$, we can calculate the correct value of the binding energy of the molecule $dt\mu$:

$$\varepsilon_{11}(dt\mu) = -1.09 \text{ eV}. \quad (29)$$

It is easy to establish from Table II that the closest difference $\bar{E}_0 - E_\nu = 1.161$ eV that exceeds the value of $|\tilde\epsilon_{11}(dt\mu)|$ corresponds to a transition to the level $(K = 1, v = 4)$ of the system $[(dt\mu)d2e]$. From relation (8) we now obtain the energy value $\epsilon_0 = 0.07$ eV, at which resonant production of $dt\mu$ mesic molecules takes place.

Figure 5 shows, together with the experimental points of [15], the function $\lambda_{dd\mu}(\epsilon_T)$ calculated from the formula

$$\lambda_{dd\mu}(\epsilon_T) = \lambda_1 + \lambda_2\epsilon + \lambda_3\epsilon_0^{1/2}\epsilon_T^{-3/2}\exp(-3\epsilon_0/2\epsilon_T) \quad (30)$$

at a value $\epsilon_0 = 0.053$ eV.

In Table V are listed quantities that determine the rate λ^{res} of the resonant production of the mesic molecules $dd\mu$ and $dt\mu$. The agreement between the calculated and measured relations $\lambda_{dd\mu}(\epsilon_T)$ serves as a good guarantee of the correctness of our calculations, and in particular, the correctness of the calculated value

$$\lambda_{dt\mu}^{max} \approx 10^8 \text{ sec}^{-1}. \quad (31)$$

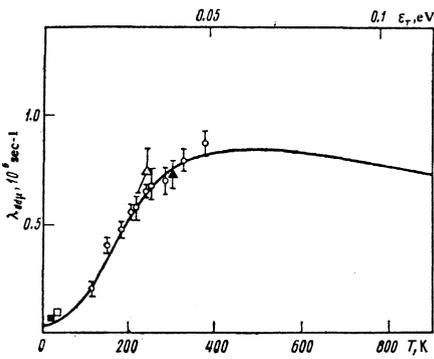448   Sov. Phys. JETP 47(3), March 1978

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FIG. 5. Dependence of the rate of production of the mesic molecule $dd\mu$ on the temperature of the medium. Experimental points were taken from the following sources: ■—Fetkovich *et al.* [11], □—Doede [12], △—Dzhelepov *et al.* [13], ▲—Bystritskiĭ *et al.* [15].

It would be extremely desirable to verify this result by direct experiment.

5. CONCLUSION

The method developed by us makes it possible to calculate the properties of a system of three bodies with a very small binding energy ($\sim 10^{-4}$ of the ground-state energy). Comparison with experiment and with variational calculations[18] demonstrates the high accuracy and effectiveness of the development calculation algorithm.

This calculation is of independent interest as a rare example of the calculation of the binding energy in a system of three charged particles, for which a reliable comparison with experiment is possible.

The calculations uncover a possibility of measuring the polarization vacuum in μ-mesic molecules. The experimental technique used in the measurement of the temperature dependence of $\lambda_{dd\mu}(\epsilon_T)$ allows us to find the polarization level shift of the mesic molecule $dd\mu$ with accuracy better than 10^{-3} eV.

A qualitatively new result of the present study is the observation of a weakly bound state of the $dt\mu$ molecule. The existence of this level leads to a very high rate of formation of the mesic molecules $dt\mu$, exceeding by two orders of magnitude the rate of decay of the free μ^- meson $\lambda_0 = 0.455 \cdot 10^6$ sec^{-1}. This means that one π^- meson in a mixture of deuterium and tritium can catalyze $\sim 10^2$ nuclear fusion reactions in accordance with the scheme

$$t\mu + d \rightarrow dt\mu \rightarrow {}^4He + n + \mu^- + 17.6 \text{ MeV}$$
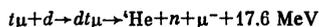
and release at the same time ~ 2 GeV of energy, or twenty times the rest energy of the muon.[32]

1) The dimensionalities of the quantities in (9) are

$$\Psi^{(i)}(r,R) \sim a_{m_e}^{-\frac{3}{2}}, \quad \Psi^{(f)}(r,R) \sim a_{m_e}^{-3}; \quad \psi^{(i)}(\rho) \sim a_0^{-\frac{1}{2}}, \quad \psi^{(f)}(\rho) \sim a_0^{-\frac{3}{2}};$$

$$\hat{H}_{int} \sim e^2 a_{m_e} a_0^{-2}, \quad \gamma(\epsilon_0, \epsilon_T) \sim a_0 e^{-2},$$
$$a_0 = \hbar^2/m_e e^2, \quad a_{m_e} = \hbar^2/m_a e^2, \quad a_\mu = \hbar^2/m_\mu e^2.$$

The range of the variables is $r \lesssim 50\, a_\mu$, $R \lesssim 50\, a_\mu$, $\rho \approx R_d \approx 300$.

2) Analogous calculations of the energy levels of the H_2 molecule, performed by us by the same method, yield values of E_ν which differ from the experimentally measured ones by not more than 10^{-3} eV, up to states $\nu = 10$. This accuracy is perfectly adequate for our purposes.

3) For this reasoning to be valid it is important that the dipole moment d_{fi} be weakly dependent on ϵ_0 and ϵ_{11}; this is in fact the case, as shown by an additional investigation.

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Influence of multiple reflections on the polarization of nonmonochromatic neutron beams

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The influence of multiple reflections from ferromagnetic mirrors on the polarization of a nonmonochromatic neutron beam was demonstrated experimentally. The polarization increased with the number of reflections. The resultant polarization of a beam of neutrons with wavelengths λ from 1 to 4 Å was $P \geq 0.99$ (polarization ratio $R' = 240$). An analysis of the experimental results is given.

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The increase in the polarization of a neutron beam on increase in the number n of successive reflections from a ferromagnetic mirror can easily be predicted by considering the expression for the polarization P_n in terms of the reflection coefficients of the components of a monochromatic neutron beam parallel a_+ and antiparallel a_- to the magnetic field:

$$P_n = \frac{a_+^n - a_-^n}{a_+^n + a_-^n} = \frac{1 - (a_-/a_+)^n}{1 + (a_-/a_+)^n}. \tag{1}$$

The formula (1) does not allow for the beam depolarization on reflection,[1] which is valid in the case of complete magnetic saturation of the reflecting layer.[2] It is clear from the formula (1) that for $a_-/a_+ < 1$ the term $(a_-/a_+)^n$ tends to zero on increase of n at a rate which increases on reduction of the ratio a_-/a_+, i.e., the limiting value of the polarization obtained as a result of multiple reflections is $P_n = 1$.

Experimental investigations of the process of increase in the polarization and of the factors governing its limiting values after multiple successive reflections are desirable from the scientific and methodological points of view. Multiple reflections may explain the high values of the neutron beam polarization obtained at the exit from polarizing neutron channels (see, for example,[3,4]) when they contain a large number of neutrons reflected repeatedly at an angle θ much smaller than the critical value θ_c so that the increase in the polariza-

tion in each reflection is small.[5] The increase in the polarization by multiple reflection can be used in the construction of magnetic monochromators based on the Drabkin resonator[6,7] and in generation of nonmonochromatic neutron beams with a high degree of polarization in systems utilizing polarized neutrons. For example, in a system for testing newly developed polarizing mirrors we can determine directly the polarizability of such mirrors if we have a beam whose polarization does not differ by more than 1% from unity.

EXPERIMENTAL RESULTS

The experiments were carried out in a channel of a water-moderated water-cooled reactor (VVR-M) using a traditional double reflection system (Fig. 1b), which included a collimator 1, polarizer 2, spin-flipper 3,[8] analyzer 4, and detector 5. The polarizer was a straight reflecting channel 1 mm wide and 1260 mm long, formed by 40 Fe-60 Co mirrors on 85 Ti-15 Gd substrates.[5] The deviation of the mirror plane from ideal did not exceed 20″ in any part of the polarizer slit. A magnetic field produced by permanent magnets was H = 500 Oe. The polarizer was mounted on a common baseplate and it was inclined at an angle $\theta_p = 11'$ to the entry beam, which ensured four reflections of the beam inside the polarizer. The height of the neutron beam was limited to 5 mm to increase the probability of spin flip of neutrons in the flipper. The spectrum of