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Optical activity of thallium and lead vapors near suppressed *M*1 transitions

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The magnitude of parity nonconservation effects in the $6p_{1/2}\to 7p_{3/2}$ transition in thallium is calculated and the feasibility of an experimental search for an optical activity is pointed out. A similar calculation is carried out for the $6p^2\,{}^3P'_0\to 6p7p\,{}^3D'_1$ transition in lead.

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Several groups of investigators are currently searching for parity nonconservation in atomic transitions.[1-5] These experiments are attracting considerable interest because the results will be critical in checking theoretical schemes describing in a unified manner both electromagnetic and weak interactions of elementary particles. The most promising approach is the search for an optical activity in heavy metal vapors,[3-5] which has already resulted in considerable narrowing of the possible ranges of the parameters of such schemes.

So far, discussions of the possibility of finding an optical activity have always been concerned with the usual *M*1 transitions between the levels in the same configuration. These discussions have been concerned specifically with the $6p_{1/2}\to 6p_{3/2}$ transitions in thallium, between the levels of the $6p^2$ configuration in lead, and those of the $6p^3$ configuration in bismuth. All the experiments have been carried out so far on bismuth.

The possibility, in principle, of searching for an optical activity near the suppressed $6p_{1/2}\to 7p_{3/2}$ transition of the *M*1 type in thallium was pointed out in [6]. However, a pessimistic view was taken there of the chance of detecting this activity. We shall report a calculation of the degree of circular polarization of photons in this transition, and of the corresponding angles of rotation of the plane of polarization of light in thallium vapor. In our view, these results indicate that the chance of observing an optical activity near this transition is quite realistic. We shall also give the results of a calculation of an optical activity in a similar transition in lead.

We shall begin with the amplitude of the *M*1 $6p_{1/2}\to 7p_{3/2}$ transition in thallium. A numerical calculation[6] shows that, in spite of the difference between the principal quantum number of the initial and final states, this amplitude is suppressed only by an order of magnitude. Its value can easily be found also analytically. The spin-orbit interaction mixes the $6p$ and $7p$ states:

$$
\begin{aligned}
|7p_{3/2}\rangle' &= |7p_{3/2}\rangle + \frac{1}{2}\,\frac{\zeta_{6,7}}{E(7p_{3/2})-E(6p_{3/2})}\,|6p_{3/2}\rangle,\\
|6p_{1/2}\rangle' &= |6p_{1/2}\rangle - \frac{\zeta_{6,7}}{E(6p_{1/2})-E(7p_{1/2})}\,|7p_{1/2}\rangle.
\end{aligned}
\tag{1}
$$

Here, $\zeta_{n,n'}$ is the radial matrix element of the spin-orbit interaction. Since the main contribution to this element comes from the region of short distances from the nucleus, where the $6p$ and $7p$ wave functions differ only in respect of the normalization, it follows that $\zeta_{6,7}\approx(\zeta_{6,6}\zeta_{7,7})^{1/2}$ (we are assuming that all the radial functions are positive in the limit $r\to 0$). Hence, using the experimental values of the energies of the relevant states, we obtain

$$
\left\langle 7p_{3/2} j_z=-\frac{1}{2}\,|M_z|\,6p_{1/2} j_z=\frac{1}{2}\right\rangle=\frac{\sqrt{2}}{3}\cdot 0.09|\mu_B|. \tag{2}
$$

The absolute value of the matrix elements is in agreement with the value given in the cited calculation[6] but the results differ in respect of the sign.

The $6p_{1/2}\to 7p_{3/2}$ transition in thallium can also be of electric quadrupole nature. A numerical calculation gives $\langle 7p_{3/2}|\,r^2\,|6p_{1/2}\rangle=-6.7a_B^2$.[6] Consequently, the

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reduced matrix element of the operator

$$Q_{\alpha\beta}=-\frac{\sqrt{3}e\omega}{4c}\left(r_\alpha r_\beta-\frac{1}{3}r^2\delta_{\alpha\beta}\right)$$

is

$$\langle 7p_{3/2}\|Q\|6p_{1/2}\rangle=-\frac{e\omega}{c\sqrt{15}}\langle 7p_{3/2}|r^2|6p_{1/2}\rangle=-0.55|\mu_B|. \qquad (3)$$

A calculation of the parity nonconservation effects in this transition is a relatively simple matter. The matrix element of the P-odd weak interaction of an electron with a nucleus is[7]

$$\langle s_{1/2}|H|p_{1/2}\rangle=iq\frac{Gm_e^2\alpha^2Z^2R}{\pi\sqrt{2}}\frac{1}{(\nu_s\nu_{p_{1/2}})^{3/2}}\frac{m_ee^4}{2\hbar^2}, \qquad (4)$$

where $G=10^{-5}/m_p^2$ is the Fermi constant, ν_s and $\nu_{p_{1/2}}$ are the effective principal quantum numbers of electrons, R is the relativistic factor ($R_{\text{Tl}}=8.5$, $R_{\text{Pb}}=8.9$), q is a dimensionless constant, which should be found experimentally. To be specific, we shall use the Weinberg model with $\sin^2\theta=0.32$, i.e., we shall assume[7]

$$q=1-A/2Z-2\sin^2\theta\approx-0.9.$$

A calculation of the admixed $E1$ amplitudes is carried out in the standard way.[7] The final value of the matrix element of the operator for a $E1$ transition is

$$\left\langle 7p_{3/2}j_z=\frac{1}{2}\middle|D_z\middle|6p_{1/2}j_z=\frac{1}{2}\right\rangle=i\cdot\frac{\sqrt{2}}{3}\cdot 2.3\cdot 10^{-10}|e|a_B. \qquad (5)$$

It should be pointed out that in the calculation of the matrix elements D_z the contributions of the various impurity states largely balance out so that the error in the calculations can be quite large.

The quantity

$$P_0=-2\,\text{Im}\frac{\langle D_z\rangle}{\langle M_z\rangle}$$

for the $6p_{1/2}\to 7p_{3/2}$ transition in thallium is -1.4×10^{-6}. In this case the large contribution of the electric quadrupole makes the value of P_0 generally different from the degree of the circular polarization P of photons

$$P=P_0\frac{{}^1/_3\langle F'\|M\|F\rangle^2}{{}^1/_3\langle F'\|M\|F\rangle^2+{}^1/_5\langle F'\|Q\|F\rangle^2} \qquad (6)$$

and it varies from one hyperfine transition to another. Using Eqs. (2) and (3), we find that

$$P(0\to1)=P_0=-1.4\cdot10^{-6},\quad P(0\to2)=0,$$
$$P(1\to1)={}^1/_{55}P_0=-2.5\cdot10^{-8},\quad P(1\to2)={}^5/_{59}P_0=-1.2\cdot10^{-7}. \qquad (7)$$

In view of the small hyperfine splitting of the $7p_{3/2}$ level the resolution of its hyperfine structure may be impossible. In this case, for any value of F we find that

$$P={}^1/_{19}P_0=-0.74\cdot10^{-7}. \qquad (8)$$

It should be noted that, in spite of the fact that our values of $\langle M_z\rangle$, $\langle D_z\rangle$, and $\langle r^2\rangle$ agree with those reported by Neuffer and Commins[6] (at least apart from the sign), the values of P from Eqs. (7) and (8) differ greatly from the value predicted by these authors[6]: $P=-1.67\times10^{-8}$. It may be that this underestimate of the degree of the circular polarization accounts for the pessimistic estimate of the possibility of the relevant experiments given by Neuffer and Commins.[6]

In addition to the $6p_{1/2}\to7p_{3/2}$ transition, there is some interest in an optical activity in $6p_{1/2}\to8p_{3/2}, 9p_{3/2}$ transitions. The degree of circular polarization in such transitions is roughly of the same order as that in the $6p_{1/2}\to7p_{3/2}$ transition, because the values of $\langle M\rangle$, $\langle D\rangle$, and $\langle Q\rangle$ decrease in approximately the same way with increase in the principal quantum number of the upper level.

In the absence of the quadrupole absorption, the suppressed $6p^2\,{}^3P_0'\to6p7p\,{}^3D_1'$ transition of the $M1$ type in lead has some advantages over the above transitions in thallium. (In our investigations[8,9] this transition is identified wrongly as strongly forbidden.) For this transition, using the wave function of the ground state $6p^2\,{}^3P_0'$ from our earlier paper[7] and assuming the states ${}^3D_1'$ to be pure $jj({}^6p_{1/2}\,{}^7p_{3/2})_1$, exactly as in thallium, we find

$$\langle 6p7p\,{}^3D_1'|M_z|6p^2\,{}^3P_0'\rangle=-{}^2/_3\cdot0.085|\mu_B|. \qquad (9)$$

The matrix element of the admixed $E1$ transition was found by us earlier[8]:

$$\langle 6p7p\,{}^3D_1'|D_z|6p^2\,{}^3P_0'\rangle=-i\cdot{}^2/_3\cdot1.9\cdot10^{-10}|e|a_B. \qquad (10)$$

The degree of the circular polarization of the radiation is then

$$P=-1.2\cdot10^{-4}. \qquad (11)$$

We shall now consider the optical activity of thallium and lead vapors. For light of frequency ω near a line of frequency ω_0 the absorption coefficient α and the angle of rotation of the plane of polarization per unit length ψ are given by

$$\alpha=\frac{4\pi}{\hbar c}\frac{N}{(2I+1)(2J+1)}\frac{\omega}{\Delta_D}\left[\frac{1}{3}\langle F'J'\|M\|FJ\rangle^2+\frac{1}{5}\langle F'J'\|Q\|FJ\rangle^2\right]f(u,v),$$
$$\psi=-\frac{2\pi}{\hbar c}\frac{N}{(2I+1)(2J+1)}\frac{\omega}{\Delta_D}\frac{1}{3}\langle F'J'\|M\|FJ\rangle^2P_0g(u,v), \qquad (12)$$

where I is the moment of a nucleus, J and J' are the initial and final moments of electrons, F and F' are the initial and final moment of an atom, N is the density of atoms, $\Delta_D=(2kT/m_ac^2)^{1/2}\omega$, $u=(\omega-\omega_0)/\Delta_D$, is the detuning $v=\Gamma/2\Delta_D$, Γ is the line width, and, finally, $f(u,v)$ and $g(u,v)$ are dimensionless functions which describe the Doppler line broadening:

$$\left\langle\frac{\Delta_D}{\omega-\omega_0+i\Gamma/2}\right\rangle=g(u,v)-if(u,v). \qquad (13)$$

We can easily find, with the aid of Eq. (12), that at 1200°C (thallium vapor pressure 100 Torr, lead vapor pressure 17 Torr[10]) the angles of rotation of the plane of polarization may reach 10^{-6} rad/m in the wing of a line when the absorption length is 1 m. A random magnetic field simulating this effect should not exceed $\sim 10^{-4}$ G in thallium and $\sim 10^{-3}$ G in lead. We shall conclude by pointing out that the precision achieved in the experiments on bismuth is quite sufficient to measure rotation angles $\sim 10^{-6}$ rad/m. Therefore, in a situation when the search for parity nonconservation in the strongly forbidden $M1$ $6p_{1/2} \to 7p_{1/2}$ transition in thallium is already under way,[2] the proposed experimental detection of an optical activity of thallium vapor in the same frequency range seems realistic.

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Optical resonators with periodic boundaries

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Results are presented of theoretical and experimental investigations of optical resonators with periodic boundaries (ORPB). The natural oscillations for various periodic structures are obtained by solving a parabolic equation. It is shown that at definite resonator lengths there exist singular types of natural oscillations—periodic modes. The spatial distributions of the fields in the near and far zones are analyzed. Results are presented of an experimental investigation of a resonator with periodic modulation of the reflection coefficient in a neodymium-glass laser; these results are in satisfactory agreement with the theory. It is shown that ORPB make possible the shaping of extremely narrow directivity patterns.

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INTRODUCTION

Open optical resonators shape the spatial structure and the directivity pattern of laser radiation. The laser fields are natural modes of the resonator oscillations, and the configuration of these modes is determined by the resonator geometry. A resonator with plane-parallel mirrors[1] has made it possible to obtain for the first time coherent emission in the optical band. The mode fields of stable resonators[2,3] are concentrated in a limited volume, so that the diffractive divergence of the modes is determined by small apertures $2a \sim (\lambda L)^{1/2}$, where λ is the wavelength and L is the resonator length. This is why multimode lasing, which affects the directivity pattern adversely, takes place in lasers with large apertures.

From the energy point of view it is preferable to have large-volume laser media, so that an important problem is to generate oscillation modes whose fields occupy the entire exit aperture. From this point of view, definite advantages are offered by unstable resonators[4] such as the telescopic one.[5]

A new interesting possibility is uncovered by optical resonators in which at least one of the mirrors is a two-dimensional grid with periodically varying reflection coefficient. The first results of the investigation of such resonators were reported recently.[6] It was shown that these resonators ensure good filling of the active medium, and the divergences of the individual light spots in the far zone can reach the diffraction value over the total aperture of the reticular mirror. These features of such systems were not noted in earlier experiments.[7-9]

It was indicated[8,9] that the theoretical investigation of diffraction resonators is a complex task. This is apparently the reason why these resonators were treated in some papers by simplified methods.[10,11] One study[12] enhanced the interest in the study of such sys-

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