Multimode generation in semiconductor lasers

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Sufficiently strong pumping of electrons and holes by an external source can cause population inversion. At a nonzero dipole moment corresponding to interband transition and at coinciding extrema of the conduction and valence bands, photon instability takes place at a frequency $\Omega=\mu_e-\mu_h$, where μ_e and μ_h are the Fermi quasilevels of the electrons and holes. Simultaneous Bose condensation of electron-hole pairs and photons corresponds to a transition of the system to the lasing regime. However, a state with one photon condensate at a frequency Ω, with small electron and photon damping, turns out to be unstable. The system can go over into a state with several photon condensates, corresponding to multimode lasing. The paper deals with a three-mode lasing regime. The separation between the modes turns out to be of the order of the energy gap $2\Delta_0$ in the electron spectrum, and the amplitude of the sideband modes is much less than the amplitude of the fundamental mode. The possibility of transition of the system into a regime with five and more modes is qualitatively considered.

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1. There are presently several theoretical models that describe the processes leading to instability of the single-frequency regime in semiconductor lasers (in particular, multimode lasing, mode switching, and automodulation phenomena). These models can be subdivided[1] into stationary, i.e., those admitting of stable excitations of many modes, and nonstationary, in which multimode generation is connected with radiation pulsations. Different processes that contribute to multimode generation depend on the radiation intensity and come into play at different excesses above the lasing threshold. In strong fields (at an intensity $\approx 10^6 \mathrm{W/cm^2}$) the energy spectrum of the electrons of the semiconductor acquires a gap, the state density past the edge of the gap is increased, and, as shown by Elesin[2] this makes it possible for a second, third, etc. mode to be excited in the laser. In Elesin's model, stable multimode generation is possible at frequencies ω such that $|\omega-\Omega|>2\Delta_0$, where Ω is the frequency of the fundamental mode and $2\Delta_0$ is the value of the gap in the spectrum. Other multimode generation mechanisms, connected with the inhomogeneous broadening of the gain band, the spatial inhomgeneity of the inversion of the nonlinear absorption, etc., were analyzed in[1]. In contrast to[2], these mechanisms are characterized by a lower radiation intensity and by a smaller excess above the lasing threshold, at which the single-mode regime becomes, unstable, and also by a large interval between the different modes.

It was shown earlier[3] that in the absence of electron and photon damping the single-mode generation regime of a semiconductor laser is characterized by the appearance of an instability due to the presence of the gap in the electron spectrum, but via a mechanism that is different from that considered in[2]. If the damping is less than a certain value (see[4]), introduction of one photon Bose condensate (i.e., one generated mode) does not make the system stable. The cause of this behavior is that under nonequilibrium conditions the Bose condensation of photons at one frequency Ω does not lift the phase degeneracy, in contrast to the equilibrium case[5], when the phase of the static charge-density wave (CDW) of the ions becomes fixed on account of the realignment of the crystal structure.

In this paper we investigate the previously considered[5] instability due to the transition of the system to the multimode generation regime. New modes can appear at frequencies ω such that $|\omega-\Omega|<2\Delta_0$ (in contrast to[2]). The onset of new modes at these frequencies was observed experimentally by many workers (see, e.g.,[6]). For simplicity we consider a regime with the minimum number of modes (three, inasmuch as the modes appear in our model in pairs on the two sides of the fundamental mode). The generalization to the case of five and more modes entails considerable computational difficulties, but does not change the results qualitatively.

2. The dispersion equation for the photons in the presence of one photon Bose condensate, obtained in[3], is given by

$$(\varepsilon-\Omega_{\mathbf{k}}-\Sigma_{11}(\mathbf{k},-\varepsilon))(\varepsilon+\Omega_{2\mathbf{k}_0-\mathbf{k}}+\Sigma_{11}(2\mathbf{k}_0-\mathbf{k},\varepsilon)) +\Sigma_{20}(\mathbf{k},-\varepsilon)\Sigma_{20}(2\mathbf{k}_0-\mathbf{k},\varepsilon)=0. \tag{1}$$

Here $\Omega_{\mathbf{k}}=ck-\Omega$; Σ_{11} and Σ_{20} are polarization operators whose explicit form is given in[3]. Equation (1) was solved for $\epsilon\ll 2\Delta_0$ and it was shown that $\mathrm{Im}\,\epsilon\neq 0$ for all $k\neq k_0$; the fact that the imaginary part of the spectrum is double-valued attests to instability of a system with one photon condensate at the frequency Ω. However, Eq. (1) can be solved also for the case $\epsilon\approx 2\Delta_0$. We neglect in this case the dependence of Σ_{11} and Σ_{20} on the wave vector k. As shown in[3], allowance for the dependence of Σ_{11} and Σ_{20} on k leads to the appearance of the terms $v_F(\mathbf{k}-\mathbf{k}_0)$ and $(k-k_0)^2/2m$. At $\mathrm{Re}\,\epsilon\approx c|\mathbf{k}-\mathbf{k}_0|\approx 2\Delta_0$, and $\mathrm{Im}\,\epsilon\ll\mathrm{Re}\,\epsilon$, however, these terms reduce to $2\Delta_0 v_F/c$ and $4\Delta_0^2/2mc^2$, respectively, which we neglect since $v_F\ll c$ and $\Delta_0\ll mc^2$.

For the quantities Σ_{11} and Σ_{20} contained in (1) we can obtain several useful relations. We write down explic-

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itly $\Sigma_{11}(\mathbf{k}_0, \epsilon)$ and $\Sigma_{20}(\mathbf{k}_0, \epsilon)$:

$$\Sigma_{11}(\mathbf{k}_0, \varepsilon) = M^2 N(0) \int_0^{\tilde\infty} \frac{4(2\xi^2+\Delta_0^2)\,d\xi}{(\xi^2+\Delta_0^2)^{1/2}(\varepsilon^2-4\xi^2-4\Delta_0^2)},$$
$$\Sigma_{20}(\mathbf{k}_0, \varepsilon) = -M^2 N(0) \int_0^{\tilde\infty} \frac{4\Delta_0^2 d\xi}{(\xi^2+\Delta_0^2)^{1/2}(\varepsilon^2-4\xi^2-4\Delta_0^2)}. \quad (2)$$

We introduce the notation

$$I = \int_0^{\tilde\infty} \frac{4d\xi}{(\xi^2+\Delta_0^2)^{1/2}(\varepsilon^2-4\xi^2-4\Delta_0^2)}, \quad (3)$$

with the aid of which we can conveniently write

$$(\Sigma_{20}-\Sigma_{11}-\Omega_{\mathbf{k}_0}) = 2M^2N(0)I\varepsilon^2/4, \quad (\Sigma_{20}+\Sigma_{11}+\Omega_{\mathbf{k}_0}) = 2M^2N(0)I(\Delta_0^2-\varepsilon^2/4). \quad (4)$$

$M = ev_{cv}(2\pi/\omega_{\mathbf{k}_0})^{1/2}$ is the electron-photon interaction constant and $N(0)$ is the state density on the Fermi level. At $\epsilon < 2\Delta_0$ the integral (3) can be calculated in explicit form:

$$I = 4\,\mathrm{arctg}[\varepsilon/(4\Delta_0^2-\varepsilon^2)^{1/2}]/\varepsilon(4\Delta_0^2-\varepsilon^2)^{1/2}. \quad (5)$$

Using relations (4) and (5), we reduce (1) to the form

$$\varepsilon^2-2c(k-k_0)\varepsilon+c^2(k-k_0)^2 = -M^4N^2(0)I^2\varepsilon^2(4\Delta_0^2-\varepsilon^2)/4 \quad (6)$$

or, substituting (5), to the form

$$\varepsilon = c(k-k_0) \pm 2iM^2N(0)\,\mathrm{arctg}[\varepsilon/(4\Delta_0^2-\varepsilon^2)^{1/2}]. \quad (7)$$

In the limit $|\epsilon| \ll 2\Delta_0$ we easily obtain from (7) the results of[3]:

$$\varepsilon = \frac{c(k-k_0) \pm ib^{1/2}c|k-k_0|}{1+b}, \quad (8)$$

where $b = M^4N^2(0)/\Delta_0^2$.

We consider Eq. (7) at $\mathrm{Re}\epsilon \approx c|k-k_0| \approx 2\Delta_0$ and $\mathrm{Im}\epsilon \ll \mathrm{Re}\epsilon$. It is then easy to show that, accurate to terms proportional to $b^{3/4}$, we have

$$|\mathrm{Im}\,\varepsilon| = \pi b^{1/2}\Delta_0. \quad (9)$$

To find the wave vector $\mathbf{k}_1$ at which $\mathrm{Im}\epsilon$ reaches a maximum, it is necessary to take into account terms of higher order in $b^{1/2}$. Simple calculations yield:

$$c|\mathbf{k}_1-\mathbf{k}_0| = 2\Delta_0[1-(\pi^{1/2}b^{1/2})/2^{1/2}]. \quad (10)$$

Using (9) and (10), we can obtain the important relation:

$$2\Delta_0 - c|\mathbf{k}_1-\mathbf{k}_0| = 2^{1/2}(\mathrm{Im}\,\varepsilon)^{1/2}/\pi^{1/2}\Delta_0^{1/2}. \quad (11)$$

It will be shown in Sec. 3 that this relation enables us to obtain the solution of the self-consistency equation for the order parameters that describe the appearance of the sideband radiation modes and the corresponding CDW of the electrons. The presence of three order parameters in the system corresponds to formation of photon Bose condensates at the frequencies Ω and $\Omega \pm c|k_1-k_0|$. But the system of three Bose condensate will also apparently be unstable in the absence of damping, i.e., two more sideband modes well produced, etc. We confine ourselves to allowance for the first pair of sideband modes. This is correct, strictly speaking, at dampings γ such that $b\Delta_0 < \gamma < b^{1/2}\Delta_0$. But since $b \ll 1$, it follows that the higher-order modes will make a small contribution even at lower values of the damping, proportional to $b^{n/2}$ (n is the number of the mode), in the expression for the amplitudes of the fundamental and first two sideband modes, so that our results remain in force in this case.

3. We write the Hamiltonian of the investigated in the usual form

$$H = \sum_{\mathbf{p}} \varepsilon(\mathbf{p})(a_{c\mathbf{p}}^+a_{c\mathbf{p}} - a_{v\mathbf{p}}^+a_{v\mathbf{p}}) + \sum_{\mathbf{k}} \omega_{\mathbf{k}}c_{\mathbf{k}}^+c_{\mathbf{k}}$$
$$+\Big\{\sum_{\mathbf{p},\mathbf{k}} M_{\mathbf{k}}a_{c\mathbf{p}}^+a_{v\mathbf{p}-\mathbf{k}}c_{\mathbf{k}}\exp(-i\nu_{\mathbf{k}}t) + \text{c.c.}\Big\}; \quad (12)$$
$$\mathbf{k} = \mathbf{k}_0,\ \mathbf{k}_0 \pm \Delta\mathbf{k}.$$

Here $\epsilon(\mathbf{p}) = p^2/2m + E_g/2$, $M_{\mathbf{k}} = ev_{cv}(2\pi/\omega_{\mathbf{k}})^{1/2}$; $\omega_{\mathbf{k}} = ck$; $a_{c\mathbf{p}}$ and $a_{v\mathbf{p}}$ are the electron-annihilation operators in the conduction and valence band, and $c_{\mathbf{k}}$ is the photon-annihilation operator. We do not take into account in (12) the Coulomb interaction of the electrons and holes, of the density–density type, which renormalizes the interaction constant but does not alter qualitatively the results;

$$\nu_{\mathbf{k}_0} = \Omega, \quad \nu_{\mathbf{k}_0\pm\Delta\mathbf{k}} = \Omega \pm c\Delta k, \quad \Delta k = |\mathbf{k}_1-\mathbf{k}_0|, \quad \mathbf{k}_1 \| \mathbf{k}_0.$$

Usually in the investigation of the single-mode regime the Hamiltonian (12) is subjected to the unitary transformation[7]

$$U(t) = \exp\Big\{i\frac{\Omega t}{2}\sum_{\mathbf{p}}(a_{c\mathbf{p}}^+a_{c\mathbf{p}} - a_{v\mathbf{p}}^+a_{v\mathbf{p}}) + i\Omega t\sum_{\mathbf{k}} c_{\mathbf{k}}^+c_{\mathbf{k}}\Big\}, \quad (13)$$

which makes it possible to eliminate subsequently the explicit dependence on the time. For the multimode case, the transformation (13) does not eliminate the explicit time dependence. It is nevertheless convenient to change over with the aid of (13) to a representation in which the photon frequency is reckoned from the frequency Ω of the fundamental mode. The Hamiltonian (12) takes in this case the form

$$H(t) = \sum_{\mathbf{p}} \xi(\mathbf{p})(a_{c\mathbf{p}}^+a_{c\mathbf{p}} - a_{v\mathbf{p}}^+a_{v\mathbf{p}}) + \sum_{\mathbf{k}} \Omega_{\mathbf{k}}c_{\mathbf{k}}^+c_{\mathbf{k}}$$
$$+\Big\{\sum_{\mathbf{p},\mathbf{k}} M_{\mathbf{k}}a_{c\mathbf{p}}^+a_{v\mathbf{p}-\mathbf{k}}c_{\mathbf{k}}\exp(-i(\nu_{\mathbf{k}}-\Omega)t) + \text{c.c.}\Big\}; \quad (14)$$
$$\xi(\mathbf{p}) = \varepsilon(\mathbf{p}) - \Omega/2, \quad \Omega_{\mathbf{k}} = \omega_{\mathbf{k}} - \Omega, \quad \mathbf{k} = \mathbf{k}_0, \mathbf{k}_0 \pm \Delta\mathbf{k}.$$

To investigate the system (14) we shall need the following Green's functions, which are introduced in the usual manner:

$$G_c(\mathbf{p},\mathbf{p}',t,t') = -i\langle Ta_{c\mathbf{p}}(t)a_{c\mathbf{p}'}^+(t')\rangle,$$
$$G_{vc}(\mathbf{p},\mathbf{p}',t,t') = -i\langle Ta_{v\mathbf{p}}(t)a_{c\mathbf{p}'}^+(t')\rangle. \quad (15)$$

The determination of the Green's functions (15) in the case of several radiation modes leads to an infinite coupled system of equations. Besides the anomalous functions that correspond to the appearance of CDW with wave vectors $\mathbf{k}_0$ and $\mathbf{k}_0 \pm \Delta\mathbf{k}$, there appear anomalous functions corresponding to CDW with wave vectors $\mathbf{k}_0 \pm m\Delta\mathbf{k}, m > 1$. We assume that all the modes except the fundamental one and two sideband modes are suppressed. Then at $b \ll 1$ it suffices to determine the Green's functions with $\mathbf{k}_0$ and $\mathbf{k}_0 \pm \Delta\mathbf{k}$ and neglect all others. The system of equations turns out to be closed and to admit of an analytic solutions.

We seek the Green's functions in the form

$$\begin{gathered}
G_c(\mathbf{p},\mathbf{p},t,t') = G_c(\mathbf{p},t-t'),\quad G_{vc}(\mathbf{p},\mathbf{p}+\mathbf{k}_0,t,t') = G_{vc}(\mathbf{p},\mathbf{p}+\mathbf{k}_0,t-t'),\\
G_{vc}(\mathbf{p}+\Delta\mathbf{k},\mathbf{p}+\mathbf{k}_0,t,t') = G_{vc}^{-}(\mathbf{p}+\Delta\mathbf{k},\mathbf{p}+\mathbf{k}_0,t-t')\exp[-i\Sigma(t+t')/2],\\
G_{vc}(\mathbf{p}-\Delta\mathbf{k},\mathbf{p}+\mathbf{k}_0,t,t') = G_{vc}^{+}(\mathbf{p}-\Delta\mathbf{k},\mathbf{p}+\mathbf{k}_0,t-t')\exp[i\Sigma(t+t')/2],\\
G_c(\mathbf{p}+\mathbf{k}_0+\Delta\mathbf{k},\mathbf{p}+\mathbf{k}_0,t,t') = G_c^{-}(\mathbf{p}+\mathbf{k}_0+\Delta\mathbf{k},\mathbf{p}+\mathbf{k}_0,t-t')\exp[-i\Sigma(t+t')/2].\\
G_c(\mathbf{p}+\mathbf{k}_0-\Delta\mathbf{k},\mathbf{p}+\mathbf{k}_0,t,t') = G_c^{+}(\mathbf{p}+\mathbf{k}_0-\Delta\mathbf{k},\mathbf{p}+\mathbf{k}_0,t-t')\exp[i\Sigma(t+t')/2].
\end{gathered} \tag{16}$$

We recognize furthermore that

$$\begin{gathered}
\langle c_{\mathbf{k}_0+\Delta\mathbf{k}}(t)\rangle = \langle c_{\mathbf{k}_0+\Delta\mathbf{k}}\rangle e^{-i\Sigma t},\\
\langle c_{\mathbf{k}_0-\Delta\mathbf{k}}(t)\rangle = \langle c_{\mathbf{k}_0-\Delta\mathbf{k}}\rangle e^{i\Sigma t},\quad \Sigma = c\Delta k.
\end{gathered} \tag{17}$$

The system of equations for the Fourier components of the functions (15) can be written, with (16) and (17) taken into account, in the form

$$\begin{gathered}
(\omega+\Sigma+\xi)G_{vc}^{-}(\omega+\Sigma/2) = \Delta_-^{*}G_c(\omega)+\Delta_0^{*}G_c^{-}(\omega+\Sigma/2),\\
(\omega+\xi-\Sigma)G_{vc}^{+}(\omega-\Sigma/2) = \Delta_+^{*}G_c(\omega)+\Delta_0^{*}G_c^{+}(\omega-\Sigma/2),\\
(\omega-\xi+\Sigma)G_c^{-}(\omega+\Sigma/2) = \Delta_0G_{vc}^{-}(\omega+\Sigma/2)+\Delta_+G_{vc}(\omega),\\
(\omega-\xi-\Sigma)G_c^{+}(\omega-\Sigma/2) = \Delta_0G_{vc}^{+}(\omega-\Sigma/2)+\Delta_-G_{vc}(\omega),\\
(\omega+\xi)G_{vc}(\omega) = \Delta_0^{*}G_c(\omega)+\Delta_+^{*}G_c^{-}(\omega+\Sigma/2)+\Delta_-^{*}G_c^{+}(\omega-\Sigma/2),\\
(\omega-\xi)G_c(\omega) = 1+\Delta_0G_{vc}(\omega)+\Delta_+G_{vc}^{+}(\omega-\Sigma/2)+\Delta_-G_{vc}^{-}(\omega+\Sigma/2),\\
\Delta_\pm = -i\sum_{p,\omega}(M_{\mathbf{k}_0}^2/\Omega_{\mathbf{k}_0})G_{vc}^{\pm}(\omega),\quad \Delta_0 = -i\sum_{p,\omega}(M_{\mathbf{k}_0}^2/\Omega_{\mathbf{k}_0})G_{vc}(\omega).
\end{gathered} \tag{18}$$

We have neglected here the dependence of the matrix elements $M_{\mathbf{k}_0\pm\Delta\mathbf{k}}$ of the interaction on $\Delta\mathbf{k}$, and have set them equal to $M_{\mathbf{k}_0}$.

To find the electron spectrum we write out in explicit form the function $G_c(\omega)$:

$$\begin{gathered}
G_c(\omega) = \frac{\omega+\xi-\Psi(-\Sigma,-\xi)}{[\omega-\xi-\Psi(\Sigma,\xi)][\omega+\xi-\Psi(-\Sigma,-\xi)]-|\tilde{\Delta}|^2}\\
\Psi(\Sigma,\xi) = |\Delta_-|^2\frac{\omega+\Sigma-\xi}{(\omega+\Sigma)^2-\xi^2-|\Delta_0|^2}+|\Delta_+|^2\frac{\omega-\Sigma-\xi}{(\omega-\Sigma)^2-\xi^2-|\Delta_0|^2}\\
\tilde{\Delta} = \Delta_0^{*}\Delta_+\Delta_-\left[\frac{1}{(\omega+\Sigma)^2-\xi^2-|\Delta_0|^2}+\frac{1}{(\omega-\Sigma)^2-\xi^2-|\Delta_0|^2}\right]+\Delta_0^{*}.
\end{gathered} \tag{19}$$

We assume furthermore that the order parameters Δ_0, Δ_+, and Δ_- are real, and furthermore that $\Delta_+ = \Delta_- = \Delta$; equating next to zero the denominator of the function $G_c(\omega)$ and solving the dispersion equation at $\Delta \ll \Delta_0$, we can obtain three groups of roots corresponding to the different branches of the electron spectrum:

$$\begin{gathered}
\omega_1 \approx \pm[\Sigma+(\xi^2+\Delta_0^2)^{1/2}]+\Delta^2/[\Sigma+(\xi^2+\Delta_0^2)^{1/2}],\\
\omega_2 \approx \pm\{\Sigma/2+([\Sigma/2-(\xi^2+\Delta_0^2)^{1/2}]^2+\Delta^2)^{1/2}\},\\
\omega_3 \approx \pm\{\Sigma/2-([\Sigma/2-(\xi^2+\Delta_0^2)^{1/2}]^2+\Delta^2)^{1/2}\}.
\end{gathered} \tag{20}$$

The last equation for ω_3 is valid for all ξ except those located near the point $\xi' = (\Sigma^2-\Delta_0^2)^{1/2}$. At $\xi \approx \xi'$ we have

$$\omega_3 \approx \pm([\Sigma-(\xi^2+\Delta_0^2)^{1/2}]^2+\Delta^4/\Sigma^2). \tag{21}$$

We note that allowance for the anomalous Green's functions that describe CDW with momenta $\mathbf{k}_0 \pm m\Delta\mathbf{k}$, $m > 1$ adds to ω_1, ω_2, and ω_3 corrections proportional to $(\Delta/\Sigma)^{2(m-1)}$. Besides the roots (20) there appear also $2(m-1)$ groups of roots, but these make a small contribution compared with ω_1 and $\omega_{2,3}$ when the self-consistency equations are solved. It can also be demonstrated that, in accordance with the rule for going around the poles of the Green's function, that when the self-consistency equation for Δ is solved we can take ω_3 in the form (20) also at $\xi \approx \xi'$, and the resultant error is proportional to $(\Delta/\Sigma)^2$.

To find the order parameter Δ we write out explicitly, for example, $G_{vc}^{-}(\omega)$:

$$\begin{aligned}
G_{vc}^{-}(\omega) = {}&\Delta\left[\left(\omega-\xi+\frac{\Sigma}{2}\right)\left(\omega+\xi-\frac{\Sigma}{2}\right)+\Delta_0^2\right]\\
&\times\varphi\left(\omega-\frac{3\Sigma}{2}\right)\Big/\operatorname{Det}\left(\omega-\frac{\Sigma}{2}\right)\\
&-\Delta^3\left\{1+\left[\left(\omega-\xi+\frac{\Sigma}{2}\right)\left(\omega+\xi-\frac{3\Sigma}{2}\right)-\Delta_0^2/\varphi\left(\omega-\frac{3\Sigma}{2}\right)\right]\right\}\\
&\times\varphi\left(\omega-\frac{3\Sigma}{2}\right)\Big/\operatorname{Det}\left(\omega-\frac{\Sigma}{2}\right),
\end{aligned} \tag{22}$$

where

$$\begin{gathered}
\varphi(\omega) = \omega^2-\xi^2-\Delta_0^2,\\
\operatorname{Det}(\omega) = (\omega^2-\omega_1^2)(\omega^2-\omega_2^2)(\omega^2-\omega_3^2).
\end{gathered} \tag{23}$$

Substituting (22) in the self-consistency equation for Δ, taking (11) into account, and performing cumbersome calculations we obtain, accurate to terms $(\Delta/\Sigma)^2$,

$$\Delta \approx \Sigma_{20}B^2/2,\quad B \approx 2.5 \approx (2\pi)^{1/2}. \tag{24}$$

We estimated the distance between modes and the amplitudes of the sideband modes under the assumption that $b \ll 1$. In real lasers usually $b \gtrsim 1$, although in principle the parameter b can also be small. It is seen from (10) that the distance between modes decreasing with increasing b, and the amplitude of the sideband modes increases. At $b \gtrsim 1$ the distance between modes can become much smaller than $2\Delta_0$, as is in fact observed in experiment.[8] A quantitative analysis of the system at $b \gtrsim 1$ is mathematically quite complicated, for in this case Δ/Σ cannot be regarded as a small parameter and the system (18) can not be used.

In the derivation of the system (18) we have assumed that all the higher harmonics ($\mathbf{k} = \mathbf{k}_0 \pm m\Delta\mathbf{k}$) are suppressed by damping. If this condition is not satisfied, then the system (18) must, strictly speaking, be supplemented also by equations for the anomalous functions corresponding to CDW with wave vectors $\mathbf{k}_0 \pm m\Delta\mathbf{k}$. It can be shown, however, that the contribution of the m-th harmonic to the basic system (18) results in a correction of order b^m, i.e., our results remain valid at $b \ll 1$, with the indicated accuracy. The higher harmonics of order m can be completely disregarded of the damping in the system is $\gamma_m > b^{m/2}\Delta_0$ (here $\Delta_m \sim b^{m/2}\Delta_0$ is the amplitude of the m-th harmonic).

The three-mode regime considered by us is stationary. However, an investigation of the limits and conditions of its stability is fraught with considerable mathematical difficulties even at $b \ll 1$. It appears that such a regime will be unstable, at least at small values of the damping. New modes can appear at a distance $\lesssim 2\Delta$

from the first sideband mode. Since the ratio Σ_{20}/Δ is by no means small, it is quite probable the amplitude of the new mode will be of the same order as that of the first sideband mode. In the case $b \gtrsim 1$ we can expect an entire group of modes to appear, with small distances between the modes and with amplitudes of comparable magnitude.

Let us discuss in greater detail the criterion for the appearance of the laser mode, $\gamma/\Delta_0 < 1$. For simplicity we confine ourselves to the single-mode regime. By γ is customarily meant the summary damping due to recombination and to electron-electron and electron-phonon collisions.[7] We shall show that the criterion indicated above is a necessary but generally speaking not a sufficient condition for the onset of generation.

It is known that the presence of instability of the photon subsystem to Bose condensation in a state with frequency Ω can be revealed by the appearance of the imaginary pole $i\Delta_0$ in the photon Green's function. In exactly the same manner, the presence of instability of the electron-hole subsystem to Bose condensation of excitons is attested by the appearance of an imaginary pole in the electron Green's function.

Assume that the criterion $\gamma/\Delta_0 < 1$ is satisfied and that the damping in the electron Green's function is negligibly small (here $\Delta_0 = \tilde{\omega}\exp(-\Omega_{\mathbf{k}_0}/M^2N(0))$. In the photon Green's function we take into account the damping due to the losses in the resonator:

$$D(\mathbf{k}_0, \omega) = (\omega - \Omega_{\mathbf{k}_0} - i\gamma_{\text{res}} - \pi_{12}(\mathbf{k}_0, \omega))^{-1}. \tag{25}$$

Here $\gamma_{\text{res}} = \Omega/Q$, Q is the figure of merit of the resonator,

$$\pi_{12}(\mathbf{k}_0, \omega) = iM^2N(0)\int G_c^0(\varepsilon, \mathbf{p})\, G_v^0(\varepsilon-\omega, \mathbf{p}-\mathbf{k}_0)\, d\varepsilon\, d\mathbf{p}, \tag{26}$$

$G_c^0(\epsilon, \mathbf{p})$ and $G_v^0(\epsilon, \mathbf{p})$ are the Green's functions in the absence of electron-hole pairing (the Coulomb interaction that can lead to electron-hole pairing will be disregarded for simplicity). The equation for the pole (25) is easily obtained after calculating (26):

$$-\omega + \Omega_{\mathbf{k}_0} + i\gamma_{\text{res}} = -M^2N(0)\left[\ln\frac{|\omega|}{\tilde{\omega}} + i\varphi - i\frac{\pi}{2}\right]. \tag{27}$$

From this we obtain readily

$$\omega = |\omega| e^{i\varphi}, \quad |\omega| = \Delta_0, \quad \varphi = \pi/2 - \gamma_{\text{res}}/M^2N(0). \tag{28}$$

If $\gamma_{\text{res}} > M^2N(0)\pi/2$, then the imaginary part of the pole of the Green's function reverses sign and the photon system remains stable, i.e., no lasing occurs in this case.

In the presence of Coulomb interaction and damping in the electron Green's function $\gamma < \Delta_{\text{Coul}}$ ($\Delta_{\text{Coul}} = \tilde{\omega} \times \exp(-1/gN(0))$ and g is the Coulomb-interaction constant), Bose condensation of the excitons can set in. In the photon Green's function at $\gamma_{\text{res}} > M^2N(0)\pi/2$, however, the pole remains as before in the lower half-plane. Thus, the presence of an exciton Bose condensate does not lead, generally speaking to the appearance of a photon Bose condensate. The reason is that the excitons can condense into a state with zero momentum, but the photons can condense in the single-mode regime only into a state with momentum $\mathbf{k}_0$. At $\gamma_{\text{res}} > M^2N(0)\pi/2$ the photon mode with momentum $\mathbf{k}_0$ is suppressed, and the exciton Bose condensate with zero momentum is conserved.

In real lasers at $\Delta \sim (10^{11} - 10^{13})\,\text{sec}^{-1}$ there should be satisfied the relation

$$Q\Omega_{\mathbf{k}_0}/\Omega \gg 1, \tag{29}$$

at which $\varphi \approx \pi/2$. The condition (29) is as a rule satisfied, so that the indicated possible existence of an exciton condensate in a nonequilibrium system without the appearance of a photon condensate can be realized only by a special choice of the laser parameters.

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