[7]A. M. Afanas'ev and V. D. Gorobchenko, Zh. Eksp. Teor. Fiz. 66, 1406 (1974) [Sov. Phys. JETP 39, 690 (1974)].
[8]H. H. F. Wegener, B. Braunecker, G. Ritter and D. Seyboth, Z. Phys. 262, 149 (1973).
[9]G. K. Shenoy, B. D. Dunlap, S. Dattagupta and L. Asch, Phys. Rev. Lett. 37, 539 (1976).
[10]A. Abragam and B. Bleaney, Electron Paramagnetic Resonance of Transition Ions, Oxford University Press, 1970, Part 2 (Russ. transl. "Mir", M., 1973).
[11]F. Hartmann-Boutron, Ann. Phys. (Paris) 9, 285 (1975).
[12]Yu. M. Kagan, A. M. Afanas'ev and V. K. Voitovetskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. 9, 155 (1969) [JETP Lett. 9, 91 (1969)].
[13]G. T. Trammell and J. P. Hannon, Phys. Rev. 180, 337 (1969).
[14]F. E. Wagner, B. D. Dunlap, G. M. Kalvius, H. Schaller, R. Felscher and H. Spieler, Phys. Rev. Lett. 28, 530 (1972).
[15]L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika (Quantum Mechanics) Chap. 18, Fizmatgiz, M., 1963 (English translation published by Pergamon Press, Oxford, 1965).
[16]M. Blume and J. Hubbard, Phys. Rev. B 1, 3815 (1970).
[17]M. Engelsberg and Nai-Cheng Chao, Phys. Rev. B 12, 5043 (1975).
[18]P. A. Fedders, Phys. Rev. B 12, 3933 (1975).
[19]M. A. Kleinin and M. Blume, Phys. Rev. B 14, 235 (1976).
[20]M. T. Evans and C. G. Windsor, J. Phys. C 6, 495 (1973).
[21]A. Abragam, The Principles of Nuclear Magnetism, Oxford University Press, 1961 (Russ. Transl. IIL, M., 1963).
[22]S. A. Al'tshuler and B. M. Kozyrev, EPR soedineniĭ elementov promezhutochnykh grupp (EPR of Compounds of Transition-Group Elements), "Nauka", M., 1972.

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Nonlinear electromagnetic absorption of short-wave sound in metals

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An analysis is made of the electromagnetic contribution to the nonlinear coefficient of absorption of short-wave sound in a metal, when the wavelength is much smaller than the free-path length of the electrons. It is shown that, depending on the value of the ratio of the wavelength of the sound to the skin depth (with anomalous skin effect), there are two possible types of dependence of the absorption coefficient on the sound intensity. In the high-frequency range, the deformation contribution to the absorption dominates; the absorption coefficient decreases, with increase of the sound intensity S, as $S^{-1/4}$. In the low-frequency range, the electromagnetic contribution to the nonlinear absorption coefficient dominates, although in the linear range the electromagnetic and deformational contributions are of the same order of magnitude. The sound-absorption coefficient initially increases with increase of the sound intensity as $S^{1/2}$ but then begins to decrease as $S^{1/4}$. The value of the nonlinear absorption coefficient may exceed the value of the linear coefficient. This type of dependence is due to the nonlinear character of the shielding of the priming eddy currents, which lead to the electromagnetic absorption, by conductivity currents. The estimates made show that the predicted dependences are completely accessible for experimental investigation.

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INTRODUCTION

The propagation in metals of short-wave sound, whose wavelength $2\pi/q$ is much smaller than the free-path length l of the electrons, has been repeatedly investigated both experimentally and theoretically. In an overwhelming majority of the experiments, what was studied was the propagation of sound of small intensity, so that the experimental situation was well described by a theory, developed in a number of papers, that is linear in the sound-wave amplitude. By comparison of the experimental results with the theory, the basic mechanisms responsible for sound absorption have been established. For longitudinal sound, a typical mechanism is the so-called deformation mechanism,[1] due to modulation of the energy of the electrons in the field of the sound wave. In the case of transverse sound, along with the deformational absorption, an important role is played by the so-called electromagnetic absorption,[2] caused by Joule losses during flow in the metal of eddy currents produced by the sound wave. In metals with a complicated Fermi surface, and also in propagation of mixed sound modes containing longitudinal and transverse components, the two mechanisms of absorption may compete.

Experimentalists are now able to introduce into metallic crystals sound waves of sufficiently large intensity for study fo the characteristics of their propagation in the nonlinear range. It has therefore become timely to develop a nonlinear theory of absorption of short-wave sound in the range

$$ql \gg 1. \quad (1)$$

For the deformation mechanism of absorption, such a theory has already been developed.[3-5] A qualitative picture describing nonlinear deformational absorption consists of the following. It is well known[1] that what is

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responsible for the absorption of short-wave sound is a group of resonance electrons, small in phase volume, the projections of whose velocities in the direction of the wave vector $\mathbf{q}$ of the sound are close to the phase velocity w of the sound. In consequence, the absorption is very sensitive to the form of the distribution of the electrons in the resonance region of $\mathbf{p}$-space. On the other hand, a traveling sound wave is accompanied by a traveling wave of potential energy of the electrons, whose amplitude is proportional to the strain potential. This wave, acting on the resonance electrons, distorts their distribution, leading to nonlinear absorption. This mechanism, resulting from distortion of the distribution of electrons with respect to momentum in the resonance region of $\mathbf{p}$-space, has received the name "momental" nonlinearity. Experimentally, momental nonlinearity has been observed by Ivanov *et al.*[6] in the piezosemiconductor n-InSb and by Fil' *et al.*[7] in superpure gallium.

The purpose of the present paper is to study the manifestation of momental nonlinearity in a situation in which the electromagnetic contribution to the absorption is significant. We shall show that redistribution of the electrons in $\mathbf{p}$-space may significantly modify the transverse component of the electrical-conductivity tensor of the metal, which is responsible for the shielding of vortical electromagnetic fields. This modification of the shielding may in turn significantly affect the Joule losses due to eddy currents and consequently the sound absorption. In particular, in the region of sufficiently low frequencies the nonlinear absorption coefficient may increase with increase of the sound intensity. In this region, the nonlinear electromagnetic contribution may significantly exceed the nonlinear deformational, even though in the linear range these contributions are of the same order of magnitude.

2. DISTRIBUTION FUNCTION OF THE ELECTRONS, AND EXPRESSION FOR THE COEFFICIENT OF SOUND ABSORPTION

The kinetic equation for the distribution function n' of the electrons, in a coordinate system K_u attached to a deformed lattice, has the form[8-10]

$$\frac{\partial n'}{\partial t}+\frac{\partial \varepsilon'}{\partial \mathbf{p}'}\frac{\partial n'}{\partial \mathbf{r}'}-\frac{\partial \varepsilon'}{\partial \mathbf{r}'}\frac{\partial n'}{\partial \mathbf{p}'}+I'(n')=0, \tag{2}$$

where

$$\varepsilon'(\mathbf{p}',\mathbf{r}')=\lambda_{ik}(\mathbf{p}')u_{ik}(\mathbf{r}')-\mathbf{v}'\left(\frac{e}{c}\mathbf{A}(\mathbf{r}')+m_0\dot{\mathbf{u}}(\mathbf{r}')\right)+\varepsilon_0(\mathbf{p}')+e\varphi, \tag{3}$$

$\lambda_{ik}(\mathbf{p}')$ is the strain-potential tensor, dependent on the electron quasimomentum $\mathbf{p}'$; $u_{ik}=\frac{1}{2}(\partial u_i/\partial x_k+\partial u_k/\partial x_i)$ is the strain tensor; $\mathbf{u}$ is the displacement vector of the lattice in the sound wave; $\mathbf{v}'$ is the velocity of an electron, e the electron charge, m_0 the mass of a free electron, and $\epsilon_0(\mathbf{p}')$ the energy spectrum of the undeformed metal. The scalar potential φ and vector potential $\mathbf{A}$ of the electromagnetic field must be determined from the electrical neutrality condition and Maxwell's equations. We choose a gauge in which div $\mathbf{A}=0$ ($\mathbf{A}\perp\mathbf{q}$).

From analysis of the electrical-neutrality condition it is easily shown that, except for corrections proportional to the small parameters w/v_F (v_F is the velocity of an electron on the Fermi surface) and $(ql)^{-1}$, the scalar potential may be considered equal to $-e^{-1}\overline{\lambda}_{ik}(\mathbf{p}')$, where the bar denotes an average over the Fermi surface. With the same accuracy, in the term $m_0\mathbf{v}'\dot{\mathbf{u}}$ it is permissible to retain only $\dot{\mathbf{u}}_\perp$, where $\mathbf{u}_\perp$ is the component of the vector $\mathbf{u}$ in the place perpendicular to $\mathbf{q}$. We shall suppose below that the tensor λ_{ik} is so defined that $\overline{\lambda}=0$.

The operator $\hat{I}'$ for collisions of electrons with impurities has the standard form

$$I'(n')=\frac{2\pi}{\hbar}\sum_{\mathbf{p}_1',\sigma}|V_{\mathbf{p}'\mathbf{p}_1'}|^2[n'(\mathbf{p}',\mathbf{r}')-n'(\mathbf{p}_1',\mathbf{r}')]\delta[\varepsilon'(\mathbf{p}',\mathbf{r}')-\varepsilon'(\mathbf{p}_1',\mathbf{r}')]. \tag{4}$$

Equation (2) was derived on the assumption

$$u_{ik}\ll 1, \tag{5}$$

which we shall suppose satisfied. We shall see that nonlinear effects of electronic origin will be appreciable even when this condition is satisfied.

We shall direct the axis $0X$ along the direction of sound propagation $\mathbf{q}$. Since the energy of an electron is an even function of its quasimomentum, the part of the strain-potential tensor that is odd in v_x must be odd also in $\mathbf{v}_\perp$($\mathbf{v}_\perp=(v_y, v_z)$). Therefore the strain addition to the energy of an electron may be represented in the form

$$\lambda_{ik}u_{ik}=\varepsilon_F d_{ik}(\mathbf{v}')u_{ik}+m_0\Lambda(\mathbf{v}')\left(v_x'\frac{\partial}{\partial x}\right)(\mathbf{v}_\perp'\mathbf{u}), \tag{6}$$

where $d_{ik}(\mathbf{v}')$ and $\Lambda(\mathbf{v}')$ are dimensionless functions, even in v_x' and $\mathbf{v}_\perp'$, of order of magnitude unity. By virtue of the electrical-neutrality condition, $\overline{d}_{ik}=0$.

For what follows, we find it convenient to transform to an auxiliary coordinate system K_Λ in which the part of the stain potential that is odd in v_x is absent. We accomplish this transformation by means of the generating function[1)]

$$\Phi(\mathbf{r}',\mathbf{p},t)=\mathbf{p}\mathbf{r}'-m_0\Lambda(\mathbf{v})(\mathbf{v}_\perp\mathbf{u});\ \mathbf{v}\equiv\mathbf{v}(\mathbf{p}). \tag{7}$$

The transformation has the form

$$p_i'=\frac{\partial\Phi}{\partial r_i'}=p_i-\Lambda m_0\frac{\partial}{\partial r'}(\mathbf{v}_\perp,\mathbf{u}), \tag{8}$$

$$r_i=\frac{\partial\Phi}{\partial p_i}=r_i'-m_0\frac{\partial}{\partial p_i}[\Lambda(\mathbf{v})(\mathbf{v}_\perp\mathbf{u})], \tag{9}$$

$$\varepsilon(\mathbf{p},\mathbf{r})=\varepsilon'(\mathbf{p}',\mathbf{r}')+\frac{\partial\Phi}{\partial t}=\varepsilon_0(\mathbf{p})-\frac{e}{c}\mathbf{A}^e\mathbf{v}+\varepsilon_F d_{ik}u_{ik}, \tag{10}$$

where

$$\mathbf{A}^e(\mathbf{v})=\mathbf{A}+e^{-1}m_0c(1+\Lambda(\mathbf{v}))\dot{\mathbf{u}}_\perp. \tag{11}$$

By use of the condition (5), the kinetic equation for the distribution function n_k, dependent on the "kinematic" momentum

$$\mathbf{k}(\mathbf{v})=\mathbf{p}-e\mathbf{A}^e/c, \tag{12}$$

can be represented in the form

$$\frac{\partial n_k}{\partial t}+v_x\frac{\partial n_k}{\partial x}+\left(e\mathbf{E}^e+\frac{e}{c}[\mathbf{v}\times\mathbf{H}^e]-\nabla U\right)\frac{\partial n_k}{\partial \mathbf{k}}+\hat{I}_\Lambda(n_k)=0. \quad (13)$$

Here $\mathbf{E}^e=-c^{-1}\dot{\mathbf{A}}^e$, $\mathbf{H}^e=\text{curl }\mathbf{A}^e$, $U=\epsilon_F d_{ik}u_{ik}$, and the collision integral $\hat{I}_\Lambda$ contains in the argument of the δ-function

$$\varepsilon_0(\mathbf{k})+U+m_0\Lambda(\mathbf{v})(\mathbf{v}_\perp\dot{\mathbf{u}}).$$

A solution of the kinetic equation is conveniently sought in the form

$$n_k=n_0[\varepsilon_0(\mathbf{k})+U+m_0\Lambda(\mathbf{v})(\mathbf{v}_\perp\dot{\mathbf{u}})]+\left(-\frac{\partial n_0}{\partial\varepsilon_0}\right)\chi_k, \quad (14)$$

where n_0 is the Fermi function. It is easily shown that the formal expression for χ_k has the form

$$\chi_k=\Lambda(\mathbf{v})m_0(\mathbf{v}_\perp\dot{\mathbf{u}})+\hat{B}^{-1}(e\mathbf{E}^e\mathbf{v}-m_0\hat{I}_\Lambda\Lambda(\mathbf{v})(\mathbf{v}_\perp\dot{\mathbf{u}})+\dot{U}), \quad (15)$$

where $\hat{B}^{-1}$ is the operator inverse to the kinetic-equation operator

$$\hat{B}=\frac{\partial}{\partial t}+v_x\frac{\partial}{\partial x}+\left(e\mathbf{E}^e+\frac{e}{c}[\mathbf{v}\times\mathbf{H}^e]-\nabla U\right)\frac{\partial}{\partial\mathbf{k}}+\hat{I}_\Lambda. \quad (16)$$

Thus in order to determine the nonequilibrium part of the distribution function χ_k, it is necessary to determine the operator $\hat{B}^{-1}$; that is, essentially, to solve the kinetic equation.

On the other hand, if one knows the function n_k and also the fields $\mathbf{E}^e$ and $\mathbf{H}^e$, it is possible to determine the absorbed power. In fact, the absorbed power P is[8]

$$P=\left\langle\int\frac{2d^3p'}{(2\pi\hbar)^3}\dot{\varepsilon}'(\mathbf{p}',\mathbf{r}')n_{p'}(\mathbf{r}')\right\rangle, \quad (17)$$

where the angular brackets denote averaging over a period of the wave in time or over a wavelength in space.

On transforming to the variables of the system K_Λ and using equations (8)–(11) and the condition (1), we have

$$P=\int\frac{dS_F}{v}[\langle\dot{U}\hat{B}^{-1}\dot{U}\rangle+e^2\langle E_i^eE_k^ev_i\hat{B}^{-1}v_k\rangle+m_0^2\langle\dot{u}_{i\perp}\dot{u}_{k\perp}\Lambda v_{i\perp}\hat{I}_\Lambda\Lambda v_{k\perp}\rangle], \quad (18)$$

where $\int dS_F$ denotes integration over the Fermi surface. The first term in (18) describes the deformation absorption, analyzed in detail in Refs. 3–5. The second describes Joule losses due to the presence of alternating electromagnetic fields. The third term is due to incomplete entrainment of the electronic lattice.[2] In the limit of frequent collisions, it would vanish. In our approximation, this term is proportional to the sound intensity and leads to a linear absorption.

Thus in order to solve the absorption problem, it is necessary to construct the operator $\hat{B}^{-1}$ (that is, to solve the nonlinear kinetic equation) and to calculate the field $\mathbf{E}^e$ from Maxwell's equations.

3. SOLUTION OF THE KINETIC EQUATION

Thus we must solve the equation

$$\hat{B}\chi_k^{(1)}=e\mathbf{E}^e\mathbf{v}+\dot{U}. \quad (19)$$

The function $\chi^{(1)}$, as well as $\mathbf{E}^e$, $\mathbf{H}^e$, and U, depends on the wave coordinate $\xi=qx-\omega t$; here, as follows from Maxwell's equation curl $\mathbf{E}=-\dot{\mathbf{H}}/c$, the dependences of $\mathbf{E}^e$ and of $\mathbf{H}^e$ on ξ are identical. But in general the $\mathbf{H}^e$ and U waves have different phases, and this considerably complicates the analysis. For this reason, we shall analyze a number of limiting cases. The case in which deformation absorption dominates was studied in Refs. 3–5. Here we shall consider the opposite limiting case, in which the electromagnetic absorption is important. Thus for transverse sound in a metal with an isotropic spectrum, $U=0$. In general, the condition for control of the electron motion by the electromagnetic forces can be easily derived by comparison of the various terms in the force that acts on an electron. Since

$$\mathbf{E}^e+\frac{1}{c}[\mathbf{v}\times\mathbf{H}^e]=\frac{\mathbf{q}}{\omega}(\mathbf{E}^e\mathbf{v})-\mathbf{E}^e\frac{\mathbf{q}\mathbf{v}-\omega}{\omega},$$

this condition has the form

$$E_0^e\gg\frac{m_0w}{e}qv_Fu_{ik}^0, \quad (20)$$

where $\mathbf{E}_0^e$ and u_{ik}^0 are, respectively, the amplitude values of the field $\mathbf{E}^e$ and of the strain tensor u_{ik}. The part $e\mathbf{E}^e(\omega-\mathbf{q}\cdot\mathbf{v})/\omega$, as is easily shown, is small with respect to the parameters w/v_F, $(ql)^{-1}$, and eE_0^e/p_F; below, we shall neglect it. We note that the fields $\mathbf{H}_0^e$ and $\mathbf{E}_0^e$ themselves depend on u_{ik}^0; therefore the conditions (20) constitute a limitation on the range of frequencies and of intensities of the sound. We shall introduce this limitation in explicit form below, after analysis of the electrodynamic part of the problem.

Thus we neglect terms containing U in equation (19) and represent $\mathbf{E}^e(\xi)$ in the form $\mathbf{E}_0^e\varphi(\xi)$, where φ is a periodic function, $\langle\varphi\rangle=0$, and $\langle\varphi^2\rangle=1$. Similarly to the manner in which this was done in Ref. 4, one can show that in our formulation the problem reduces to a one-dimensional one, and the operator $\hat{I}_\Lambda$ to an inverse drift relaxation time (in the case that we are considering, of scattering by short-range impurities, the drift and transport relaxation times coincide). Furthermore, in the functions $\Lambda(\mathbf{v})$ and $d_{ik}(\mathbf{v})$ one may set $v_x=0$, since the function is small except in the region of small v_x.

If we measure the x component of velocity in units

$$\tilde{v}=\left[\frac{e|\mathbf{E}_0^e\mathbf{v}_\perp|m_{xx}^{-1}}{\omega}\right]^{1/2}, \quad (21)$$

where $m_{xx}^{-1}=\partial v_x/\partial p_x$, it is easy to derive the following equation:

$$\hat{L}_\pm\tilde{\chi}^\pm=-\varphi(\xi), \quad (22)$$

where the functions $\tilde{\chi}^\pm$ are defined by the relation $\chi=e\omega_0^{-1}(\mathbf{E}_0^e\cdot\mathbf{v})\tilde{\chi}^\pm$; the signs ± correspond to the sign of $(\mathbf{E}_0^e\cdot\mathbf{v})$,

$$\hat{L}_\pm=s\frac{\partial}{\partial\xi}\pm\varphi\frac{\partial}{\partial s}+\frac{1}{\omega_0\tau}, \quad (23)$$

where $s=(v_x-w)/\tilde{v}$ is the dimensionless x component of the velocity, and $\omega_0\equiv q\tilde{v}$. The quantity $\omega_0\tau$ plays the role of nonlinearity parameter.

All the quantities of interest to us contain, as is easily shown,

$$\left\langle \varphi \int\limits_{-\infty}^{0} dp_y v_y \chi \right\rangle = -\int\limits_{0}^{\infty} dp_y v_y^2 \langle \varphi (\hat{L}_+^{-1} + \hat{L}_-^{-1}) \varphi \rangle.$$

By starting from the properties of the operator $\hat{L}$, one can easily show that $\langle \varphi \hat{L}_+^{-1} \varphi \rangle = \langle \varphi \hat{L}_- \varphi \rangle$; therefore it is sufficient for us to limit ourselves to analysis of the operator $\hat{L}_+^{-1}$, that is to the solution of Eq. (22) with the sign +. This equation essentially coincides with the equation analyzed in detail in Ref. 4, and we shall use the results of this analysis. The characteristics of Eq. (22) are determined by the one-dimensional "energy" integral

$$s^2(\xi)/2 + V(\xi) = \mathscr{E} = \text{const}, \tag{24}$$

where $\varphi(\xi) = -dV/d\xi$, $\langle V \rangle = 0$. The function $V(\xi)$ plays the role of dimensionless potential energy (its extreme values $V_{\min}$, $V_{\max} \sim 1$), and $\mathscr{E}$ of dimensionless total energy. Depending on the value of $\mathscr{E}$, the electrons are divided into two groups: transient ($\mathscr{E} > V_{\max}$) and trapped ($V_{\max} > \mathscr{E} > V_{\min}$).

The solutions of Eq. (22) look simplest in the case of strong nonlinearity, $\omega_0 \tau \gg 1$. We shall consider this case below. When $\omega_0 \tau \gg 1$, the solution of the kinetic equation can be expanded as a series in powers of $(\omega_0 \tau)^{-1}$. The lowest order of the expansion that gives a contribution to the current density $\mathbf{j}$ and to the sound absorption has the form[4]

$$\bar{\chi}^+ = \frac{1}{\omega_0 \tau} \int\limits_{V_{\min}}^{\infty} d\mathscr{E} \int\limits_{-\infty}^{\infty} d\xi' \varkappa(\mathscr{E}, \xi, \xi') \varphi(\xi'), \tag{25}$$

where

$$\varkappa(\mathscr{E}, \xi, \xi') = -\frac{1}{2} A(\xi, \xi') \left[\frac{A(\xi, \xi')}{A(0, 2\pi)} - 1 \right] G(\mathscr{E}, \xi') \gamma(\xi, \xi + 2\pi, \xi') \tag{26}$$

when $\mathscr{E} > V_{\max}$ (untrapped particles), and where

$$\varkappa(\mathscr{E}, \xi, \xi') = -\frac{1}{2} G(\mathscr{E}, \xi') \left[\frac{A_2(\xi_1, \xi)}{A(\xi_1, \xi_2)} \gamma(\xi_1, \xi_2, \xi') - 2A(\xi', \xi) \gamma(\xi_1, \xi, \xi') \right] \tag{27}$$

when $V_{\min} < \mathscr{E} < V_{\max}$ (trapped particles).

In equations (26) and (27)

$$G(\mathscr{E}, \xi) = [2(\mathscr{E} - V(\xi))]^{-1/2}, \quad A(x, y) = \int\limits_{x}^{y} G(\xi) d\xi, \quad \gamma(x, y, \xi) = \theta(\xi - x) \theta(y - \xi),$$

$\theta(x)$ is the unit step function, and $\xi_{1,2}$ are the roots of the equation $V(\xi) = \mathscr{E}$.

4. ELECTRODYNAMIC PART OF THE PROBLEM

Knowing the solution of the kinetic equation, one can determine the electric current density (in the laboratory coordinate system) and, by substituting the expression for it in Maxwell's equations, can determine the field $\mathbf{E}^e$. We shall restrict ourselves to the case in which the condition $qc \ll \omega_p$ is satisfied, where ω_p is the plasma frequency.[2)] Then, as is easily shown, the second term in the expression (11) is small in comparison with the first, and $\mathbf{E}^e$ coincides with the electric field $\mathbf{E}$.

The electric current density in the system is

$$j_i = em_0 \int \frac{dS_F}{v} \Lambda(\mathbf{v}_\perp) v_i (\mathbf{v}_\perp \dot{\mathbf{u}}) + e^2 \int \frac{dS_F}{v} v_i \hat{B}^{-1} v_k E_k$$
$$= C_{ik} e N \dot{u}_k + \hat{\sigma}_{ik} E_k, \quad \mathbf{j} = (j_y, j_z), \tag{28}$$

where N is the concentration of the electrons and C_{ik} is a tensor with components of order unity; the meaning of the quantities C_{ik} and $\hat{\sigma}_{ik}$ is clear from equation (28). From equations (25)–(27) it is easily perceived that the relation of the current to the electric field E is nonlocal; the norm of the operator $\hat{\sigma}$ is of the order

$$\frac{1}{\bar{\omega}_0 \tau} \frac{Ne^2}{qp_F}, \quad \bar{\omega}_0 \tau = q\tau \left[\frac{eE_0 m_{xx}^{-1} v_F}{\omega} \right]^{1/2}.$$

The specific expression for the operator $\hat{\sigma}$ depends on the form of the function $\varphi(\xi)$. The latter must be determined by simultaneous solution of Maxwell's equations and of the dispersion equation for a sound wave. We represent $\dot{\mathbf{u}}(\xi)$ in the form $\dot{\mathbf{u}}_0 b(\xi)$, where $\langle b^2 \rangle = 1$, and we direct the axes $0Y$ and $0Z$ along the principal axes of the tensor $\hat{\sigma}_{ik}$. Then Maxwell's equation can be put into the form

$$-\varphi' + \frac{\delta_i^2}{\bar{\omega}_0 \tau} \hat{\varkappa} \varphi = \frac{4\pi e N w C_{ik} \dot{u}_{0k}}{E_{0i} qc^2} b(\xi), \tag{29}$$

where

$$\hat{\varkappa} \varphi = \int d\mathscr{E} \int d\xi' \varkappa(\mathscr{E}, \xi, \xi') \varphi(\xi'), \quad \delta_i = f_i \frac{\omega_p}{qc} \left(\frac{3\pi w}{4v_F} \right)^{1/3},$$

f_i are numbers of order unity (in the isotropic case they are equal to unity). The parameter δ_i is in order of magnitude equal to the cube root of the ratio of the length of the sound wave to the skin depth (for the anomalous skin effect). Equation (29) must be solved with periodic boundary conditions; the relation of $\mathbf{E}_0$ to $\mathbf{u}_0$ is determined by the normalization condition $\langle \varphi^2 \rangle = 1$. The form of the sound wave (form of the function $b(\xi)$) depends on a number of circumstances. As is easily shown from formulas (25)–(27), even with a sinusoidal $\varphi(\xi)$ there are higher harmonics of the current of the same order of magnitude as the fundamental. In consequence, higher harmonics of the displacement are produced in the sound wave. The relative value of these harmonics is determined by the ratio of the nonlinear terms to the characteristic dispersion or the frequency-dependent part of the attenuation. Analysis shows that the electronic contribution to the dispersion of sound in a metal is small and cannot guarantee smallness of the higher harmonics. Therefore the higher harmonics develop at distances of the order of the nonlinear attenuation distance. At this distance, the higher harmonics may attain values of the order of the fundamental (they have no literal smallness parameter). But from the form of the distribution function (25)–(27) it can be seen that the electronic response depends little on the form of the function $\varphi(\xi)$ and hence on the form of the sound wave. The relation between the amplitude of the field $\mathbf{E}_0$ and the amplitude of the velocity of motion $\mathbf{u}_0$ in the sound wave can be obtained by multiplying (29) first by φ and then by φ', averaging over the wavelength, and combining the formulas obtained. This relation can be put into the form

$$E_{0i}^2=a_{1i}\left(\frac{4\pi eNC_{ik}\dot{u}_{0k}w}{qc^2}\right)^2[1+2a_{2i}\delta_i^4(\bar{\omega}_0\tau)^{-2}]^{-1}. \quad (30)$$

To determine $a_{1i}=[\langle\varphi_i b\rangle^2+\langle\varphi_i b'\rangle^2]/\langle\varphi_i'^2\rangle$ and $a_{2i}=\langle\varphi_i\varkappa\varphi_i\rangle/2\langle\varphi_i'^2\rangle$, it is necessary to solve the following equation, which follows from (29) and (30):

$$-\varphi_i'+\frac{\delta_i^2}{\bar{\omega}_0\tau}\hat{\varkappa}\varphi_i=[1+2a_{2i}\delta_i^4(\bar{\omega}_0\tau)^{-2}]^{1/2}b(\xi). \quad (31)$$

It is evident that the values of a_{1i} and a_{2i} depend only on the single parameter $\delta_i^2/\bar{\omega}_0\tau$ and vary little, remaining always of order of magnitude unity. We shall consider them to be constants and shall treat formula (30) as an interpolation formula.

By use of the expression (30), it is easy to relate the nonlinearity parameter $\bar{\omega}_0\tau$ to the sound intensity $S=a_3\rho|\mathbf{u}_0|^2$, where $a_3=\langle b^2\rangle+\langle b'^2\rangle\sim 1$. This relation has the form

$$(\bar{\omega}_0\tau)^4=\left(\frac{4\pi e^2Nm_{xx}^{-1}\tau^2v_F}{c^2}\right)^4\frac{S}{a_3\rho|\dot{u}_0|^2}\sum_i\frac{a_{1i}(C_{ik}\dot{u}_{0k})^2}{1+2a_{2i}\delta_i^4(\bar{\omega}_0\tau)^{-2}}. \quad (32)$$

The function $\bar{\omega}_0\tau(S)$ looks simplest in the case when $a_{2y}=a_{2z}=a_2$, $\delta_y=\delta_z=\delta_2$. Then

$$\bar{\omega}_0\tau=\delta^2a_2^{1/2}[(1+S/S_0)^{1/2}-1]^{1/2}, \quad (33)$$

where the characteristic intensity S_0 is

$$S_0=a_4(\delta/ql)^4\rho w^3, \quad (34)$$

and where the constant $a_4\sim 1$ is a combination of the constants a_{1-3}. In general, the qualitative behavior of the function $\bar{\omega}_0\tau(S)$ is similar to that described by formula (33).

The electromagnetic part of the nonlinear absorption coefficient, Γ_e, is according to (18)

$$\Gamma_e=a_5\Gamma_l\delta^2\frac{((1+S/S_0)^{1/2}-1)^{1/2}}{(1+S/S_0)^{1/2}+1}, \quad (35)$$

where $a_5\sim 1$, and where

$$\Gamma_l=4NCqp_F/3\pi\rho w \quad (36)$$

is a quantity of the order of the linear absorption coefficient of longitudinal sound in the metal, or order of magnitude qw/v_F; C is a characteristic value of the components of the tensor C_{ik}. The linear electromagnetic contribution to the absorption (in our notation) is[8]

$$\Gamma_{0e}=\Gamma_l\delta^4/(1+\delta^4), \quad (37)$$

the contribution of incomplete entrainment, described by the third term in (18), is of the order of $\Gamma_n\sim\Gamma_l(ql)^{-1}$.

We turn now to the condition (20), which determines the possibility of neglecting the deformational forces in comparison with the electromagnetic. In accordance with the formulas obtained above, the condition (20) can be rewritten in the form

$$\delta^2[((1+S/S_0)^{1/2}-1)/((1+S/S_0)^{1/2}+1)]\gg 1. \quad (38)$$

We shall therefore consider three limiting cases.

$$\text{I.}\ 1\gg S/S_0\gg\delta^{-4}. \quad (39)$$

The second inequality in (39) is necessary for satisfaction of the condition for strong nonlinearity, $\bar{\omega}_0\tau\gg 1$. Case I can be realized when $\delta^4\gg 1$; that is, at sufficiently low frequencies.

$$\text{II.}\ S/S_0\gg 1,\ \delta\gg 1; \quad (40)$$

$$\text{III.}\ \delta\ll 1. \quad (41)$$

Case I. In this case, condition (38) turns out to be satisfied automatically. Since $\delta^4\gg 1$, $\Gamma_{0e}\sim\Gamma_l\gg\Gamma_n$. For the ratio Γ_e/Γ_{0e} we have

$$\Gamma_c/\Gamma_{0c}\sim\delta^2(S/S_0)^{1/2}\gg 1, \quad (42)$$

the electromagnetic absorption coefficient increases with increase of the sound intensity and exceeds the value of the linear absorption coefficient Γ_{0e}. It is easy to show that in Case I the electromagnetic and deformational forces vary in phase. Therefore the analysis of the deformation absorption is completely analogous to that presented in Ref. 4. The deformation absorption, described by the first term in (18), is of the order

$$\Gamma_d\sim\Gamma_l/\omega_0\tau\sim\Gamma_l/\delta^2(S/S_0)^{1/2}. \quad (43)$$

Thus in Case I the nonlinear electromagnetic absorption dominates over the deformational, despite the fact that the contributions to the linear absorption are of the same order. The frequency dependence of the absorption coefficient is $\Gamma_e\sim\omega^3$, since $\Gamma_{0e}\sim\omega$, $\delta^2\sim\omega^{-2}$, and $S_0\sim\omega^{-8}$. Thus in Case I we have $\Gamma\sim\omega^3S^{1/2}$.

Case II. In Case II, the condition (38) is also satisfied. The nonlinear coefficient of electromagnetic absorption is

$$\Gamma_e\sim\Gamma_l\delta^2(S/S_0)^{-1/4}, \quad (44)$$

$\Gamma_e\sim\omega^{-3}S^{-1/4}$. We note that the nonlinear absorption coefficient may exceed the linear, $\sim\Gamma_l$ in the range $(S/S_0)^{1/4}\lesssim\delta^2$. The nonlinear coefficient of deformational absorption is

$$\Gamma_d\sim\Gamma_l/\bar{\omega}_0\tau\sim\Gamma_l/\delta^2(S/S_0)^{1/4}. \quad (45)$$

Therefore $\Gamma_e/\Gamma_d\sim\delta^4\gg 1$: the electromagnetic contribution dominates in the nonlinear range, although in the linear range the contributions are of the same order.

Thus when $\delta\gg 1$, the absorption coefficient initially increases (in region I) with increase of the sound intensity, and then begins to decrease (in region II). Such behavior of the absorption coefficient is easy to understand qualitatively. The electromagnetic absorption of sound resulting from Joule losses in eddy currents is proportional to the product of the appropriate component of the electrical conductivity tensor by the square of the effective electric field. The latter in turn depends substantially on the degree of shielding of the priming eddy currents by conductivity currents. The degree of this shielding is determined by the ratio of the length of the sound wave to the skin depth—the parameter δ. If $\delta \gg 1$, then the shielding is significant in the linear range, and the sound absorption is actually inversely proportional to the electrical conductivity. With increase of the sound intensity, the latter decreases; the coefficient of sound absorption increases. But with further increase of the sound intensity, the degree of shielding decreases (in the nonlinear range, the shielding is determined by the parameter $\delta^2/\overline{\omega}_0\nu$). Therefore, begnining with a certain intensity, the shielding becomes insignificant, the absorption coefficient becomes directly proportional to the electrical conductivity, and it decreases with increase of the sound intensity.

Case III. In this case, condition (38) is not satisfied, and deformational forces play a decisive role in the dynamics of the electrons. The appropriate nonlinearity parameter $\overline{\omega}_0\tau$ for deformational interaction was obtained in Ref. 3 and is equal in order of magnitude to

$$\overline{\omega}_0\tau=\delta(S/S_0)^{1/4}. \qquad (46)$$

Thus in region III the nonlinear deformational absorption is of the order

$$\Gamma_l/\delta\left(\frac{S}{S_0}\right)^{1/4}.$$

The coefficient of electromagnetic absorption is

$$\Gamma_e\sim\Gamma_l\delta^3\bigg/\left(\frac{S}{S_0}\right)^{1/4}. \qquad (47)$$

By virtue of condition (46), $\Gamma_e \ll \Gamma_{0e}=\Gamma_l\delta^4$; the ratio of the electromagnetic absorption to the deformational $\sim\delta^4 \ll 1$. Therefore in range III the deformational absorption dominates, and the dependence of the total absorption on the intensity is $\Gamma\sim S^{-1/4}$. An exception is the case of purely transverse sound in a metal with an isotropic spectrum. In this case, deformational absorption is absent, and the total absorption coefficient is described by formula (47) as long as $(S/S_0)^{1/4}<\delta^3 ql$. At higher intensities, the principal contribution comes from the third term in (18); $\Gamma\sim\Gamma_l(ql)^{-1}$ and is independent of the sound intensity.

Thus, depending on the frequency of the sound (the parameter δ), there are two types of variation of the nonlinear absorption with the sound intensity: when $\delta \ll 1$, we have $\Gamma\sim S^{-1/4}$; but when $\delta \gg 1$, the absorption coefficient initially increases $\sim S^{1/2}$ and then decreases $\sim S^{-1/4}$.

In conclusion, we shall present representative estimates of the possibility of observing nonlinear electromagnetic attenuation of short-wave sound.

The ratio δ/ql, which occurs in the expression for S_0, has for typical metallic parameters the order of magnitude $10^5/q^{2l}$ (if q is measured in cm^{-1} and l in centimeters); the value of $\rho w^3\sim 4\cdot 10^9\ W/cm^2$. Thus if we set $l=10^{-1}$ cm and $q=2\cdot 10^4\ cm^{-1}$ (frequency of order 1 GHz), we have $S_0\sim 0.2\ W/cm^2$. The parameter δ under these conditions ~5. It is evident that it is possible to realize experimentally both Case I and Case II. In range I, on assuming $S\sim S_0/4$ we have $\overline{\omega}_0\tau\sim 10$; in II, with $S\sim 5S_0$ we get $\overline{\omega}_0\tau\sim 30$. Case III can be realized at higher frequencies, since $\delta\sim q^{-1}$.

We remark that it is possible to separate the nonlinear deformational and electromagnetic contributions to the attenuation of sound in metals that transform to a superconducting state, by studying the change of the nonlinear absorption coefficient during the superconducting transition. In the temperature range below T_c, in consequence of the Meissner shielding of the eddy currents, the electromagnetic absorption disappears. We propose to investigate later the nonlinear electromagnetic absorption in the vicinity of T_c

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[1)]We note that the coordinate systems K_Λ are in general different for electrons with different momenta **p**. It is easily shown that for free electrons $\Lambda=-1$, and the system K_Λ coincides with the laboratory system.

[2)]In the opposite limiting case, the contribution of electromagnetic effects is small.

[1]A. I. Akhiezer, M. I. Kaganov, and G. Ya. Lyubarskiĭ, Zh. Eksp. Teor. Fiz. 32, 837 (1957) [Sov. Phys. JETP 5, 685 (1957)].

[2]A. B. Pippard, Phil. Mag. 46, 1104 (1955).

[3]Yu. M. Gal'perin and V. D. Kagan, Zh. Eksp. Teor. Fiz. 59, 1657 (1970) [Sov. Phys. JETP 32, 903 (1971)].

[4]Yu. M. Gal'perin, V. D. Kagan, and V. I. Kozub, Zh. Eksp. Teor. Fiz. 62, 1521 (1972) [Sov. Phys. JETP 35, 798 (1972)].

[5]P. E. Zil'berman, Zh. Eksp. Teor. Fiz. 60, 1943 (1971) [Sov. Phys. JETP 33, 1048 (1971)].

[6]S. N. Ivanov, I. M. Kotelyanskiĭ, G. D. Mansfel'd, and E. N. Khazanov, Pis'ma Zh. Eksp. Teor. Fiz. 13, 283 (1971) [JETP Lett. 13, 201 (1971)].

[7]V. D. Fil', V. I. Denisenko, and P. A. Bezuglyĭ, Fiz. Nizk. Temp. 1, 1217 (1975) [Sov. J. Low Temp. Phys. 1, 584 (1975)].

[8]V. L. Gurevich, I. G. Lang, and S. T. Pavlov, Zh. Eksp. Teor. Fiz. 59, 1679 (1970) [Sov. Phys. JETP 32, 914 (1971)].

[9]V. M. Kontorovich, Zh. Eksp. Teor. Fiz. 45, 1638 (1963) [Sov. Phys. JETP 18, 1125 (1964)].

[10]Yu. M. Gal'perin, Zh. Eksp. Teor. Fiz. 67, 2195 (1974) [Sov. Phys. JETP 40, 1088 (1974)].

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