Renormalization in a one-dimensional electron-phonon system

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A one-dimensional electron-phonon system at $T=0$ K, describable by the Fröhlich Hamiltonian, is investigated by the renormalization-group method. Because of certain features of the given model—in particular, the presence of two cutoff parameters in the problem, the renormalization-group analysis is performed using normalization parameters and is formulated as a theory with dimensional constants. It is shown that, up to the third approximation of the renormalization group, for the given model the dimensional constants of the problem drop out in the low-energy region, the theory becomes "massless" and there is a finite renormalization of the coupling constant. Under these conditions the Green functions have power asymptotic forms in the low-energy region, corresponding to a scaling picture.

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1. Interest in the study of quasi-one-dimensional systems has grown recently.[1] These objects possess a whole series of properties that are intrinsic only to one-dimensional systems, the prime example of these properties being the Peierls transition.[1] Because of the large fluctuation effects, it is not possible to give a sufficiently complete description of the properties of quasi-one-dimensional systems within the framework of some or other variant of mean-field theory. In view of this the need arises for a more correct solution of the purely one-dimensional problem than that given by the mean field approximation. The renormalization-group method has often been used to study the one-dimensional problem.[2-4] In these papers a four-fermion interaction model with two[2,3] or more[4] coupling constants was considered. In the present paper the renormalization-group method is used to study a one-dimensional electron-phonon system. Analysis of this model is of undoubted interest, since phenomena such as superconductivity and the Peierls transition are directly connected with the electron-phonon interaction.

The model under consideration and those investigated previously are examples of models with one or more dimensional coupling constants and several dimensional constants, some of which appear as natural cutoff parameters in the perturbation-theory expansions. These dimensional constants can be regarded as certain "masses" in the theory. The renormalization-group method makes it possible, primarily, to investigate the asymptotic behavior of various Green functions in a certain region of energies or momenta. It may be assumed that the pattern of the asymptotic behavior is determined by two factors[5]: 1) whether or not the dimensional constants of the theory drop out of the corresponding equations in the asymptotic limit, and 2) the behavior of the invariant coupling constants or invariant charges. The most favorable situation corresponds to the case when the dimensional constants drop out and we can then speak of a "massless" theory. In such a theory, in the case of a finite renormalization of the coupling constants, for example, the asymptotic forms of the Green functions have a purely power behavior.

Such behavior was established for the models considered in Refs. 2–4. The investigation of a one-dimensional electron-phonon system at $T=0$ K carried out below on the basis of the Fröhlich Hamiltonian makes it possible to elucidate the distinctive features of this model and shows that, in the low-energy region, to within the third approximation of the renormalization group, the Green functions have a power-law behavior for this model too.

2. We shall consider a one-dimensional electron-phonon system at $T=0$ K. We assume that the electrons are well described in the tight-binding approximation. We disregard the Coulomb interaction between the electrons, assuming it to be sufficiently weak. It may be supposed that this assumption is justified for the whole series of quasi-one-dimensional compounds in which the magnitude of the Coulomb interaction is small because of the large polarizability of the molecules composing the crystal.[1] In this case the system can be described by the Fröhlich Hamiltonian[6]

$$H=\sum_{k,\sigma}\varepsilon(k)a_{k,\sigma}^{+}a_{k,\sigma}+\sum_{q}\omega_0(q)b_q^{+}b_q + \sum_{k,q,\sigma}g_0(q)a_{k+q,\sigma}^{+}a_{k,\sigma}(b_q+b_{-q}^{+}), \tag{1}$$

where $a_{k,\sigma}^{+}$ ($a_{k,\sigma}$) is the creation (annihilation) operator for an electron with momentum k and spin σ, b_q^{+} (b_q) is a phonon creation (annihilation) operator, and $g_0(q)$ is the electron-phonon interaction constant. We take $\hbar=1$. We consider the case of a half-filled band; the Fermi momentum k_F is equal to $k_F=\pi/2a$, where a is the period of the chain. For the electron energy $\epsilon(k)$, the frequency $\omega_0(q)$ of the bare acoustic phonons, and the interaction constant $g_0(q)$, we use the expressions

$$\varepsilon(k)=-\varepsilon_0\cos ka,\quad \omega_0(q)=\omega_0|\sin(qa/2)|, \quad g_0^{2}(q)=\rho_0|\sin(qa/2)|, \tag{2}$$

where ϵ_0 is the half-width of the band, ω_0 is the value of the phonon frequency at the edge of the Brillouin zone, and ρ_0 is a certain constant. The zeroth-order electron and phonon Green functions have the form

$$G_0(k,\omega)=\frac{1}{\omega-\varepsilon(k)-i\delta\operatorname{sign}\varepsilon(k)},\quad D_0(q,\nu)=\frac{2\omega_0(q)}{\nu^2-\omega_0^{2}(q)+i\delta}, \tag{3}$$

where ω and ν are the electron and phonon energies,

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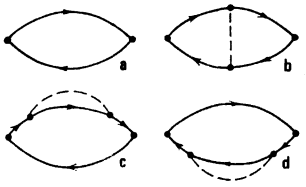

FIG. 1.

respectively.

By investigating the perturbation-theory diagrams for the electron self-energy part $\Sigma(k,\omega)$, the phonon self-energy part $\Pi(q,\nu)$ and the vertex function $\Gamma(k,k+q,q;\omega,\omega+\nu,\nu)$, it can be established that the diagrams corresponding to values $q\simeq 2k_F$ of the external phonon momentum and values $k\simeq k_F$ of the external electron momentum give the most singular contributions. Therefore, as in the previous papers,[2-4] we can confine ourselves to investigating the behavior of the quantities of interest to us as functions only of the energy variables ω, $\omega+\nu$ and ν, fixing the external phonon and electron momenta: $q=2k_F$, $k=k_F$. We shall consider the structure of the perturbation-theory expansions for the quantities $\Sigma(k_F,\omega)=\Sigma(\omega)$, $\Pi(2k_F,\nu)=\Pi(\nu)$ and $\Gamma(2k_F,k_F;\omega,\omega+\nu,\nu)=\Gamma(\omega,\omega+\nu,\nu)$. The diagrams for the quantities Π and Σ in the chosen approximation are given in Figs. 1 and 2, where the solid lines correspond to electrons and the dashed lines to phonons. Diagrams for the quantity Γ are given, e.g., in Fig. 55 in Ref. 7. We shall investigate the expansion for Π. The contribution of the diagram in Fig. 1a is equal to

$$4\omega_0 g_0\left[\frac{1}{2}\ln\left|\frac{2\varepsilon_0-(4\varepsilon_0^2-\nu^2)^{1/2}}{2\varepsilon_0+(4\varepsilon_0^2-\nu^2)^{1/2}}\right|-i\frac{\pi}{2}\right]\approx 4\omega_0 g_0\left(\ln\left|\frac{\nu}{4\varepsilon_0}\right|-i\frac{\pi}{2}\right),\tag{4}$$

where it is convenient, henceforth, to denote the bare dimensionless coupling constant by $g_0=g_0^2(2k_F)/2\pi a\epsilon_0\omega_0$. The contribution of the diagram in Fig. 1b is equal to (we give the leading term)

$$8\omega_0 g_0^2\left(\ln\left|\frac{\nu}{2\omega_0}\right|-i\frac{\pi}{2}\right)^2.\tag{5}$$

It can be seen from (4) and (5) that the expansion for the quantity Π contains two cutoff parameters: ϵ_0 and ω_0. An analogous situation obtains for the quantities Σ and Γ. The parameter ϵ_0 comes from the integration of electron-hole loops with no internal phonon insertions. The contributions of diagrams with internal phonon insertions contain the cutoff parameter ω_0. We consider now the expression for Σ. The contribution of the diagram in Fig. 2a is equal to

$$2\omega\alpha g_0\left(\ln\left|\frac{\omega}{2\varepsilon_0}\right|-i\frac{\pi}{2}\right)-2\omega g_0,\tag{6}$$

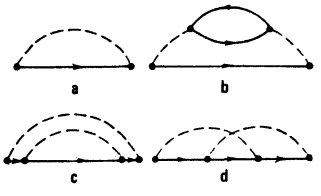

FIG. 2.

where $\alpha=\omega_0/\epsilon_0$. This behavior of the quantity Σ means that in the present problem we have the analog of the so-called "infrared catastrophe" noted in Ref. 8, which, in the present case, consists in the fact that an electron near the Fermi level can emit or absorb an arbitrary number of phonons with small momenta. Terms of this type would play the leading role in a study of the region of momenta $q\simeq 0$, $k\simeq k_F$. The problem in this case is analogous to the investigation of the infrared asymptotic forms in electrodynamics.[5] However, since, in the model under consideration, $g_0(q)\to 0$ as $q\to 0$, this problem is not of interest for the given model. In the case of interest to us ($q=2k_F$, $k=k_F$), terms of this type lead to the appearance of the additional small expansion parameter α. The contribution of the second-order diagram for the vertex function Γ is equal to (we give the leading terms)

$$-g_0(2k_F)g_0\left[\frac{2\omega_0^2}{(\omega+\nu/2)^2-\omega_0^2}\left(\ln\left|\frac{\nu}{2\omega_0}\right|-i\frac{\pi}{2}\right)-\alpha\left(\ln\left|\frac{\omega}{2\varepsilon_0}\right|+\ln\left|\frac{\omega+\nu}{2\varepsilon_0}\right|-i\pi\right)\right].\tag{7}$$

Thus, the perturbation-theory expansions for the functions Σ, Π and Γ contain two cutoff parameters ω_0 and ϵ_0, and as an expansion parameter we have not only the dimensionless coupling constant g_0 but also the adiabatic parameter α. Taking into account the smallness of the parameter α, in the expansions for the quantities Σ, Π and Γ we shall confine ourselves below to terms linear in α. From (7) and from the expression for the exact phonon Green function

$$D(\nu)=\frac{2\omega_0}{\nu^2-\omega_0^2-2\omega_0\Pi(\nu)}\tag{8}$$

it follows that the structure of the perturbation-theory expansions for the functions D and Γ depends on the relative magnitudes of the quantities ν, ω and ω_0; e.g., in the region $\omega\ll\omega_0$, $\nu<\omega_0$, in addition to the purely logarithmic terms these expansions will also contain terms proportional to ν^2/ω_0^2. Of most interest to us is the region $\nu\ll\omega_0$, $\omega\ll\omega_0$, since phenomena such as superconductivity and the Peierls transition are linked with the behavior of the corresponding Green functions at small values of the electron and phonon energies. Below we shall investigate the region $\nu\ll\omega_0$, $\omega\ll\omega_0$. In this region, when only the leading terms in the parameter α are taken into account, the perturbation-theory expressions for the quantities Σ, Π and Γ in the chosen approximation have the form

$$\Sigma=2\omega\alpha g_0\left(\ln\left|\frac{\omega}{\varepsilon_0}\right|-i\frac{\pi}{2}\right)+12\omega g_0^2\left(\ln\left|\frac{\omega}{\omega_0}\right|-i\frac{\pi}{2}\right)+O(g_0^2),$$

$$\Pi=4\omega_0 g_0\left(\ln\left|\frac{\nu}{\varepsilon_0}\right|-i\frac{\pi}{2}\right)+8\omega_0 g_0^2\left(\ln\left|\frac{\nu}{\omega_0}\right|-i\frac{\pi}{2}\right)^2-4\omega_0 g_0^2\left(\ln\left|\frac{\nu}{\omega_0}\right|-i\frac{\pi}{2}\right)+O(g_0^2),$$

$$\Gamma=g_0(2k_F)\left\{1+2g_0\left(\ln\left|\frac{\nu}{\omega_0}\right|-i\frac{\pi}{2}\right)+\alpha g_0\left(\ln\left|\frac{\omega}{\varepsilon_0}\right|+\ln\left|\frac{\omega+\nu}{\varepsilon_0}\right|-i\pi\right)+14g_0^2\left(\ln\left|\frac{\nu}{\omega_0}\right|-i\frac{\pi}{2}\right)^2-16g_0^2\left(\ln\left|\frac{\nu}{\omega_0}\right|-i\frac{\pi}{2}\right)\left(\ln\left|\frac{\nu}{\varepsilon_0}\right|-i\frac{\pi}{2}\right)-4g_0^2\left(\ln\left|\frac{\nu}{\omega_0}\right|-i\frac{\pi}{2}\right)+8\alpha g_0^2\left(\ln\left|\frac{\omega}{\varepsilon_0}\right|+\ln\left|\frac{\omega+\nu}{\varepsilon_0}\right|-i\pi\right)\left(\ln\left|\frac{\nu}{\omega_0}\right|-i\frac{\pi}{2}\right)+O(g_0^2)\right\}.\tag{9}$$

It follows from (9) that in the low-energy region the perturbation-theory expansions have a logarithmic structure and we can apply the renormalization-group method to investigate the behavior of the quantities of interest to us in this region. The essential point is that the logarithmic theory is valid only in the asymptotic region, and therefore the renormalization-group analysis must be performed, using the general approach of Ref. 5, by introducing additional quantities—normalization parameters—into the theory. Using the renormalization-group method we shall investigate the behavior of the electron and phonon Green functions.

3. Below we shall indicate perturbation-theory results by the index 0. The perturbation-theory functions $G^{(0)}$, $D^{(0)}$ and $\Gamma^{(0)}$ depend not only on the variables ω, ν and $\omega+\nu$ but also on the bare quantities ω_0, $g_0(2k_F)$ and $\epsilon(0)$. From the perturbation-theory functions, using a renormalization transformation, we can construct another set of functions—the so-called renormalized functions G, D and Γ. The latter are defined to within arbitrary renormalization constants z_i. This arbitrariness can be fixed by introducing a normalization parameter λ with the dimensions of energy and subjecting the renormalized functions to normalization conditions. We assume that the renormalized functions depend not only the variables ω, ν and $\omega+\nu$ but also on the renormalized coupling constant $g(2k_F)$ and the bare quantities ω_0 and ϵ_0. We note that, by following the general approach of Ref. 5, we can analyze the dependence of the functions G, D and Γ on renormalized quantities $\bar{\omega}(2k_F)$ and $\bar{\epsilon}_0$. The latter can be defined as the values of the poles of the corresponding Green functions of perturbation theory, and the values of these poles should be calculated to the required accuracy in the coupling constant. However, as will be seen, there is no need for this in the case under consideration; in the final results, at least, it is always possible to make the replacement $\omega_0 \to \bar{\omega}$, $\epsilon_0 \to \bar{\epsilon}_0$.

From the three dimensional quantities ω_0, ϵ_0 and $g(2k_F)$ we can extract one dimensionless renormalized coupling constant $g=g^2(2k_F)/2\pi a\epsilon_0\omega_0$ and one dimensionless parameter—the adiabatic parameter $\alpha=\omega_0/\epsilon_0$. We shall regard the quantity ω_0 as the dimensional constant of the theory. In place of the functions Γ and $\Gamma^{(0)}$ it is convenient to introduce the dimensionless quantities

$$\Gamma=\frac{\Gamma^2}{2\pi a\varepsilon_0\omega_0}, \quad \Gamma^{(0)}=\frac{\Gamma^{(0)2}}{2\pi a\varepsilon_0\omega_0}. \tag{10}$$

We subject the renormalized functions to the following normalization conditions:

$$\left.\frac{\partial G^{-1}}{\partial\omega}\right|_{\omega^2=-\lambda^2}=1, \quad D|_{\nu^2=-\lambda^2}=\frac{2\omega_0}{-\lambda^2-\omega_0^2},$$
$$\Gamma|_{\omega^2=-\lambda^2,\,(\omega+\nu)^2=-\lambda^2,\,\nu^2=-\lambda^2}=g. \tag{11}$$

Normalization at a point ($\nu^2=-\lambda^2$) is chosen in order to fulfill the condition that the constants z_i be real. The renormalization-group transformation can now be written in the form

$$G^{(0)}(\omega,\omega_0,\alpha,g_0)=z_2G(\omega,\omega_0,\alpha,g,\lambda),$$
$$D^{(0)}(\nu,\omega_0,\alpha,g_0)=z_3D(\nu,\omega_0,\alpha,g,\lambda),$$
$$\Gamma^{(0)}(\omega,\omega+\nu,\nu,\omega_0,\alpha,g_0)=z_2^{-2}z_3^{-1}\Gamma(\omega,\omega+\nu,\nu,\omega_0,\alpha,g,\lambda),$$
$$g_0=z_1^2z_2^{-2}z_3^{-1}g. \tag{12}$$

In (12) the quantities z_i are functions of the form $z_i=z_i(\omega_0,\alpha,g_0,\lambda)$.

Differentiating the equation for D and $D^{(0)}$ in (12) with respect to λ at constant ω_0, α and g_0, and using the homogeneity of the function D, we obtain the following equation for $D(\nu,\omega_0,\alpha,g,\lambda)$[5]:

$$\left[\frac{\partial}{\partial t}-\beta\frac{\partial}{\partial g}+\omega_0\frac{\partial}{\partial\omega_0}-(d_D+\gamma_D)\right]D=0. \tag{13}$$

Here $t=\ln(\nu/\nu_0)$, where ν_0 is a certain characteristic energy, $d_D=-1$ is the canonical dimension of D, and the functions $\beta(g,\alpha,\omega_0/\lambda)$ and $\gamma_D(g,\alpha,\omega_0/\lambda)$ are equal to

$$\beta=\lambda\frac{\partial g}{\partial\lambda}, \quad \gamma_D=\lambda\frac{\partial\ln z_3}{\partial\lambda}. \tag{14}$$

In (14), after the differentiation, the quantity g_0 is expressed in terms of g. The equation for $G(\omega,\omega_0,\alpha,g,\lambda)$ has the form (13) with the replacements $\nu\to\omega$, $\gamma_D\to\gamma_G$ and $d_D\to d_G=-1$, where the quantity $\gamma_G(g,\alpha,\omega_0/\lambda)$ is equal to

$$\gamma_G=\lambda\frac{\partial\ln z_2}{\partial\lambda}. \tag{15}$$

The solution of (13) has the form

$$D(\nu,\omega_0,\alpha,g,\lambda)=(\nu/\nu_0)^{d_D}D(\nu_0,\bar{\omega}_0(t),\alpha,\bar{g}(t),\lambda)$$
$$\times\exp\left[\int_0^t\gamma_D(\bar{g}(t'),\alpha,\bar{\omega}_0(t')/\lambda)\,dt'\right], \tag{16}$$

where the functions $\bar{g}(t)$ and $\bar{\omega}_0(t)$ obey the equations

$$\frac{d\bar{g}(t)}{dt}=\beta(\bar{g}(t),\alpha,\bar{\omega}_0(t)/\lambda), \quad \frac{d\bar{\omega}_0(t)}{dt}=-\bar{\omega}_0(t) \tag{17}$$

with the boundary conditions

$$\bar{g}(0)=g, \quad \bar{\omega}_0(0)=\omega_0. \tag{18}$$

The quantity $\bar{g}(t)$, determined by Eq. (17), is called the invariant coupling constant or invariant charge. The quantity $\bar{\omega}_0(t)$ is an effective parameter of the theory.

Using the perturbation-theory results, we shall determine the functions β, γ_D and γ_G in the low energy region. We define the latter by the conditions

$$|\omega|\ll\omega_0, \quad |\nu|\ll\omega_0. \tag{19}$$

In this region we choose a normalization point λ satisfying the condition

$$|\lambda|\ll\omega_0. \tag{20}$$

We shall consider the region of variation of the variables ω and ν such that the ratios ω/λ and ν/λ take the following values:

$$0\leqslant|\omega/\lambda|\leqslant1, \quad 0\leqslant|\nu/\lambda|\leqslant1. \tag{21}$$

In accordance with the condition (21) we put the quantity ν_0 equal to $\nu_0=|\lambda|$. In the low-energy region the per-

turbation-theory functions $G^{(0)}$ and $D^{(0)}$ are defined in the natural way in terms of the expressions (9) for the quantities Σ and Π. The function $\tilde{\Gamma}^{(0)}$ in the same region is determined, in accordance with (10), from the expression (9) for $\Gamma^{(0)}$. In the approximation of interest to us, i.e., the second approximation of the renormalization group, it is necessary to take into account the terms of order g_0^2 in the perturbation-theory expressions. In this case, taking into account the smallness of the parameter α, we shall neglect terms of order αg_0^2, i.e., in the expression (9) for $\Gamma^{(0)}$ we shall not take into account the penultimate term, which couples the variables ω, ν and $\omega+\nu$.

Using the normalization conditions (11) for the functions D and G, from (12), taking (20) into account, we obtain

$$\begin{aligned} z_3&=1-4g_0\ln v+8g_0\eta+12g_0^2\ln^2 v+4g_0^2\ln v\\ &\quad -64g_0^2\eta\ln v+64g_0^2\eta^2,\\ z_2&=1+\alpha g_0\ln v-2\alpha g_0\eta+6g_0^2\ln v,\quad v=\lambda^2/\omega_0^2, \end{aligned} \tag{22}$$

where $\eta=\ln(1/\alpha)$. In an analogous way, using the normalization condition (11) for the function $\tilde{\Gamma}$ with the aid of formulas (22), we find

$$\begin{aligned} g&=g_0[1-2(1-2\alpha)g_0\ln v+8g_0\eta+4g_0^2\ln^2 v\\ &\quad +12g_0^2\ln v-32g_0^2\eta\ln v+64g_0^2\eta^2]. \end{aligned} \tag{23}$$

Eqs. (22) and (23) determine the quantities z_3, z_2 and g as functions of g_0, ω_0, α and λ. Substituting the formulas (22) and (23) into (14) and (15), differentiating, and expressing the quantity g_0 in terms of g with the aid of Eq. (23), we obtain

$$\begin{aligned} &\gamma_D=-8g+8g^2,\quad \gamma_G=2\alpha g+12g^2,\\ &\beta=-4(1-2\alpha)g^2+24g^3. \end{aligned} \tag{24}$$

It follows from (24) that the functions β, γ_D and γ_G do not contain the quantity ω_0/λ that plays the role of the "mass variable" in the given problem. The dimensional constants have dropped out of the basic equations of the problem, and in the asymptotic limit we have a "massless" theory.

We shall consider the first approximation of the renormalization group, retaining the first terms in the right-hand sides of Eqs. (24). Below we shall need the form of the functions $D(\nu_0,\tilde{\omega}_0(t),\alpha,\tilde{g}(t),\lambda)$ and $G(\nu_0,\tilde{\omega}_0(t),\alpha,\tilde{g}(t),\lambda)$. Since we have taken $\nu_0=|\lambda|$, according to the conditions (11) these functions are equal to

$$G(\nu_0)=\frac{1}{\nu_0},\quad D(\nu_0,\tilde{\omega}_0(t))=\frac{2\tilde{\omega}_0(t)}{\nu_0^2-\tilde{\omega}_0^2(t)}. \tag{25}$$

From (17) we find, in the first approximation,

$$\tilde{g}=g[1-4(1-2\alpha)g\ln|\lambda/\nu|]^{-1}. \tag{26}$$

According to (17), the effective parameter $\tilde{\omega}_0(t)$ of the problem is equal to

$$\tilde{\omega}_0(t)=\omega_0 e^{-t}=\omega_0\nu_0/\nu. \tag{27}$$

Taking (24)–(27) and the condition (20) into account, from the expression (13) for D and the analogous expression for G we obtain, in the first approximation,

$$\begin{aligned} D&=(-2/\omega_0)[1-4(1-2\alpha)g\ln|\lambda/\nu|]^{-2},\\ G&=(1/\omega)[1-4(1-2\alpha)g\ln|\lambda/\omega|]^{\alpha}. \end{aligned} \tag{28}$$

The functions $\tilde{g}$ and D have a pole at $4(1-2\alpha)g\ln|\lambda/\nu|\to 1$. The appearance of the pole is associated with the instability of the system under consideration with respect to a Peierls doubling of the period. In this connection it is useful to compare the position of the pole in the expression (26), calculated by summing the leading logarithmic terms, with the position of the pole determined in the mean-field approximation. For this, in (26) we express the asymptotic coupling constant g in terms of g_0 with the aid of Eq. (23); as a result, we have

$$\tilde{g}=g_0[1+4g_0\eta-4(1-2\alpha)g_0\ln|\varepsilon_0/\nu|]^{-1}. \tag{29}$$

It is convenient to compare the values of the corresponding critical temperatures T_{p1} and T_{p0}. Putting $\nu=T_{p1}$ in (29), we obtain

$$T_{p1}=\varepsilon_0\exp\left[-\frac{1+4g_0\eta}{4(1-2\alpha)g_0}\right]. \tag{30}$$

The quantity T_{p0}, determined in the mean-field approximation, is equal to[9]

$$T_{p0}=\varepsilon_0\exp[-1/8g_0]. \tag{31}$$

The quantities T_{p1} and T_{p0} are written to within a numerical coefficient in the pre-exponential factor. It can be seen that the value of T_{p1} differs from T_{p0} principally by the presence of a factor of two in the exponent.

The validity of the first approximation is limited by the condition $\tilde{g}<1$, and, thus, the result of the first approximation only indicates that we go outside the weak-coupling picture at large values of $\ln|\lambda/\nu|$. Because of this it is necessary to consider the second approximation. The important point is that the function β (24) in the second approximation has a zero at the point

$$\tilde{g}=g^*=\varkappa/6,\quad \varkappa=1-2\alpha. \tag{32}$$

From (17) we obtain the following equation for $\tilde{g}$:

$$\varkappa\ln\left|\frac{\lambda}{\nu}\right|=\frac{1}{g}-\frac{1}{\tilde{g}}+\frac{1}{g^*}\ln\left|\frac{\tilde{g}(g^*-g)}{g(g^*-\tilde{g})}\right|. \tag{33}$$

It follows from (33) that for $\nu\to 0$ the quantity $\tilde{g}$ behaves like

$$\tilde{g}=g^*(1-|\nu/\lambda|^{\varkappa/3}). \tag{34}$$

Thus, in the asymptotic limit the invariant charge tends in a power-law manner to its limiting value g^*, and, in the model under consideration, to within terms of order g^3 there is a finite renormalization of the coupling constant.

The finite renormalization of the coupling constant leads, in the framework of the "massless" theory under consideration, to power-law asymptotic forms of the functions G and D for $\omega\to 0$, $\nu\to 0$; according to (13), (24), (27) and (34), these are equal to

$$G \approx (1/\omega)\,|\omega/\lambda|^{\tau}, \quad \tau = \alpha\varkappa/3 + \varkappa^2/3,$$
$$D \approx (-2/\omega_0)\,|\nu/\lambda|^{\delta}, \quad \delta = -4\varkappa/3 + 2\varkappa^2/9, \tag{35}$$

where the power exponents τ and δ depend through the adiabatic parameter α on the dimensional constants ω_0 and ϵ_0 of the problem. It follows from (35) that the singularities that exist at finite values of ν in the first approximation are shifted in the second approximation to the point $\nu = 0$, indicating the absence of a phase transition at finite temperatures in the model under consideration.

4. In conclusion we shall discuss briefly the results obtained. The investigation carried out has shown that the renormalization-group method can be applied to analyze a one-dimensional electron-phonon system and has exhibited the distinctive features of the given model that give rise to the specific form of application of this method. Amongst the most important features we can include the fact that for the given model the logarithmic theory holds only in the asymptotic region, and also the presence of two cutoff parameters in the problem. As a result, as has been explained, the renormalization-group analysis of this model and other models of this kind should be performed with the introduction of normalization parameters and should be formulated from the outset as a theory with several dimensional constants. For example, for the given model in the case of an arbitrarily filled band, apart from the dimensional constant ω_0 a further constant μ (the chemical potential) appears in the problem. The character of the behavior of the different Green functions is determined primarily by whether or not the dimensional constants of the theory drop out of the corresponding equations in the limit of interest to us. In the case when dimensional constants remain in the problem, as can be seen from the results of Sec. 3, they play the role of certain effective parameters of the problem, dependent on the energies or momenta. As a result the solution of the renormalization-group equations becomes more complicated and the pattern of pure power behavior of the asymptotic forms of the Green functions no longer obtains, although if, for example, the dependence on the dimensional constants is logarithmic, the Green functions can possess quasi-power asymptotic forms.

For the model under consideration the Green functions have power asymptotic forms in the low-energy region. An analogous result can be obtained if we investigate the temperature dependences of the given quantities; in this case, power-law behavior corresponds to the region of temperatures $T \ll \theta_D$, where θ_D is the Debye temperature. For the present it is not possible to reach concrete conclusions about the character of the behavior of the system in the region of energies $\nu \lesssim \omega_0$ or temperatures $T \lesssim \theta_D$, since in this region the renormalization-group method ceases to be effective. The results presented make it possible to suppose that for real quasi-one-dimensional systems the character of the behavior with temperature should depend on the relative magnitudes of the quantities θ_D and T_0—the phase-transition temperature with allowance for the coupling between the chains. It may be supposed that in the temperature region $T_0 < T \ll \theta_D$ a power-law behavior close to that which follows from the solution of the purely one-dimensional problem will be displayed. It is necessary to note that the results obtained are valid to within terms of order g^3, but it may be hoped that, for the given model, as for the four-fermion interaction model,[10] the results given will also be preserved in higher approximations of the renormalization group.

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