Two-level system in a sinusoidal electric field

N. E. Nevesskiĭ

Institute of Chemical Physics, USSR Academy of Sciences, Moscow
(Submitted 26 August 1977)
Zh. Eksp. Teor. Fiz. 74, 1569–1578 (May 1978)

The behavior of a two-level system in a continuous sinusoidal electric field of arbitrary amplitude and frequency is investigated. The exact solution of the initial equations is obtained at moments $t_N = 2\pi N/\omega$. The quasienergy spectrum is found and the probabilities of many-photon transitions are calculated. The excitation of the upper lever near many-photon resonances is considered. A study is made of the behavior of a two-level system in a pulsed field and in the case of two alternately acting pulsed fields. It is shown that for certain values of the parameters characterizing these fields the probability of excitation of the upper level may reach unity even in weak and nonresonant fields.

PACS numbers: 41.10.Hv, 42.65.Bp

INTRODUCTION

The very first theoretical investigations of the behavior of an atom in a strong electromagnetic field have revealed that, firstly, the problem is complex even in the simplest case of a two-level system used simulating a real atom and, secondly, the solution of this problem opens up possibilities of describing basically new phenomena associated with many-photon absorption.[1]

The phenomenon of many-photon absorption by a two-level system has been studied primarily by the methods of the resonance perturbation theory.[2,3] However, in spite of the fact that this theory imposes serious restrictions on the amplitude of the external field, it is not free of basic difficulties.[4] The problem has been solved for moderately strong fields by a different method avoiding the difficulties of the resonance perturbation theory.[4,5] In this way probabilities have been found of many-photon transitions and values have been calculated of the quasienergy, governing the spectrum of the reemitted energy under resonance and nonresonance conditions.

The case of a strong field when the perturbation theory is known to be inapplicable was considered by Melikyan,[6] who obtained the quasienergy spectrum without imposing restrictions on the external field intensity but did not determine the transition probabilities. The same case was considered by Zaretskiĭ and Kraĭnov[7,8] subject to an additional condition $\omega \ll \omega_{12}$. The adiabatic perturbation theory was used to determine the quasienergy spectrum identical with that obtained by Melikyan[6] and the probabilities of many-photon transitions near resonances in the limit of weak fields, which were found to be identical with the probabilities deduced from the resonance perturbation theory.[3]

In addition to these investigations, in which the results were obtained by the analytic method, there have been several numerical calculations carried out on a computer. This applies particularly to the work of Shirley[9] who carried out a machine calculation of the average probabilities of many-photon transitions in strong fields, and also to investigations of the behavior of a two-level system in a field of frequency ω close to the natural frequency of the system ω_{12}.[10,11]

The present paper is concerned with the same problem. In the first section we shall give the exact solution (at discrete moments in time) of the equations describing the interaction of a two-level system with a sinusoidal electric field. In the second section we shall discuss the behavior of a two-level system in the field of a modulated wave and we shall show that, under certain conditions, the probability of a many-photon electron transition may reach unity even in weak and nonresonant external fields.

§ 1. INVESTIGATIONS OF EQUATIONS DESCRIBING THE BEHAVIOR OF A TWO-LEVEL SYSTEM IN A SINUSOIDAL ELECTRIC FIELD

The initial system of equations is

$$\begin{aligned} i\dot{\alpha}_2 &= \varepsilon_2\alpha_2 + \alpha_1 d_{12}^* E_0 \sin \omega t, \\ i\dot{\alpha}_1 &= \varepsilon_1\alpha_1 + \alpha_2 d_{12} E_0 \sin \omega t, \end{aligned} \tag{1}$$

where α_1 and α_2 are the amplitudes of the probability of finding an electron at the first and second levels, respectively; E_0 is the amplitude of the external field; ϵ_1 and ϵ_2 are the energies of the lower and upper levels; d_{12} is the dipole matrix element.

A change of variables reduces the system (1) to the dimensionless integral form

$$\begin{aligned} x_2(\xi) &= x_2(\xi_0) - iq^* \int_{\xi_0}^{\xi} \sin \xi_1 \exp[i\lambda\xi_1] x_1(\xi_1)\, d\xi_1, \\ x_1(\xi) &= x_1(\xi_0) - iq \int_{\xi_0}^{\xi} \sin \xi_1 \exp[-i\lambda\xi_1] x_2(\xi_1)\, d\xi_1, \end{aligned} \tag{2}$$

where the following notation is used:

$$\xi = \omega t, \quad \lambda = (\varepsilon_2 - \varepsilon_1)/\omega, \quad q = E_0 d_{12}/\omega, \quad x_j(\xi) = \alpha_j(\xi) \exp[i\varepsilon_j \xi/\omega]. \tag{3}$$

The application of successive iterations to these two integral relationships readily yields the formal result:

$$\mathbf{x}(\xi)=\hat{M}(\xi,\xi_0)\mathbf{x}(\xi_0),\tag{4}$$

$$\mathbf{x}(\xi)=\begin{pmatrix}x_2(\xi)\\ x_1(\xi)\end{pmatrix},\quad \hat{M}(\xi,\xi_0)=\begin{pmatrix}M_{11} & M_{12}^*\\ -M_{12} & M_{11}^*\end{pmatrix}.\tag{5}$$

Elements of the matrix $\hat{M}(\xi,\xi_0)$ can be expressed in terms of the integral series

$$\begin{aligned}M_{11}(\xi,\xi_0)&=1-|q|^2\int_{\xi_0}^{\xi}d\xi_1\exp[i\lambda\xi_1]\sin\xi_1\int_{\xi_0}^{\xi_1}d\xi_2\exp[-i\lambda\xi_2]\sin\xi_2+\ldots,\\ M_{12}(\xi,\xi_0)&=-\frac{iq^*}{|q|}\Big\{|q|\int_{\xi_0}^{\xi}d\xi_1\exp[i\lambda\xi_1]\sin\xi_1-|q|^3\int_{\xi_0}^{\xi}d\xi_1\\ &\times\sin\xi_1\exp[i\lambda\xi_1]\int_{\xi_0}^{\xi_1}d\xi_2\exp[-i\lambda\xi_2]\sin\xi_2\int_{\xi_0}^{\xi_2}d\xi_3\exp[i\lambda\xi_3]\sin\xi_3+\ldots\Big\}.\end{aligned}\tag{6}$$

It follows from the definition (4) that the matrix $\hat{M}(\xi,\xi_0)$ has the following property:

$$\hat{M}(\xi_1+\xi_2,\xi_0)=\hat{M}(\xi_1+\xi_2,\xi_1)\hat{M}(\xi_1,\xi_0).\tag{7}$$

If we take the initial moment to be $\xi_0=0$ and the running time as $\xi=2\pi N$, where N is an integer, we obtain the following expression for $\mathbf{x}(2\pi N)$:

$$\mathbf{x}(2\pi N)=\prod_{k=N}^{k=1}\hat{M}(2\pi k,2\pi(k-1))\mathbf{x}(0).\tag{8}$$

Direct calculations readily show that

$$\begin{aligned}M_{11}(2\pi k,2\pi(k-1))&=M_{11}(2\pi,0),\\ M_{12}(2\pi k,2\pi(k-1))&=M_{12}(2\pi,0)e^{2\pi i\lambda(k-1)}.\end{aligned}\tag{9}$$

Introducing the matrix $\hat{U}_k$

$$\hat{U}_k=\begin{pmatrix}e^{i\lambda\pi k} & 0\\ 0 & e^{-i\lambda\pi k}\end{pmatrix},\tag{10}$$

we obtain

$$\hat{M}(2\pi k,2\pi(k-1))=\hat{U}_{k-1}\hat{M}(2\pi,0)\hat{U}_{k-1}^+.\tag{11}$$

Substituting Eq. (11) into Eq. (8), we find that

$$\mathbf{x}(2\pi N)=\hat{U}_N[\hat{U}_1^+\hat{M}(2\pi,0)]^N\mathbf{x}(0),\tag{12}$$

and, consequently,

$$\alpha(2\pi N)=\hat{Q}(2\pi N)\hat{U}_N[U_1^+\hat{M}(2\pi,0)]^N\alpha(0);\tag{13}$$

$$\hat{Q}(\xi)=\begin{pmatrix}\exp(-i\varepsilon_2\xi/\omega) & 0\\ 0 & \exp(-i\varepsilon_1\xi/\omega)\end{pmatrix}.\tag{14}$$

Elements of the matrix $\hat{M}(2\pi,0)$, which we shall simply call $\hat{M}(q)$, contain all the information on the action of an external field on a two-level system. We shall write them in the form

$$M_{11}(2\pi,0)=\mathcal{H}_\lambda(q),\quad M_{12}=(2\pi,0)=-iq^*\mathcal{A}_\lambda(q)/|q|.\tag{15}$$

The functions $\mathcal{H}_\lambda(q)$ and $\mathcal{A}_\lambda(q)$ represent, according to Eq. (6), series of multiple integrals

$$\begin{aligned}\mathcal{H}_\lambda(q)&=1-|q|^2\int_0^{2\pi}d\xi_1\exp[i\lambda\xi_1]\sin\xi_1\int_0^{\xi_1}d\xi_2\exp[-i\lambda\xi_2]\sin\xi_2+\ldots,\\ \mathcal{A}_\lambda(q)&=|q|\int_0^{2\pi}d\xi_1\exp[i\lambda\xi_1]\sin\xi_1-|q|^3\int_0^{2\pi}d\xi_1\exp[i\lambda\xi_1]\sin\xi_1\cdot\\ &\int_0^{\xi_1}d\xi_2\exp[-i\lambda\xi_2]\sin\xi_2\int_0^{\xi_2}d\xi_3\exp[i\lambda\xi_3]\sin\xi_3+\ldots\end{aligned}\tag{16}$$

In view of the fact that the matrix $\hat{M}(q)$ is unitary, we have

$$|\mathcal{H}_\lambda(q)|^2+|\mathcal{A}_\lambda(q)|^2=1.\tag{17}$$

Let a two-level system be in the ground state at $t=0$. Taking the matrix $\hat{U}_1^+\hat{M}(q)$ to the power N, we find that $\alpha(2\pi N)$ becomes

$$\alpha(2\pi N)=\hat{Q}(2\pi N)\hat{U}_N\frac{\sin N\varphi}{\sin\varphi}\begin{pmatrix}-\dfrac{iq^*}{|q|}\mathcal{A}_\lambda e^{-i\lambda\pi}\\ -\mathcal{H}_\lambda e^{-i\lambda\pi}+\dfrac{\sin(N+1)\varphi}{\sin N\varphi}\end{pmatrix},\tag{18}$$

$$\varphi=\arccos[\mathrm{Re}(\mathcal{H}_\lambda e^{-i\lambda\pi})].\tag{19}$$

Since the operator $\hat{Q}(2\pi N)\hat{U}_N$ alters only the phases of the probability amplitudes α_1 and α_2 but does not affect their moduli, it follows that on the basis of Eq. (16) we can find the probability of an electron transition to the upper level up to a time $t_N=2\pi N/\omega$:

$$P_N=\frac{|\mathcal{A}_\lambda|^2}{|\mathcal{A}_\lambda|^2+\mathrm{Im}^2(\mathcal{H}_\lambda e^{-i\lambda\pi})}\sin^2\left\{\frac{\omega t_N}{2\pi}\arcsin\left(|\mathcal{A}_\lambda|^2+\mathrm{Im}^2(\mathcal{H}_\lambda e^{-i\lambda\pi})\right)^{1/2}\right\}\tag{20}$$

Thus, the probability of a λ-photon transition oscillates in time at a frequency

$$\Omega_\lambda=\frac{\omega}{2\pi}\arcsin[|\mathcal{A}_\lambda|^2+\mathrm{Im}^2(\mathcal{H}_\lambda e^{-i\pi\lambda})]^{1/2},\tag{21}$$

varying from zero to the maximum value

$$P_{N\,\max}=\frac{|\mathcal{A}_\lambda|^2}{|\mathcal{A}_\lambda|^2+\mathrm{Im}^2(\mathcal{H}_\lambda e^{-i\lambda\pi})}.\tag{22}$$

The functions $\mathcal{A}_\lambda$ and $\mathcal{H}_\lambda$ occurring in the final results are defined by the system (16) and depend on $|q|$ and λ, i.e., on the amplitudes and frequencies of the external field. For arbitrary values of $|q|$ and λ these functions clearly cannot be expressed in terms of the known functions and have to be tabulated.

The frequencies Ω_λ represent essentially the quasienergies calculated by Melikyan.[6] In the $|q|\ll 1$ case, i.e., if an external field is such that the inequality $E_0|d_{12}|/\omega\ll 1$ applies, the frequencies Ω_λ can be calculated if we confine our expansions of $\mathcal{H}_\lambda$ and $\mathcal{A}_\lambda$ in powers of $|q|$ to just the first few terms. For the odd values of λ, we obtain

$$\Omega_{\lambda\neq 1,3}=\frac{\omega}{2}|q|^2\frac{\lambda}{\lambda^2-1}\left\{1-\frac{|q|^2}{4}\frac{1+3\lambda^2}{(\lambda^2-1)^2}+\ldots\right\},$$

$$\Omega_3=\omega|q|^2\frac{3}{16}\left\{1-\frac{55}{576}|q|^2+\ldots\right\}, \quad (23)$$

$$\Omega_1=\frac{\omega}{2}|q|\left\{1-\frac{1}{8}\left(\frac{\pi^2}{3}+\frac{1}{4}\right)|q|^2+\ldots\right\}.$$

For $\lambda\neq 3$ the expressions (23) are identical with the Melikyan results.[6] If $\lambda\gg 1$, the expression for Ω_λ is an expansion in terms of a small parameter $|q|^2/\lambda$, as expected on the basis of the work of Zaretskiĭ and Kraĭnov[7].

We shall go back to the expression (20) for P_N. It follows from this expression that when the function $\mathrm{Im}(\mathscr{H}_\lambda e^{-i\lambda\pi})$ vanishes, the value of $P_{N\max}$ becomes unity, i.e., the average population of the upper level reaches 0.5. Thus, zeros of the function $\mathrm{Im}(\mathscr{H}_\lambda e^{-i\pi\lambda})$, which do not coincide with the zeros of the function $\mathscr{A}_\lambda$, govern many-photon resonances. If there is a resonance for $\lambda=\lambda^*$, then near λ^* we have

$$P_N\approx\frac{|\mathscr{A}_\lambda|^2}{|\mathscr{A}_\lambda|^2+\gamma^2}\sin^2\left[\frac{\omega t_N}{2\pi}\arcsin(|\mathscr{A}_\lambda|^2+\gamma^2)^{1/2}\right], \quad (24)$$

where

$$\gamma=\frac{\partial\,\mathrm{Im}(\mathscr{H}_\lambda e^{-i\lambda\pi})}{\partial\lambda}\bigg|_{\lambda=\lambda^*}(\lambda-\lambda^*) \quad (25)$$

is the detuning from resonance.

If $|q|\ll 1$ the zeros of the function $\mathrm{Im}(\mathscr{H}_\lambda e^{-i\lambda\pi})$ are located near integral values of λ. They can be found in the form of expansions in powers of $|q|^2$:

$$\lambda_k^*=k-\frac{k}{k^2-1}|q|^2-\frac{1}{4}\frac{k(k^2+3)}{(k^2-1)^3}|q|^4+\ldots, \quad (26)$$

provided $k\neq 1$ or 3. The zeros of the function $\mathrm{Im}(\mathscr{H}_\lambda\times e^{-i\pi\lambda})$ located near even k are also zeros of the function $\mathscr{A}_\lambda$ and, therefore, they do not govern many-photon resonances. The average population of the upper level near even values of k is $\sim 2|q|^2/(k^2-1)^2+\ldots$. For the odd k, Eq. (26) gives the positions of many-photon resonances. These resonances are shifted relative to integral values of k (Stark shift).

Near the k-th resonance and for $|q|\ll 1$, the expression for the probability P_N becomes

$$P_N\approx\frac{|\mathscr{A}_\lambda|^2}{|\mathscr{A}_\lambda|^2+\gamma^2}\sin^2\left[\frac{\omega t_N}{2\pi}(|\mathscr{A}_\lambda|^2+\gamma^2)^{1/2}\right], \quad (27)$$

$$\gamma=\pi(\lambda-\lambda^*),\qquad \mathscr{A}_\lambda=-\frac{2\pi}{[(k-1)!!]^2}\left(\frac{i|q|}{2}\right)^k+\ldots, \quad (28)$$

which agrees with the result obtained by Kraĭnov.[8]

The average intensity of the nonresonant excitation is, to within $|q|^2$, given by

$$\langle P_N\rangle_{\text{nonres}}=2|q|^2/(\lambda^2-1)^2.$$

Since the value of Ω_λ in the nonresonance case is equal, with the same precision, to

$$\Omega_\lambda=\frac{\omega}{2}\left[\frac{\lambda}{\lambda^2-1}|q|^2+\lambda\right],$$

the probability of a λ-photon transition without allowance for resonances is (for $\lambda=1$)

$$P_N=\frac{4|q|^2}{(\lambda^2-1)^2}\sin^2\left[\frac{\omega}{2}\left(\lambda+\frac{\lambda}{\lambda^2-1}|q|^2\right)t_N\right].$$

This result agrees with that obtained by Sen Gupta,[5] who solved this problem to within second order in the magnitude of the interaction.

§ 2. EXCITATION OF A TWO-LEVEL SYSTEM BY A PULSED ELECTRIC FIELD

1. We shall now return to the expression (20) for P_N. It should be noted that oscillation of the excitation probability P_N in time is not self-evident. The question arises as to why the probability having reached a maximum begins to fall and drops again to zero. In fact, even when the probability rises and then later when it falls the same external field is responsible for the change in the probability. Since the action of this field on a two-level system is not always the same, it is governed not only by the field itself because its nature does not change, but also by the state of the two-level system itself, which should vary with time either for internal reasons or under the action of the external field. The state of a two-level system can be described in general by the quantity α:

$$\alpha=\begin{pmatrix} b^{1/2}\exp[iF_2]\\ (1-b)^{1/2}\exp[iF_1]\end{pmatrix}, \quad (29)$$

i.e., after subtracting the normalization condition and the fact that α is generally defined only to within an arbitrary phase factor characterized by two numbers: $b^{1/2}$, indicating the ratio of the moduli of the probability amplitudes, and by the phase difference F_1-F_2.

There are no other characteristics of the state of the system so that these are responsible for the fact that the action of the external field does not always increase the probability of excitation of the upper level but periodically produces the opposite effect. Moreover, in a weak external field the time dependence of the transition probability characterized by $b^{1/2}$ is negligible but this is not true of the change in the phase difference F_1-F_2, i.e., special attention should be paid specifically to the phase characteristic of the state of a two-level system.

2. We shall now illustrate these by formulas. We shall first consider an unexcited two-level system. Let us assume that in a time $t_{N0}=2\pi N_0/\omega$ the probability of excitation of the upper level reaches its maximum. According to Eq. (20), by the time $t=2t_{N0}$ this probability vanishes again, i.e.,

$$P_{2N_0}=|\alpha_2(2\pi 2N_0)|^2=0.$$

On the other hand,

$$\alpha(2\pi 2N_0)=\hat{Q}(2\pi 2N_0)\hat{U}_{2N_0}(\hat{U}_1{}^+\hat{M})^{N_0}U_{N_0}{}^+Q^+(2\pi N_0)\alpha(2\pi N_0), \quad (30)$$

where $\hat{U}_{N0}+\hat{Q}+(2\pi N_0)\alpha(2\pi N_0)$ can be represented using Eq. (18) in the form of Eq. (29):

$$U_{N_0}{}^+Q^+(2\pi N_0)\alpha(2\pi N_0)=\begin{pmatrix}[b(N_0)]^{1/2}\exp[iF_2(N_0)]\\ [1-b(N_0)]^{1/2}\exp[iF_1(N_0)]\end{pmatrix}, \quad (31)$$

where $b(N_0)=|\mathscr{A}_\lambda|^2/[|\mathscr{A}_\lambda|^2+\mathrm{Im}^2(\mathscr{H}_\lambda e^{-i\pi\lambda})]$ and the phase difference $F_1(N_0)-F_2(N_0)$ can be found from Eq. (18).

The fact that this phase difference has a very different value is responsible for the vanishing of P_{2N0}. In fact, if we calculate P_{2N0} from Eqs. (30) and (31), we obtain from the formulas derived in the first section

$$P_{2N_0}=|\alpha_2(2\pi 2N_0)|^2=\frac{\sin^2 N_0\varphi}{\sin^2\varphi}\Big\{b(N_0)\Big|-(\mathscr{H}_\lambda e^{-i\lambda\pi})^*$$
$$+\frac{\sin(N_0+1)\varphi}{\sin N_0\varphi}\Big|^2-(1-b(N_0))|\mathscr{A}_\lambda|^2-[b(N_0)(1-b(N_0))]^{1/2}\Big[\Big[-\mathscr{H}_\lambda e^{-i\lambda\pi}$$
$$+\frac{\sin(N_0+1)\varphi}{\sin N_0\varphi}\Big]\frac{iq^*}{|q|}\mathscr{A}_\lambda e^{-i\pi\lambda}\exp[i(F_1(N_0)-F_2(N_0))]$$
$$-\Big[-(\mathscr{H}_\lambda e^{-i\lambda\pi})^*+\frac{\sin(N_0+1)\varphi}{\sin N_0\varphi}\Big]\frac{iq}{|q|}\mathscr{A}_\lambda{}^*e^{i\lambda\pi}\exp\{-i(F_1(N_0)-F_2(N_0))\}\Big]\Big\}. \quad (32)$$

It is clear from Eq. (32) that the phase difference accumulated by the time t_{N0} and equal to $F_1(N_0)-F_2(N_0)$ does indeed occur in the expression for P_{2N0} and has a very strong influence on this probability. It represents the phase state of the system at the moment t_{N0} and governs the subsequent reaction of the system to the field acting on it.

By way of proof we may quote the following general arguments. The state of a two-level system is described by two parameters: b and F_1-F_2, both of which have definite physical meaning. The parameter b governs the ratio of the probabilities that an electron is at the first or second level, whereas the parameter F_1-F_2 represents the phase state of the system and governs the relationship between the system and the external field.

3. If the reaction of a two-level system to the field acting on it is governed by its phase state, i.e., by the quantity F_1-F_2, and if the latter quantity changes under the action of the field, we can attempt to pump the upper level by acting on a two-level system alternately with two different nonresonant fields, the first of which increases the excitation probability and the second (weaker) alters the phase state in a special way so that the excitation probability continues to rise on the next application of the first field. We shall use the term pump and phase fields for the first and second fields, respectively. Naturally, the pump field alters also the phase state and the action of the phase field affects the excitation probability but since the fields are different and the excitation probability and phase state are two independent characteristics of a two-level system, a suitable selection of the field intensities and pulse durations may ensure the expected continuation of the rise of the excitation probability.

We shall now study this possibility. We shall assume that the two-level system is subjected to alternate pulses of two different fields. Let the frequencies of these fields be identical and equal to ω and let their amplitudes and pulse durations be different. Moreover, let us assume that the fields are weak. Up to a moment $t=k(N_1+N_2)2\pi/\omega$ the alternate action of k pulses of the pump field $|q_1|$ of duration N_1 periods and k pulses of the phase field $|q_2|$ of duration N_2 periods the amplitude of a two-level system α becomes, according to Eq. (13),

$$\alpha[2\pi k(N_1+N_2)]$$
$$=Q[2\pi k(N_1+N_2)]U_{k(N_1+N_2)}\{(U_1{}^+\mathfrak{M}(q_2))^{N_2}(U_1{}^+\mathfrak{M}(q_1))^{N_1}\}^k\alpha(0). \quad (33)$$

Taking the matrices in Eq. (33) to the required power and using the definitions (15) of the matrix elements $M(q)$, we can find the excitation probability of an initially unexcited two-level system by the time $t_{k(N_1+N_2)}$:

$$P_{k(N_1+N_2)}=\frac{\sin^2 N_1\varphi_1}{\sin^2\varphi_1}\frac{\sin^2 N_2\varphi_2}{\sin^2\varphi_2}\frac{1}{\sin^2\beta}$$
$$\times\Big|[i\,\mathrm{Im}(\mathscr{H}_\lambda(q_2)e^{-i\lambda\pi})+\mathrm{ctg}\,N_2\varphi_2\sin\varphi_2]\frac{q_1^*}{|q_1|}\mathscr{A}_\lambda(q_1)$$
$$+[-i\,\mathrm{Im}(\mathscr{H}_\lambda(q_1)e^{-i\lambda\pi})+\mathrm{ctg}\,N_1\varphi_1\sin\varphi_1]\frac{q_2^*}{|q_2|}\mathscr{A}_\lambda(q_2)\Big|^2$$
$$\times\sin^2\Big[\frac{\omega\beta}{2\pi(N_1+N_2)}t_{k(N_1+N_2)}\Big], \quad (34)$$

where

$$\cos\beta=\frac{\sin N_2\varphi_2}{\sin\varphi_2}\frac{\sin N_1\varphi_1}{\sin\varphi_1}\mathrm{Re}\Big\{[i\,\mathrm{Im}(\mathscr{H}_\lambda(q_2)e^{-i\lambda\pi})$$
$$+\mathrm{ctg}\,N_2\varphi_2\sin\varphi_2][i\,\mathrm{Im}(\mathscr{H}_\lambda(q_1)e^{-i\lambda\pi})+\mathrm{ctg}\,N_1\varphi_1\sin\varphi_1]$$
$$-\frac{q_2^*q_1}{|q_2||q_1|}\mathscr{A}_\lambda(q_2)\mathscr{A}_\lambda{}^*(q_1)\Big\}, \quad (35)$$

and the quantities φ_1 and φ_2 are given, in accordance with the above, in the form

$$\cos\varphi_j=\mathrm{Re}\,[\mathscr{H}_\lambda(q_j)e^{-i\pi\lambda}]. \quad (36)$$

Thus, the transition probability varies with time as $\sin^2\Omega t$ and its frequency is

$$\Omega=\frac{\omega}{2\pi}\frac{\beta}{N_1+N_2}, \quad (37)$$

the maximum value depending of the fields q_1 and q_2, and also on the pulse durations N_1 and N_2. We shall show that a suitable selection of these parameters can ensure that the maximum value of the probability reaches unity. We shall assume that the acting fields are related by

$$q_2=\varkappa q_1, \quad (38)$$

where $\varkappa<1$. Assuming, for simplicity, that λ is not an integer, we shall write out $\mathscr{A}_\lambda$ and $\mathscr{H}_\lambda$ with just the first terms:

$$\mathscr{A}_\lambda(q)=|q|\frac{1}{\lambda^2-1}(e^{2\pi i\lambda}-1)+\ldots,$$
$$\mathrm{Re}(\mathscr{H}_\lambda e^{-i\pi\lambda})=\cos\pi\lambda-|q|^2\frac{\lambda\pi}{\lambda^2-1}\sin\pi\lambda+\ldots, \tag{39}$$
$$\mathrm{Im}(\mathscr{H}_\lambda e^{-i\pi\lambda})=-\sin\pi\lambda-|q|^2\frac{1}{\lambda^2-1}\left[\lambda\pi\cos\lambda\pi-\frac{2}{\lambda^2-1}\sin\lambda\pi\right]+\ldots$$

Since the fields are weak, $|q_2|<|q_1|\ll 1$, the above terms in the expansions are quite sufficient.

Substituting Eq. (39) into Eq. (34) we easily find that in the case when the field intensities $|q_1|$ and $|q_2|$ and pulse durations N_1 and N_2 are such that

$$N_1\varphi_1+N_2\varphi_2=\pi n, \tag{40}$$

where n is an integer, the value of $P_{\max}$ becomes unity. The excitation probability considered as a function of time is then

$$P_{k(N_1+N_2)}=\sin^2\Omega t_{k(N_1+N_2)},$$

where Ω is given by Eq. (37) and amounts to

$$\Omega=\frac{\omega}{\pi}\frac{1}{N_1+N_2}\frac{|q_1|}{(\lambda^2-1)}(1-\varkappa)|\sin N_1\varphi_1|. \tag{41}$$

The condition (40) represents the pumping condition. It is valid for any value of λ.

We have thus shown that indeed it is possible to pump the upper level of a two-level system by a weak nonresonant field if this field is modulated in a suitable manner. The effect is due to the fact that the phase field alters the phase state of the system in the interval between the two consecutive pump field pulses and at the beginning of each new pulse of the pump field the system is in a state most suitable for excitation. We may also attempt to achieve a similar effect by using other modulated fields, for example, an alternating field with smooth variation of the amplitude or frequency-modulated fields. In any case, the behavior of a two-level system under the action of a modulated external field is very different from its behavior in a monochromatic field.

4. Since the essence of the problem reduces to a suitable change in the phase state of a two-level system in the time intervals between the pump field pulses, one might try to leave simply the system to itself during these intervals. In fact, the phase state of the system varies with time without any external influence and the phase difference F_1-F_2 is a linear function of time simply because of the difference between the level energies. If the intervals between the pump field pulses are selected suitably, we can achieve a situation in which a two-level system tunes itself in a suitable manner at the beginning of each new pulse. In this way we can pump the upper level with one pulsed field $|q|$. The pumping condition can easily be obtained using the results obtained above for the general case of two alternately acting fields assuming that $q_2=0$ and $\hat{M}(q_2)=1$. This condition becomes

$$N_1\varphi_1+\lambda\pi N_2=n\pi. \tag{42}$$

The probability of excitation of a two-level system up to the moment $t_{k(N_1+N_2)}$ oscillates at the frequency

$$\Omega=\frac{\omega}{\pi}\frac{|q_1|}{\lambda^2-1}\frac{|\sin N_1\varphi_1|}{N_1+N_2}, \tag{43}$$

varying from zero to unity. The upper level can be pumped by a pulse field only for fractional values of $N_2\lambda$. If $N_2\lambda$ is an integer, the change in the phase difference of a two-level system in the interval between two consecutive field pulses is a multiple of 2π, i.e., at the beginning of each new pulse the system is in exactly the same state as at the end of the preceding pulse.

The author regards it as a pleasant duty to thank V. P. Kraĭnov for his valuable advice and help in the analysis of the material and preparation of the manuscript.

[1]N. B. Delone, V. P. Kraĭnov, and V. A. Khodovoĭ, Usp. Fiz. Nauk 117, 189 (1975) [Sov. Phys. Usp. 18, 750 (1975)].

[2]É. Ya. Manykin, Prokhozhdenie izlucheniyz cherez veshchestvo (Passage of Radiation Through Matter), Atomizdat, M., 1966, p. 72.

[3]R. H. Pantell and H. E. Puthoff, Fundamentals of Quantum Electronics, Wiley, New York, 1969 (Russ. Transl., Mir, M., 1972).

[4]N. D. Sen Gupta, J. Phys. A 3, 618 (1970).

[5]N. D. Sen Gupta, J. Phys. A 5, 401 (1972).

[6]A. O. Melikyan, Zh. Eksp. Teor. Fiz. 68, 1228 (1975) [Sov. Phys. JETP 41, 610 (1975)].

[7]D. F. Zaretskiĭ and V. P. Kraĭnov, Zh. Eksp. Teor. Fiz. 66, 537 (1974) [Sov. Phys. JETP 39, 257 (1974)].

[8]V. P. Kraĭnov, Zh. Eksp. Teor. Fiz. 70, 1197 (1976) [Sov. Phys. JETP 43, 622 (1976)].

[9]J. H. Shirley, Phys. Rev. 138, B979 (1965).

[10]J. V. Moloney, M. K. Ali, and W. J. Meath, Phys. Lett. A 49, 207 (1974).

[11]W. R. Salzman, Phys. Rev. Lett. 26, 220 (1971).

Translated by A. Tybulewicz