[JETP Lett. 23, 86 (1976)]; Preprint Fiz. Inst. Akad. Nauk No. 178, 1975.
[3]T. A. Astemirov, V. S. Bagaev, L. I. Paduchikh, and A. G. Poyarkov, Pis'ma Zh. Eksp. Teor. Fiz. 24, 225 (1976) [JETP Lett. 24, 200 (1976)].
[4]T. A. Astemirov, V. S. Bagaev, L. I. Paduchkikh, and A. G. Poyarkov, Fiz. Tverd. Tela (Leningrad) 19, 937 (1977) [Sov. Phys. Solid State 19, 547 (1977)].
[5]B. M. Ashkinadze and I. M. Fishman, Fiz. Tekh. Poluprovodn. 11, 301 (1977) [Sov. Phys. Semicond. 11, 174 (1977)].
[6]T. C. Damen and J. M. Worlock, Proc. Third Intern. Conf. on Light Scattering on Solids, Campinas, Brazil, 1975, p. 183; J. M. Worlock, Proc. First Soviet-American Symposium on Theory of Light Scattering in Solids, Moscow, 1975, p. 195.
[7]M. Combescot, Phys. Rev. B 12, 1591 (1975).
[8]A. D. Durandin, N. N. Sibel'din, V. B. Stopachinskiĭ, and V. A. Tsvetkov, Pis'ma Zh. Eksp. Teor. Fiz. 26, 395 (1977) [JETP Lett. 26, 272 (1977)].
[9]V. V. Katyrin, N. N. Sibel'din, V. B. Stopachinskiĭ, and V. A. Tsvetkov, Fiz. Tverd. Tela (Leningrad) 20, No. 5 (1978) [Sov. Phys. Solid State 20, No. 5 (1978)].
[10]J. C. Hensel, T. G. Phillips, and T. M. Rice, Phys. Rev. Lett. 30, 227 (1973).
[11]R. M. Westervelt, T. K. Lo, J. L. Staehli, and C. D. Jeffries, Phys. Rev. Lett. 32, 1051 (1974).
[12]C. Benoit à la Guillaume, M. Capizzi, B. Etienne, and M. Voos, Solid State Commun. 15, 1031 (1974).
[13]T. A. Astemirov, V. S. Bagaev, L. I. Paduchikh, and A. G. Poyarkov, Kratk. Soobshch. Fiz. No. 11, 3 (1976).
[14]B. M. Ashkinadze and I. M. Fishman, Fiz. Tekh. Poluprovodn. 11, 408 (1977) [Sov. Phys. Semicond. 11, 235 (1977)].
[15]P. S. Gladkov, B. G. Zhurkin, and N. A. Penin, Fiz. Tekh. Poluprovodn. 6, 1919 (1972) [Sov. Phys. Semicond. 6, 1649 (1973)].
[16]B. M. Ashkinadze, N. N. Zinov'ev, and I. M. Fishman, Zh. Eksp. Teor. Fiz. 70, 678 (1976) [Sov. Phys. JETP 43, 349 (1976)].
[17]L. D. Landau and E. M. Lifshitz, Élektrodinamika sploshnykh sred (Electrodynamics of Continuous Media), Fizmatgiz, 1959, §84. [Pergamon, New York, (or Oxford, 1968].
[18]V. S. Bagaev, N. V. Zamkovets, L. V. Keldysh, N. N. Sibel'din, and V. A. Tsvetkov, Zh. Eksp. Teor. Fiz. 70, 1501 (1976) [Sov. Phys. JETP 43, 783 (1976)]; Preprint Fiz. Inst. Akad. Nauk No. 139, 1975.
[19]J. L. Staehli, Phys. Status Solidi B 75, 451 (1976).
[20]R. M. Westervelt, Phys. Status Solidi B 74, 727 (1976).
[21]V. A. Zayats, V. N. Murzin, I. N. Salganik, and K. S. Shifrin, Zh. Eksp. Teor. Fiz. 73, 1422 (1977) [Sov. Phys. JETP 46, (1977)]; Preprint Fiz. Inst. Akad. Nauk No. 38, 1977.

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# Damping of sound in antiferromagnets of the easy-plane type with a high Néel temperature

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The effect of the magnon system on the damping of sound is considered for high-temperature ($T_N > \Theta_D$) antiferromagnets with anisotropy of the "easy plane" type. It is shown that over a broad frequency range, the damping is determined by exchange-amplified relativistic phonon-magnon interaction. The damping of ultrasound has a peculiar frequency, temperature, and field dependence; in the hypersound range, the coupling of the magnetic and elastic subsystems leads to a peak in the frequency dependence of the damping. The calculation is carried out by the diagram method. The applicability of the phenomenological approach to the calculation of the relaxation times of coupled magnetoelastic waves is discussed.

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## INTRODUCTION

As is well known, damping of sound in dielectrics is caused by interaction of it with thermal phonons by virtue of the anharmonicity of the oscillations of the crystal lattice.[1,2] In magnetic dielectrics, the damping of sound may be determined by interaction with magnons.[3-5] The magnon subsystem plays an especially important role in shaping the acoustic properties of high-temperature antiferromagnets with anisotropy of the "easy plane" type (AFEP), such as $\alpha-Fe_2O_3$ and $FeBO_3$, because the magnetoelastic coupling of relativistic nature in AFEP is significantly amplified by intersublattice exchange interaction.[6] In relatively weak magnetic fields $H$ (for $\alpha-Fe_2O_3$, when $H \leq 1$ kOe), the coupling is so strong that the experimentally observed magnetic corrections to the second-order dynamic moduli of elasticity amount to tens of percent.[7,8] The strong mixing of phonons with magnons that occurs in AFEP, even in the absence of an intersection of the acoustic and spin-wave spectra, renders significant some channels for relaxation of phonons that do not, as a rule, play a role in ferromagnets. In particular, the nonlinearity of the magnetic subsystem and the magnetoelastic coupling introduce into the elastic subsystem of AFEP an additional anharmonicity that may significantly exceed the elastic anharmonicity of ordinary solids.[9]

In the analysis of sound damping, one usually distinguishes two characteristic cases, corresponding to different relations between the frequency of the sound wave being considered, $\Omega$, and the mean lifetime $\tau$ of the intermediate states that take part in the relaxation process. When $\Omega\tau \gg 1$, the finiteness of the lifetime of the intermediate states is, as a rule, unimportant, and the sound damping can be calculated, for example, by the usual quantum-mechanical formula for the transi-

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tion probability, in the first nonvanishing order of perturbation theory.[1] For the contrary case $\Omega\tau \ll 1$, a method of calculation has been proposed[2] that is based on the kinetic equation, and in which the sound wave is treated as an external nonstationary field modulating the spectrum of quasiparticles of the crystal. There has also been proposed a modified perturbation theory, suitable also in the intermediate range $\Omega\tau \sim 1$, corresponding to allowance, in the energy-conservation law, for an indeterminacy of order $\tau^{-1}$ in the energies of the intermediate states.[10] But the most systematic approach, in the analysis of relaxation phenomena, is the diagrammatic approach, which makes it possible in a unique manner to treat relaxation processes at arbitrary frequencies, and also to elucidate the conditions for applicability of the methods mentioned above.[5,11]

In the present paper, the diagram method is used, with allowance for the finite life of the intermediate states, to investigate the effect of magnetoelastic interaction on sound damping in AFEP, in the example of the weakly ferromagnetic phase of hematite ($\alpha$–$Fe_2O_3$). In the first section of the paper, the Hamiltonian of the magnonphonon interaction is given. In the second, the damping of ultrasound caused by relativistic magnon-phonon interactions is calculated. In the third section, the damping in the hypersonic range of frequencies is calculated, and questions are discussed that relate to the applicability of a phenomenological calculation of the damping of magnetoelastic waves. In the fourth section, the influence of exchange magnon-phonon interaction on sound damping is discussed.

## 1. INTERACTION OF MAGNETOELASTIC WAVES IN AFEP

The Hamiltonian of AFEP consists of magnetic, elastic, and magnetoelastic parts:

$$\mathcal{H}=\mathcal{H}_m+\mathcal{H}_{ph}+\mathcal{H}_{m-ph}.$$

In the second-quantization representation, the part of the Hamiltonian $\mathcal{H}$ that is bilinear in the magnon and phonon creation and annihilation operators can be represented in the form[6]

$$\mathcal{H}_0=\sum_{jk}\varepsilon_{jk}c_{jk}^+c_{jk}+\sum_{pk}\omega_{pk}b_{pk}^+b_{pk}+\sum_{jpk}[D_{jpk}c_{jk}^+(b_{pk}+b_{p-k}^+)+\text{h.c.}], \tag{1}$$

where $\epsilon_{jk}=(\epsilon_{j0}^2+s^2k^2)^{1/2}$ is the energy of magnons of the two branches of the spectrum ($j=1,2$), $\omega_{pk}$ is the energy of the phonons ($p=1,2,3$), and $D_{jpk}$ are the magnetoelastic coupling parameters, whose form for crystals of rhombohedral structure, of the type of $\alpha$–$Fe_2O_3$, $FeBO_3$, and $MnCO_3$, has been given.[12]

We consider crystals in which there is no intersection of the magnon and phonon spectra. Then the relativistic magnetoelastic coupling is most significant in the range of small wave vectors $\mathbf{k}$, in which $\epsilon_{1k} \ll \epsilon_{2k}$. In consequence, it is sufficient to take account solely of the coupling of phonons with the low-activation magnon branch $\epsilon_{1k}$.

Three-particle interactions of magnons of the lower branch with phonons are described by the Hamiltonian

$$\mathcal{H}_{m-ph}^{(3)}=N^{-1/2}\sum_{\substack{\mathbf{kq}\\p}}\psi_{m-ph}(\mathbf{k},\mathbf{q})(b_{\mathbf{k}p}^++b_{-\mathbf{k}p})(c_{1\mathbf{q}}^++c_{1-\mathbf{q}})(c_{1-\mathbf{k}-\mathbf{q}}^++c_{1\mathbf{k}+\mathbf{q}}), \tag{2}$$

where

$$\psi_{m-ph}(\mathbf{k},\mathbf{q})=\frac{i}{2}\Theta\left(\frac{\omega_{p\mathbf{k}}}{2Mc_p^2}\right)^{1/2}\frac{J_0}{(\varepsilon_{1\mathbf{q}}\varepsilon_{1\mathbf{k}+\mathbf{q}})^{1/2}}\mathbf{e}_p\mathbf{a}', \tag{3}$$

$$\mathbf{a}_\mathbf{k}'=(\xi n_x,\ \xi n_y+n_z,\ n_y),\ \mathbf{n}=\mathbf{k}/|\mathbf{k}|,\ \xi=(B_{11}-B_{12})/2B_{14},$$

$\Theta=2B_{14}v_0$ is a characteristic energy of magnetoelastic interaction ($\Theta \sim 10$ K), $c_p(\mathbf{n})$ is the velocity of sound with polarization $\mathbf{e}_p(\omega_p=c_pk)$; $B_{11}$, $B_{12}$, and $B_{14}$ are magnetoelastic constants, $v_0$ and $M$ are the volume and mass of the elementary cell, and $J_0$ is the exchange constant. Later, we shall also use a characteristic temperature $\Theta_N=s/a$, coinciding in order of magnitude with the Néel temperature; $a$ is a quantity of the order of magnitude of the period of the lattice. The external field is assumed to be oriented in the basal plane of the crystal, parallel to the binary axis $x$.

The Hamiltonian (1) can be diagonalized by transforming from the operators $b_{\mathbf{k}p}^+$, $b_{\mathbf{k}p}$, $c_{1\mathbf{k}}^+$, and $c_{1\mathbf{k}}$ to the creation and annihilation operators of magnetoelastic waves, $d_{i\mathbf{k}}^+$ and $d_{i\mathbf{k}}$ ($i=0,1,2,3$) (see Appendix). Then the conditon $\epsilon_{1\mathbf{k}} \gg \omega_{\mathbf{k}p}$, which is valid for $\alpha$–$Fe_2O_3$ and $FeBO_3$, permits us to separate the magnetoelastic waves into quasimagnons with energies $\Omega_{0\mathbf{k}} \approx \epsilon_{1\mathbf{k}}$, corresponding to the operators $d_\mathbf{k}^+$ and $d_\mathbf{k}$, and quasiphonons with energies $\Omega_{\lambda\mathbf{k}} \ll \epsilon_{1\mathbf{k}}$, corresponding to the operators $d_{\lambda\mathbf{k}}^+$ and $d_{\lambda\mathbf{k}}$ ($\lambda=1,2,3$ is the index of polarization of the quasiphonon). We note that it is the quasiphonons that are normal acoustic modes of the crystal.

The interaction Hamiltonian (2), transformed to the operators $d_{i\mathbf{k}}^+$ and $d_{i\mathbf{k}}$, describes three-particle interactions, of which the only ones that contribute significantly to sound damping are interactions of quasiphonons with each other and of a single quasiphonon with two quasimagnons. The processes of the first type are analogous to those processes of phonon interaction that are caused by anharmonicity of the oscillations of the crystal lattice. In the present case the effective anharmonicity is provided by the nonlinear magnon-phonon coupling and is described by the Hamiltonian

$$V_{ph-ph}^{(3)}=N^{-1/2}\sum_{\substack{\mathbf{kq}\\l\lambda\nu}}\psi_{l\lambda\nu}^{ph}(\mathbf{k},\mathbf{q})(d_{\lambda\mathbf{k}}^++d_{\lambda-\mathbf{k}})(d_{\nu\mathbf{q}}^++d_{\nu-\mathbf{q}})(d_{l-\mathbf{k}-\mathbf{q}}^++d_{l\mathbf{k}+\mathbf{q}}), \tag{4}$$

where

$$\psi_{l\lambda\nu}^{ph}(\mathbf{k},\mathbf{q})=-\frac{\Theta}{2^{1/2}}\left(\frac{\Omega_{\lambda\mathbf{k}}\Omega_{\nu\mathbf{q}}\Omega_{l\mathbf{k}+\mathbf{q}}}{M^3c_\lambda^2c_\nu^2c_l^2}\right)^{1/2}\frac{\Theta^2J_0^2}{\varepsilon_{1k}^2\varepsilon_{1q}^2}(\mathbf{a}_\mathbf{k}\mathbf{e}_\lambda)(\mathbf{a}_\mathbf{q}\mathbf{e}_\nu)(\mathbf{a}_{\mathbf{k}+\mathbf{q}}'\mathbf{e}_l), \tag{5}$$

$$\mathbf{a}_\mathbf{k}=(\xi n_y+n_z,\ \xi n_x,\ n_x).$$

Here $c_\lambda(\mathbf{k})$ are renormalized sound velocities determined by the relations $\Omega_{\lambda\mathbf{k}}=c_\lambda k$.

Important in the analysis of relaxation processes is the difference of the effective anharmonicity from the usual elastic anharmonicity; this difference is due to dispersion of the spin waves whose energies enter into the amplitude (5). In the long-wave range ($c_p k \ll \epsilon_{10}$), the effective anharmonicity may appreciably exceed the strictly elastic (for $\alpha-Fe_2O_3$, by two orders of magnitude at $H \approx 1$ kOe[9]). The effective anharmonicity makes practically no contribution to the interaction of sound with short-wave thermal quasiphonons. In consequence, the main contribution to sound damping from processes described by the Hamiltonian (2)–(5) is made not by thermal quasiphonons but by those of much longer wavelength, with wave vectors $q^* \sim \epsilon_{10}/a\Theta_N$.

Processes involving participation of a single quasiphonon and two quasimagnons, with $\epsilon_{i\mathbf{k}} \gg \omega_{p\mathbf{k}}$, are described by a Hamiltonian similar to (2)–(3),

$$\hat{V}^{(3)}_{ph-m} = N^{-1/2} \sum_{\substack{\mathbf{k}\mathbf{q} \\ \lambda}} \bar{\psi}_{ph-m}(\mathbf{k},\mathbf{q}) (d_{\lambda\mathbf{k}}^{+} + d_{\lambda-\mathbf{k}}) (d_{\mathbf{q}}^{+} + d_{-\mathbf{q}}) (d^{+}_{-\mathbf{k}-\mathbf{q}} + d_{\mathbf{k}+\mathbf{q}}), \qquad (6)$$

where

$$\bar{\psi}_{ph-m}(\mathbf{k},\mathbf{q}) = -\frac{\Theta}{2}\left(\frac{\Omega_{\lambda\mathbf{k}}}{2Mc_\lambda^2}\right)^{1/2} \frac{J_0}{(\varepsilon_{1\mathbf{q}}\varepsilon_{1\mathbf{k}+\mathbf{q}})^{1/2}} (\mathbf{a}_{\mathbf{k}}'\mathbf{e}_\lambda).$$

## 2. DAMPING OF ULTRASOUND BECAUSE OF RELATIVISTIC MAGNETOLEASTIC INTERACTION

In this section we shall consider the damping of ultrasound, with frequency $\Omega \ll \epsilon_{10}$, as a result of relativistic magnetoelastic interaction, to which corresponds: a) effective anharmonicity of quasiphonons, $V^{(3)}_{\text{ph-ph}}$; b) phonon-magnon interaction, $V_{\text{ph-}m}$.

a) First we shall consider damping of sound because of effective anharmonicity. At the frequencies being considered, the processes of decay of a sound quantum into two make a negligibly small contribution to the damping, and the main processes are ones of fusion; that is, the damping is determined by the imaginary part of the mass operator represented in Fig. 1, where a circle denotes a symmetrized amplitude of the effective anharmonicity $\psi^{\text{ph}}(\mathbf{k},\mathbf{q})$ and a wavy line denotes the Green's function of a quasiphonon.

For high frequencies, $\Omega \gg \tau^{*-1}_{\text{ph}}$ ($\tau^*_{\text{ph}}$ is the lifetime of the phonons that make the principal contribution to the sound damping), one can neglect broadening of the spectrum of intermediate states, and for calculation of the sound damping it is sufficient to consider only the second order of perturbation theory (the first term of the series of Fig. 1):

$$\gamma_{ph}(\mathbf{k}) = 9\pi \frac{\Omega_{\mathbf{k}}}{T} v_0 \int \frac{d\mathbf{q}}{(2\pi)^3} \frac{|\psi_{ph}(\mathbf{k},\mathbf{q})|^2}{\operatorname{sh}^2(\Omega_{\mathbf{q}}/2T)} \delta(\Omega_{\mathbf{k}} + \Omega_{\mathbf{q}} - \Omega_{\mathbf{k}+\mathbf{q}}). \qquad (7)$$

It is easy to see that because of the strong dependence of the amplitude $\psi^{\text{ph}}(\mathbf{k},\mathbf{q})$ on the wave vector $\mathbf{q}$, the main contribution to the integral (7) is made by phonons with wave vectors $q^* \sim \epsilon_{10}/a\Theta_N$ and, correspondingly, energies $\Omega_{\mathbf{q}*} \sim \epsilon_{10}\Theta_D/\Theta_N$.

Analysis shows that the lifetime $\tau^*_{\text{ph}}$ of phonons with energies $\Omega_{\mathbf{q}*} \sim \epsilon_{10}\Theta_D/\Theta_N$, which make the principal contribution to the sound damping, is determined by Akhiezer's mechanism of damping as a result of ordinary elastic anharmonicity: $\tau^{*-1}_{\text{ph}} \sim (\epsilon^2_{10}/T)(\Theta_D/\Theta_N)^2$ (the calculation of the damping of quasiparticles by Akhiezer's method corresponds to summation of ladder diagrams in the calculation of the mass operator).

At low frequencies, $\Omega \ll \tau^{*-1}_{\text{ph}}$, it is not permissible to retain only the second order of perturbation theory in the calculation of sound damping, because the diagram series for the mass operator diverges with respect to the parameter $(\Omega\tau^*_{\text{ph}})^{-1} \gg 1$. Physically, this is due to the necessity for taking into account the finite lifetime $\tau^*_{\text{ph}}$ of the intermediate particles. In the general case, also, renormalization of the interaction amplitude is possible. (Characteristic graphs for a vertex are shown in Fig. 2, a and b. A straight line denotes the Green's function of the magnons.) Estimates show, however, that the renormalized effective elastic anharmonicity is negligible. Thus the relative renormalization determined by Fig. 2a is small in proportion to the smallness of the parameter $(\epsilon_{10}/T)(\Theta_D/\Theta_N)^3 \ll 1$; the renormalization determined by Fig. 2b can be neglected at magnetic fields satisfying the condition

$$\left(\frac{\varepsilon_{10}^2}{\mu H \varepsilon_{20}}\right)^2 \frac{\Theta_D}{\Theta_N} \ll 1$$

(for $\alpha-Fe_2O_3$, at $H \gtrsim 1$ kOe).

Thus for calculation of the sound damping with frequency $\Omega \ll \tau^{*-1}_{\text{ph}}$, it is sufficient to renormalize the Green's functions of the phonons; this is equivalent to

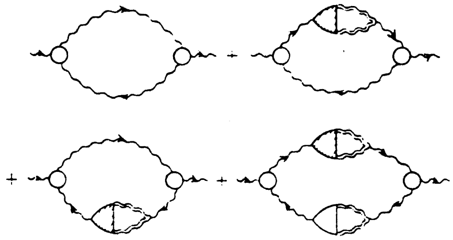

FIG. 1. Diagram series for the mass operator of quasiphonons. A cross-hatched vertex corresponds to the total amplitude of interaction of phonons by virtue of ordinary elastic anharmonicity.

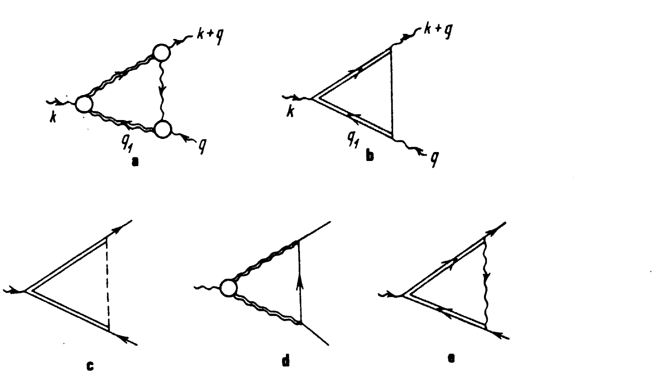


FIG. 2. Simplest diagrams of renormalizations of the amplitudes $\psi_{\text{ph-ph}}$ and $\psi_{\text{ph-}m}$: a) by quasiphonon processes; b) by processes of interaction of quasiphonons with quasimagnons; c) by magnon-magnon processes involving participation by magnons of the upper branch of the spectrum; d), e) by processes of interaction of quasiphonons with quasimagnons.

replacement of the δ function by a Lorentz function in the expression (7) for the damping:

$$\gamma_{ph}(\mathbf{k})=9\frac{\Omega_{\mathbf{k}}}{T}v_0\int\frac{d\mathbf{q}}{(2\pi)^3}\frac{|\psi^{ph}(\mathbf{k},\mathbf{q})|^2}{\mathrm{sh}^2(\Omega_{\mathbf{q}}/2T)}\frac{2\tau_{ph}^{-1}(\mathbf{q})}{(\Omega_{\mathbf{k}}+\Omega_{\mathbf{q}}-\Omega_{\mathbf{k}+\mathbf{q}})^2+4\tau_{ph}^{-2}(\mathbf{q})}. \quad (8)$$

The relation (8) describes the sound damping caused by the effective anharmonicity at arbitrary frequencies. In the limiting cases $\Omega\tau^*\ll 1$ and $\Omega\tau^*\gg 1$ (when $\Omega\tau^*\gg 1$, the relation (8) is obviously identical with (7)),

$$\gamma_{ph}(\mathbf{k})=\frac{\beta_1}{2^5}\Omega_{\mathbf{k}}\frac{\Theta^6 T\Theta_N}{\varepsilon_{10}{}^5(Mc^2)^3},\quad \Omega_{\mathbf{k}}\gg\omega_1{}^{ph}=\tau_{ph}^{*-1}, \quad (9a)$$

$$\gamma_{ph}(\mathbf{k})=\beta_2\frac{\Omega_{\mathbf{k}}{}^2}{\varepsilon_{10}}\left(\frac{\Theta}{\varepsilon_{10}}\right)^6\frac{T^2\Theta_N}{(Mc^2)^3},\quad \Omega_{\mathbf{k}}\ll\omega_1{}^{ph}, \quad (9b)$$

where $\beta_1$ and $\beta_2$ are functions of the direction of propagation of the sound wave and of its polarization and are of order of magnitude unity.

The unusual feature of the damping mechanism considered is that the lifetime of the intermediate quasiparticles that interact with the sound wave cannot be determined in the second order of perturbation theory, in contrast to the cases considered in previous papers.[2,5] Analysis of such situations in the language of kinetic equations is of definite interest but is not within the scope of the present paper.

b) We shall now consider the channel for relaxation of sound that is due to its interaction with magnons (see (6)). Analysis of the diagrams of Figs. 2 c, d, and e for the renormalizations of the amplitude of the phonon-magnon interaction shows that they may be neglected under the condition

$$\varepsilon_{10}\Theta\gg(\mu H)^2\gg\Theta^2\left(\frac{\varepsilon_{10}}{\varepsilon_{20}}\right)^2\frac{\Theta_N}{Mc^2}.$$

For the sound damping in this field interval, which is of the greatest interest from the experimental point of view, one can obtain, by a derivation similar to that of (8), the expression

$$\gamma_{ph-m}(\mathbf{k})=\frac{\Omega_{\mathbf{k}}}{4T}v_0\int\frac{d\mathbf{q}}{(2\pi)^3}\frac{|\psi_{m-ph}(\mathbf{k},\mathbf{q})|^2}{\mathrm{sh}^2(\varepsilon_{1\mathbf{q}}/2T)}\frac{2\tau_m{}^{-1}(\mathbf{q})}{(\Omega_{\mathbf{k}}+\varepsilon_{1\mathbf{q}}-\varepsilon_{1\mathbf{k}+\mathbf{q}})^2+4\tau_m{}^{-2}(\mathbf{q})}. \quad (10)$$

In the case of high frequencies $\Omega_{\mathbf{k}}\gg\tau_m^{-1}$, when the lifetime $\tau_m$ of the intermediate magnons may be neglected, we have

$$\gamma_{ph-m}(\mathbf{k})=\frac{|\mathbf{a}_{\mathbf{k}}{}'\mathbf{e}_{\lambda\mathbf{k}}|^2}{8\pi}\Omega_{\lambda\mathbf{k}}\frac{T}{\varepsilon_{10}}\left(\frac{\Theta}{\Theta_N}\right)^2\frac{\Theta_D}{Mc_\lambda{}^2}. \quad (11)$$

In the case of low frequencies ($\Omega\lesssim\tau_m^{-1}$), the finite lifetime of the intermediate magnons is important in the calculation of the integral (10). The principal contribution of the integral (10) is made by magnons with wave vectors $q^*\sim\epsilon_{10}/a\Theta_N$. Analysis shows that their lifetime $\tau_m(\mathbf{q}^*)$ is determined by processes of fusion of two magnons of the lower branch into a magnon of the upper branch[13]:

$$\tau_{m-m}^{-1}(\mathbf{q}^*)=\frac{1}{8\pi}\frac{(\mu H)^2}{\Theta_N}\frac{T}{\Theta_N}\left(\frac{\varepsilon_{20}}{\varepsilon_{10}}\right)^2\frac{J_0}{\Theta_N}. \quad (12)$$

From the expression (10), with use of the relation (12), we get for the damping of sound with frequencies $\Omega\ll\omega_1^m\equiv\tau_m^{-1}(\mathbf{q}^*)\Theta_D/\Theta_N$

$$\gamma_{ph-m}(\mathbf{k})=\frac{\pi}{4}|\mathbf{a}_{\mathbf{k}}{}'\mathbf{e}_{\lambda\mathbf{k}}|^2\varepsilon_{10}\left(\frac{\Omega_{\lambda\mathbf{k}}}{\varepsilon_{20}}\right)^2\left(\frac{\Theta}{\mu H}\right)^2\frac{J_0}{Mc_\lambda{}^2}. \quad (13)$$

In the case of weak magnetic fields,

$$\mu H\leqslant\Theta\frac{\varepsilon_{10}}{\varepsilon_{20}}\left(\frac{\Theta_N}{Mc^2}\right)^{1/2}$$

the lifetime $\tau_m(\mathbf{q}^*)$ of the intermediate magnons is determined not solely by three-magnon processes (see (12)), but also by a contribution from relativistic magnon-phonon interactions (6):

$$\tau_{m-ph}^{-1}(\mathbf{q}^*)=\frac{\varepsilon_{1\mathbf{q}^*}}{4T}v_0\int\frac{d\mathbf{k}}{(2\pi)^3}\frac{|\psi_{m-ph}(\mathbf{q}^*,\mathbf{k})|^2}{\mathrm{sh}(\varepsilon_{\mathbf{k}+\mathbf{q}^*}/2T)\,\mathrm{sh}(\Omega_{\mathbf{k}}/2T)}\delta(\varepsilon_{1\mathbf{q}^*}+\Omega_{\mathbf{k}}-\varepsilon_{1\mathbf{k}+\mathbf{q}^*})$$
$$=\frac{T}{4\pi}\frac{\Theta}{Mc^2}\frac{\Theta}{\Theta_N}\left(\frac{J_0}{\Theta_N}\right)^2. \quad (14)$$

Here the renormalization of the amplitude of the phonon-magnon interaction, determined by the diagram of Fig. 2e, is not small, but it does not change the order of magnitude of $\psi_{\text{ph-}m}$. The damping of low-frequency sound in this case is determined by the relation (13) with an additional factor $(1+\tau^*_{m\text{-}m}/\tau^*_{m\text{-ph}})^{-1}$, which takes account of renormalization of the Green's functions of the magnons not only by mangon-magnon but also by magnon-phonon interactions.

From the form of the amplitude $\tilde{\psi}_{m\text{-ph}}(\mathbf{k},\mathbf{q})$ in (6) it follows that for certain isolated acoustic modes (such that $\mathbf{a}\cdot\mathbf{e}_\lambda=0$) this amplitude, and along with it the damping caused by interaction of sound with magnons, vanishes. Thus for such modes, there is only the first of the two channels of relaxation considered: that due to the effective anharmonicity.1) For the remaining modes, the damping is determined by both channels; but comparison of (9) with (11) and (13) shows that the principal channel for relaxation of sound is that due to its interaction with magnons.

Comparison of the expressions (9), (11), and (13) with the sound attenuation $\gamma_{\text{ph}}^A\sim\Omega^2/T$ due to ordinary elastic anharmonicity (the Akhiezer mechanism) shows that relativistic phonon-magnon interactions amplified by intersublattice exchange play a decisive role in the relaxation of ultrasound with frequencies $\Omega<\omega_2^{\text{ph}}$, $\omega_2^m$, where

$$\omega_2{}^{ph}=\Theta\left(\frac{\Theta}{\varepsilon_{10}}\right)^5\frac{T^2\Theta_N}{(Mc^2)^3},\quad \mathbf{a}_{\mathbf{k}}{}'\mathbf{e}_{\lambda\mathbf{k}}=0,$$
$$\omega_2{}^m=\Theta\frac{\Theta}{\varepsilon_{10}}\left(\frac{T}{\Theta_N}\right)^2\frac{\Theta_D}{Mc^2},\quad \mathbf{a}_{\mathbf{k}}{}'\mathbf{e}_{\lambda\mathbf{k}}\neq0. \quad (15)$$

The general behavior of the frequency variation of the sound damping is shown in Fig. 3. We note that the ratio of the damping due to magnon-phonon interactions to the damping $\gamma_{\text{ph}}^A$, when $\Omega<\omega_1$ (the section where there is quadratic variation with frequency), is of the order of $\omega_2/\omega_1$. According to estimates for hematite at room temperature and at $H\sim 1$ kOe, this ratio is $\omega_2^{\text{ph}}/\omega_1^{\text{ph}}\sim 10$, $\omega_2^m/\omega_1^m\sim 10^2$. Estimation of the frequencies $\omega_2^{\text{ph}}$ and $\omega_2^m$ gives $\omega_2^{\text{ph}}\sim 10^6$ sec$^{-1}$, $\omega_2^m\sim 10^7$ sec$^{-1}$.

In measurements of sound absorption in hematite[14] at $\Omega\gtrsim\omega_2^m$, there was observed a sensitivity of the damping decrement to the direction of the magnetic field, and also the presence of a contribution to the damping

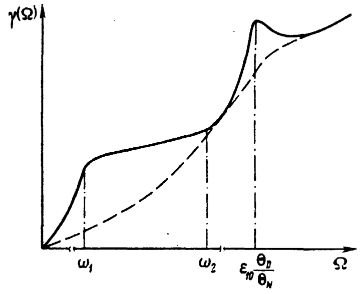


FIG. 3. Frequency variation of the damping coefficients of sound (the dotted curve shows the function $\gamma(\Omega)$ without allowance for phonon-magnon interactions).

proportional to the temperature when $T>270$ K. In principle, these results can be explained by the contribution of the relativistic phonon-magnon channel of relaxation (see the expression (11)). But definitive conclusions require data from measurements in the frequency range $\Omega<\omega_2^m$.

## 3. DAMPING OF HYPERSOUND

The well-known phenomenological method of calculation of the damping of coupled magnetoelastic waves reduces to taking account of the finite relaxation times of interacting phonons and magnons in the solution of the dispersion equation that determines the frequencies of magnetoelastic waves.[15] In application to our case, such a calculation would give the following relation for the damping coefficient of the quasiphonons:

$$\gamma_{ph}(\mathbf{k})=\gamma_{ph}{}^0(\mathbf{k})+\Delta\gamma_{ph}(\mathbf{k}),\quad \Delta\gamma_{ph}(\mathbf{k})\approx\frac{G_{\mathbf{k}}{}^2}{\omega_{p\mathbf{k}}\varepsilon_{1\mathbf{k}}}\gamma_m{}^0(\mathbf{k}),$$

and $\gamma_{ph}^0(\mathbf{k})$ and $\gamma_m^0(\mathbf{k})$ are the damping coefficients of the interacting phonons and magnons, respectively. As is well known, in AFEP at $sk\lesssim\epsilon_{10}$ the value of $\gamma_m^0(\mathbf{k})$ is determined by processes of fusion of magnons of the lower branch of the spectrum with formation of a magnon of the upper branch[13] (see the expression (12)). Thus the phenomenological method of calculation corresponds physically in the present case to taking account of three-magnon channels of relaxation. But as will be shown below, a systematic calculation of the contribution to relaxation of quasiphonons from three-particle processes of interaction of magnons of the two branches of the spectrum leads to results essentially different from the phenomenological.

This magnon interaction is described by the Hamiltonian

$$\mathscr{H}_m^{(3)}=N^{-1/2}\sum_{\mathbf{kq}}[\psi_m{}^+(\mathbf{k},\mathbf{q})c_{1\mathbf{k}}c_{1\mathbf{q}}\overset{+}{c}_{2\mathbf{k}+\mathbf{q}}+2\psi_m{}^-(\mathbf{k},\mathbf{q})c_{1\mathbf{k}}{}^+c_{1-\mathbf{q}}c_{2\mathbf{k}+\mathbf{q}}+\text{h.c.}].\quad (16)$$

where

$$\psi_m{}^\pm(\mathbf{k},\mathbf{q})=\frac{\mu H}{8}\left(\frac{2J_0}{\varepsilon_{1\mathbf{k}}\varepsilon_{1\mathbf{q}}\varepsilon_{2\mathbf{k}+\mathbf{q}}}\right)^{1/2}(\varepsilon_{2\mathbf{k}+\mathbf{q}}+\varepsilon_{1\mathbf{k}}\pm\varepsilon_{1\mathbf{q}}).\quad (17)$$

By transforming from magnon operators to creation and annihilation operators for magnetoelastic waves, one can obtain the Hamiltonian for interaction of quasiphonons with quasimagnons[2)] of the lower and magnons of the upper branch of the spectrum. Since for high-temperature ($\Theta_N>\Theta_D$) antiferromagnets the process of splitting of a phonon into two magnons is forbidden, we shall write only that part of the interaction that corresponds to a process of fusion of a phonon with a magnon of one branch and creation of a magnon of the other branch:

$$\mathscr{H}_{m-ph}^{(3)}=N^{-1/2}\sum_{\mathbf{kq}\lambda}[\psi^+_{\lambda,m-ph}(\mathbf{k},\mathbf{q})d_{\lambda\mathbf{k}}{}^+c_{1\mathbf{k}}{}^+c_{2\mathbf{k}+\mathbf{q}}+\psi^-_{\lambda,m-ph}(\mathbf{k},\mathbf{q})d_{\lambda-\mathbf{k}}c_{1\mathbf{q}}{}^+c_{2\mathbf{k}+\mathbf{q}}+\text{h.c.}],$$

$$\psi^\pm_{\lambda,m-ph}(\mathbf{k},\mathbf{q})=-\frac{\mu H}{2}\left(\frac{J_0}{2\varepsilon_{1\mathbf{q}}\varepsilon_{2\mathbf{k}+\mathbf{q}}\Omega_{\lambda\mathbf{k}}}\right)^{1/2}\nu(\Omega_{\lambda\mathbf{k}})\left[\varepsilon_{1\mathbf{k}}\pm\frac{\Omega_{\lambda\mathbf{k}}(\varepsilon_{1\mathbf{q}}+\varepsilon_{2\mathbf{k}+\mathbf{q}})}{\varepsilon_{1\mathbf{k}}}\right].\quad (18)$$

The processes being considered make an appreciable contribution to sound damping only in the hypersound frequency range $\Omega_{\lambda\mathbf{k}}\gtrsim\epsilon_{10}$, where it is permissible to retain only the second order of perturbation theory in calculating the mass operator. Then the value of $\Delta\gamma_{ph}(\mathbf{k})$ is determined by the relation

$$\Delta\gamma_{ph}(\mathbf{k})=\frac{\zeta(3)}{2\pi}\left(\frac{\Omega_{\lambda\mathbf{k}}}{\varepsilon_{1\mathbf{k}}}\right)^2\frac{T^2}{sk}\left(\frac{\mu H}{\Theta_N}\right)^2\frac{J_0}{\Theta_N}\nu^2(\Omega_{\lambda\mathbf{k}}).\quad (19)$$

From the expression (19) it follows that the sound damping has a pronounced maximum in the hypersonic frequency region $\Omega\sim\epsilon_{10}c/s$. An estimate for hematite at room temperature and at

$$\mu H\gg\varepsilon_{10}\left(\frac{\varepsilon_{10}}{T}\right)^{1/2}\frac{\Theta_N}{\Theta}\left(\frac{Mc^2}{\Theta_N}\right)^{1/2}\frac{s}{c}$$

shows that at the maximum, $\Delta\gamma_{ph}\gtrsim\gamma_{ph}^A$, where $\gamma_{ph}^A$ is the phonon-phonon sound damping produced by the Akhiezer mechanism. Thus the relaxation channel being considered will show up in the form of a peak of the damping of hypersound at frequencies $\Omega\sim\epsilon_{10}c/s$ against a background of ordinary phonon-phonon damping.

The contribution to sound damping from three-magnon processes, as determined by the relation (19), differs substantially from the result of the phenomenological calculation, but with respect to the temperature and field dependence and with respect to the frequency dependence. One of the reasons for the incorrectness of the phenomenological method is that taking part in the elementary acts of interaction of quasiparticles are not magnons coupled with phonons, but quasiphonons, having a substantially different energy. Thus this method can have meaning only near an intersection of the magnon and phonon spectra. In a treatment of the damping of magnetoelastic waves in AFEP, however, there are also other reasons for the unsuitability of the phenomenological method, namely: the amplitudes $\psi^\pm_{\lambda,m-ph}(\mathbf{k},\mathbf{q})$ in (18) are linear combinations of the amplitudes $\psi_m^\pm(\mathbf{k},\mathbf{q})$, so that in a calculation of the damping (19), interference of the amplitudes $\psi_m^\pm(\mathbf{k},\mathbf{q})$ is important. At the same time, processes of fusion of magnons of different branches of the spectrum with formation of a magnon of the lower branch, which are described by the second term in (16), are forbidden and make no contribution to the relaxation of magnons of the branch $\epsilon_{1\mathbf{k}}$. Therefore in a phenomenological approach, their contribution to the sound damping can not be taken into account.

## 4. DAMPING OF ULTRASOUND BECAUSE OF EXCHANGE MAGNON-PHONON INTERACTION

Besides the relativistic interaction of phonons with magnons, treated in Secs. 1–3, in magnetic materials there is a magnon-phonon interaction of exchange nature, caused by coupling of the lattice strains with significantly nonuniform oscillations of the magnetic moments. We shall write, with application to AFEP, the part of this interaction that corresponds to processes of fusion of a phonon with magnons, which make the most important contribution to the sound damping:

$$V_{m-ph}^{(ex)}=N^{-1/2}\sum_{\substack{\mathbf{kq}\\ p}}[\psi_{1p}(\mathbf{k},\mathbf{q})c_{1\mathbf{q}}^{+}c_{1\mathbf{q}-\mathbf{k}}b_{p\mathbf{k}}+\psi_{2p}(\mathbf{k},\mathbf{q})c_{2\mathbf{q}}^{+}c_{2\mathbf{q}-\mathbf{k}}b_{p\mathbf{k}}]. \quad (20)$$

We need the values of the amplitudes of the magnon-phonon interaction at large wave vectors of the magnons, $q\gg k\sim a^{-1}\epsilon_{10}/\Theta_N$:

$$\psi_1(\mathbf{k},\mathbf{q})=\psi_2(\mathbf{k},\mathbf{q})=-i2^{1/2}\left(\frac{\Omega_{p\mathbf{k}}}{Mc_p^2}\right)^{1/2}sq\left[\frac{\mathbf{ek}}{k}+\frac{(\mathbf{kq})(\mathbf{eq})}{kq^2}\right]. \quad (21)$$

At large wave vectors of the magnons, the interaction of a sound wave with magnons of different branches gives the same contribution to the sound damping, in contrast to the situation considered in the third section.

At high frequencies, $\Omega\gg\Theta_D/\Theta_N\tau_m$, the sound damping was calculated earlier in the second order of perturbation theory[16] and agrees, with respect to its frequency and temperature variation, with the Landau–Rumer[1] phonon-phonon damping $\gamma_{L\text{-}R}(k)$; the ratio of the dampings $\gamma_{ph}(k)/\gamma_{L\text{-}R}(k)\sim(\Theta_D/\Theta_N)^n\ll 1$ (for transverse sound, $n=6$; for longitudinal, $n=4$). Thus in the high-frequency range, the exchange channel of phonon-magnon relaxation makes no significant contribution to the damping of sound. But at low frequencies, $\Omega\lesssim\Theta_D/\Theta_N\tau_m$, it is necessary to take into account the finite lifetime $\tau_m$ of the intermediate thermal magnons, which is determined by four-magnon processes.[17] Then one can obtain

$$\gamma_{ph}(\Omega)\approx 10^2\frac{\Omega^2}{T}\frac{\Theta_D}{Mc_p^2}.$$

The contribution obtained has a frequency and temperature variation that agrees with the analogous variations of the Akhiezer phonon-phonon damping, and it has the same order of magnitude.

Thus the specific frequency, temperature, and field dependence of the coefficient of sound damping in high-temperature AFEP is due to relativistic phonon-magnon interaction (amplified by inter-sublattice exchange[6]), and not to the phonon-magnon interaction that is caused by the dependence of the exchange integrals on the lattice strains.

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## APPENDIX

Since the energy of the high-activation branch of the spin waves, $\epsilon_{2\mathbf{k}}$, in the wave-vector range $k\lesssim a^{-1}\epsilon_{10}/\Theta_N$ satisfies the condition $\epsilon_{2\mathbf{k}}\gg\epsilon_{1\mathbf{k}}$, only the interaction of phonons with the lower branch $\epsilon_{1\mathbf{k}}$ of the spectrum is important. Then the canonical transformation that diagonalizes the bilinear part of the Hamiltonian (1) gives the following relation of the spin-wave and phonon operators to the creation and annihilation operators of magnetoelastic waves:

$$c_{1\mathbf{k}}^{+}\pm c_{1-\mathbf{k}}=i\left(\frac{\varepsilon_{1\mathbf{k}}}{\Omega_{0\mathbf{k}}}\right)^{\pm 1/2}\nu(\Omega_{0\mathbf{k}})(d_{-\mathbf{k}}\mp d_{\mathbf{k}}^{+})-\sum_{\lambda=1}^{3}\left(\frac{\varepsilon_{1\mathbf{k}}}{\Omega_{\lambda\mathbf{k}}}\right)^{\mp 1/2}\nu(\Omega_{\lambda\mathbf{k}})(d_{\lambda\mathbf{k}}^{+}\pm d_{\lambda-\mathbf{k}}),$$

$$b_{p\mathbf{k}}+b_{p-\mathbf{k}}^{+}=-\left(\frac{\varepsilon_{1\mathbf{k}}}{\Omega_{0\mathbf{k}}}\right)^{1/2}\frac{G_{p\mathbf{k}}\omega_{p\mathbf{k}}}{\omega_{p\mathbf{k}}^2-\Omega_{0\mathbf{k}}^2}\nu(\Omega_{0\mathbf{k}})(d_{\mathbf{k}}-d_{-\mathbf{k}}^{+})$$

$$-i\sum_{\lambda=1}^{3}\left(\frac{\varepsilon_{1\mathbf{k}}}{\Omega_{\lambda\mathbf{k}}}\right)^{1/2}\frac{G_{p\mathbf{k}}\omega_{p\mathbf{k}}}{\omega_{p\mathbf{k}}^2-\Omega_{\lambda\mathbf{k}}^2}\nu(\Omega_{\lambda\mathbf{k}})(d_{\lambda\mathbf{k}}+d_{\lambda-\mathbf{k}}),$$

where

$$\nu(\Omega)=\left[1+\varepsilon_{1\mathbf{k}}\sum_{p}\omega_{p\mathbf{k}}G_{p\mathbf{k}}^2(\omega_{p\mathbf{k}}^2-\Omega^2)^{-2}\right]^{-1/2},$$

$$G_{p\mathbf{k}}=2\,\mathrm{Im}\,D_{1p\mathbf{k}}=\Theta\left(\frac{\omega_{p\mathbf{k}}}{\varepsilon_{1\mathbf{k}}}\frac{J_0}{Mc_p^2}\right)^{1/2}\mathbf{a}\mathbf{e}_p,\quad \mathbf{a}=(\xi n_y+n_z,\ \xi n_x,\ n_x).$$

The energy $\Omega$ of the magnetoelastic waves satisfies the dispersion equation

$$\varepsilon_{1\mathbf{k}}^2-\Omega^2-\sum_{p=1}^{3}G_{p\mathbf{k}}^2\frac{\omega_{p\mathbf{k}}\varepsilon_{1\mathbf{k}}}{\omega_{p\mathbf{k}}^2-\Omega^2}=0.$$

For a hematite crystal, $\epsilon_{1\mathbf{k}}\gg\omega_{p\mathbf{k}}$, there is no intersection of the phonon and magnon branches of the spectrum. Then it is convenient to separate the quasimagnon branch of the spectrum $\Omega_{0\mathbf{k}}\sim\epsilon_{1\mathbf{k}}$ from the quasiphonon branches $\Omega_{\lambda\mathbf{k}}\ll\epsilon_{1\mathbf{k}}$. The latter inequality enables us to introduce unit vectors of polarization of the quasiphonons:

$$\mathbf{e}_{\lambda\mathbf{k}}=\sum_{p}\left(\frac{\varepsilon_{1\mathbf{k}}}{\omega_{p\mathbf{k}}}\right)^{1/2}\frac{G_{p\mathbf{k}}\omega_{p\mathbf{k}}}{\omega_{p\mathbf{k}}^2-\Omega_{\lambda\mathbf{k}}^2}\mathbf{e}_{p\mathbf{k}}.$$

[1]We note that for quasiacoustic modes with $\mathbf{a}'\cdot\mathbf{e}=0$, the renormalization of the amplitude of the effective anharmonicity, determined by Fig. 2 b, is identically zero.

[2]Under the condition $\epsilon_{1\mathbf{k}}\gg\omega_{p\mathbf{k}}$, which is valid for $\alpha$-$Fe_2O_3$, the difference of quasimagnons from "pure" magnons is unimportant and will hereafter be disregarded.

[1]L. D. Landau and G. Rumer, Phys. Z. Sowjetunion 11, 18 (1937) (in German). English translation in: Collected Papers of L. D. Landau (D. ter Haar, ed.), Pergamon Press, 1965, p. 187.
[2]A. I. Akhiezer, Zh. Eksp. Teor. Fiz. 8, 1318 (1938).
[3]M. I. Kaganov and Ya. M. Chikvashvili, Fiz. Tverd. Tela 3, 275 (1961) [Sov. Phys. Solid State 3, 200 (1961)].
[4]B. I. Shklovskiĭ, Fiz. Tverd. Tela 9, 1917 (1967) [Sov. Phys. Solid State 9, 1512 (1968)].
[5]V. S. Lutovinov, O. A. Ol'khov, and S. P. Semin, Zh. Eksp. Teor. Fiz. 72, 2275 (1977) [Sov. Phys. JETP 45, 1196 (1977)].
[6]M. A. Savchenko, Fiz. Tverd. Tela 6, 864 (1964) [Sov. Phys. Solid State 6, 666 (1964)].
[7]M. H. Seavey, Solid State Commun. 10, 219 (1972).
[8]P. P. Maksimenkov and V. I. Ozhogin, Zh. Eksp. Teor. Fiz. 65, 657 (1973) [Sov. Phys. JETP 38, 324 (1974)].
[9]V. I. Ozhogin and V. I. Preobrazhenskiĭ, Zh. Eksp. Teor. Fiz. 73, 988 (1977) [Sov. Phys. JETP 46, 523 (1977)].
[10]S. Simons, Proc. Phys. Soc. Lond. 82, 401 (1963).
[11]L. É. Gurevich and B. I. Shklovskiĭ, a) Fiz. Tverd. Tela 9,

526 (1967) [Sov. Phys. Solid State 9, 401 (1967)]; b) Zh. Eksp. Teor. Fiz. 53, 1726 (1967) [Sov. Phys. JETP 26, 989 (1968).
[12]M. A. Savchenko, Candidate's dissertation, FTI, Kharkov, 1964.
[13]V. G. Bar'yakhtar and V. L. Sobolev, Fiz. Tverd. Tela 15, 2651 (1973) [Sov. Phys. Solid State 15, 1764 (1974)].
[14]Y. Shapira, Phys. Rev. 184, 589 (1969).
[15]A. I. Akhiezer, V. G. Bar'yakhtar, and S. V. Peletminskiĭ, Spinovye volny (Spin Waves), "Nauka", 1967 (translation, North-Holland, 1968).
[16]V. L. Sobolev, Fiz. Tverd. Tela 16, 1238 (1974) [Sov. Phys. Solid State 16, 802 (1974)].
[17]V. G. Bar'yakhtar, V. L. Sobolev, and A. G. Kvirikadze, Zh. Eksp. Teor. Fiz. 65, 790 (1973) [Sov. Phys. JETP 38, 392 (1974)].

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# Investigation of the dynamic NMR frequency shift for $^{57}Fe$ in $FeBO_3$

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NMR of $^{57}Fe$ in $FeBO_3$ is investigated by pulsed magnetic resonance techniques in the 2 to 70 K temperature range. A dynamic NMR frequency shift is observed which is due to coupling between the NMR oscillations and the low-frequency AFMR mode. The magnitude of the shift exceeds only slightly the micro-inhomogeneous NMR-line broadening caused by the spread of the values of hyperfine field at the nuclei. Under these conditions the nuclear spin system possesses a number of unusual properties. Thus, the broadening of the magnetic resonance line is homogeneous, the homogeneous broadening depends on the angle between the nuclear magnetization and the equilibrium axis, and an echo signal is detected which is similar to the "solid echo" in substances with dipole broadening of the magnetic resonance line.

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## 1. INTRODUCTION

Recently, significant progress has been achieved in the investigation of coupled nuclear-electron oscillations and the dynamics of spin motion in magnetically-ordered substances. If sufficiently large coupling exists in these substances between the electron and nuclear spin systems, their magnetic resonance spectra are restructured. The AFMR spectrum acquires a hyperfine spectral gap proportional to the nuclear magnetization, and a dynamic frequency shift (DFS, pulling) arises in the NMR spectrum. Under these conditions, the nuclear spin system possesses a number of unusual properties, such as the existence of nuclear spin waves, a dependence of the nuclear-precession frequency on the precession amplitude, deviations of the nuclei from the equilibrium axis, etc. Previously, DFS has been observed only on $^{55}Mn$ nuclei in weakly anisotropic antiferromagnets.[1,2,3] We have succeeded in observing DFS on the $^{57}Fe$ nuclei in $FeBO_3$ and investigated its properties.

Iron borate is a rhombohedral antiferromagnet with weak ferromagnetism.[4] Its magnetic properties are known quite well. NMR of $^{57}Fe$ in $FeBO_3$ was investigated by the spin-echo technique on polycrystal samples at helium temperatures[5] and on single-crystal samples at nitrogen temperatures.[6]

## 2. SAMPLES AND MEASUREMENT PROCEDURE

The measurements were made on $FeBO_3$ single-crystal plates of thickness ~0.1 mm and area ~9 mm² and parallel to the magnetization easy plane. The single crystals were grown in the Physics Institute, Siberian Division, USSR Academy of Sciences by a method described earlier.[7] Because gluing the $FeBO_3$ single-crystal samples leads to sharp broadening of the low-frequency AFMR line, our samples were mounted without adhesive by securing the samples between two soft foamed plastic plates. The constant magnetic field $H$ and the RF field $h$ were perpendicular to each other in the easy plane of the sample. The principal investigations were carried out in an external magnetic field of 20 Oe. It was shown[6] that in this field the sample can be considered as being in a one-domain state.

The NMR of the $^{57}Fe$ nuclei was investigated by the two-pulse spin-echo technique and from the fall-off of the free induction signal (FI) following a short RF pulse. The experiments were carried out with a Bruker SXP 4-100 pulse spectrometer in the Vichuri Physics Laboratury, Turku University (Finland).

## 3. INVESTIGATION OF THE FREE-INDUCTION SIGNAL

Petrov *et al.*[5] pointed out a number of nonlinear effects in the pulsed NMR of the $^{57}Fe$ nuclei in $FeBO_3$ at helium temperatures, which indirectly indicated the existence of DSF. To determine the dynamic characteristics of the $^{57}Fe$ nuclear spin systems, we carried out an investigation of the FI signal following a short RF pulse of duration $\tau_i = 1\,\mu$sec. In all experiments, the FI signal fall-off was close to exponential with a time constant $T_2^*$. At the same time, the following conditions were

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